

Received September 6, 2019, accepted September 20, 2019, date of publication September 30, 2019, date of current version October 9, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2944414

Compact Model-Free Adaptive Control Algorithm for Discrete-Time Nonlinear Systems

XIAOFEI ZHANG¹, HONGBIN MA^{1,2}, XINGHONG ZHANG³, AND YOU LI⁴

¹School of Automation, Beijing Institute of Technology, Beijing 100081, China

²State Key Laboratory of Intelligent Control and Decision of Complex Systems, Beijing Institute of Technology, Beijing 100081, China

³School of Electrical Engineering and Automation, Henan Institute of Technology, Xinxiang 453003, China

⁴National Key Laboratory of Human Factors Engineering, China Astronaut Research and Training Center, Beijing 100094, China

Corresponding author: Hongbin Ma (mathmhb@qq.com)

This work was supported in part by the Ministry of Science and Technology of People's Republic of China under Grant 2017YFF0205306 and Grant WQ20141100198, and in part by the National Natural Science Foundation of China under Grant 91648117.

ABSTRACT In model free adaptive control (MFAC), a virtual equivalent dynamic linearized model is built. The linearization length constants (LLCs) of the virtual equivalent dynamic linearized model are selected by the practitioner based on experience. In this paper, the optimal LLCs are investigated, and compact model free adaptive control (CMFAC) is introduced for a class of unknown discrete-time nonlinear systems. Compared with MFAC, the proposed CMFAC does not need to consider the values of LLCs, and the optimal LLCs are decided by the desired tracking error of systems. Simulation experiments are taken, and the simulation results indicate that the proposed control algorithm is effective and can achieve asymptotic tracking.

INDEX TERMS Adaptive algorithm, control design, discrete time systems.

I. INTRODUCTION

With the development of control theory, many concepts and algorithms [1]–[16] have been proposed, such as model free adaptive control (MFAC), iterative learning control (ILC), fuzzy control, adaptive control, sliding mode control (SMC), etc. Besides, some algorithms based on reinforcement learning [17], [18] have been investigated for control systems. Nowadays the production technologies and processes become more and more complex, and it is difficult to obtain an accurate mechanism model of a physical system due to its complexity. Besides, the information of systems may be incomplete, imprecise or inadequate, even establishing a simplified model of systems is also impossible. MFAC [19]–[24] is a class of data-driven control (DDC) [25], which uses the input and output (I/O) data of controlled systems and does not need to consider mechanism models of systems.

The design of MFAC algorithm is directly based on pseudo-partial-derivatives (PPD), and the values of PPD can be derived on-line from the I/O information of systems using parameter estimation algorithms, such as projection

algorithm, recursive least squares, etc. MFAC has gained a large amount of interests in the recent years, and MFAC is used to deal with some unknown discrete-time nonlinear systems. In MFAC, a virtual equivalent dynamic linearized model is built by using a dynamic linearization technique, and those linearization length constants (LLCs) of the virtual equivalent dynamic linearized model should be set to reasonable values.

Most researches of MFAC are focused on improving the accuracy of control systems by obtaining accurate values of PPD and using other methods [26]–[28]. Large LLCs make the controller based on MFAC technique contain more information, and large LLCs could improve the control performances of systems. In [29], a simulation experiment was done for demonstrating the influence on the control performances with respect to the choice of LLCs, and simulation results show that the control performances of systems cannot be improved so much by increasing the values of LLCs when LLCs are large enough. Besides, large LLCs may require more calculation time. It is meaningful to investigate the optimal LLCs of MFAC methods for unknown discrete-time nonlinear systems, such as the position control system of a manipulator end-actuator.

The associate editor coordinating the review of this manuscript and approving it for publication was Qi Zhou.

The LLCs are selected by the practitioner based on experience, and the existing examples show that they could always be chosen quite low. Up to now, we have not found any introduction about optimal LLCs of MFAC methods for unknown discrete-time nonlinear systems. Although, MFAC has been widely used in control systems, some matters of MFAC need to be further investigated [30], such as the optimal LLCs, the relation between the change speed of PPD and system stability, etc. In this paper, the optimal LLCs are investigated, and inspired by broad learning system [31] and error minimized extreme learning machine [32], compact model free adaptive control (CMFAC) is introduced for the position control system of a manipulator end-actuator. This paper is structured as follows: the full form dynamic linearization of plant (FFDL_p) and MFAC are briefly introduced in Section II. Then the proposed CMFAC is stated in Section III. Section IV and V present the simulation results and conclusion, respectively.

II. MODEL FREE ADAPTIVE CONTROL

Consider the single input single output (SISO) unknown discrete-time nonlinear plant

$$y(k+1) = f(y(k), \dots, y(k-L_y), u(k), \dots, u(k-L_u)) \quad (1)$$

where $f(\cdot)$ represents an unknown nonlinear function, L_y is the unknown order of output $y(k)$, and L_u is the unknown order of input $u(k)$.

To make further study, the following assumptions are used.

Assumption 1: The system (1) is observable and controllable in following meaning, that is, to the expected bounded system output signal $y^*(k+1)$, there exist a bounded feasible control input signal which drives the system output equal to the expected output.

Assumption 2: $f(\cdot)$ is a smooth nonlinear function, and the partial derivatives of $f(\cdot)$ with respect to $u(k), \dots, u(k-L_u)$ and $y(k), \dots, y(k-L_y)$ are continuous.

Assumption 3: The system (1) is generalized Lipschitz, that is, satisfying

$$|\Delta y(k+1)| \leq L_b \|\Delta \theta(k)\|, \quad (2)$$

and

$$\Delta \theta(k) = [\Delta y(k), \Delta u(k)] \quad (3)$$

where

$$\Delta y(k+1) = y(k+1) - y(k), \quad (4)$$

$$\begin{aligned} \Delta y(k) &= [\Delta y(k), \dots, \Delta y(k-L_y)], \\ \Delta u(k) &= [\Delta u(k), \dots, \Delta u(k-L_u)], \end{aligned} \quad (5)$$

here L_b is a constant, and $L_b > 0$.

Assumption 4: $\eta(y(k-1), \dots, y(k-L_y-1), u(k-1), \dots, u(k-L_u-1))$ is a vector-valued function, and define $\eta(y(k-1), \dots, y(k-L_y-1), u(k-1), \dots, u(k-L_u-1)) \triangleq f(y(k-1), \dots, y(k-M_y-1), y(k-M_y-1), \dots, y(k-L_y), u(k-1), \dots, u(k-M_u-1))$,

$$\begin{aligned} &u(k-M_u-1), \dots, u(k-L_u)) \\ &-f(y(k-1), \dots, y(k-M_y-1), y(k-M_y-2), \\ &\dots, y(k-L_y-1), u(k-1), \dots, u(k-M_u-1), \\ &u(k-M_u-2), \dots, u(k-L_u-1)) \end{aligned} \quad (6)$$

where L_y is the unknown order of output $y(k)$, and L_u is the unknown order of input $u(k)$; M_y and M_u are LLCs of the virtual equivalent dynamic linearized model. Suppose that $\eta(y(k-1), \dots, y(k-L_y-1), u(k-1), \dots, u(k-L_u-1))$ is bounded.

