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Compact Form of Expressions for Inductance Calculation of Meander Inductors

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Abstract: The quality of modelling and analysing of RF IC with planar inductors extremely depends on accuracy of expression for inductance. In this paper efficient methods for total inductance calculation of meander inductor, are given. By using proposed algorithm, we are able to predict correctly all inductance variations introduced by varying geometry parameters. Our developed program gives possibility for fast and accurate inductance calculation for meander inductor, while new expression in monomial form is useful for optimisation of this inductor. The results validations, given by proposed software tool are conformed by comparison with measured data from literature.

Keywords: Equivalent electrodes method, Electric filed, Cub electrode.

1 Introduction

Rapid development of radio-frequency integrated circuits (RF IC) and incredible development and spread of market wireless-cell, i.e. mobile communications, have also made a hasty interest for monolithic inductors. The inductors of spiral shape (square, octagonal, circular) are most frequently applied, although for their fabrication more metal levels are needed. In earlier published papers [1], [2] are presented the simulators for analytical calculation of inductance of spiral inductors, or inductance calculation in interconnect structures [3], [4], but there is no a software tool for quick and accurate calculation of meander's inductance.

Layout of meander inductor is given in Figure 1. Although in comparison to inductors of other shapes it gives weaker performances, i.e. it gives smaller inductance per equal chip surface and has smaller quality factor (Q-factor) for the equal inductance due to longer conductor (greater DC resistance), meander inductors are used because of simple process of production. Hence, in addition to a simple layout, it is not needed to make metal contacts of inductors in two levels, so providing way to avoid two levels of photolithography what simplifies technological process [5].

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Commercial electromagnetic simulators can be use to calculate inductance of meander inductors. However, they are computationally intensive and time consuming. To quickly calculate inductance for design guidance, simple analytical formulae are desired.

In order to be implemented in circuits, meander inductors have to be supported by models. The major problem for implementation of a compact lumped inductor model was the lack of an accurate and simple expression for meander inductance. In this work, we have explained how we have generated such an expression, which can be directly implemented in the RF circuit simulator.

In this paper are presented the results of our software tool INDCAL (***IND****uctance* ***CAL****culation*) that serves for fast and accurate inductance calculation of the meander inductors. In the section 2 are given analytical expressions for the self-inductance and mutual inductance of meander and in clear (tabular) way are summarized expressions suitable for implementation into a computer program. The simple monomial expression for inductance that is suitable for application in the optimisation procedures is given in the Section 3. In the Section 4 are presented the most important results of proposed program while in the Section 5 are given conclusions of this work.

2 Inductance Calculation

The starting point for the derivation of proposed formula is *Greenhouse* theory [6]. *Greenhouse* decomposed inductor into its constituent segments. Basically, meander inductor is divided into straight conductive segments. Then the total inductance of the meander inductor is a sum of self-inductances of all segments and the negative and positive mutual inductances between all combinations of straight segments. In addition to *Greenhouse's,* the great contribution to the inductance calculation is given by *Grover* [7]. He has considered the concept of partial inductance, i.e. contribution of individual segments to the overall inductance, and he also has introduced the idea of the geometric mean distance, called GMD. In this paper are studied inductors manufactured by means of MEMS technology, having a final thickness (8μm) and width of several tenths of μm, so that in calculation of self-inductance as one of the factors also appears GMD. When studying of mutual inductance as filament we take the middle lines of the observed meander as in Fig. 1. The filaments are the straight segments of the meander inductor having neglectable cross section in relation to the length.

