

# Compact Thermal Models of Semiconductor Devices – a Review

Krzysztof Górecki, Janusz Zarębski, Paweł Górecki and Przemysław Ptak

**Abstract**—In the paper the problem of modelling thermal properties of semiconductor devices with the use of compact models is presented. This class of models is defined and their development over the past dozens of years is described. Possibilities of modelling thermal phenomena both in discrete semiconductor devices, monolithic integrated circuits, power modules and selected electronic circuits are presented. The problem of the usefulness range of compact thermal models in the analysis of electronic elements and circuits is discussed on the basis of investigations performed in Gdynia Maritime University.

**Keywords**—compact thermal models, thermal phenomena, semiconductor devices, modelling

## I. INTRODUCTION

SEMICONDUCTOR devices have been basic components of electronic circuits for dozens of years. During operation of these devices conversion of electrical energy into heat is observed. This heat causes a rise of internal temperature  $T_j$  of semiconductor devices above ambient temperature  $T_a$  as a result of self-heating phenomena [1 - 6].

The rise of internal temperature causes a change of the course of characteristics of semiconductor devices and a change in the value of their exploitive parameters [3, 5, 7, 8, 9, 10]. On the other hand, it is known that each semiconductor device is characterised by a certain value of the maximum admissible internal temperature  $T_{jmax}$ , after exceeding which the producer does not guarantee that the values of catalogue exploitive parameters will be maintained [11-14]. It is known also that a rise of temperature  $T_j$  causes shortening of life time of the semiconductor device [15-18].

Because of the mentioned above information about values of internal temperature of semiconductor devices during their operation is of great importance. Therefore, already in the seventies of 20<sup>th</sup> century scientific investigations concerning modelling thermal phenomena in semiconductor devices were conducted. These investigations led to formulation of thermal models of the considered devices of different accuracy.

In the classical theory of heat flow the equation of heat conduction is used. This equation for homogeneous area has the following form [5, 19, 20].

$$\operatorname{div}(\lambda \cdot \operatorname{grad}(T)) - c \cdot \rho \cdot \frac{\partial T}{\partial t} = -g(x, y, z, t) \quad (1)$$

The project financed within the program of the Ministry of Science and Higher Education called "Regionalna Inicjatywa Doskonałości" in the years 2019 – 2022, the project number 006/RID/2018/19, the sum of financing 11 870 000 PLN.

Krzysztof Górecki, Janusz Zarębski, Paweł Górecki and Przemysław Ptak are with Department of Marine Electronics, Gdynia Maritime University, Gdynia, Poland (e-mail: k.gorecki@we.umg.edu.pl, zarebski@am.gdynia.pl, p.gorecki@we.umg.edu.pl, p.ptak@we.umg.edu.pl).

where  $\lambda$  denotes proper thermal conductance of this area,  $c$  – its specific heat,  $\rho$  – density of the area, while  $g(x, y, z, t)$  is the waveform of voluminal density of the generated power.

As it is visible from the equation form (1), the equation of heat conduction enables calculating time-spatial distribution of temperature inside the semiconductor device. To solve this equation suitable set of initial and border conditions characterising heat abstraction from the surface of the investigated device resulting from convections and radiation is indispensable [20, 21].

As it results among other from the paper [20], apart from idealised cases, the equation of heat conduction is not soluble analytically. Therefore, numerical methods are universally applied to its solutions and accuracy of the obtained solution depends among others on the discretisation network used [22 - 25]. Most often a microscopic thermal model in the form of the equation of heat conduction is used to analyse the heat flow between semiconductor structure and the case of the semiconductor device [5]. Such a model allows producers of semiconductor devices to optimise the process of assembly of semiconductor structure to the case and proper selection of the used materials to construct cases of semiconductor devices.

Microscopic thermal models make possible calculations of time-spatial distribution of temperature of the analysed semiconductor device, but they need long duration time of calculations. This time rapidly rises while increasing the surface of the analysed area, e.g. the semiconductor device together with the heat sink or the printed circuit board (PCB). Therefore, practically microscopic models are not needed in the analysis of electronic circuits or semiconductor devices with complex cooling systems. In such a case typically compact thermal models are used [26].

Compact thermal models are formulated with the assumption foundation that in the active part of the semiconductor device the uniform distribution of temperature occurs. This means that it is possible to talk about one internal temperature of the semiconductor device  $T_j$ . Such models were described in the literature in the seventies of the 20<sup>th</sup> century [27, 28], and in Gdynia Maritime University investigations in this range were conducted at the beginning of the eighties of the 20<sup>th</sup> century. In this paper the idea of formulating compact thermal models of the considered devices is described. Then development of these models over the space of past dozens of years is shown, and a manner of estimating parameters and examples of such models of selected semiconductor devices are presented.

## II. BASIC FORM OF COMPACT THERMAL MODEL

The compact thermal model of the semiconductor device can be formulated by solving the equation of heat conduction for the semi-infinity rod. On the end of this rod the semiconductor

structure of the thickness  $l$  and the area  $S$ , in which heat is generated, is characterized, is situated and it is characterised by power  $P_0$ . As a result of solution of such an equation for the homogeneous rod one obtains a thermal model of the form [3]

$$T_j(t) - T_a = R_{th} \cdot P_0 \cdot \left[ 1 - \frac{8}{\pi^2} \cdot \sum_{n=1}^{\infty} \frac{\exp\left(-\frac{(2 \cdot n - 1)^2 \cdot t}{\tau_{th}}\right)}{(2 \cdot n - 1)^2} \right] \quad (2)$$

where  $T_a$  denotes ambient temperature,  $R_{th}$  – thermal resistance, and  $\tau_{th}$  – thermal time constant described with the formula

$$\tau_{th} = \frac{4 \cdot c \cdot \rho \cdot l^2}{\pi^2 \cdot \lambda} \quad (3)$$

The network representation of the equation (2) is a RC circuit, containing the parallelly connected resistor and capacitor, excited from the current source generating the signal corresponding to power dissipated in the modelled device. Voltage on this circuit corresponds to the excess of internal temperature of the device  $T_j$  over ambient temperature  $T_a$ . Resistor  $R$  represents thermal resistance  $R_{th}$  given with the formula

$$R_{th} = \frac{l}{\lambda \cdot S} \quad (4)$$

and capacitor  $C$  - heat capacitance  $C_{th}$  is described with the expression of the form

$$C_{th} = V \cdot \rho \cdot c \quad (5)$$

In the equation (5)  $V$  denotes volume of the element of the heat flow path.