For the nonlinear system (1), satisfying assumptions (1)-(3), there must be $\chi(k)$. When $\|\Delta \theta(k)\| \neq 0$, Equation (1) can be rewritten as

$$\begin{aligned} y(k+1) &= y(k) + \Delta u(k)\chi_u(k) + \Delta y(k)\chi_y(k) \\ &= y(k) + \Delta \theta(k)\chi(k) \end{aligned} \quad (7)$$

where

$$\begin{aligned} \chi(k) &= [\chi_y(k), \chi_u(k)]^T \\ &= [\chi_1(k), \dots, \chi_{L_u+L_y}(k)]^T, \end{aligned} \quad (8)$$

and Equation (7) is also called full form dynamic linearization of system (1).

Proof: Using (1) and Cauchy differential mean value theorem, we can get that

$$\begin{aligned} \Delta y(k+1) &= f(y(k), \dots, y(k-L_y), u(k), \dots, u(k-L_u)) \\ &\quad - f(y(k-1), \dots, y(k-L_y-1), u(k-1), \dots, \\ &\quad u(k-L_u-1)) \\ &= \frac{\partial f^*}{\partial y(k)} \Delta y(k) + \dots + \frac{\partial f^*}{\partial y(k-L_y)} \Delta y(k-L_y) \\ &\quad + \frac{\partial f^*}{\partial u(k)} \Delta u(k) + \dots + \frac{\partial f^*}{\partial u(k-L_u)} \Delta u(k-L_u). \end{aligned} \quad (9)$$

Define

$$\chi(k) \triangleq \left[\frac{\partial f^*}{\partial y(k)}, \dots, \frac{\partial f^*}{\partial y(k-L_y)}, \frac{\partial f^*}{\partial u(k)}, \dots, \frac{\partial f^*}{\partial u(k-L_u)} \right]^T, \quad (10)$$

then we can get (7).

A. DESIGN OF MFAC

Lemma 1: If A, X and $CA^{-1}B + X^{-1}$ are reversible, then

$$[A + BXC]^{-1} = A^{-1} - A^{-1}B[CA^{-1}B + X^{-1}]^{-1}CA^{-1} \quad (11)$$

where A, B, X and C are matrices.

Consider the following cost function with an additional penalty on the abrupt change of estimated parameter:

$$J(u(k)) = |y^*(k+1) - y(k+1)|^2 + \lambda |u(k) - u(k-1)|^2 \quad (12)$$

where $y^*(k+1)$, $y(k+1)$ and $u(k)$ indicate the desired output signal, output signal and control signal, respectively, and λ is a constant.

Substituting (7) into (12), differentiating (12) with respect to $u(k)$ and setting it to zero yields

$$u(k) = u(k-1) + \frac{\chi_{L_y+1}(k)}{\lambda + |\chi_{L_y+1}(k)|^2} \times (y^*(k+1) - y(k) - \Delta\theta'(k)\chi'(k)) \quad (13)$$

where

$$\Delta\theta'(k) = [\Delta y(k), \dots, \Delta y(k - L_y), \Delta u(k-1), \dots, \Delta u(k - L_u)], \quad (14)$$

and

$$\chi'(k) = [\chi_1(k), \dots, \chi_{L_y}(k), \chi_{L_y+2}(k), \dots, \chi_{L_y+L_u}(k)]^T. \quad (15)$$

Consider the following cost function

$$Q(\chi) = \frac{1}{2} \sum_{k=1}^N (\Delta y(k+1) - \Delta\theta(k)\chi)^2 + \frac{1}{2} (\chi - \hat{\chi}(0))^T P_0^{-1} (\chi - \hat{\chi}(0)). \quad (16)$$

Set

$$Y(k+1) = [\Delta y(1), \Delta y(2), \dots, \Delta y(k+1)]^T, \quad (17)$$

and

$$\psi(k) = [\Delta\theta(0), \Delta\theta(1), \dots, \Delta\theta(k)]^T. \quad (18)$$

Then the cost function (16) can be rewritten as

$$Q(\chi) = \frac{1}{2} [Y(k+1) - \psi(k)\chi]^T [Y(k+1) - \psi(k)\chi] + \frac{1}{2} (\chi - \hat{\chi}(0))^T P_0^{-1} (\chi - \hat{\chi}(0)). \quad (19)$$

Define

$$P^{-1}(k) \triangleq (\psi^T(k)\psi(k) + P_0^{-1}) \quad (20)$$

where

$$\psi^T(k) = [\psi^T(k-1), \Delta\theta^T(k)]. \quad (21)$$

Substituting (21) into (20), the following equation can be obtained

$$P(k) = \left[P^{-1}(k-1) + \Delta\theta^T(k)\Delta\theta(k) \right]^{-1}. \quad (22)$$

Using Lemma 1, the update formula of $P(k)$ is

$$P(k) = P(k-1) - \frac{P(k-1)\Delta\theta^T(k)\Delta\theta(k)P(k-1)}{1 + \Delta\theta(k)P(k-1)\Delta\theta^T(k)}. \quad (23)$$

Differentiating (19) with respect to χ and setting it to zero yields

$$\left[\psi^T(k)\psi(k) + P_0^{-1} \right] \chi = P_0^{-1} \hat{\chi}(0) + \psi^T(k)Y(k+1). \quad (24)$$

Then we can get the estimated value of χ at time $k+1$, and

$$\hat{\chi}(k+1) = \left[\psi^T(k)\psi(k) + P_0^{-1} \right]^{-1} \left[P_0^{-1} \hat{\chi}(0) + \psi^T(k)Y(k+1) \right]$$

$$\begin{aligned} &= P(k) [P_0^{-1} \hat{\chi}(0) + \psi^T(k-1)Y(k) + \Delta\theta^T(k)\Delta y(k+1)] \\ &= P(k) \left[P^{-1}(k-1)\hat{\chi}(k) + \Delta\theta^T(k)\Delta y(k+1) \right] \\ &= P(k) \left[P^{-1}(k) - \Delta\theta^T(k)\Delta\theta(k) \right] \hat{\chi}(k) \\ &\quad + P(k)\Delta\theta^T(k)\Delta y(k+1) \\ &= \hat{\chi}(k) + P(k)\Delta\theta^T(k)(\Delta y(k+1) - \Delta\theta(k)\hat{\chi}(k)). \end{aligned} \quad (25)$$

Replace $\chi(k)$ with $\hat{\chi}(k)$, then we can get that the adaptive control law of MFAC is

$$u(k) = u(k-1) + \frac{\hat{\chi}_{L_y+1}(k)}{\lambda + |\hat{\chi}_{L_y+1}(k)|^2} (y^*(k+1) - y(k) - \Delta\theta'(k)\hat{\chi}'(k)) \quad (26)$$

where

$$\Delta\theta'(k) = [\Delta y(k), \dots, \Delta y(k - L_y), \Delta u(k-1), \dots, \Delta u(k - L_u)], \quad (27)$$

and

$$\hat{\chi}'(k) = [\hat{\chi}_1(k), \dots, \hat{\chi}_{L_y}(k), \hat{\chi}_{L_y+2}(k), \dots, \hat{\chi}_{L_y+L_u}(k)]^T. \quad (28)$$

The updating algorithm of $\hat{\chi}(k+1)$ and $P(k)$ are

$$\hat{\chi}(k+1) = \hat{\chi}(k) + \frac{P(k-1)\Delta\theta^T(k)}{1 + \Delta\theta(k)P(k-1)\Delta\theta^T(k)} \times (\Delta y(k+1) - \Delta\theta(k)\hat{\chi}(k)) \quad (29)$$

and

$$P(k) = P(k-1) - \frac{P(k-1)\Delta\theta^T(k)\Delta\theta(k)P(k-1)}{1 + \Delta\theta(k)P(k-1)\Delta\theta^T(k)}. \quad (30)$$

