Compact Dielectric Particles as a Building Block for Low-Loss Magnetic Metamaterials

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We characterize experimentally a compact dielectric particle that can be used to design very low-loss artificial electromagnetic materials (metamaterials). Focusing on magnetic media, we show that the particle can behave almost identically to the well-known split-ring resonators (SRRs) widely used in present designs, without suffering from the Ohmic losses that can limit the applicability of SRRs especially at high frequencies. We experimentally compare qualitatively and quantitatively the dielectric particle with a typical split-ring resonator of the same size built on a low-loss dielectric substrate and show that at GHz frequencies the quality factor of the dielectric particle is more than 3 times bigger than that of its metallic counterpart. Low-loss and simple geometry are significant advantages compared to conventional metal SRRs.

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The prospect of implementing exotic devices such as perfect lenses [1], perfect tunneling devices [2,3], or even invisibility cloaks [4,5], leads to significant research on the artificial electromagnetic materials (metamaterials) capable of achieving the large range of material parameters needed to implement these devices. Most current metamaterial designs obtain the desired electromagnetic parameters (i.e., permittivity ε and/or permeability μ) by using periodic arrays of highly subwavelength metallic inclusions such as split-ring resonators [6], wires [7], electric-field-coupled resonators (ELC) [8] and other similar structures. A limitation of this approach is the significant loss encountered in these resonant particles. As was pointed out before [9], a significant part of this loss is represented by Ohmic losses in the metallic components of the particles. These Ohmic losses become larger with frequency which explains why metallic particle designs are not suitable for high frequency applications.

An alternative way to devise metamaterials, which avoids using metallic inclusions, is to employ high dielectric constant (high-ε) materials. While strong electric response is obtained naturally in these dielectrics, the challenge, from the metamaterial point of view, is to obtain a strong magnetic response, and, for this reason, we will focus here on magnetic metamaterials. Theoretical analysis of arrays of small dielectric spheres [10,11] or thin cylinders of infinite extent [12] showed that Mie resonances in these structures can be used, in principle, to generate negative index of refraction metamaterials (i.e., both ε and μ negative [13]), which, in turn, is indirect evidence that a magnetic response can, indeed, be obtained with these structures. A recent experiment involving waves traveling through a prism made of thin dielectric cylinders showed the negative angle of refraction indicative of negative index media [14]. However, the experiment also showed that this approach requires a huge electric response to generate only a modest magnetic response, which in turn created a severe impedance mismatch between the cylinder-based metamaterial and the surrounding region of air. In addition, the infinite extent of the cylinders and the very large permittivity needed ($\varepsilon_r = 600$) increases the difficulty of implementing this design.

In this Letter we take a different approach, and demonstrate quantitatively through experiments that compact dielectric particles (CDP) having a reasonably high ε can be designed to provide a strong magnetic response while keeping the electric response at minimum. Moreover, we show that in terms of losses, even at lower frequencies, the dielectric particle has a quality factor significantly higher than that of the traditional copper split-ring resonator (SRR) typically employed in metamaterial design, while it keeps essentially the same functionality as the SRR. The advantages of the dielectric particle over its metallic counterpart suggest it as a replacement not only at high frequencies but also at the lower microwave frequencies.

Consider a rectangular block of dielectric of relative permittivity ε_r, width w, height h, and thickness d smaller than both w and h [see Fig. 1(a)]. The physics of these structures have been analyzed in great detail before (for example, [15,16]), as has their resonant behavior in the context of electrically small antennas [17] and microwave devices such as oscillators and filters [16]. Unlike a regular SRR, which is an LC resonator, the CDP is a cavity resonator (its faces can be roughly approximated with perfect magnetic conductors); however, as we will see below, the behavior of the two types of resonances is very similar.

In this Letter we focus on the lowest order resonant mode supported by the dielectric block (called $H_{11\delta}$ in [15] and TE^z_{111} in [18]), due to its property of having the electric field oriented in a loop around the block, as illustrated in Fig. 1(a) (obtained in a numerical simulation performed in Ansoft HFSS: the particle was illuminated with a plane wave having the magnetic field perpendicular on the largest face). Similar to the SRR, below the resonant frequency the displacement current loop (consequently the magnetic moment generated) is in phase with the applied external magnetic field, while above the resonant frequency the displacement current becomes 180 degrees