If in fact that the real semiconductor device consists of many layers embracing the semiconductor structure, soldered or glue jointed, the copper basis and the case will be taken into account. Simultaneously if it is assumed that these is one-dimensional heat flow, then the compact thermal model of such a device in the form of the classical Cauer shown in Fig.1 will be obtained.

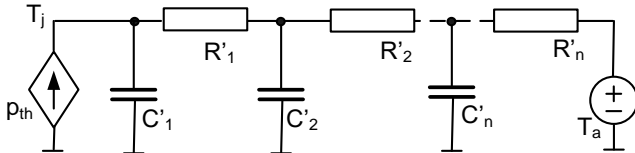


Fig. 1. Network representation of a compact thermal model of the semiconductor device in the form of the Cauer network

Each pair of  $R_i C_i$  elements, visible in Fig. 1, represents thermal resistances and thermal capacitances corresponding to each constructional layer of the semiconductor device, whereas voltage on each resistor corresponds to a difference of temperature between both sides of each layer. The thermal model corresponding to the diagram shown in Fig. 1 can be described with the formula of the form [2, 3, 5, 29, 30, 31]

$$T_j(t) = T_a + \int_0^t Z'_{th}(t - \tau) \cdot p(\tau) \cdot d\tau \quad (6)$$

where  $Z'_{th}(t)$  denotes time derivative of transient thermal impedance, typically described with the following formula [2, 3, 5, 29, 30, 31]

$$Z_{th}(t) = R_{th} \cdot \left[ 1 - \sum_{i=1}^N a_i \cdot \exp\left(-\frac{t}{\tau_{thi}}\right) \right] \quad (7)$$

where  $a_i$  is the weight coefficient corresponding to thermal time constant  $\tau_{thi}$ . The number of thermal time constants  $N$  typically ranges from 3 to 5 and it depends on the applied cooling system [32].

The form of the equation (6) corresponds to the convolution integral, which can be calculated using so called memoryless algorithms described among others in papers [29, 30]. The network analogue of equations (6 - 7) in the form Foster network shown in Fig.2 could be also applied.

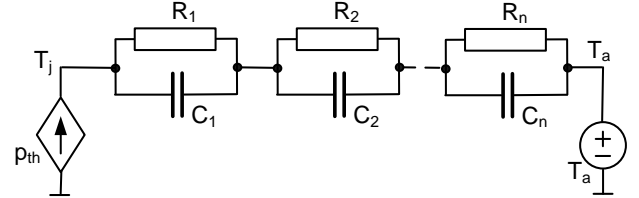


Fig. 2. Network representation of a thermal model of the semiconductor device in the form of the Foster network

As it results among others from papers [31, 33], thermal models shown in Fig. 1 and Fig. 2 are fully adequate from the point of view of node  $T_j$ . An advantage of the thermal model in the form of the Cauer network is a possibility of calculations of temperatures of each structural component of the semiconductor device, whereas a drawback of this model is the complicated manner of estimating values of RC elements for the well-known waveform of transient thermal impedance  $Z_{th}(t)$  [34]. In turn, voltages in nodes of the Foster network have not simple physical interpretation, but values of each RC element could be estimated using simple formulas of the form

$$R_i = a_i \cdot R_{th} \quad (8)$$

$$C_i = \frac{\tau_{thi}}{R_i} \quad (9)$$

The presented classical compact thermal models can be applied to the simplest constructions of semiconductor devices in which the heat flow can be described as one-dimensional. In the case when the cooling system of the device contains such elements as the heat-sink, the ventilator, the PCB, the case of the device etc. this flow is 3 Dimensional. In order to obtain high accuracy of the model the multi-way heat flow should be taken into account.

### III. THERMAL MODELLING WITH MULTI-WAY HEAT TRANSFER TAKEN INTO ACCOUNT

Removal of the heat generated in the semiconductor device is realised with the use of three mechanisms: conduction, convections and radiation [5, 19, 26]. Conduction is accountable for transport of heat through structural components of the semiconductor device made from solids - semiconductors and metals. Convection occurs on the surface of metals that contact a cooling factor (liquid or air). In turn, radiation appears on the surface of each element of the heat flow path. The participation of each mechanism of heat transfer is different for different cooling systems.

In papers [11, 33] compact thermal models of power semiconductor devices, in which three-dimensional heat transfer should be taken into account, are presented. Particularly, different intensity of heat dissipation from the bottom and top surface of the case of the semiconductor device is taken into account. Also in the paper [35] a compact thermal

model taking into account three-dimensional heat transfer in the IGBT module is proposed. However, the cited papers take into account the multi-way heat flow only in the section between the semiconductor structure and the device case.

In turn, in the paper [36] structures of compact thermal models of power semiconductor devices co-operating with different cooling systems are presented. In these models the multi-way heat flow between the semiconductor structure and the environment is taken into account. For example, in Fig. 3 a thermal model of the cooling system of the semiconductor device situated on the heat-sink inside the case of the equipment is presented.