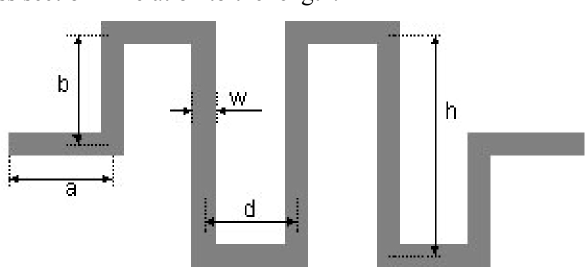


Fig. 1 - *Meander inductor with characteristic dimensions.*

2.1 Self-inductance

Expression for self-inductance of the conductive segment, as in Fig. 2, is given by the following equation

$$L = 0.002 \cdot l \cdot [\ln(2l/\mathrm{GMD}) - 1.25 + \mathrm{AMD}/l + (\mu/4) \cdot T], \tag{1}$$

with the following:

L – inductance in micro henries, l – length of conductive segment in centimetres, GMD and AMD represent geometrical and arithmetical mean distance of the conductor's cross-section, μ – conductor's permeability, and T is frequency dependant correction factor.

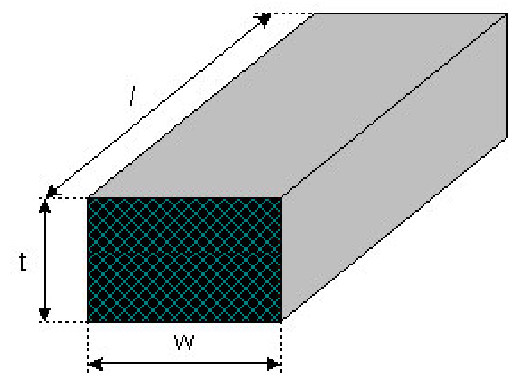


Fig. 2 – *Cross-section of the conductive segment of the meander inductor.*

If the expressions given in literature [5] for GMD and AMD for the conductor of rectangular cross-section are replaced, then the expression for self-inductance is

$$L = 0.002 l \left\{ \ln[2l/0.2232(w+t)] - 1.25 + [(w+t)/3l] + (\mu/4)T \right\}, \tag{2}$$

i.e.

$$L = 0.002 l \left\{ \ln[2l/(w+t)] + 0.50049 + [(w+t)/3l] \right\}, \tag{3}$$

where w and t are dimensions of the cross-section, as it is shown in Fig. 2. The last equation is valid in case when the magnetic permeability is 1 and when the frequency dependant factor T is not taken into account, i.e. $T = 1$.

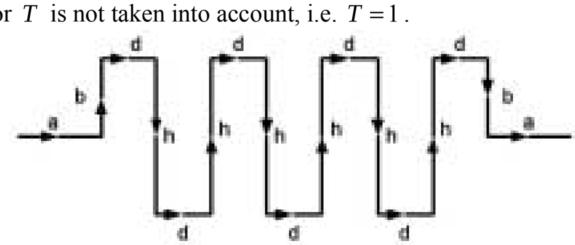


Fig. 3 – *Presentation of characteristic geometrical dimensions of the meander inductor.*

The expression (3) will be used as an analytical expression for calculation of self-inductance of individual straight segments (shown in Fig. 3), and by that also for calculation of total inductance of the meander inductor. The total self-inductance L_{selftot} is calculated as a sum of self-inductances of all individual line segments, which form meander inductor,

$$L_{\text{selftot}} = 2 \cdot L_a + 2 \cdot L_b + N \cdot L_{\boldsymbol{h}} + (N+1) \cdot L_d \,. \tag{4}$$

This is a generalized equation where $L_{a,b,h,d}$ are self-inductances of segments where the length is $l = a, b, h, d$ (respectively) and are calculated by means of the expression for self-inductance (3). In the equation (4), N represents a number of segments of the greatest length h (for the example given in Fig. 3 it is evident that $N = 6$).

