Title: Compacted dimensions & singular plasmonic surfaces

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Abstract: In advanced field theories there can be more than four dimensions to space, the excess dimensions described as compacted and unobservable on everyday length scales. We report a simple model, unconnected to field theory, for a compacted dimension realised in a metallic metasurface periodically structured in the form of a grating comprising a series of singularities. An extra dimension of the grating is hidden, and the surface plasmon excitations, though localised at the surface, are characterised by three wave vectors rather than the two of typical two-dimensional metal grating. We propose an experimental realisation in a doped graphene layer.

One Sentence Summary: Plasmonic excitations of a singular metallic grating serve as a model for compacted dimensions.

Main Text: A conventional two dimensional object is characterised by two quantum numbers. For example the frequencies of surface plasmons on a periodic surface are labelled by the components of their momentum projected onto the surface axes. We describe theoretically systems that instead require three quantum numbers to label them: the two conventional in-plane momenta plus a third momentum corresponding to a compacted dimension hidden from view inside a singularity. Compacted dimensions are ingredients of advanced string theories (*1,2*) where the extra dimensions in a 4+N dimensional theory are said to be compacted and so not directly observed on everyday length scales. As far as we know our singular surfaces are the only physically realisable model of this curious effect. We give two instances of how this might be done.

We make use of the technique of transformation optics (*3-5*) which exploits the invariance of Maxwell's equations under a coordinate transformation: only the values of ε,μ are affected by the transformation. We use this theory to compact a dimension through a singular transformation that compresses one of the dimensions of a 3D system into one or more singular points. An example of the process is given (Fig. 1) for a 3D system (Fig. 1A), periodic in one of the dimensions and translationally invariant in the two other directions. The blue shaded areas are metallic and support surface plasmons *(6)* whose spectrum is characterised by three wave vectors: k_x, k_y, k_u where k_u is the wave vector heading out of the plane of the paper.

Our intent is to show that the x dimension can be hidden using 2D conformal transformations where the x, y coordinates are represented by a complex number $z = x + iy$. Conformal transformations in 2D have the property of conserving the permittivity and permeability, ε,μ, in the plane of the transformation so that in this plane we are working with the same materials in all coordinate frames. Under each successive transformation the spectral properties are preserved, and the modes once calculated in the initial frame can be found in the other frames through the properties of the transformation.

In the first step we compress $x = -\infty$ to a point at the origin,

$$z' = \exp(z) \tag{1}$$

which gives rise to Fig. 1B. This transformation produces knife edges which have been discussed previously *(7)* and Davis has commented on the continuous nature of the spectrum *(8)*. Next we compress $z' = +\infty$ to a point by inverting about $z' = -a$ giving rise to a structure shown in Fig. 1C that is finite in the x'', y'' plane,

$$z'' = \frac{1}{z' + a} \tag{2}$$

Finally we add a further transformation to create a periodic metasurface (Fig. 1D).

$$z''' = \frac{d}{2\pi} \ln\left[z'' - \frac{1}{2a} \right] \tag{3}$$

The metasurface shown in Fig. 1D has some unusual properties. The modes of Fig. 1A are truly three dimensional being spread out over the whole structure and, are characterised by the three wave vectors, k_x, k_y, k_u. In contrast, these modes when transformed through to Fig. 1D are now found to be exponentially localised at the interface and at first sight would seem to be two-dimensional objects. However, this is not the case: the structure inherits the spectral characterisation of the mother structure and therefore is labelled by the same three wave vectors, k_x, k_y, k_u. This affects the response of the metasurface to external stimulae. A conventional grating, free of any singularities such as sharp edges, is characterised by only two wave vectors which are defined by the angle of incidence of external radiation. This leads to a discrete excitation spectrum. In other words ordinary gratings appear coloured to the eye. In contrast, external radiation incident on our metasurface defines only two of the three wave vectors, the third being selected by the frequency. Thus, the modes form a continuum and can be excited whatever the incident angle or frequency. Our singular metasurfaces are not coloured: they are grey, or black in the limit of strong coupling as we shall show.

For these transformed modes, radiation is captured in the broad smooth portions of the metasurface, and travels towards the cusps becoming increasingly compressed, but never reaching the cusp. The effect of compression is to increase the field strength which is inversely proportional to the local group velocity and in an ideal loss free system would rise to infinity at the cusp. However, in a realistic system losses would intervene and result in a finite but very large field enhancement as in a SERS experiment.

In the following, we provide detailed calculations made using the techniques of transformation optics *(9)* which we have successfully deployed on non-singular gratings.

The dispersion of the modes for Fig. 1A are calculated assuming a Drude form for the metallic component,

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2} \tag{4}$$

The dielectric separating the metal layers is taken to be vacuum.