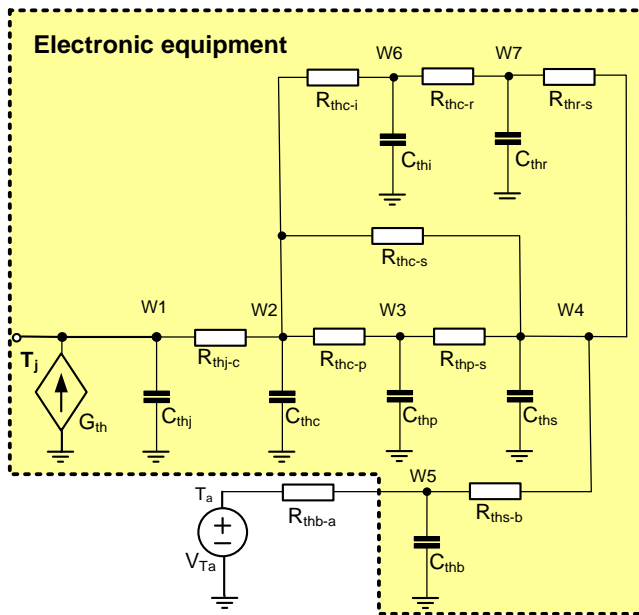


Fig. 3. Thermal model of the semiconductor device situated on the heat-sink inside the case of equipment

In this model the following nodes are marked and they represent:

- a) internal device temperature (node W1),
- b) case temperature of the device (node W2),
- c) temperature of soldering pads on PCB (node W3),
- d) temperature of thermal interface between device case and the heat-sink (node W6),
- e) temperature of the heat-sink (node W7),
- f) temperature of the air inside the case of equipment (node W4),
- g) temperature of the case of equipment (node W5),
- h) ambient temperature (node T<sub>a</sub>).

The multi-way heat transport appears between the case of the semiconductor device and with air in the equipment case. In the considered case even 3 possible ways of the heat flow appear. The first of these ways models the heat flow from the device case through the soldering pad to the air in the case of equipment. The second way models direct heat transfer from the device case to the air in the case of equipment. In turn, the third way of heat dissipation from the device case leads through the thermal interface and the heat-sink to the air in the case of equipment. Depending on the value of power dissipated in the semiconductor device and dimensions of soldering pads and the heat-sink, as well as emissivity of the surface of the heat-sink and the case, differences in

participation of the mentioned ways in heat transport are observed. Usefulness of the presented models in practice depends on a possibility of estimating parameters values of such a model.

#### IV. COMPACT THERMAL MODELS OF POWER MODULES

Except discrete semiconductor devices more and more commonly monolithic integrated circuits and power modules are used. In integrated circuits typically dimensions of the semiconductor structure are so small in relation to dimensions of the device case that satisfactory results can be obtained using compact thermal models of the identical structure as for discrete semiconductor device [37 - 40].

More and more often, among others in power electronics and in the lighting technique, power modules are used. Such modules contain from several to a dozen semiconductor structures situated on the common basis MCPCB (LED modules) or in the common case (MOS modules, IGBT modules).

Due to the fact that semiconductor devices are on the common basis, mutual thermal couplings appear between these devices. Because internal temperature of each such semiconductor device depends on ambient temperature and the excess of internal temperature caused by self-heating phenomena in this device and mutual thermal couplings with other devices situated on the same base should be taken into account [41, 42].

In the paper [32] the thermal model of the IGBT module with the diagram of connections shown in Fig. 4, is proposed.

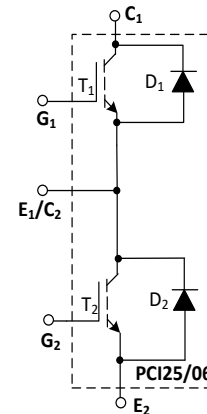


Fig. 4. Diagram of connections of the considered IGBT module

As it is visible in Fig. 4, the considered module contains 4 semiconductor devices situated in the common case.

The network representation of the thermal model of the considered IGBT module is shown in Fig.5 [32]. This model makes it possible to calculate internal temperatures of both the transistors and both the diodes included in the considered IGBT module with a self-heating phenomenon and mutual thermal couplings between all the components of this module taken into account.

In this model current sources represent powers dissipated in each component of the considered module. Voltages in nodes T<sub>T1</sub>, T<sub>T2</sub>, T<sub>D1</sub> and T<sub>D2</sub> represent internal temperatures of the transistors and the diodes included in the module. Independent voltage sources represent ambient temperature, controlled voltage sources - the excess of temperature of each component of the module caused by mutual thermal couplings with other

components of this module. Values of these increases are calculated by means of circuits visible in the right-hand part of the figure with the use of RC elements representing suitable mutual transient thermal impedances. In circuits visible in the left-hand part of the figure values of internal temperatures of

each element of the module are calculated, and self-heating phenomena are modelled with the use of RC elements describing self transient thermal impedances of the considered devices.