III. DESIGN OF CMFAC

The precondition of designing controllers is that L_y and L_u are known, however in actual systems it is difficulty to get the values of L_y and L_u . In some cases, the values of L_y and L_u are changeable, and the values of L_y and L_u also may be very big. In order to design MFAC, a compact full form dynamic linearization (CFFDL) is investigated by some scholars. For the unknown nonlinear system (1), satisfying assumptions (1)-(4), Equation (1) can be rewritten as

$$\Delta y(k+1) = \Delta\bar{\theta}(k)\bar{\chi}(k) \quad (31)$$

where

$$\Delta\bar{\theta}(k) = [\Delta y(k), \dots, \Delta y(k - M_y), \Delta u(k), \dots, \Delta u(k - M_u)], \quad (32)$$

and

$$\bar{\chi}(k) = [\chi_1(k), \chi_2(k), \dots, \chi_{M_y+M_u}(k)]^T, \quad (33)$$

and $\bar{\chi}(k)$ is called pseudo-partial-derivative vector.

Proof: Equation (1) gives

$$\begin{aligned} \Delta y(k+1) &= f(y(k), \dots, y(k - L_y), u(k), \dots, u(k - L_u)) \\ &\quad - f(y(k-1), \dots, y(k - L_y - 1), u(k-1), \dots, \\ &\quad u(k - L_u - 1)) \end{aligned}$$

$$\begin{aligned}
 &= f(y(k), \dots, y(k - M_y), y(k - M_y - 1), \\
 &\quad \dots, y(k - L_y), u(k), \dots, u(k - M_u), \\
 &\quad u(k - M_u - 1), \dots, u(k - L_u)) \\
 &\quad - f(y(k - 1), \dots, y(k - M_y - 1), y(k - M_y - 1), \\
 &\quad \dots, y(k - L_y), u(k - 1), \dots, u(k - M_u - 1), \\
 &\quad u(k - M_u - 1), \dots, u(k - L_u)) \\
 &\quad + f(y(k - 1), \dots, y(k - M_y - 1), y(k - M_y - 1), \\
 &\quad \dots, y(k - L_y), u(k - 1), \dots, u(k - M_u - 1), \\
 &\quad u(k - M_u - 1), \dots, u(k - L_u)) \\
 &\quad - f(y(k - 1), \dots, y(k - L_y - 1), u(k - 1), \dots, \\
 &\quad u(k - L_u - 1)) \\
 &= \frac{\partial f^*}{\partial y(k)} \Delta y(k) + \dots + \frac{\partial f^*}{\partial y(k - M_y)} \Delta y(k - M_y) \\
 &\quad + \frac{\partial f^*}{\partial u(k)} \Delta u(k) + \dots + \frac{\partial f^*}{\partial u(k - M_u)} \Delta u(k - M_u) \\
 &\quad + \eta(y(k - 1), \dots, y(k - L_y - 1), \\
 &\quad u(k - 1), \dots, u(k - L_u - 1)). \tag{34}
 \end{aligned}$$

Because $\eta(y(k - 1), \dots, y(k - L_y - 1), u(k - 1), \dots, u(k - L_u - 1))$ is bounded, there must be $\bar{\chi}^*(k)$, and

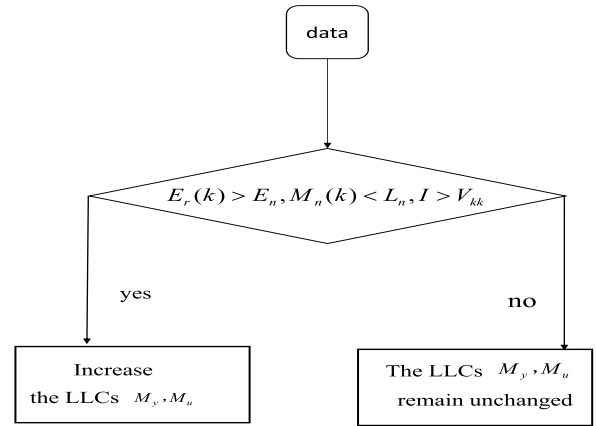
$$\eta(y(k - 1), \dots, y(k - L_y - 1), u(k - 1), \dots, u(k - L_u - 1)) = \Delta \theta(k) (\bar{\chi}^*(k))^T. \tag{35}$$

Then

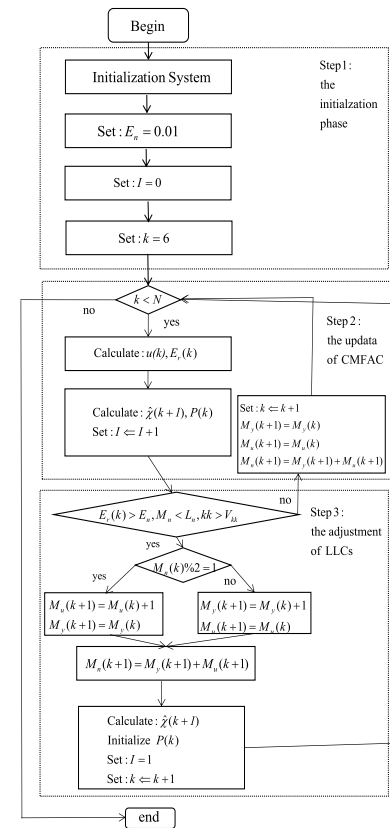
$$\begin{aligned}
 \bar{\chi}(k) = & \left[\frac{\partial f^*}{\partial y(k)}, \dots, \frac{\partial f^*}{\partial y(k - M_y)}, \frac{\partial f^*}{\partial u(k)}, \dots, \right. \\
 & \left. \times \frac{\partial f^*}{\partial u(k - M_u)} \right]^T + (\bar{\chi}^*(k))^T, \tag{36}
 \end{aligned}$$

hence (31) can be obtained.

M_y and M_u are called LLCs of the virtual equivalent dynamic linearized model (31), and the LLCs M_y and M_u could be set to be values of L_y and L_u , respectively, if the order L_y and L_u are known *a priori*. Otherwise, the LLCs M_y and M_u should be set to be reasonable values according to complexity of plants. The LLCs M_y and M_u are selected by the practitioner based on experience, and the existing examples show that they could always be chosen quite low. The control performances of systems cannot be improved so much by increasing the values of LLCs when LLCs are large enough. Besides, large LLCs may require more calculation time. In this paper, CMFAC is introduced, and the optimal LLCs M_y and M_u are decided by the desired tracking error of systems. In order to get rid of the trouble that the proposed CMFAC need initial data of systems and accelerate the speed for system identification, the variable forgetting factor $a(k)$ is used in the proposed CMFAC. Fig. 1 shows the process of the proposed CMFAC. The proposed CMFAC can be divided into three phases, the first phase is the initialization of systems, the second phase is the updating of CMFAC, and the third phase is the adjustment of the LLCs M_y and M_u .



(a)



(b)

FIGURE 1. The process of the proposed CMFAC: (a) the simplified process of the proposed CMFAC, (b) the detailed process of the proposed CMFAC.