Fig. 2A shows the dispersion of the modes at $k_y = k_u = 0$ with respect to the hidden variable, k_x calculated for the structure in Fig. 1A. The ratio of metal to vacuum is 1:4 which makes the angle of the cusp in Fig. 1D to be $72°$. Fig. 2B shows the same calculation extended to include k_y, whilst $k_u = 0$. There are two modes, one symmetric the other antisymmetric about the point of the cusp in Fig. 1D, the lower mode being dark.

The singularity harvests photons incident on the metasurface concentrating their density at the tip. Fig. 3 shows the phase and amplitude of the electric field at the metal dielectric interface for a mode with hidden variable $k_x = 10$ at $k_y = k_u = 0$. In a lossless system the amplitude rises to infinity at the singular points and the phase oscillates infinitely rapidly resulting from compression of the infinite hidden dimension into a singular point. In a system with loss, this is still the case until a critical value of the loss is exceeded at which point the amplification mechanism breaks down. The total energy content remains finite at all times.

In the second realisation we start from a thin slab of conductor, which could represent doped graphene, and use transformations previously reported *(10)* to transform the slab into a 2D grating (Fig. 4).

$$z' = \frac{d'}{2\pi d} \ln\left[\frac{1}{e^{z+x_0} - iw_0} + iy_0 \right] \tag{5}$$

where x_0, y_0, w_0 are parameters that can be used to adjust to shape of the grating. Then we add a further step taking the limit where the thickness of the grating vanishes at the minima marked by asterisks separated by period d'. The corresponding points marked in A are separated by d. If we choose to keep the grating period and maximum thickness constant during the limiting process, this implies that $d \to \infty$ and the asterisks recede to $\pm\infty$. This implies that the whole of the continuous spectrum of the infinite y dimension in A is compressed into the finite segment length d' in C, the infinity vanishing into the singularity. At the same time y' remains infinite in extent so we have in fact compressed two dimensions into one. The original y dimension and its spectrum is hidden in the singularities, and C is still outwardly a 2D system characterised by three wave vectors, k_y for the hidden dimension, $k_{y'}$ the new dimensions and k_u the out of plane dimension. These steps comprise our second route to hidden dimensions through a different sort of singularity.

We propose an experimental realisation: doped graphene supports plasmonic excitations in the THz regime and doping can be controlled by the proximity of a charged surface which attracts or repels electrons to or from that region of the graphene. In this way graphene can be periodically doped to form a grating. Since graphene is very thin, the variation in conductivity can be modelled as a constant permittivity material varying in thickness and hence mapped onto the system illustrated in Fig. 4. Transformation optics has been shown to be a powerful tool for studying graphene *(10)* and as we have shown in a previous paper *(11)*, the grating can strongly couple incident radiation to plasmons leading to strong absorption at the resonant frequency. In this sense graphene patterned with a smooth grating is 'coloured': absorption happens only at discrete frequencies. However we can model our singular grating by increasing the modulation until the electron density and therefore the conductivity approaches zero at a singular point following the sort of profile shown in Fig. 4. Our simulations for a succession of narrowing gaps show a series of peaks in the transmissivity merging into a continuous absorption spectrum at the singularity.

We calculate the electromagnetic response of the graphene metasurface by considering a plane wave incident normal ($k_{y''} = k_u = 0$) to the graphene layer with modulated conductivity and transforming it to a frame where graphene is homogenously doped. The homogeneously doped graphene can be modelled as a thin slab of constant thickness (Fig. 4A). Then we apply boundary conditions for the electromagnetic fields in the slab frame and include the radiative reaction of the graphene grating as it couples to external radiation. In order to approach the singularity smoothly, we use the transformation shown in equation (5). By allowing the grating minimum to approach a touching point, we can model periodic gratings that tend towards a singular metasurface.

Evolution of the transmissivity for three graphene metasurfaces as the singularity is approached is shown in Figs. 4C-E(solid lines correspond to our analytical modelling based on transformation optics, dots represent full electrodynamic simulations). In our calculations we use the conductivity of graphene from the random phase approximation, which depends on frequency, chemical potential, mobility (we use 10^4 $cm^2V^{-1}s^{-1}$ for the blue lines in C-E) and temperature (we use T=300K). For all of them the period is 2.5μm and the periodic conductivity profile peaks at $2.13\times10^{-3}\Omega^{-1}$ at a frequency of 15 THz while the minimum doping value approaches zero from 4C to 4E (see 4B). Our calculations show that, as the metasurfaces tend to the singular case, the transmissivity peaks are crushed closer together, finally merging into a continuum. Every point in the transmission spectrum defines a hidden wave vector, k_y, through the dispersion relationship $\omega(k_y)$. In this way a discrete spectrum with absorption peaks at the Wood's anomaly positions is broadened into a continuum. We find it quite remarkable that one layer of a singular graphene metasurface can reject almost half of the THz radiation incident upon it over a broad band width.