2.2 Mutual Inductance

In order to get the final expression for the total mutual inductance, we are observing two cases that appear in the process of manufacture of inductors, with even number and odd number of the longest segments (length h). In Fig. 4 there is presented characteristic situations of the position of two segments that can appear in meander inductors, i.e. two parallel conductors of the equal length at the corresponding mutual distance. The equation (5), taken from the book of *Grover* [5] calculates mutual inductance (let sign it with M_c) of the segments with equal length and l , at distance r and are place opposite one to another as in Fig. 4.

$$M_c(l,r) = \pm \frac{\mu_0}{2 \cdot \pi} \cdot l \cdot \left[\ln\left(\frac{l}{r} + \sqrt{1 + \left(\frac{l}{r}\right)^2} \right) - \sqrt{1 + \left(\frac{r}{l}\right)^2} + \frac{r}{l} \right]. \tag{5}$$

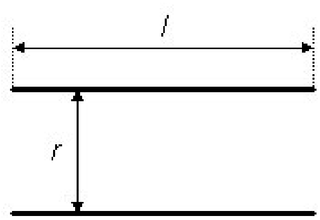


Fig. 4 – *Two equal parallel straight filaments.*

Mutual inductance of another segment positions can be expressed by linear combination of expression for calculation of mutual inductance of segments with equal length, at distance r and which are positioned one opposite the other (expression (5)).

Equation for the segment combinations as given in Fig. 5 (a), for unequal parallel filaments, is

$$M_{a1}(l_1, l_2, r, s) = 0.5 \cdot [M_c(l_1 + l_2 + s, r) + M_c(s, r) - M_c(l_1 + s, r) - M_c(l_2 + s, r) \,. \tag{6}$$

For the situation in the Fig. 5(b), where filaments with their ends in the common perpendicular, valid equation is as the following

$$M_{a2}(l_1,l_2,r)=0.5\cdot\left[M_c(l_1,r)+M_c(l_2,r)-M_c(l_1-l_2,r)\right]. \tag{7}$$

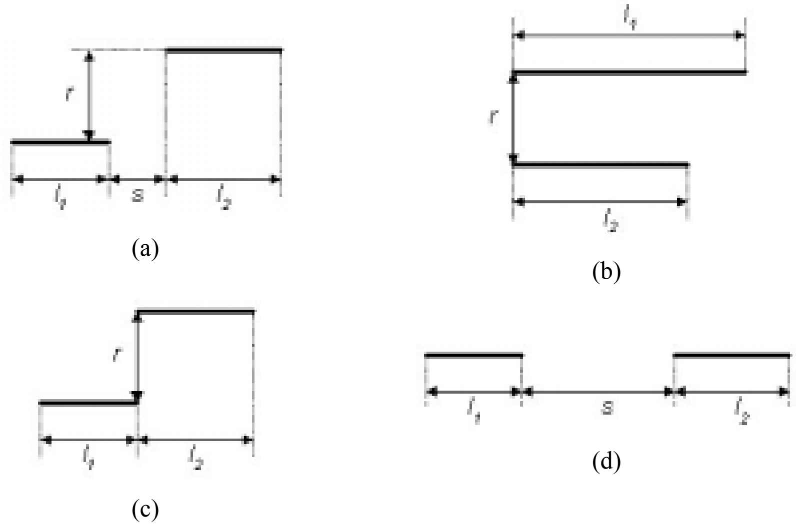


Fig. 5 – *Presentation of four typical segment combinations in meander inductors*.

For the situation in Fig. 5 (c), appearing only in case when N-odd for the couple of filaments with length b, the following equation is used,

$$M_{a3}(l_1,l_2,r)=0.5[M_c(l_1+l_2,r)-M_c(l_1,r)-M_c(l_2,r)]. \tag{8}$$

For the position as in Fig. 5 (d) when the two segments are in the same axis (segments pairs $a-a$ and $d-d$) at the corresponding distance between each other the expression given in the equation (9) designed with M_b is given,

$$M_b(l_1,l_2,s)=\frac{\mu_0}{4\pi}[(l_1+l_2+s)\ln(l_1+l_2+s)-(l_1+s)\ln(l_1+s)-(l_2+s)\ln(l_2+s)+s\ln(s)]. \tag{9}$$

Thus, now we can present the final equations for self-inductance and mutual inductance by which we calculate accurate values for inductances of meander inductors, and which are suitable for programming. In order to make it clearer, in Table I are given the characteristic situations of all obtained expressions for calculation of the total inductance.