In order to simplify the calculation of the proposed CMFAC, Equation (31) can be rewritten as

$$\Delta y(k + 1) = \Delta \theta^1(k) \chi^1(k) \tag{37}$$

where

$$\Delta \theta^1(k) = [\Delta y(k), \Delta u_3(k) \dots, \Delta y(k - M_y), \Delta u_3(k - M_u)], \tag{38}$$

$$\chi^1(k) = \left[\chi_1^1(k), \chi_2^1(k), \dots, \chi_{M_y + M_u}^1(k) \right]^T, \tag{39}$$

$u_3(k)$ is the control signal of CMFAC, and $u_3(k)$ is defined in below.

A. INITIALIZATION PHASE

- 1) Initialize systems: set $y(5) = y(4) = y(3) = y(2) = y(1) = 0$, $u_3(5) = u_3(4) = u_3(3) = u_3(2) = u_3(1) = 0$, $M_y(6) = M_y(5) = M_y(4) = M_y(3) = M_y(2) = M_y(1) = 1$, and $M_u(6) = M_u(5) = M_u(4) = M_u(3) = M_u(2) = M_u(1) = 1$.
- 2) Set $M_n(5) = M_y(5) + M_u(5)$, $P_I = 1000$, $E_n = 0.01$, and $I = 0$.
- 3) Set $\lambda = 0.01$, $a_0 = 0.99$, $a(5) = 0.95$, $V_{kk} = 30$, $L_n = 20$, $V_E = 20$,

$$P(5) = P_I \times I_{M_n(5)}, \quad (40)$$

and

$$\hat{\chi}^1(6) = [0.01, 0.01]^T \quad (41)$$

where $I_{M_n(5)}$ denotes $M_n(5) \times M_n(5)$ identify matrix.

B. THE UPDATING OF CMFAC

- 1) Set: $I \leftarrow I + 1$.
- 2) Calculate $u_3(k)$

$$u_3(k) = u_3(k-1) + \frac{\hat{\chi}_2^1(k)}{\lambda + |\hat{\chi}_2^1(k)|^2} (y^*(k+1) - y(k) - \Delta\theta^{11}(k)\hat{\chi}^{11}(k)) \quad (42)$$

$$\Delta\theta^{11}(k) = \begin{cases} M & \text{if } \text{mod}(M_n(k), 2) = 1 \\ N & \text{if } \text{mod}(M_n(k), 2) = 0 \end{cases} \quad (43)$$

where

$$M = [\Delta y(k), \Delta y(k-1), \Delta u_3(k-1), \dots, \Delta y(k - M_y)], \quad (44)$$

$$N = [\Delta y(k), \Delta y(k-1), \Delta u_3(k-1), \dots, \Delta u_3(k - M_u)], \quad (45)$$

$$\hat{\chi}^{11}(k) = [\hat{\chi}_1^1(k), \hat{\chi}_3^1(k), \dots, \hat{\chi}_{M_y+M_u-1}^1(k)]^T. \quad (46)$$

- 3) Calculate $E_r(k)$,

$$E_r(k) = V/V_E \quad (47)$$

where

$$V = \sum_{i=k-V_E}^{k-1} (y^*(i) - y(i)).$$

- 4) Update $\hat{\chi}^1(k+1)$ and $P(k)$

$$\hat{\chi}^1(k+1) = \hat{\chi}^1(k) + \frac{P(k-1)(\Delta\theta^1(k))^T}{a(k-1) + \Delta\theta^1(k)P(k-1)(\Delta\theta^1(k))^T} \times (\Delta y(k+1) - \Delta\theta^1(k)\hat{\chi}^1(k)), \quad (48)$$

and

$$P(k) = \frac{1}{a(k-1)} [P(k-1) - \frac{P(k-1)\chi\Delta\theta^1(k)^T\Delta\theta^1(k)P(k-1)}{a(k-1) + \Delta\theta^1(k)P(k-1)\chi\Delta\theta^1(k)^T}]. \quad (49)$$

- 5) Update the forgetting factor $a(k)$

$$a(k) = a_0 a(k-1) + 1 - a_0. \quad (50)$$

- 6) When the tracking error does not meet the requirements of systems, the LLC M_y or the LLC M_u will be adjusted. The core of the proposed CMFAC is the adjustment of the LLCs M_y and M_u . if $E_r(k) \geq E_n \wedge M_n(k) \leq L_n \wedge I \geq V_{kk}$ then execute II; or I.
I) $k \leftarrow k + 1$, $M_y(k+1) = M_y(k)$, $M_u(k+1) = M_u(k)$, $M_n(k+1) = M_y(k+1) + M_u(k+1)$, and go to Subsection III-B.
II) Go to Subsection III-C.

C. THE ADJUSTMENT OF THE LLCs M_y AND M_u

- 1) When the LLC M_y or the LLC M_u are increased, it is equivalent to add a new column to $\Delta\theta^1(k)$. if $\text{mod}(M_n(k), 2) = 1$ then execute ii; or i.
i)

$$\Delta\delta = \Delta y(k - M_y - 1), \quad (51)$$

and

$$\begin{aligned} M_u(k+1) &= M_u(k) \\ M_y(k+1) &= M_y(k) + 1. \end{aligned} \quad (52)$$

ii)

$$\Delta\delta = \Delta u_3(k - M_u - 1), \quad (53)$$

and

$$\begin{aligned} M_u(k+1) &= M_u(k) + 1 \\ M_y(k+1) &= M_y(k). \end{aligned} \quad (54)$$

- 2)

$$\Delta\theta^1(k+1) = [\Delta\theta^1(k) \mid \Delta\delta]. \quad (55)$$

- 3) The pseudo inverse of the new $(\Delta\theta^1(k+1))^+$ is

$$(\Delta\theta^1(k+1))^+ = \begin{bmatrix} (\Delta\theta^1(k))^+ & -db \\ & b \end{bmatrix} \quad (56)$$

where

$$d = (\Delta\theta^1(k))^+ \Delta\delta, \quad (57)$$

$$b = \begin{cases} c^{-1} & \text{if } c \neq 0 \\ (1 + d^T d)^{-1} d^T (\Delta\theta^1(k))^+ & \text{if } c = 0, \end{cases} \quad (58)$$

and

$$c = \Delta\delta - \Delta\theta^1(k)d. \quad (59)$$

Again, calculate $\check{\chi}^1(k+1)$

$$\check{\chi}^1(k+1) = \begin{bmatrix} \hat{\chi}^1(k+1) - db\Delta y(k+2) \\ b\Delta y(k+2) \end{bmatrix}. \quad (60)$$

In actual systems, the value of $\Delta y(k+2)$ can not be obtained at time k . In this paper, we replace $\Delta y(k+2)$

with a historical data $\Delta\delta$ of $Y(k+1)$, Equation (60) can be rewritten as

$$\check{\chi}^1(k+1) = \begin{bmatrix} \hat{\chi}^1(k+1) - db\Delta\delta \\ b\Delta\delta \end{bmatrix}, \quad (61)$$

then the new $\hat{\chi}^1(k+1)$ is

$$\hat{\chi}^1(k+1) = \check{\chi}^1(k+1). \quad (62)$$

4) Initialize $P(k)$,

$$M_n(k+1) = M_y(k+1) + M_u(k+1), \quad (63)$$

and

$$P(k) = P_I \times I_{M_n(k+1)} \quad (64)$$

where $I_{M_n(k+1)}$ denotes $M_n(k+1) \times M_n(k+1)$ identify matrix.