Some qualifications should be made. We have assumed a local form for ε, ignoring the fact that at very short distances this assumption breaks down *(12,13,14)* which will remove the possibility of a perfect singularity as postulated here. Systems also have resistive loss as we have discussed and to some extent the two effects compensate for one another: non locality by removing the perfect singularity will tend to produce a discrete spectrum; on the other hand loss will smear a discrete but still dense spectrum into a continuum leaving the broad band absorption intact. This is shown in Fig 4E where the green line corresponds to a lower mobility of $5x10^3$ $cm^2V^{-1}s^{-1}$

The model system described here is a realisation of the exotic phenomenon of a compacted dimension. The hidden dimension has a dramatic effect on electromagnetic properties of the system: transmission through a single sheet of graphene structured according to our prescription shows strong broad band absorption of THz radiation, as opposed to the isolated absorption peaks of a conventional grating.

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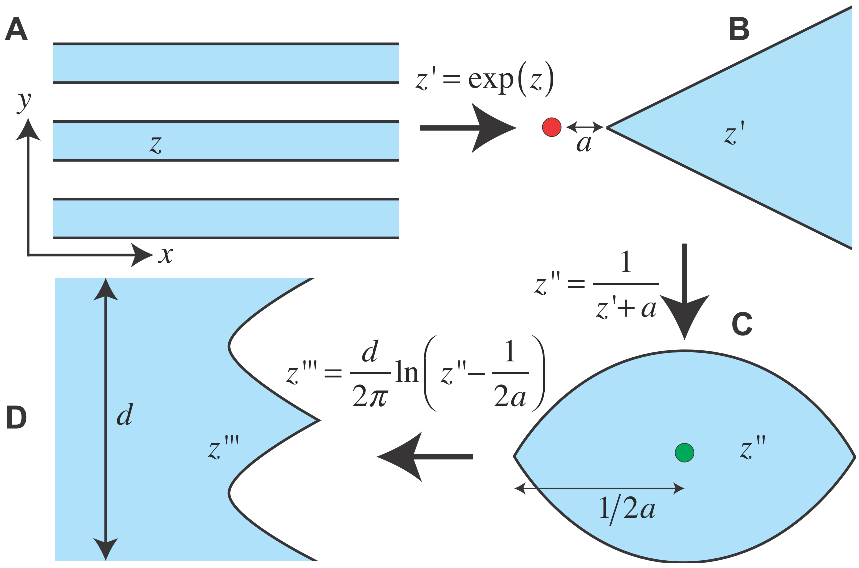
Figure Captions

Fig. 1. A series of transformations compacts 3D into 2D. The infinite dimension along x in panel A is transformed into singular points in panel D.The u axis lies out of the plane.

Fig. 2. Dispersion with respect to the hidden variable. (A) showing k_x alone. (B) showing k_x and k_y for $k_u = 0$.

Fig. 3. Profile of the grating and associated mode. Left: phase of a mode for $k_x = 10$ at $k_y = k_u = 0$. Right: field amplitude on the surface of the grating.

Fig. 4. Merging the spectrum of graphene gratings into a continuum. Panel A shows the transformation to a periodically doped grating. Panels C-E show transmittance through gratings whose doping level approaches zero at the grating minimum as given in B.



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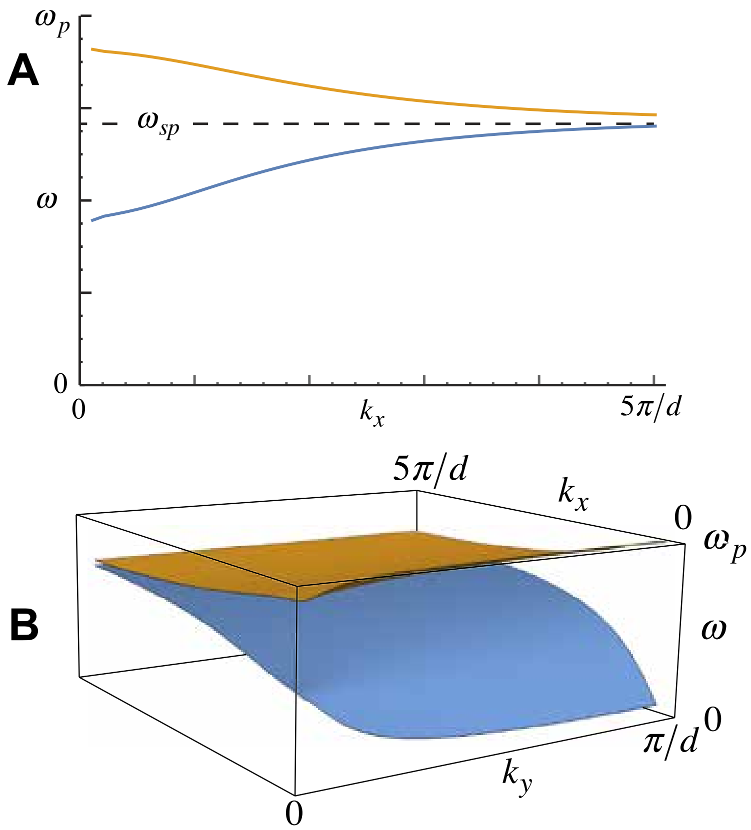