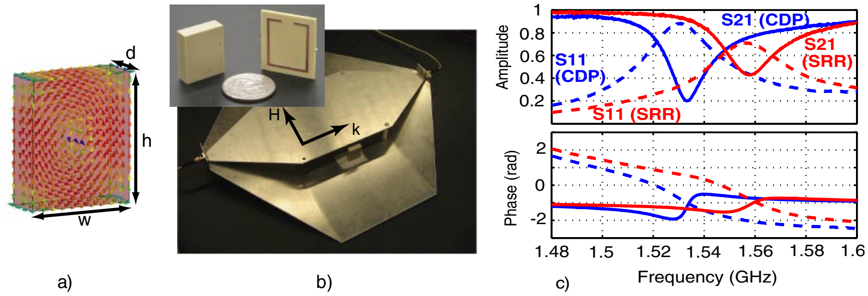

FIG. 1 (color online). (a) The electric field of the fundamental mode of a rectangular dielectric particle. (b) Picture of the dielectric particle (inset left), split-ring resonator (inset right), and the microstrip used to measure their properties. Notice the position of the dielectric particle with respect to the microstrip (shown for clarity at the edge of the waveguide) placed such that the magnetic field excites its fundamental mode. (c) The measured S_{21} (solid) and S_{11} (dashed) for the CDP (blue) and SRR (red lines).

out of phase. This sign change means that both negative and positive permeabilities can be obtained, as for SRRs. The fields generated inside the particle are symmetric with respect to an axis parallel to the magnetic field, which indicates that the TE^z_{111} mode is excited by the magnetic field. We expect the external electric field to have a negligible influence on the CDP response, because the E field is perpendicular to the axis of symmetry. We should note that the response to the electric field becomes significant at a much higher frequency, at the second resonant mode. These observations are confirmed by numerical simulations and experiments: if the CDP is rotated 90 degrees such that the magnetic and electric fields are both perpendicular on the first mode's axis of symmetry, the magnetic response of the particle vanishes (i.e., the mode is not excited). If the permittivity of the dielectric is big enough, the displacement current density inside the particle, $J_d = j\omega\varepsilon E_{\mathrm{internal}}$, can be increased together with the magnetic field generated by the current loop (i.e., the magnetic response of the particle). Moreover, the displacement current distribution inside the dielectric is analogous to that of the conduction current in the metallic conductors of a regular SRR; therefore, we expect the behavior of the SRR and CDP to be very similar. Our purpose is to quantify the electromagnetic response of the dielectric particle and compare it with the response of a typical metallic SRR. In order to do this comparison, we observe that the similarities between the CDP and SRR allow us to model a CDP-based metamaterial in the same way we model its SRR-based counterpart: such a metamaterial is anisotropic and can be described by effective μ and ε tensors. We neglect small, second order effects that are encountered in some SRRs (such as bianisotropy). The symmetry of the field distribution inside the CDP suggests that these effects are even smaller than in SRRs.

As it was pointed out in the past [6,19], the effective magnetic susceptibility (defined as $\chi_m \equiv \mu_r - 1$) of an SRR-based medium can be described using

$$\chi_m = \frac{F\omega^2}{\omega_0^2 - \omega^2 + j\omega\omega_0/Q}, \tag{1}$$

where ω_0 is the angular resonant frequency, F is a geometry factor that determines the bandwidth of the medium, and Q is the quality factor that determines the loss inside the material. Based on the similarity between the current distributions inside the CDP and SRR, and also on the generality of the above formula, which applies to a wide range of resonators, we expect Eq. (1) to also hold for the dielectric block. We can determine Q from χ_m using the following procedure. First, we compute $|\chi_m|^2/\omega^4$:

$$\frac{|\chi_m|^2}{\omega^4} = \frac{F^2}{(\omega^2 - \omega_0^2)^2 + (\omega\omega_0/Q)^2}. \tag{2}$$

Second, we notice that, under the approximation $Q^2 \gg 1$ (as it is always desirable), $|\chi_m|^2/\omega^4$ has a maximum at $\omega = \omega_0$, and decreases to half the maximum at $\omega = \omega_0 \pm \omega_0/(2Q)$. Therefore, Q is simply the inverse of the half peak bandwidth of $|\chi_m|^2/\omega^4$ normalized to the resonant frequency. Next we use this approach to design and experimentally characterize the dielectric particle and compare it with a traditional SRR.

Our target resonant frequency is 1.5 GHz. To keep the particle dimensions small the material must have a high dielectric constant. We use K-100, a commercial ceramic manufactured by TCI Ceramics, which has $\varepsilon_r = 100$ and a specified loss tangent of 0.001 at 9 GHz. We want our resonant dielectric block to have a size of at most $\lambda/8$ at resonance; therefore, we chose $w = 20$ mm, and $h = 24$ mm. The last dimension, d, is a function of w, h, ε_r, and the desired ω_0 and can be determined analytically using one of the many models developed for rectangular dielectric resonators. For the purpose of our design, we used the simple method outlined in [18] which predicts

$$d = \frac{2}{k_z}\tan^{-1}\frac{k_{z0}}{k_z}, \tag{3}$$

where $k_z = \sqrt{\varepsilon_r(\omega_0/c)^2 - (\pi/w)^2 - (\pi/h)^2}$ and $k_{z0} = \sqrt{(\varepsilon_r - 1)(\omega_0/c)^2 - k_z^2}$. We obtain $d = 6$ mm. Past experiments showed that the interaction with the experimental setup slightly increases the resonant frequency of the isolated particle, so that, for the final design presented on the left of Fig. 1(b), we chose $d = 7$ mm.