5) Set $I = 1$, and go to the updating of MFAC III-B.

IV. ANALYSIS OF EXPERIMENTAL RESULTS

In order to indicate the effectiveness of the proposed CMFAC algorithm, three simulation experiments are done. The first simulation experiment is done for the purpose of testing the effectiveness of the proposed CMFAC algorithm in obtaining the optimal LLCs. In the first simulation experiment, $L_y = 4$, and $L_u = 3$. In the second simulation experiment, the proposed CMFAC algorithm is considered for the position control system of a manipulator end-actuator. In the third simulation experiment, the proposed CMFAC is compared with the MFAC based on RLS [30, Equation.5.49] for a unknown discrete-time nonlinear system. In order to show the effectiveness of algorithms, the integral square error (ISE) index of predicted output (65) is introduced

$$e_{ISE} = \sum_{j=1}^{2000} (y^*(j) - y(j))^2. \quad (65)$$

A. THE FIRST EXPERIMENT

In the first simulation experiment, the following discrete-time system is considered

$$y(k+1) = -3y(k) + 2y(k-1) - y(k-2) + 1.5y(k-3) + 2.1u(k) - 2u(k-1) + 1.2u(k-2). \quad (66)$$

In order to simplify the calculation of the proposed CMFAC, the system (66) is rewritten as

$$\begin{aligned} \Delta y(k+1) &= -3\Delta y(k) + 2.1\Delta u(k) + 2\Delta y(k-1) \\ &\quad - 2\Delta u(k-1) - \Delta y(k-2) + 1.2\Delta u(k-2) \\ &\quad + 1.5\Delta y(k-3). \end{aligned} \quad (67)$$

The desired reference signal is

$$y^*(k+1) = 2\sin(0.5\pi kT_s) + 1.5\cos(\pi kT_s) \quad (68)$$

where T_s denotes sampling time, and $T_s = 0.01$.

The control law is designed as (42).

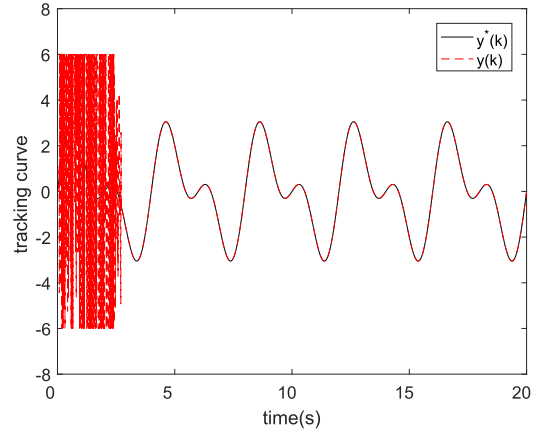


FIGURE 2. The tracking curves of simulation experiment 1.

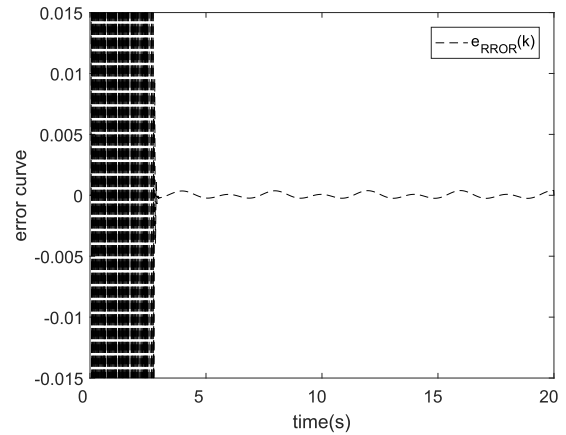


FIGURE 3. The tracking error of simulation experiment 1.

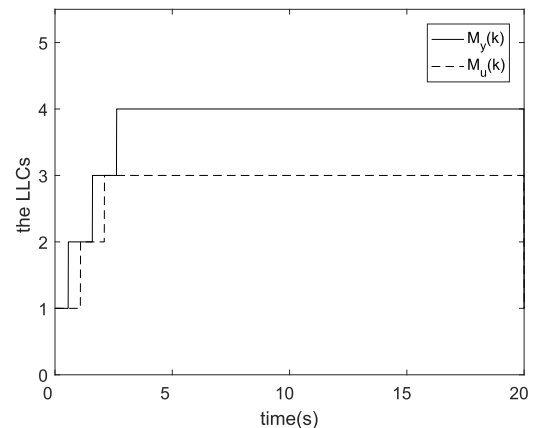


FIGURE 4. The LLCs of the proposed CMFAC.

Fig. 2-Fig. 5 denote the first simulation experimental results. Fig. 2 plots the tracking curves. In Fig. 3, the curve denotes tracking error change trend. Fig. 4 plots the change curves of the LLCs M_y and M_u . When $k = 1$, we set $M_y = M_u = 1$. The LLCs M_y, M_u of the proposed CMFAC can be adjusted automatically, and finally $M_y = L_y = 4$, and $M_u = L_u = 3$. Fig. 4 indicates that the proposed CMFAC

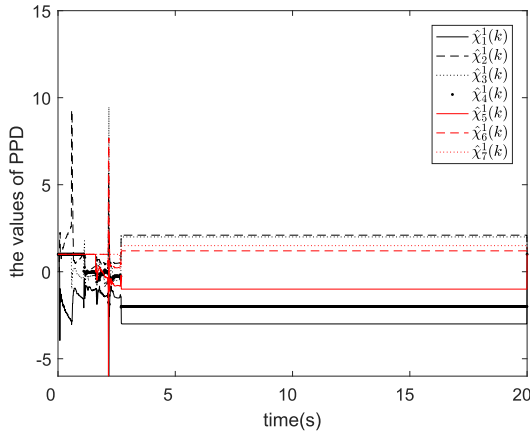


FIGURE 5. The PPD of simulation experiment 1.

TABLE 1. The values of PPD.

the values of PPD	actual value	estimated value	estimated error
χ_1^1	-3	-2.9993	-0.0007
χ_2^1	2.1	2.0995	0.0995
χ_3^1	2	1.9994	-0.0006
χ_4^1	-2	-1.9995	-0.0005
χ_5^1	-1	-0.9998	-0.0002
χ_6^1	1.2	1.1997	0.0003
χ_7^1	1.5	1.4996	0.0004

is effective in obtaining the optimal LLCs. Fig. 5 plots the changing curves of the values of PPD. The comparison results of the actual values of PPD and the estimated values of PPD are show in Table 1. The comparison results of Table 1 and the change curves of Fig. 4 and Fig. 5 show that the proposed CMFAC also is effective in estimating the values of PPD.