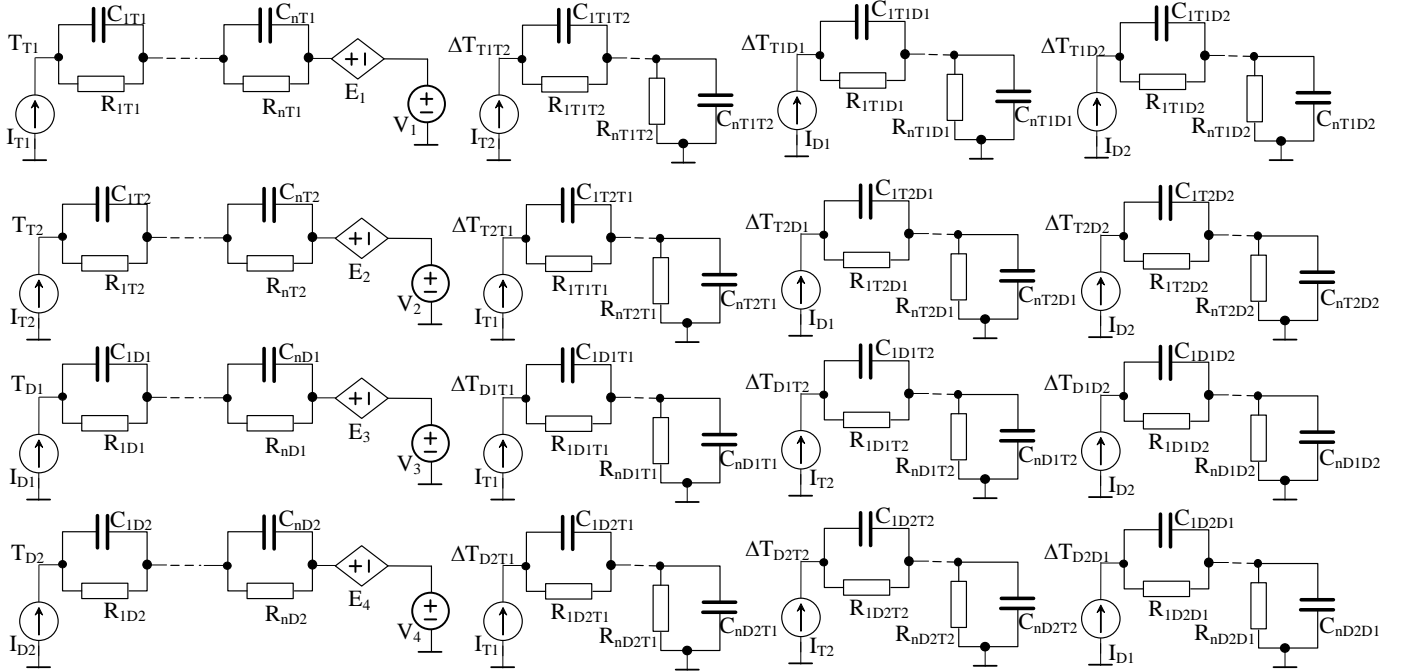


Fig. 5. Network representation of the thermal model of the IGBT module [32]

In the description of the thermal model of the considered module 4 self transient thermal impedances and 12 mutual transient thermal impedances appear. Thermal capacitance and thermal resistance between each element of the heat flow path depend on parameters of materials occurring in this path and their geometrical dimensions. Therefore, heat transfer between every pair of components of the module in both directions can be characterised by identical mutual transient thermal impedances. Because heat transfer between each pair of semiconductor devices included in the considered module is characterised by the same mutual transient thermal impedances, only six (instead of 12) mutual transient thermal impedances are necessary in the model.

In turn, in the case of LED modules typically diodes of the same type situated on the common basis MCPCB are used. In the paper [43] the thermal model of the LED module of the form shown in Fig. 6 is proposed. The model forms is presented on the example of a module including 3 diodes connected in series and mounted on the common base.

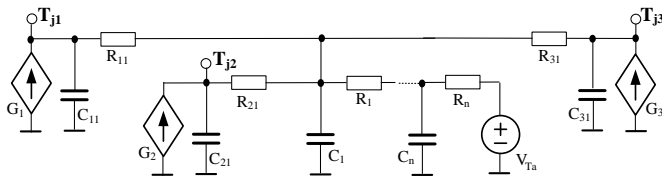


Fig. 6. Network representation of the thermal model of the LED module.

The thermal model takes into account self-heating phenomena in each diode and mutual thermal coupling between all the diodes.

This compact thermal model makes it possible to calculate internal temperature of every diode included in the considered

module. Values of this temperature correspond to values of voltage in nodes  $T_{j1}$ ,  $T_{j2}$  and  $T_{j3}$ , respectively. Controlled current sources  $G_1$ ,  $G_2$  and  $G_3$  represent thermal power dissipated in each diode in the considered module. RC networks visible in Fig.6 represent their self and mutual transient thermal impedances in the considered module. Elements  $R_{11}$ ,  $C_{11}$ ,  $R_{21}$ ,  $C_{21}$  and  $R_{31}$ ,  $C_{31}$  model the heat flow between each solid-state structure and the MCPCB, while the remaining RC elements characterise the heat flow between the MCPCB and the surrounding. The number of RC elements depends on the cooling system used. In the model, capacitances  $C_{11}$ ,  $C_{21}$ ,  $C_{31}$  and resistances  $R_{11}$ ,  $R_{21}$ ,  $R_{31}$  have the same values equal to thermal resistance between the junction and the case of each LED. The voltage source  $V_{Ta}$  represents ambient temperature  $T_a$ .

Thermal power dissipated in each diode is equal to the difference between electrical power supplying the considered diode and power of the emitted light [41, 44]

The method of measurements of transient thermal impedances of power LEDs is described in papers [45, 46]. Furthermore, the method of measurements of mutual transient thermal impedance between power LEDs is presented in [43].

## V. NONLINEAR COMPACT THERMAL MODELS

The presented in previous sections compact thermal models belong to groups of linear models. In such models dependence of efficiency of heat removal, characterised by transient thermal impedances on power dissipated in the modelled device and on its internal temperature is omitted. Meanwhile, as it results among others from papers [5, 26] thermal conductance and the convection coefficient depend on temperature. A macroscopic effect of this dependence is

described among other in many papers [47-54]. The decreasing dependence of thermal resistance on dissipated power is observed. This dependence is taken into account in the non-linear thermal model of the semiconductor device described in the paper [26].

This model is based on the linear Cauer network shown in Fig. 1. Whereas, in the model presented in the paper [26] dependence of thermal resistances and thermal capacitances occurring in this model on power dissipated in the modelled device is taken into account. The network representation of this model is shown in Fig. 7. In this model  $\Delta T_j$  means the excess of internal temperature of the device over ambient temperature. The current source  $p_{th}$  represents power dissipated in this device, controlled current sources model non-linear heat capacitances, and controlled voltage sources - non-linear thermal resistances.

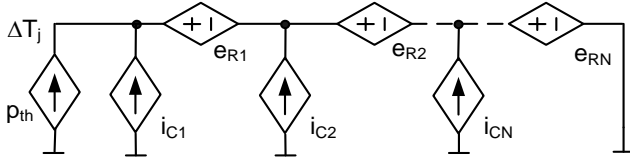


Fig. 7. Network representation of the nonlinear thermal model given in [26]

The output values of controlled sources existing in the diagram shown in Fig. 7 are expressed by

$$i_{C_i} = C_i \cdot \frac{dv_i}{dt} \quad (10)$$

$$e_{R_i} = R_i \cdot i_{R_i} \quad (11)$$

where  $v_i$  denotes voltage on capacitor  $C_i$ , whereas  $i_{R_i}$  is current flowing through resistor  $R_i$ . Values of derivative  $du/dt$  are calculated with the use of the DDT function.  $C_i$  and  $R_i$  are described by the following formulas [26]

$$C_i = C_{i0} \cdot \left[ 1 + a_{i1} \cdot \exp\left(-\frac{p_{th} - p_{i1}}{b_{i1}}\right) + a_{i2} \cdot \exp\left(-\frac{p_{th} - p_{i2}}{b_{i2}}\right) \right] \quad (12)$$

$$R_i = R_{i0} \cdot \left[ 1 + d_{i1} \cdot \exp\left(-\frac{p_{th} - p_{i3}}{e_{i1}}\right) + d_{i2} \cdot \exp\left(-\frac{p_{th} - p_{i4}}{e_{i2}}\right) \right] \quad (13)$$

where  $C_{i0}$ ,  $a_{i1}$ ,  $a_{i2}$ ,  $b_{i1}$ ,  $b_{i2}$ ,  $d_{i1}$ ,  $d_{i2}$ ,  $e_{i1}$ ,  $e_{i2}$ ,  $R_{i0}$ ,  $p_{i1}$ ,  $p_{i2}$ ,  $p_{i3}$ ,  $p_{i4}$  are model parameters.