It is necessary to emphasize that formulae given in Table I make a closed form of the expression for inductance of the meander. Their advantage is also in that to the designer of circuit they give information on how the geometrical parameters of the inductor influence upon the self-inductance, positive or negative mutual inductance.

3 Fitting Technique

In reference [8] there already exist expressions in a simple monomial form for the square and octagonal planar inductors. This form of the expression (10) for calculation of inductance is very suitable and necessary for application in the optimisation procedure of the inductor by means of geometrical programming method. While in reference [8] this expression has been obtained by fitting according to a great number of fabricated inductors, in this paper fitting techniques has been made according a great number of numerical values of inductance using the expressions described in section 2. We have taken five variables because there are five characteristic geometrical dimensions that in fact determine inductance of the inductor itself. Let's take the following $a \equiv x_1, h \equiv x_2, N \equiv x_3, d \equiv x_4, \omega \equiv x_5$. In Fig. 1 all these values can be seen,

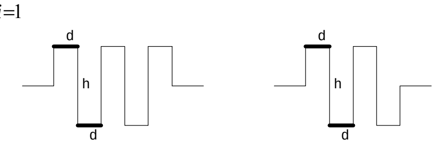
$$L = \beta \cdot x_1^{\alpha_1} \cdot x_2^{\alpha_2} \cdot x_3^{\alpha_3} \cdot x_4^{\alpha_4} \cdot x_5^{\alpha_5} . \tag{10}$$

However, this simple form of expression for inductance does not exist in literature for the meander inductor.

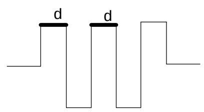
The expressions for calculation of the total inductance, presented in the previous section we have implemented into the computer program which served for generating of a great number of results, based on the given input geometrical and technological parameters. The fitting was done by means of the Lsm2000 program [9]. The fitting using the method of the least squares finds the parameters of the equation by which are minimized the sum of squares of the error between the accurate data and fitted equation. This program serves for fitting (adjusting) parameters, analytically given equations (curves, surfaces), so to suit the given points. Here, will be presented only final expression in the monomial form, which is suitable for optimisation via geometric programming.

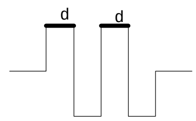
Table I

Presentation of all expressions for calculation of total inductance for N -even and N - odd.

$L = 0.002 l \{\ln[2l/(\omega+t)] + 0.50049 + [(\omega+t)/3l]\}$
$L_{\text{selftot}} = 2 \cdot L_a + 2 \cdot L_b + N \cdot L_h + (N+1) L_d$
$M_1 = \sum_{i=1}^{N/2} (2N+4-4i) \cdot M_{\text{ul}}(d, d, h, (2i-2)d)$, for N even

$M_1 = \sum_{i=1}^{(N+1)/2} (2N+4-4i) \cdot M_{a1}(d, d, h, (2i-2)d)$, for N odd

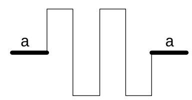
$$M_2 = \sum_{i=1}^{N/2} (2N + 2 - 4i) \cdot M_b(d, d, (2i-1)d) \text{, for } N \text{ even}$$



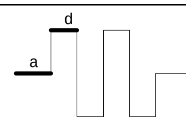


$$M_2 = \sum_{i=1}^{(N-1)/2} (2N + 2 - 4i) \cdot M_b(d, d, (2i-1)d) \text{, for } N \text{ odd}$$

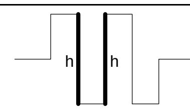
$$M_3 = 2 \cdot M_b(a, a, (N+1)d)$$



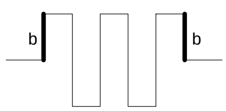
$$M_4 = \sum_{i=0}^{N} 4 \cdot M_{a1}(a, d, b, id)$$