The particle is measured using the procedure described in [20] inside a TEM microstrip waveguide consisting of two parallel metallic plates situated 3 cm apart. The top plate has a width of 15 cm and narrows towards the coaxial-to-microstrip adapter to maintain a 50 Ω impedance across the whole length of the waveguide. Notice that, if we neglect fringing effects, we can approximate the sides of the waveguide as perfect magnetic conductors. As a result, as in [20], we can measure only one particle inside the microstrip in order to obtain the material parameters of an equivalent medium of infinite extent in the directions perpendicular to the microstrip axis. The CDP is positioned inside the microstrip such that the magnetic field is perpendicular on its largest face in order to excite the fundamental mode, as illustrated in Fig. 1(b). The reflection and transmission parameters were measured with an Agilent 8720C vector network analyzer [see Fig. 1(c)]. Based on these measurements [21] we can recover the effective material parameters of the medium composed of periodic arrangements of unit cells that contain the dielectric block and extend the whole transverse section of the waveguide. Figure 2(a) shows the measured parameters (blue lines). We notice the Lorentzian shape of the effective μ very similar to that of a typical split-ring. Also, notice the resonant frequency within 3% of the desired resonant frequency of 1.5 GHz, which validates Eq. (3).

To compare these parameters with the parameters of a traditional split-ring-based medium, we designed a low-loss SRR that resonates at 1.5 GHz using the following well established procedure. The rectangular copper ring has the same dimensions as the dielectric ring, namely, it is 20 mm by 24 mm. The copper trace is 1 mm wide and $\approx$30 μm thick. The gap width (6 mm) has been determined iteratively by running numerical simulations in Ansoft HFSS (a finite element solver of Maxwell's equations) until the desired resonant frequency has been obtained. To reduce losses as much as possible, the ring was manufactured on a low-loss Rogers TMM3 substrate ($\varepsilon_r = 3.3$, loss tangent $\approx$0.002). As we will see shortly the performance of this design is very similar to that of split-ring-based particles reported in the literature; therefore, the SRR described above can be considered typical for its kind. We used the same procedure described above to characterize the SRR, and we plotted the results on Fig. 2(a) (red lines).

As expected, the shape of the measured effective material parameters for the dielectric particle are very similar to those of the SRR. Both particles have a predominant magnetic response, reflected in the significant, Lorentzian variation of μ with frequency and the modest change in ε. Moreover, the resonance bandwidth is also almost the same in both cases, indicative of similar F factors [6] [see Eq. (1)]. However, the most important thing to notice in Fig. 2(a) is the bigger magnetic response of the dielectric block compared to that of the SRR. This difference strongly suggests that the quality factor, Q, of the dielectric block is also higher, and consequently losses are smaller. We can estimate the quality factors from the measured susceptibility as explained above [see Eq. (2)]. The result is illustrated in Fig. 2(b). According to Eq. (2), the quality factor is the inverse of the half peak bandwidth normalized to the resonant frequency. We obtain a quality factor of 509 for the dielectric block and only 141 for the SRR. We should point out that the SRR quality factor obtained here is comparable if not exceeds that of designs published elsewhere [19,21]. These values of Q demonstrate that the CDP is lower loss than the SRR even at relatively low microwave frequencies. The small negative imaginary part of the retrieved ε (less than 0.3 in absolute value) is a typical experimental artifact cause by imperfections in the experimental setup, such as the small mismatch between the microstrip and the coax cables that connect it to the network analyzer. Ideally, as the HFSS simulation described below shows, the imaginary part is always positive [see the dotted line in Fig. 2(a)].