The simplified form of the model from the paper [26] is presented in the paper [55]. In the cited paper it is shown that heat capacitances very weakly depend on dissipated power and they can be described by means of liner elements (capacitors  $C_1$ ,  $C_2$ , ...,  $C_n$ ), whereas nonlinearity of thermal resistance is taken into account while using controlled voltage sources  $E_1$ ,  $E_2$ , ...,  $E_n$ . In this manner the thermal model of the form shown in Fig. 8 is obtained. In the paper [56] correctness of this model on the example of the IGBT is verified.

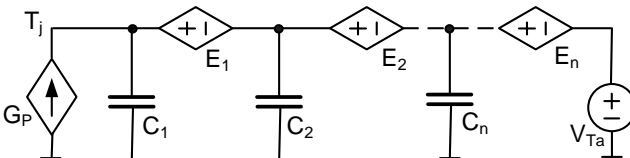


Fig. 8. Network representation of the non-linear thermal model of semiconductor devices given in [55]

In this model the device internal temperature is equal to voltage in node  $T_j$ , whereas ambient temperature  $T_a$  is represented by voltage source  $V_{Ta}$ . Values of voltage in other

nodes of this model represent temperatures of particular components of the heat flow path. Controlled current source  $G_p$  describes power dissipated in the device, controlled voltage sources  $E_1$ ,  $E_2$ , ...,  $E_n$  represent temperature differences between each element of the heat flow path.

The presented non-linear thermal models take into account influence of dissipated power on efficiency of the process of cooling the semiconductor device, but they do not take into account changes of this efficiency at changes of the value of internal temperature. This enables correct modelling of thermal properties of the semiconductor device only while stimulating with power in the form of the Heaviside's jump, not of stimulation with power of another shape, e.g. with the rectangular pulses train, where essential differences between the results of calculations and measurements can appear [56]. This drawback was eliminated in the model proposed in the paper [54].

The description of efficiencies of controlled voltage sources shown in Fig. 8 takes into account changes in the value of thermal resistance characterising the heat flow between the considered structural elements. Capacitors model thermal capacitances of each structural element belonging to the heat flow path, e.g. the semiconductor chip, the mounting base, the case, the heat-sink, the PCB, etc.

Voltage on controlled voltage sources are described by the following equation [54]

$$E_i = d_i \cdot \left( R_{th1} \cdot \exp\left(-\frac{T_j - T_a}{T_0}\right) + R_{th0} \right) \cdot i_{E_i} \quad (14)$$

where  $d_i$  denotes the quotient of  $R_i$  thermal resistance and the device thermal resistance  $R_{th}$ , whereas  $R_{th0}$ ,  $R_{th1}$  and  $T_0$  are model parameters and  $i_{E_i}$  is current of the source  $E_i$ .

The form of the equation (14) shows that thermal resistance is a decreasing function of the difference between the device internal temperature and ambient temperature. Such character of the considered dependence correlates with the presented in literature measurements results of thermal resistance of semiconductor devices on dissipated power, to which internal temperature is proportional.

## VI. ESTIMATION OF PARAMETERS VALUES OF COMPACT THERMAL MODELS

Having in mind practical applications of the presented thermal models it is indispensable to know values of parameters occurring in this model. An advantage of the compact thermal models is a possibility of estimating values of their parameters on the basis of measured waveforms of transient thermal impedances  $Z_{th}(t)$  of the modelled semiconductor device. For linear models, measuring  $Z_{th}(t)$  waveforms of selected cooling conditions and selected values of power supply of this device is sufficient. In the case of the non-linear model realization of a series of measurements  $Z_{th}(t)$  at different conditions of power supply of modelled device is indispensable. In turn, for models of power modules measuring the set of self and mutual transient thermal impedances is indispensable.

Methods of measurement  $Z_{th}(t)$  for different semiconductor devices are widely described in the literature for discrete semiconductor devices [39, 52, 54, 56 - 60], monolithic integrated circuits [47, 48, 51, 62], LED modules [62] or

power modules [32, 63, 64]. On the basis of measured suitable waveforms  $Z_{th}(t)$  values of parameters occurring in the equation (7) and values of RC elements describing the classical Cauer network are calculated. In the case of non-linear models, the coefficients occurring in equations describing each dependence of thermal resistances and thermal capacitances on power or on temperature are estimated additionally.

One of the estimation methods of the value of these parameters was worked out in Gdynia Maritime University. The algorithm ESTYM realising this method was described in detail in papers [34, 65]. The input data for the algorithm ESTYM are the measured waveforms  $Z_{th}(t)$ . The value of  $R_{th}$  corresponds to the value of  $Z_{th}(t)$  at the steady state. In order to estimate values of parameters  $a_i$  and  $\tau_{thi}$  the function  $y_i(t)$  is defined of the form

$$y_i(t) = \ln \left( 1 - \frac{Z_{th}(t)}{R_{th}} - \sum_{j=1}^{i-1} a_j \cdot \exp \left( -\frac{t}{\tau_{thj}} \right) \right) \quad (15)$$

Because thermal time constants considerably differ from each other, for large values of time  $t$ , of the waveform  $Z_{th}(t)$  decides only about the longissimus thermal time constant  $\tau_{th1}$ , and exponentially factors in the equation (7), connected with shorter time constants are ommitably small ( $t \gg \tau_{thi}$ ). Then, dependence (15) is reduced to linear dependence of the form

$$y_i(t) = -\frac{t}{\tau_{thi}} + \ln(a_i) \quad (16)$$

While estimating the value of parameters  $\tau_{thi}$  and  $a_i$  in dependence (16) one ought to use only coordinates of points lying within the range of linearity of dependence (15). Calculations are realised sequentially, beginning from the longissimus thermal time constant, to shorter and shorter thermal time constants. While estimating parameters  $\tau_{thi}$  and  $a_i$  calculated in the previous steps values of these parameters connected with longer than calculated thermal time constants are used.