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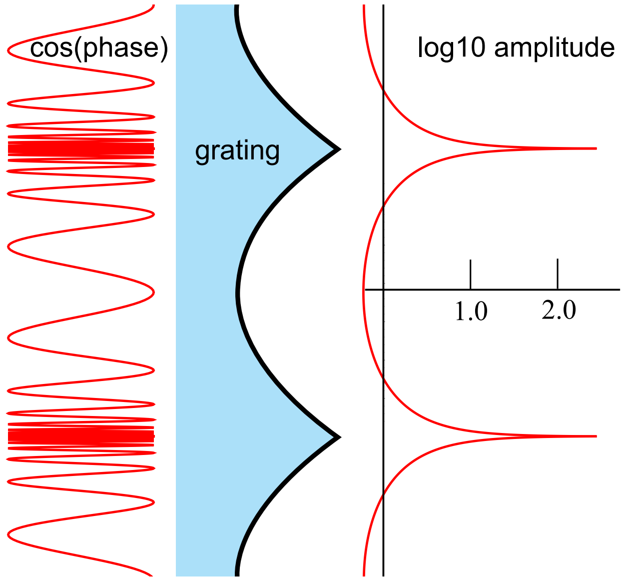

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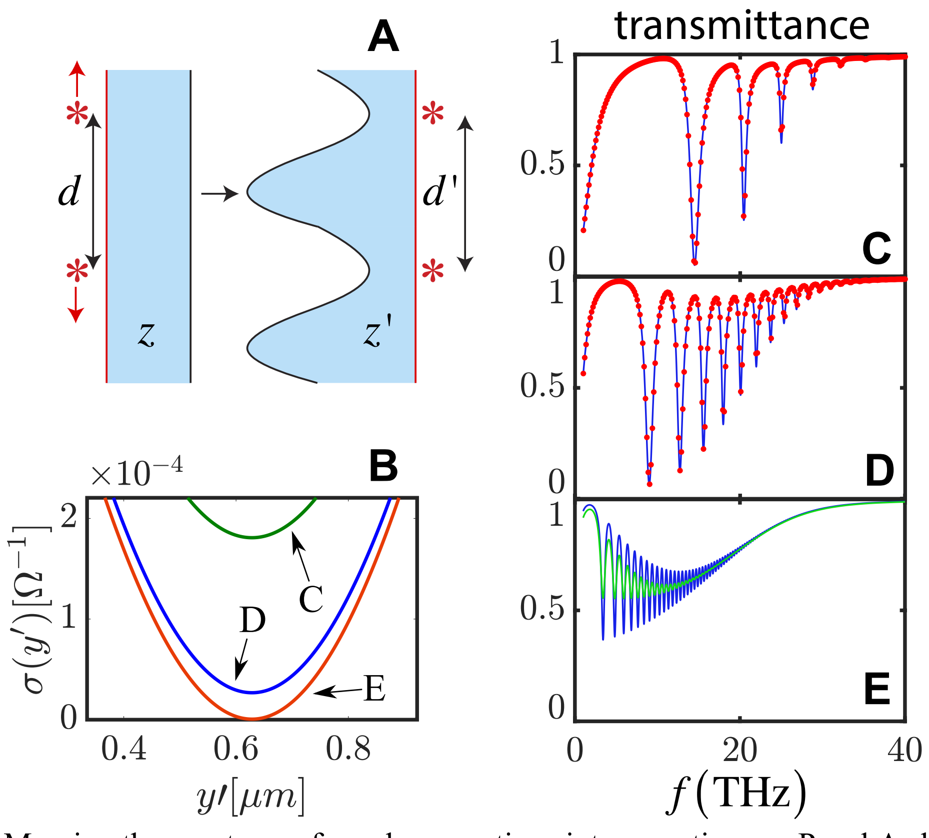

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