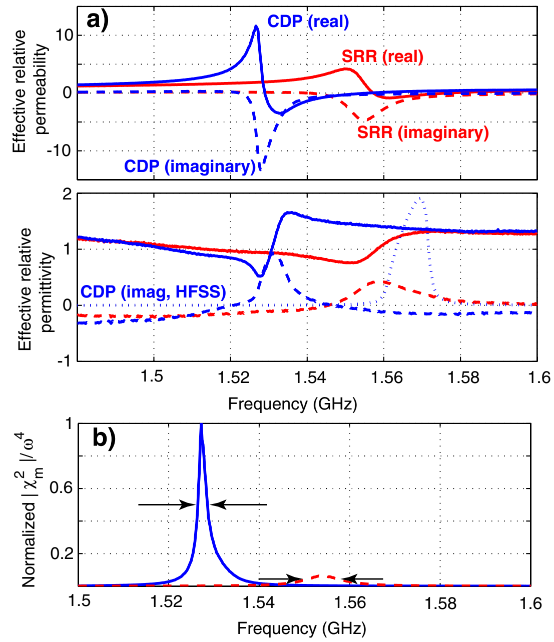

FIG. 2 (color online). (a) The retrieved real (solid) and imaginary (dashed) parts of the effective material parameters (top: permeability; bottom: permittivity) of the dielectric block (blue lines) and SRR (red lines). The small negative imaginary part of ε is an experimental artifact that does not show in numerical simulations (blue dotted line). (b) The measured normalized $|\chi_m^2|/\omega^4$ allows us to determine the quality factors of the two particles. We obtain $Q = 509$ for the dielectric block (solid line), and $Q = 141$ for the SRR (dashed line).

To further check the validity of our results we performed a numerical simulation of the dielectric block in Ansoft HFSS using a standard procedure described, for example, in [21], which follows closely the experimental setup described above. Thus, the dielectric block is positioned inside a waveguide 15 cm wide and 3 cm tall. The waveguide top and bottom walls are perfect electric conductors, while

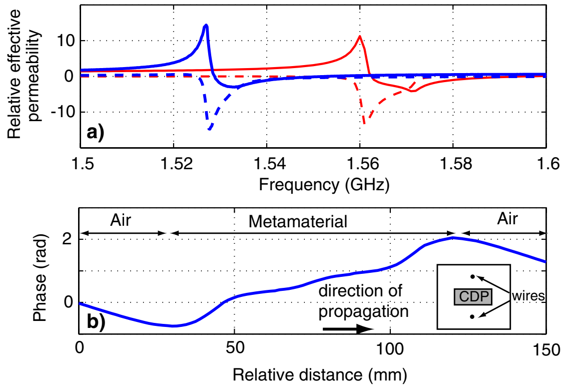

FIG. 3 (color online). (a) Comparison between the effective permeability real (solid) and imaginary (dashed) components retrieved in a numerical simulation (red) and experimentally (blue) for the CDP-based medium; (b) Phase distribution of the electric field at 1.53 GHz in front, inside, and behind a NIM made of CDPs and vertical wires. Inset: top view of unit cell.

the sides are perfect magnetic conductors. This choice of boundary conditions simulate a transversally infinite metamaterial slab. The S parameters provided by HFSS are used to extract the effective material parameters of this metamaterial. The good agreement between the simulation and the experiment is shown in Fig. 3 and validates our design.

We verified in a simulation the ability of the CDP to replace SRRs in negative index material (NIM) designs. For this purpose we used a similar setup as above with the unit cell shrunk to 3 cm by 3 cm by 3 cm. Two vertical thin wires (0.2 mm in diameter) that touched the top and bottom conducting walls were positioned inside the unit cell on each side of the CDP in order to obtain negative permittivity. The metamaterial slab is three cells long in the propagation direction. A direct way to find the sign and value of the effective refractive index is to look at the phase variation of the electric or magnetic field in the direction of propagation [22] inside the slab and in the air regions in front and behind it [see Fig. 3(b) for the phase distribution of the E field at 1.53 GHz]. The negative slope outside the metamaterial medium corresponds to the $n = 1$ refractive index in air. Inside the metamaterial, on the other hand, the slope is positive which indicates negative refractive index, $n = -1.05$. Moreover, a retrieval based on S-parameter measurements gives a similar value of $n = -1.1$. These results verify that CDPs can, indeed, replace SRRs in NIM structures.

In conclusion, we showed how rectangular dielectric particles can be designed to have a strong magnetic response at the desired frequency, and can be made highly subwavelength without requiring huge permittivities. A comparison with the traditional split-ring resonator typically employed in metamaterial designs showed that the dielectric particle is significantly lower loss than the SRR even at lower frequencies while keeping the same basic functionality. More specifically, the measured quality factor of the dielectric particle was found to be more than 3 times bigger than that of the SRR. Despite having relatively high permittivity, the dielectric particle has a relatively small electric response. From this point of view it is superior to other dielectric structures that required a strong electric response in order to obtain only a modest magnetic response [14]. This particle has a very simple geometry and does not require patterning for fabrication, and its inherent symmetry means that it has no magnetoelectric response. These qualities indicate that it may be a viable replacement for the SRR not only at high frequencies where the Ohmic loss in metallic SRRs becomes too large, but also at lower microwave frequencies in applications which require low loss.

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