B. THE SECOND EXPERIMENT

Most industrial manipulators have six degrees of freedom, and a cooperative robot is implemented by six joints (J1 to J6), respectively. The six joints (J1 to J6) all have servo motor and decelerator. In this paper, the J6 joint is discussed. In the second simulation experiment, the proposed CMFAC is considered for the position control system of a manipulator end-actuator, besides, a control algorithm based on mechanism model is used for the contrast simulation experiment. The position control system of a manipulator end-actuator can be written as

$$y(k + 1) = -2y(k) + 0.8y(k - 1) + 0.5u(k) + g(y(k)) + \tau(k + 1) \tag{69}$$

where $g(y(k))$ indicates the unmodeled dynamic characteristic, $\tau(k + 1)$ is random noise, and

$$g(y(k)) = 0.5\cos(0.5\pi k)y(k - 1). \tag{70}$$

The system (69) can be rewritten as

$$y(k + 1) = y(k) + \chi_1^1(k)\Delta y(k) + \chi_2^1(k)\Delta u(k) + \chi_3^1(k)\Delta y(k - 1) + d(k). \tag{71}$$

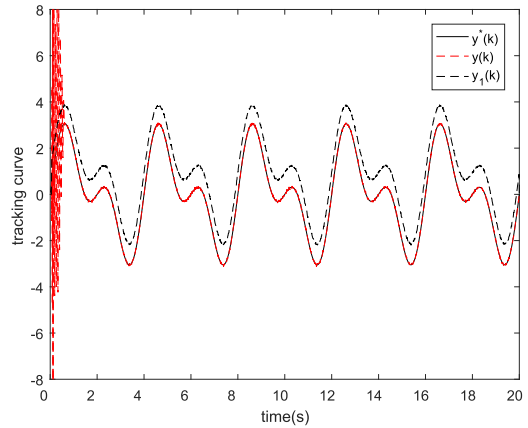


FIGURE 6. The tracking curves of simulation experiment 2.

where $d(k) = g(y(k)) - g(y(k - 1)) + \tau(k + 1) - \tau(k)$ is the disturbance signal.

The desired reference signal of the second simulation experiment is same with the desired reference signal of the first simulation experiment.

The control law of the proposed CMFAC is designed as (42).

The control law of the control algorithm based on mechanism model is

$$u_1(k) = (y^*(k + 1) + 2y(k) - 0.8y(k - 1))/(0.5). \tag{72}$$

The parameters of the proposed CMFAC are showed in Subsection III-A, we can get the main results depicted in Fig. 6-Fig. 9. $y(k)$ and $y_1(k)$ are the output of the controlled system based on the proposed CMFAC and the output of the controlled system based on mechanism model, respectively, and $e_{RROR}(k)$ and $e_{1RROR}(k)$ are the tracking errors of the proposed CMFAC and the control algorithm based on mechanism model, respectively. Fig. 6 plots the tracking curves. In Fig. 7, the curves denote tracking error change trends. Fig. 8 and Fig. 9 show the change curves of LLCs and the values of PPD, respectively. Due to the unmodeled dynamic characteristic $g(y(k))$ and $\tau(k + 1)$, the control accuracy of the control algorithm based on mechanism model becomes bad, besides, we can get that the values of $\hat{\chi}_1^1(k)$, $\hat{\chi}_2^1(k)$ and $\hat{\chi}_3^1(k)$ are changeable, and the tracking error of the proposed CMFAC is smaller than the tracking error of the control algorithm based on mechanism model.

C. THE THIRD EXPERIMENT

In this simulation experiment, the proposed CMFAC is compared with the MFAC based on RLS [30, Equation 5.49] for the following system

$$y(k + 1) = \frac{1.5y(k)y(k - 1)}{1 + y^2(k) + y^2(k - 1)} + u(k) + 0.35\sin(y(k) + y(k - 1)). \tag{73}$$

The parameters of the proposed CMFAC are showed in Subsection III-A. Fig. 10 plots the change curves of tracking

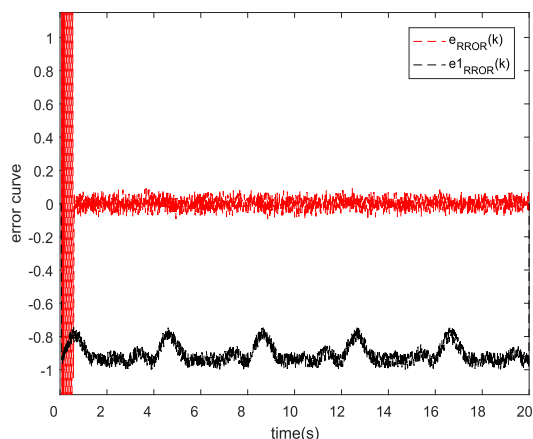


FIGURE 7. The tracking errors of simulation experiment 2.

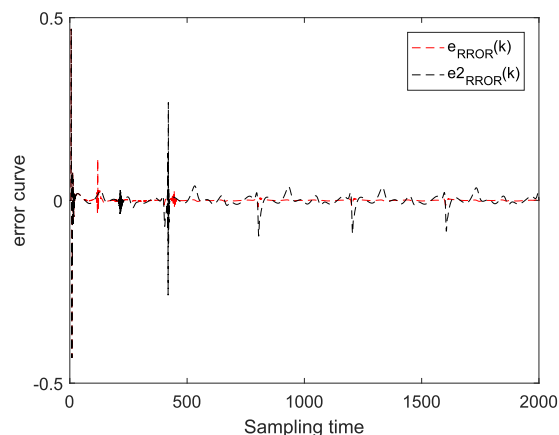


FIGURE 10. The tracking errors of simulation experiment 3.

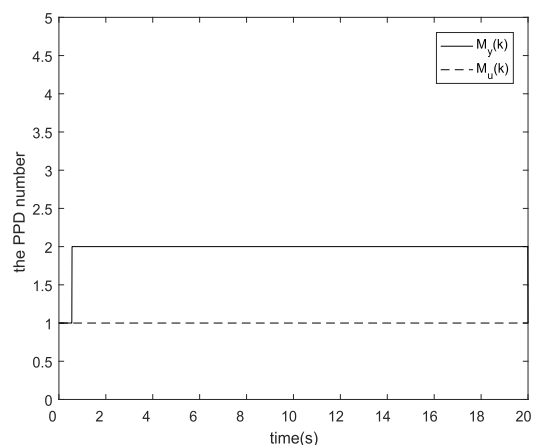


FIGURE 8. The LLCs of the proposed CMFAC.

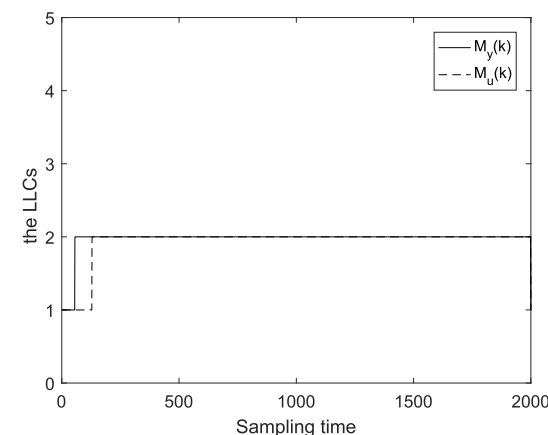


FIGURE 11. The LLCs of the proposed CMFAC.

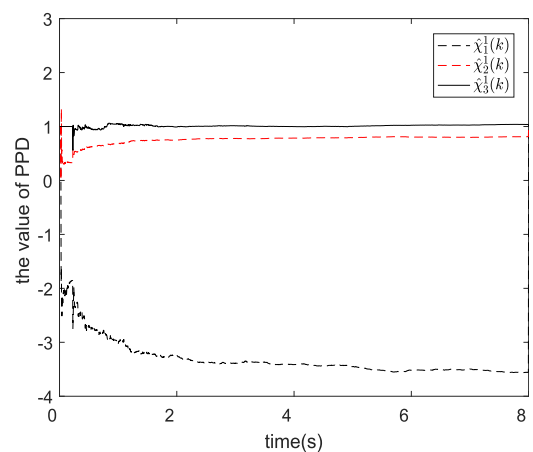


FIGURE 9. The PPD of simulation experiment 2.

errors, and $e_{RROR}(k)$ and $e2_{RROR}(k)$ are the tracking errors of the proposed CMFAC and the MFAC based on RLS [30, Equation 5.49], respectively. When $k = 1$, we set the LLCs of the proposed CMFAC are $M_y = 1$ $M_u = 1$. When the tracking error of systems does not meet requirement, the LLCs of the proposed CMFAC can be adjusted automatically. However the LLCs of the MFAC based on RLS are fixed values.