Values of parameters  $R_{th}$ ,  $a_i$  and  $\tau_{thi}$  and computable values of passive RC elements, occurring in network analogues of the thermal model estimated with the use of the algorithm ESTYM, are used. Values of elements of the Foster network are properly calculated using equations (8) and (9), whereas estimation of values of RC elements of the Cauer network is more complicated and it demands:

a) delimitations of the operator thermal impedance  $Z(s)$  of the investigated device of given by the formula [34]

$$Z(s) = R_{th} \cdot \frac{\sum_{i=1}^N \left( \frac{a_i}{\tau_{thi}} \cdot \prod_{j=1, j \neq i}^N \left( s + \frac{1}{\tau_{thj}} \right) \right)}{\prod_{j=1}^N \left( s + \frac{1}{\tau_{thj}} \right)} = \frac{L(s)}{M(s)} = \frac{\sum_{i=0}^{N-1} b_i \cdot s^i}{\sum_{i=0}^N c_i \cdot s^i} \quad (17)$$

b) the alternating division of polynomials  $L(s)$  and  $M(s)$ , where only expressions occurring at the highest powers of variable  $s$  are divided. In the case when polynomials of the same degrees are divided one obtains the value of the resistance  $R'_i$ , whereas when the degree of the numerator is greater than the degree of the denominator of 1, then one obtains the value of capacitance  $C'_i$ .

## VII. CONCLUSIONS

In the paper the problem of modelling thermal properties of semiconductor devices at the use of compact thermal models is considered. The manner of formulating such models, their classical forms and enhanced forms of such models that make possible taking into account the multi-way heat flow, mutual thermal couplings between devices situated on the common basis or nonlinearities of processes responsible for removal heat generated in semiconductor devices are presented.

It was shown that the use of non-linear compact thermal models made possible more accurate estimation of waveforms of internal temperature of the semiconductor device than in the case when the classical linear thermal models are used. As it results from papers [26, 54] the non-linear thermal model assures very good agreement between the results of calculations and measurements, whereas the difference between values of internal temperature calculated by means of the linear thermal model can differ from the results of measurements even by about 20°C.

In turn, taking into account the thermal model of the LED module [43] or the IGBT module [32] not only self-heating, but also mutual thermal couplings make it possible to correctly calculate values of temperature of each component of such modules. As it results from the cited papers the difference between values of internal temperature of such components can exceed even 10°C.

Simplicity of compact thermal models and the ease of calculating values of their parameters caused that they are universally used in analyses of electronic networks. Moreover, they enable obtaining reliable results of calculations at acceptable time duration. Compact thermal models are also an essential component of electrothermal models of semiconductor devices which allows taking into account interactions of electric and thermal phenomena occurring in these devices. The use of the described in this paper modifications of the classical compact thermal models can lead to improvements of accuracy of computer analyses of electronic circuits.

## REFERENCES

- [1] P.A. Mawby, P.M. Iqic and M.S. Towers: Physically based compact device models for circuit modelling applications, *Microelectronics Journal*, Vol. 32, No. 5-6, 2001, pp. 433-447.
- [2] V. Szekely: A New Evaluation Method of Thermal Transient Measurement Results. *Microelectronics Journal*, Vol. 28, No. 3, 1997, pp. 277-292.
- [3] J. Zarębski: Modelowanie, symulacja i pomiary przebiegów elektrotermicznych w elementach półprzewodnikowych i układach elektronicznych. *Prace Naukowe Wyższej Szkoły Morskiej w Gdyni*, Gdynia, 1996.
- [4] Z. Lisik: Zjawiska w strukturach półprzewodnikowych – metody ich modelowania. Wydawnictwo Politechniki Łódzkiej, Łódź 2005
- [5] W. Janke, Zjawiska termiczne w elementach i układach półprzewodnikowych, WNT, Warszawa, 1992
- [6] A. S. Bahman, K. Ma, P. Ghimire, F. Iannuzzo, F. Blaabjerg.: A 3D Lumped Thermal Network Model for Long-term Load Profiles Analysis in High Power IGBT Modules. *IEEE Journal of Emerging and Selected Topics in Power Electronics*, Vol. 4, No. 3, 2016, pp. 1050 - 1063. doi.org/10.1109/JESTPE.2016.2531631
- [7] V. Szekely, M.Rencz, B. Courtois: Thermal investigations of IC's and microstructures, *Microelectronics Journal*, Vol. 28, No 3, 1997, pp. 205-207.
- [8] C.J.M. Lasance, A. Poppe: *Thermal Management for LED Applications*. Springer Science+Business Media, New York, 2014.

