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COMPACT KAEHLER MANIFOLDS WITH POSITIVE RICCI TENSOR

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The purpose of the present note is to announce the following:

THEOREM 1. *A compact Kaehler manifold with positive definite Ricci tensor is simply connected.*

We say that the first Chern class of a compact Kaehler manifold is positive definite if it can be represented by a real closed (1, 1)-form which is positive in the sense of Kodaira [2]. The first Chern class of a manifold satisfying the assumption in Theorem 1 is necessarily positive definite. Theorem 1 follows from the following two theorems.

THEOREM 2. *If the first Chern class of a compact Kaehler manifold M is positive definite, then the fundamental group of M has no proper subgroup of finite index.*

THEOREM OF MYERS. *The fundamental group of a compact Riemannian manifold with positive definite Ricci tensor is finite* [3].

Theorem 2 can be proved by Kodaira's Vanishing Theorem and by the Riemann-Roch Theorem of Hirzebruch. Let g_p be the dimension of the space of holomorphic p-forms on M. Then $\chi(M)=\sum_{p=0}^{n}(-1)^p g_p$, where $n=\dim_C M$, is called the arithmetic genus of M. If M is algebraic, then $\chi(M)$ is given as the integral over M of a polynomial in Chern classes c_i of weight n, polynomial depending only on n, not on M [1]. From this follows that if M^* is a k-fold covering space of M, then $\chi(M^*)=k\cdot\chi(M)$. On the other hand, if the first Chern class is positive definite, then $g_p=0$ for $1\leqq p\leqq n$ [2] and, hence, the arithmetic genus is 1. If the first Chern class of M is positive definite, so is the first Chern class of M^*. Hence, $\chi(M^*)=\chi(M)=1$, proving that $k=1$.

Note that Theorem 2 can be rephrased as follows. If the first Chern class of M is positive definite, then every holomorphic transformation of finite period has fixed points.

In view of the fact that we know no example of a compact Kaehler manifold with positive definite first Chern class whose Ricci tensor is not positive definite, we conjecture that M is simply connected under the assumption of Theorem 2.

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