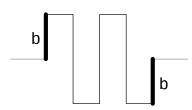


$$M_5 = \sum_{i=0}^{N-1} (-1)^i \cdot 2 \cdot (N-i) \cdot M_c(h, id)$$



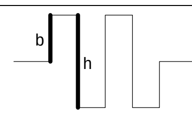
$$M_6 = -2 \cdot M_c(b, (N+1)d) \text{, for } N \text{ even}$$





$$M_6 = +2 \cdot M_{a3}(b, b, (N+1)d) \text{, for } N \text{ odd}$$

$$M_7 = \sum_{i=0}^{N} (-1)^i \cdot 4 \cdot M_{a2}(b, h, i\,d)$$



$$M_{\text{tot}} = M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_7$$

$$L_{\text{tot}} = L_{\text{selftot}} + M_{\text{tot}}$$

The monomial equation for the total inductance L_{mon}, of the meander inductor that gave the least relative error (maximally up to 12%) is

$$L_{\text{mon}} = 0.00266 \cdot a^{0.0603} \cdot h^{0.4429} \cdot N^{0.954} \cdot d^{0.606} \cdot w^{-0.173}. \qquad (11)$$

Thus, by this expression for the first time was presented the inductance of the meander inductor in the monomial form, so that the optimisation of the inductor can be done by procedure of the geometrical programming. Simplicity and relatively good accuracy are the advantages of this expression, but on the other hand the physical sense of the expression is being lost. Also, it is not possible to get information on mutual inductance or of the self-inductance of the meander inductor.

4 Results and Discussion

In order to demonstrate the validity of results of the developed software tool in this part we compared results that are obtained by means of proposed programs and experimental results taken from the literature. In the Table I are shown geometrical parameters and measured inductances for three meander inductors as found in the open literature [5].

Table II

Dimensions of three types of meander inductors and their measured inductances [5].

Name	N	w [μm]	d [μm]	L_{meas} [nH]
Ind #1	5	40	40	1.5
Ind #2	8	24	24	2
Ind #3	11	17	17	2.5

In the Section 2 in great details is described the procedure for calculation of the total inductance of the meander inductor both for the even and odd number the longest segments using the mathematical induction, and by using the *Greenhouse's* method [6]. Expressions that are summed in the Table I served as the basis to write the program for calculation of the self-inductance, positive or negative mutual inductance and total inductance for the meander inductors. The program is written in the program language Visual Basic 6.0, while the source code of program has been completely developed by our own knowledge acquired by previous scientific research and experience. The program has its graphical interface (Fig. 6), while the user gives the input data and very quickly gets the information about the self-inductance ($L_{selftot}$), positive (M^{+}) or negative (M^{-}) mutual inductance and the total inductance (L_{tot}) of the meander inductors. Simulated results of inductance for the three types of the meanders are given in the Table III. The program can also draw the layout of the simulated meander, and gives the information on the used area of the inductor.

Table III

Self-inductance, positive, negative mutual inductance and total inductance for three types of meander inductors.

Induct. name	$L_{selftot}$ [nH]	M^{+} [nH]	M^{-} [nH]	L_{tot} [nH]
Ind #1	1.481	0.601	-0.577	1.504
Ind #2	2.463	0.729	-1.217	1.974
Ind #3	3.450	0.810	-1.971	2.289

In the Section 3 is explained the procedure for obtaining the expression for calculation of the meander inductance in the monomial form and at the end of that section is given the expression used in optimisation of meander layout by use of procedure of geometrical programming (results of this procedure will be given in one of our future scientific papers). Comparison of the experimental results for the total inductance, for three meander inductors given in the Table II, with results obtained by proposed new expressions (based on *Greenhouse* theory, Table I), or monomial expression (11) is given in the Table IV.

Table IV

Comparison of obtained results for the inductance of meander inductors.