- [9] A. Poppe: Multi-domain compact modeling of LEDs: An overview of models and experimental data, *Microelectronic Journal*, Vol. 46, 2015, pp. 1138-1151.
- [10] Ł. Starzak, M. Zubert, M. Janicki, T. Torzewicz, M. Napieralska, G. Jabłoński, A. Napieralski: Behavioral approach to SiC MPS diode electrothermal model generation, *IEEE Transactions on Electron Devices*, Vol. 60, No. 2, 2013, pp. 630-638.
- [11] Infineon Technologies web-site <http://www.infineon.com>.
- [12] M.K. Kazimierzczuk: *Pulse-width Modulated DC-DC Power Converters*, John Wiley & Sons, Ltd, 2008
- [13] J. Singh: *Semiconductor Devices. Basic Principles*, John Wiley & Sons, 2001.
- [14] R. Barlik, M. Nowak: *Energoelektronika. Elementy, podzespoły, układy*. Oficyna Wydawnicza Politechniki Warszawskiej, Warszawa, 2014.
- [15] A. Castellazzi, Y.C. Gerstenmaier, R. Kraus and G.K.M. Wachutka: Reliability analysis and modeling of power MOSFETs in the 42-V-PowerNet, *IEEE Transactions on Power Electronics*, Vol. 21, 2006, No. 3, pp. 603-612.
- [16] A. Castellazzi, R. Kraus, N. Seliger, D. Schmitt-Landsiedel: Reliability analysis of power MOSFET's with the help of compact models and circuit simulation. *Microelectronics Reliability*, Vol. 42, 2002, pp.1605-1610.
- [17] M. Ciappa, F. Carbognani, P. Cora, W. Fichtner: A novel thermomechanics-based lifetime prediction model for cycle fatigue failure mechanisms in power semiconductors. *Microelectronics Reliability*, Vol. 42, 2002, pp.1653-1658
- [18] N. Narendran, Y. Gu: Life of LED-based white light sources. *Journal of Display Technology*, Vol. 1, No. 1, 2005, pp. 167- 171.
- [19] Y. Yener, S. Kakac: *Heat Conduction*. Taylor & Francis, 2008.
- [20] A. Nowakowski: Badanie procesów termicznych w przyrządach półprzewodnikowych. *Zesz. Nauk. Polit. Gdańskiej, Elektronika* Vol. 60, No. 389, 1984.
- [21] A. Napieralski: Komputerowe projektowanie układów półprzewodnikowych mocy ze szczególnym uwzględnieniem ich właściwości termicznych. *Zeszyty Naukowe Politechniki Łódzkiej*, Vol. 562, Łódź 1988.
- [22] M. Zubert: Wielowymiarowe i wielodomenowe modelowanie i symulacja zjawisk fizycznych w nowoczesnych strukturach półprzewodnikowych. *Praca doktorska, Politechnika Łódzka*, 1998.
- [23] T. Raszkowski, A. Samson, M. Zubert: Influence of temperature and heat flux time lags on the temperature distribution in modern GAAFET structure based on Dual-Phase-Lag thermal model. *Microelectronics Reliability*, Vol. 86, 2018, pp. 10-19.
- [24] M. Zubert, T. Raszkowski, A. Samson, P. Zając: Methodology of determining the applicability range of the DPL model to heat transfer in modern integrated circuits comprised of FinFETs, *Microelectronics Reliability*, Vol. 86, 2018, pp. 139-153.
- [25] P. Zając, A. Napieralski: Novel thermal model of microchannel cooling system designed for fast simulation of liquid-cooled ICs. *Microelectronics Reliability*, Vol. 86, 2018, pp. 245-258
- [26] K. Górecki, J. Zarębski: Nonlinear compact thermal model of power semiconductor devices, *IEEE Transactions on Components and Packaging Technologies*, Vol. 33, No. 3, 2010, pp. 643-647.
- [27] K. Nemeth: On the analysis of nonlinear resistive networks considering the effect of temperature. *IEEE Journal Solid-State Circuits*, Vol. SC-11, No. 4, 1976, p. 550
- [28] K.F. Fukahori, P.R. Gray: Computer simulation of integrated circuits in the presence of electrothermal interactions. *IEEE Journal Solid-State Circuits*, Vol. SC-11, No. 6, 1976, p. 834
- [29] W. Janke, G. Blakiewicz: Semi analytical recursive algorithms for convolution calculations, *IEE Proc.- Circuits Devices Syst.*, vol. 142, No. 2, 1995.
- [30] J. Zarębski, K. Górecki, Properties of some convolution algorithms for the thermal analysis of semiconductor devices. *Applied Mathematical Modelling*, Vol. 31, No. 8, 2007, pp. 1489 – 1496.
- [31] J. Zarębski, K. Górecki: Modelling CoolMOS Transistors in SPICE. *IEE Proceedings on Ciccuits, Devices and Systems*, Vol. 153, No. 1, 2006, pp. 46-52.
- [32] K. Górecki, P. Górecki, J. Zarębski: Measurements of parameters of the thermal model of the IGBT module. *IEEE Transactions on Instrumentation and Measurement* in press, doi: 10.1109/TIM.2019.2900144.
- [33] P.E. Bagnoli, C. Casarosa, M. Ciampi, E. Dallago: Thermal resistance analysis by induced transient (TRAIT) method for power electronic devices thermal characterization. *IEEE Trans. on Power Electronics, I. Fundamentals and Theory*, Vol. 13, No. 6, 1998; pp. 1208-19.
- [34] K. Górecki, J. Zarębski, „Estymacja parametrów modelu termicznego elementu półprzewodnikowego”, *Kwartalnik Elektroniki i Telekomunikacji*, Vol. 52 No. 3, 2006, pp. 347-360.
- [35] A. Bahman, K. Ma, F. Blaabjerg: A Lumped Thermal Model Including Thermal Coupling and Thermal Boundary Conditions for High Power IGBT Modules. *IEEE Transactions on Power Electronics*, vol. 33, no. 3, 2017, pp. 2518 - 2530. 10.1109/TPEL.2017.2694548.
- [36] K. Górecki, J. Zarębski: Paths of the heat flow from semiconductor devices to surrounding. *Proceedings of the 19<sup>th</sup> International Conference Mixed Design of Integrated Circuits and Systems MIXDES 2012*, Warszawa, 2012, pp. 313-318.
- [37] J. Zarębski, K. Górecki: A SPICE Electrothermal Model of the Selected Class of Monolithic Switching Regulators. *IEEE Transactions on Power Electronics*, Vol. 23, No. 2, 2008, pp. 1023 – 1026.
- [38] J. Zarębski, K. Górecki: SPICE-Aided Modelling of the UC3842 Current Mode PWM Controller with Selfheating Taken into Account. *Microelectronics Reliability*, Vol. 47, No. 7, 2007, pp. 1145-1152.
- [39] K. Górecki: The electrothermal macromodel of switching voltage regulators from L4970 family. *International Journal of Numerical Modelling Electronic Networks, Devices and Fields*, Vol. 21, No. 6, 2008, pp. 455-473.
- [40] R. Perret, *Power electronics semiconductor devices*. John Wiley & Sons, Hoboken, 2009.
- [41] K. Górecki: Modelling mutual thermal interactions between power LEDs in SPICE. *Microelectronics Reliability*, Vol. 55 No. 2, 2015, pp. 389-395.
- [42] K. Górecki, D. Bisewski, J. Zarębski, R. Kisiel, M. Myśliwiec: High-temperature properties of Schottky diodes made of silicon carbide. *Proceedings of 23<sup>rd</sup> International Conference Mixed Design of Integrated Circuits and systems MIXDES 2016, Łódź*, 2016, p. 382-386.
- [43] K. Górecki, P. Ptak: Modelling mutual thermal coupling in LED module. *Microelectronics International*, Vol. 32, No. 3, 2015, pp. 152-157.
- [44] A. Poppe, C.J.M. Lasance: On the standardization of thermal characterization of LEDs, *25<sup>th</sup> Annual IEEE Semiconductor Thermal Measurement and Management Symposium SEMI-THERM*, San Jose, 2009, pp.151- 158.
- [45] K. Górecki: Measurements of thermal resistance of power LEDs. *Microelectronics International*, Vol. 31, No. 3, 2014, pp. 217-223.
- [46] K. Górecki, P. Ptak: New method of measurements transient thermal impedance and radial power of power LEDs. *IEEE Transactions on Instrumentation and Measurement* in press, doi: 10.1109/TIM.2019.2894043.
- [47] K. Górecki, J. Zarębski and D. Bisewski: “An influence of the selected factors on the transient thermal impedance model of power MOSFET”, *Informacje MIDEM – Journal of Microelectronics, Electronic Components and Materials*, Vol. 45, No. 2, 2015, pp. 110-116.
- [48] J. Zarębski, K. Górecki: A Method of Measuring the Transient Thermal Impedance of Monolithic Bipolar Switched Regulators. *IEEE Transactions on Components and Packaging Technologies*, Vol. 30, No. 4, 2007, pp. 627 – 631.
- [49] D. Bisewski, M. Myśliwiec, K. Górecki, R. Kisiel, J. Zarębski: Examinations of selected thermal properties of packages of SiC Schottky diodes. *Metrology and Measurement Systems*, Vol. 23, No. 3, 2016, pp. 451-459.
- [50] J. Zarębski, K. Górecki: A new method for the measurement of the thermal resistance of the monolithic switched regulator LT1073. *IEEE Transactions on Instrumentation and Measurement*, Vol. 56, No. 5, 2007, pp. 2101-2104.
- [51] K. Górecki, P. Górecki: The analysis of accuracy of selected methods of measuring the thermal resistance of IGBTs. *Metrology and Measurement Systems*, Vol. 22, No. 3, 2015, pp. 455-464.
- [52] D.L. Blackburn, F.F. Oettinger: *Transient Thermal Response Measurements of Power Transistors*. *IEEE Transactions on Industrial Electronics and Control Instrumentation*, 1976, Vol. IECI-22, No. 2, pp. 134-141.
- [53] K. Górecki, J. Zarębski: Modeling the influence of selected factors on thermal resistance of semiconductor devices, *IEEE Transactions on Components, Packaging and Manufacturing Technology*, Vol. 4, No. 3, 2014, pp. 421-428.
- [54] K. Górecki, P. Górecki: A new form of the non-linear compact thermal model of the IGBT. *12<sup>th</sup> IEEE International Conference on Compatibility, Power Electronics and Power Engineering CPE-POWERENG 2018, Doha*, 2018, paper QF-004383.
- [55] P. Górecki, K. Górecki: Non-linear compact thermal model of IGBTs, *2017 21st European Microelectronics and Packaging Conference (EMPC) & Exhibition.*, Warszawa, 2017, doi: 10.23919/EMPC.2017.8346910