Fig. 11 plots the change curves of the LLCs in the proposed CMFAC. In Table 2, the ISE indexes of the proposed CMFAC and the MFAC based on RLS are compared, and Table 2 shows the ISE index of the proposed CMFAC is smaller than the ISE index of the MFAC based on RLS. Table 2 and the tracking errors of Fig. 10 show that the proposed CMFAC has a better tracking effect than the MFAC based on RLS for the system (73), and the proposed CMFAC is effective in improving system performances by using reasonable LLCs.

TABLE 2. The ISE indexes of those two algorithms.

algorithm	the final M_y	the final M_u	ISE index
the proposed CMFAC	2	2	0.517660
the MFAC based on RLS	1	1	1.057114

V. CONCLUSION

DDC uses the I/O data of the controlled systems to realize the adaptive control of a system, and it can deal with some unknown nonlinear systems. MFAC is a class of DDC, and MFAC builds a virtual equivalent dynamic linearized model by using a dynamic linearization technique for unknown nonlinear systems. In this paper, the optimal LLCs are investigated. Inspired by broad learning system and error

minimized extreme learning machine, CMFAC is introduced. The advantage of the proposed CMFAC is that we do not need to consider the LLCs, and the optimal LLCs are decided by the desired tracking error of systems. In order to indicate the effectiveness of the proposed CMFAC algorithm, three simulation experiments are done, and the simulation results show that the proposed CMFAC is effective in obtaining the optimal LLCs and improving system performances by using reasonable LLCs.

REFERENCES

- [1] W. He, T. Meng, X. He, and S. S. Ge, "Unified iterative learning control for flexible structures with input constraints," *Automatica*, vol. 96, pp. 326–336, Oct. 2018. doi: [10.1016/j.automatica.2018.06.051](https://doi.org/10.1016/j.automatica.2018.06.051).
- [2] M. Yu, W. Zhou, and B. Liu, "On iterative learning control for MIMO nonlinear systems in the presence of time-iteration-varying parameters," *Nonlinear Dyn.*, vol. 89, no. 4, pp. 2561–2571, 2017. doi: [10.1007/s11071-017-3604-0](https://doi.org/10.1007/s11071-017-3604-0).
- [3] K. Sun, S. Mou, J. Qiu, T. Wang, and H. Gao, "Adaptive fuzzy control for nontriangular structural stochastic switched nonlinear systems with full state constraints," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 8, pp. 1587–1601, Aug. 2019. doi: [10.1109/TFUZZ.2018.2883374](https://doi.org/10.1109/TFUZZ.2018.2883374).
- [4] Y.-J. Liu and S. Tong, "Adaptive fuzzy control for a class of nonlinear discrete-time systems with backlash," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 5, pp. 1359–1365, Oct. 2014. doi: [10.1109/TFUZZ.2013.2286837](https://doi.org/10.1109/TFUZZ.2013.2286837).
- [5] J. Qiu, K. Sun, T. Wang, and H. Gao, "Observer-based fuzzy adaptive event-triggered control for pure-feedback nonlinear systems with prescribed performance," *IEEE Trans. Fuzzy Syst.*, to be published. doi: [10.1109/TFUZZ.2019.2895560](https://doi.org/10.1109/TFUZZ.2019.2895560).
- [6] H. Liang, Y. Zhang, T. Huang, and H. Ma, "Prescribed performance cooperative control for multiagent systems with input quantization," *IEEE Trans. Cybern.*, to be published. doi: [10.1109/TCYB.2019.2893645](https://doi.org/10.1109/TCYB.2019.2893645).
- [7] Y.-X. Li and G.-H. Yang, "Adaptive asymptotic tracking control of uncertain nonlinear systems with input quantization and actuator faults," *Automatica*, vol. 72, pp. 177–185, Oct. 2016. doi: [10.1016/j.automatica.2016.06.008](https://doi.org/10.1016/j.automatica.2016.06.008).
- [8] Y. Sun, J. Xu, H. Qiang, C. Chen, and G. Lin, "Adaptive sliding mode control of maglev system based on RBF neural network minimum parameter learning method," *Measurement*, vol. 141, pp. 217–226, Jul. 2019. doi: [10.1016/j.measurement.2019.03.006](https://doi.org/10.1016/j.measurement.2019.03.006).
- [9] H. Wang and J. Fei, "Nonsingular terminal sliding mode control for active power filter using recurrent neural network," *IEEE Access*, vol. 6, pp. 67819–67829, 2018. doi: [10.1109/ACCESS.2018.2878892](https://doi.org/10.1109/ACCESS.2018.2878892).
- [10] F. Wang, B. Chen, C. Lin, J. Zhang, and X. Meng, "Adaptive neural network finite-time output feedback control of quantized nonlinear systems," *IEEE Trans. Cybern.*, vol. 48, no. 6, pp. 1839–1848, Jun. 2018. doi: [10.1109/TCYB.2017.2715980](https://doi.org/10.1109/TCYB.2017.2715980).
- [11] C. Sun, W. He, W. Ge, and C. Chang, "Adaptive neural network control of biped robots," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 47, no. 2, pp. 315–326, Feb. 2017. doi: [10.1109/TSMC.2016.2557223](https://doi.org/10.1109/TSMC.2016.2557223).
- [12] R. Hernández-Alvarado, L. G. García-Valdovinos, T. Salgado-Jiménez, A. Gómez-Espinoza, and F. Fonseca-Navarro, "Neural network-based self-tuning PID control for underwater vehicles," *Sensors*, vol. 16, no. 9, p. 1429, Sep. 2016. doi: [10.3390/s16091429](https://doi.org/10.3390/s16091429).
- [13] P. Du, H. Liang, S. Zhao, and C. K. Ahn, "Neural-based decentralized adaptive finite-time control for nonlinear large-scale systems with time-varying output constraints," *IEEE Trans. Syst., Man, Cybern., Syst.*, to be published. doi: [10.1109/TSMC.2019.2918351](https://doi.org/10.1109/TSMC.2019.2918351).
- [14] H. Wang, H. R. Karimi, P. X. Liu, and H. Yang, "Adaptive neural control of nonlinear systems with unknown control directions and input dead-zone," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 48, no. 11, pp. 1897–1907, Nov. 2018. doi: [10.1109/TSMC.2017.2709813](https://doi.org/10.1109/TSMC.2017.2709813).
- [15] X. H. Gao, K. I. Wong, P. K. Wong, and C. M. Vong, "Adaptive control of rapidly time-varying discrete-time system using initial-training-free online extreme learning machine," *Neurocomputing*, vol. 194, pp. 117–125, Jun. 2016. doi: [10.1016/j.neucom.2016.01.071](https://doi.org/10.1016/j.neucom.2016.01.071).
- [16] H. Liang, Z. Zhang, and C. K. Ahn, "Event-triggered fault detection and isolation of discrete-time systems based on geometric technique," *IEEE Trans. Circuits Syst., II, Exp. Briefs*, to be published. doi: [10.1109/TCSII.2019.2907706](https://doi.org/10.1109/TCSII.2019.2907706).
- [17] J. Zhang, Z. Wang, and H. Zhang, "Data-based optimal control of multiagent systems: A reinforcement learning design approach," *IEEE Trans. Cybern.*, vol. 49, no. 12, pp. 4441–4449, Dec. 2019. doi: [10.1109/TCYB.2018.2868715](https://doi.org/10.1109/TCYB.2018.2868715).
- [18] M. Zhang and M.-G. Gan, "Data-driven adaptive optimal control for linear systems with structured time-varying uncertainty," *IEEE Access*, vol. 7, pp. 9125–9224, 2019. doi: [10.1109/ACCESS.2019.2891575](https://doi.org/10.1109/ACCESS.2019.2891575).
- [19] Z. Hou, and W. Huang, "The model-free learning adaptive control of a class of SISO nonlinear systems," presented at the 16th Amer. Control Conf., Jun. 1997. doi: [10.1109/ACC.1997.611815](https://doi.org/10.1109/ACC.1997.611815).
- [20] Z. Hou and S. Jin, "A novel data-driven control approach for a class of discrete-time nonlinear systems," *IEEE Trans. Control Syst. Technol.*, vol. 19, no. 6, pp. 1549–1558, Nov. 2011. doi: [10.1109/TCST.2010.2093136](https://doi.org/10.1109/TCST.2010.2093136).
- [21] O. Tutsoy, H. Tugal, and D. E. Barkana, "Design of a completely model free adaptive control in the presence of parametric, non-parametric uncertainties and random control signal delay," *ISA Trans.*, vol. 76, pp. 67–77, May 2018. doi: [10.1016/j.isatra.2018.03.002](https://doi.org/10.1016/j.isatra.2018.03.002).
- [22] A. Safaei and M. N. Mahyuddin, "Adaptive model-free control based on an ultra-local model with model-free parameter estimations for a generic SISO system," *IEEE Access*, vol. 6, pp. 4266–4275, 2018. doi: [10.1109/ACCESS.2018.2799229](https://doi.org/10.1109/ACCESS.2018.2799229).
- [23] Z. Hou and Y. Zhu, "Controller-dynamic-linearization-based model free adaptive control for discrete-time nonlinear systems," *IEEE Trans. Ind. Informat.*, vol. 9, no. 4, pp. 2301–2309, Nov. 2013. doi: [10.1109/TII.2013.2257806](https://doi.org/10.1109/TII.2013.2257806).
- [24] X. Bo, "On Robustness of data-driven Model free adaptive control and learning control," Ph.D. dissertation, School Traffic Transp., Beijing Jiaotong Univ., Beijing, China, 2011.
- [25] Z.-S. Hou and J.-X. Xu, "On data-driven control theory: The state of the art and perspective," *Acta Automatica Sinica*, vol. 35, no. 6, pp. 650–667, Jul. 2009.
- [26] X. Bu, Z. Hou, F. Yu, and Z. Fu, "Model free adaptive control with disturbance observer," *J. Control Eng. Appl. Inf.*, vol. 14, no. 4, pp. 42–49, Sep. 2012.
- [27] M. B. Kadri, "Disturbance rejection using fuzzy model free adaptive control (FMFAC) with adaptive conditional defuzzification threshold," *J. Franklin Inst.*, vol. 351, no. 5, pp. 3013–3031, May 2014. doi: [10.1016/j.jfranklin.2014.02.003](https://doi.org/10.1016/j.jfranklin.2014.02.003).
- [28] X. Bu, Z. Hou, F. Yu, and F. Wang, "Robust model free adaptive control with measurement disturbance," *IET Control Theory Appl.*, vol. 6, no. 9, pp. 1288–1296, Jun. 2012. doi: [10.1049/iet-cta.2011.0381](https://doi.org/10.1049/iet-cta.2011.0381).
- [29] Y. Zhu and Z. Hou, "Data-driven MFAC for a class of discrete-time nonlinear systems with RBFNN," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 25, no. 5, pp. 1013–1020, May 2014. doi: [10.1109/TNNLS.2013.2291792](https://doi.org/10.1109/TNNLS.2013.2291792).
- [30] Z. Hou, *Nonparametric Model Adaptive Control Theory*. Beijing, China: Science Press, 1999.
- [31] C. L. P. Chen and Z. L. Liu, "Broad learning system: An effective and efficient incremental learning system without the need for deep architecture," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 1, pp. 10–24, Jan. 2018. doi: [10.1109/TNNLS.2017.2716952](https://doi.org/10.1109/TNNLS.2017.2716952).
- [32] G. Feng, G.-B. Huang, Q. Lin, and R. Gay, "Error minimized extreme learning machine with growth of hidden nodes and incremental learning," *IEEE Trans. Neural Netw.*, vol. 20, no. 8, pp. 1352–1357, Aug. 2009. doi: [10.1109/TNN.2009.2024147](https://doi.org/10.1109/TNN.2009.2024147).