Induct. name	Measured inductance [nH]	This work (Table I) [nH]	Monomial expression (11) [nH]
Ind #1	1.5	1.50	1.61
Ind #2	2	1.97	2.08
Ind #3	2.5	2.29	2.44

As it is seen from the Table IV, the agreement of the results is very good. It is necessary to mention again that in the scientific literature there is very small number of papers giving experimental data for the meander inductors. However, on the other hand our dealing with meander inductors and conformity that so far can be evident, gives even greater value to research in this field.

It should be noted that are no unphysical fitting factors in proposed expressions depicted in Table I. Those expressions have the advantage that it indicates to the designer how the relative contributions of self, positive, and negative mutual inductance are related to the geometrical parameters.

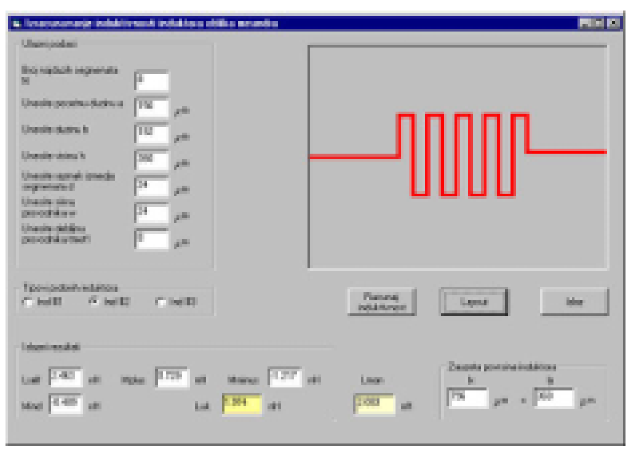


Fig. 6 - *One example of software tool interface for inductance calculation of meander inductors.*

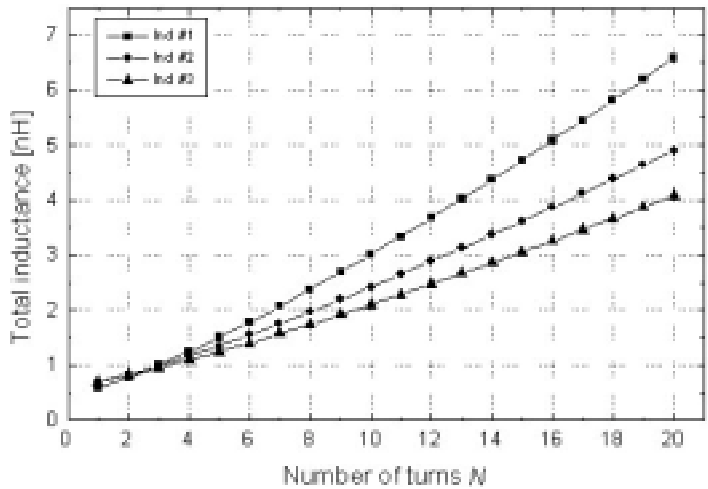


Fig. 7 - *The total inductance of meander inductors versus number of segments of the greatest length.*

How number of the longest segments N influence the total inductance for tree type of tested meander inductors is depicted in Figure 7. Hence, Ind #1 has the widest conductor's segment the total inductance rises greater with increased the number of the longest segments.

When we change one of parameters, all another take the same values as in the Table II.

The above figure demonstrates that all inductance variations by varying geometry parameters are successfully predicted using proposed software tool.

5 Conclusion

Planar meander inductors of the simple layout are inevitable component in the contemporary RF integrated circuits. The calculation of the meander's inductance has been treated in the literature with very great importance. In this paper is presented a software tool, which in a fast and efficient manner to calculate inductance of the meander inductors. There are given two ways for calculation of inductance – based on *Greenhouse* method and the form of monomial expression. The accuracy of presented results is evaluated by comparison with earlier published experimental values of inductance. The proposed closed form inductance expressions is compatible with circuit simulators.

6 Acknowledgement

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