- [56] Y. Avenas, L. Dupont, Z. Khatir: Temperature measurement of power semiconductor devices by thermo-sensitive electrical parameters – a review. *IEEE Transactions on Power Electronics*, Vol. 27, No. 6, 2012, pp. 3081-3092.
- [57] Z. Lisik: Pomiar rezystancji cieplnej bipolarnych tranzystorów mocy Darlingтона. *Kwart. Elektr. i Telekomunikacji*, Vol. 40, No. 3, 1994, s. 369.
- [58] D.L. Blackburn: Temperature Measurements of Semiconductor Devices – A Review, 20th IEEE Semiconductor Thermal Measurement and Management Symposium SEMI-THERM, San Jose, 2004, pp. 70-80.
- [59] J.W. Sofia, Analysis of thermal transient data with synthesized dynamic-models for semiconductor-devices. *IEEE Transactions on Components Packaging and Manufacturing Technology Part A*, Vol. 18, No. 1, 1995, pp. 39-47.
- [60] F.F. Oettinger, D.L. Blackburn, S. Rubin: Thermal characterization of power transistors, *IEEE Transactions on Electron Devices*, Vol. 23, No. 8, 1976, pp. 831-838.
- [61] K. Górecki, J. Zarębski: Badanie wpływu wybranych czynników na parametry cieplne tranzystorów mocy MOS. *Przegląd Elektrotechniczny*, Vol. 85, No. 4, 2009, pp. 159-164.
- [62] K. Górecki, B. Dziurdzia, P. Ptak: The influence of a soldering manner on thermal properties of LED modules. *Soldering & Surface Mount Technology*, Vol. 30, No. 2, 2018, pp. 81-86.
- [63] P. Górecki, K. Górecki, J. Zarębski, Thermal model of the IGBT module. *Journal of Physics: Conference Series*, Vol. 1033, 2018, 012001, pp. 1-7, doi: 10.1088/1742-6596/1033/1/012001.
- [64] M. Janicki, P. Kawka, G. De Mey and A. Napieralski, IGBT Hybrid Module Thermal Measurements and Simulations. 8-th International Conference Mixed Design of Integrated Circuits and Systems MIXDES 2001, Zakopane, 2001, p. 249.
- [65] K. Górecki, M. Rogalska, J. Zarębski: Parameter estimation of the electrothermal model of the ferromagnetic core, *Microelectronics Reliability*, Vol. 54, No. 5, 2014, pp. 978-984.