XIAOFEI ZHANG was born in Xingtai, Hebei, China, in 1990. He received the M.S. degree in control engineering from the Inner Mongolia University of Science and Technology, in 2016. He is currently pursuing the Ph.D. degree with the Beijing Institute of Technology.

His research interests include sliding mode variable structure control, semi-parametric adaptive control, and online sequential extreme learning machine. Besides, he knows six-DOF robots, flight simulation turntables, and flight training simulators. He participated in the dual-arm-cooperative robot competition of the 2018 World Robotics Conference, in 2018, and received the Third Prize.



HONGBIN MA was born in Zhengzhou, Henan, China, in 1978. He received the B.S. degree in basic mathematics from Zhengzhou University, in 2001, and the M.S. and Ph.D. degrees in operational research and cybernetics from the Academy of Mathematics and Systems Science, Chinese Academy of Sciences.

From 2006 to 2009, he was a Research Scientist with Temasek Laboratories, National University of Singapore. Since 2009, he has been a Professor with the Beijing Institute of Technology. He is the author of four books and more than 100 articles. His research interests include adaptive estimation and control, integrated navigation and intelligent navigation, and unmanned systems. He became a member of ACM, SIAM, and CSIAM, in 2012. His awards and honors include the Wu Wenjun Award for Artificial Intelligence Science, the China Big Data Academic Innovation Award, and the Recognition Award of the International Conference on Robots and Applications.



YOU LI was born in Hunan, China. He received the Ph.D. degree in man-machine and environmental engineering from the College of Aerospace Science and Engineering, National University of Defense Technology, Changsha, China, in 2011. He is currently an Assistant Researcher with the National Key Laboratory of Human Factors Engineering, China Astronaut Research and Training Center. His research interests include human factors engineering, human-computer interaction, and computer vision.

...



XINGHONG ZHANG received the bachelor's degree from the Henan University of Science and Technology, in 2004, the master's degree from Zhengzhou University, in 2007, and the Ph.D. degree from the School of Automation, Beijing Institute of Technology. She is currently a Lecturer with the School of Electrical Engineering and Automation, Henan Institute of Technology. Her research interests include adaptive control, multiagent systems, discrete-time systems, and nonlinear systems.