International Scholarly Research Network
ISRN Optics
Volume 2012, Article ID 243672, 5 pages
doi:10.5402/2012/243672

Research Article

Comparative Analysis of Free Optical Vibration of Lamination Composite Optical Beams Using the Boubaker Polynomials Expansion Scheme and the Differential Quadrature Method

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Received 22 November 2011; Accepted 4 January 2012

Academic Editor: D. H. Woo

The effects of stacking sequences of composite laminated optical beams on free vibration frequencies are investigated using two methods: the Boubaker Polynomials Expansion Scheme (pbes) and the Differential Quadrature Method (dqm). In the last decades, these two techniques have been separately performed for obtaining accurate numerical solutions to several initial boundary value problems (Vo et al. 2010, Li et al. 2008, Chen 2003, Hu et al. 2008, Karami et al. 2003, Malekzadeh et al. 2004, Khare et al. 2004, Della and Shu 2005, Ramtekkar et al. 2002, Adam 2003). Conjointly yielded results are compared and discussed.

1. Introduction

Free optical vibration of generally laminated beams has been of increasing interest in the last decades' literature [1–12]. Vo et al. [1] investigated free vibration of axially loaded thin-walled composite beams with arbitrary lay-ups. The proposed model was based on equations of motion for flexural-torsional coupled vibration which were derived from the Hamilton's principle. In the same context, Li et al. [2] studied the free vibration and buckling behaviors of axially loaded laminated composite beams using the dynamic stiffness method. The model took into account influences of axial force, Poisson effect, axial deformation, shear deformation, and rotary inertia. Hu et al. [4] Karami et al. [5], and Malekzadeh et al. [6] proposed a differential quadrature element method (DQEM) by using Hamilton's principle for free vibration analysis of arbitrary nonuniform Timoshenko beams on elastic supports.

Many other analytical methods of analysis have been used to study the vibration of plates, shells, and beams [7–12].

In this paper, a model on the vibration analysis of laminated composite beam has been developed and studied using two resolution protocols. For the beam used, it is assumed that Bernoulli-Euler hypothesis is valid. The results obtained by the two methods are compared. It has been concluded that all of the results are very close to each other.

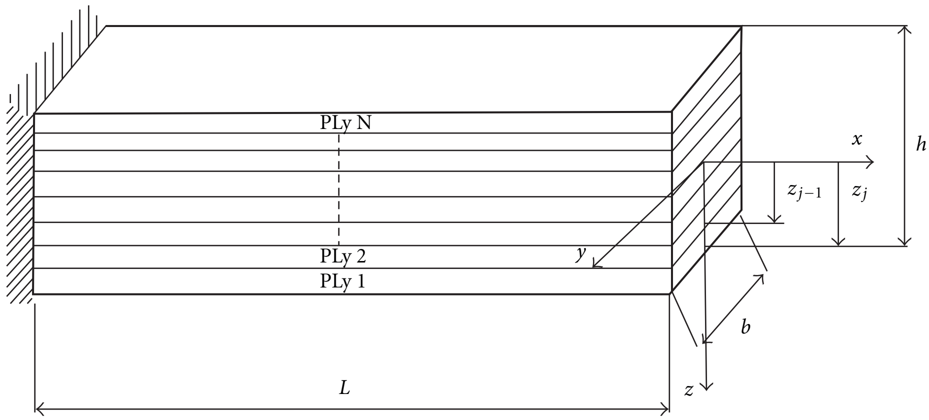

Figure 1: Cantilever composite laminated optical beam.

2. Problem Formalization

The normal stress in jth layer of a composite laminated beam shown in Figure 1 can be written in the following way:

$$(\sigma_x)_j = (E_x)_j \cdot (\varepsilon_x)_j. \tag{1}$$

According to Bernoulli-Euler hypotheses, the deformation at a certain distance from neutral plane is

$$\varepsilon_x = \frac{z}{\rho}, \tag{2}$$

where ρ is the curvature of the beam. The relationship between normal stress and bending moment is given by

$$M = 2\int_0^{h/2} \sigma_x \cdot z \cdot b \cdot dz, \tag{3}$$

or

$$M = \frac{2b}{3\rho}\sum_{j=1}^{N/2} (E_x)_j \left(z_j^3 - z_{j-1}^3\right), \tag{4}$$

where h and b are the height and the width of the beam, N is the number of layer and z_j is the distance between the outer face of jth layer, and the neutral plane. The relationship between the bending moment and the curvature can be written as follows:

$$M = \frac{E_{\text{ef}} \cdot I_{yy}}{\rho} = E_{\text{ef}} I_{yy} \frac{d^2 w}{dx^2}, \qquad E_{\text{ef}} = \frac{8}{h^3}\sum_{j=1}^{N/2} (E_x)_j \left(z_j^3 - z_{j-1}^3\right), \tag{5}$$

where E_{ef} is the effective elasticity modulus and I_{yy} is the cross-sectional inertia moment of the beam. Flexural motion of a linear elastic laminated composite beam without shear or rotary inertia effects is described by Bernoulli-Euler equation:

$$E_{\text{ef}} \cdot I_{yy} \cdot \frac{\partial^4 w}{\partial x^4} + \rho \cdot A \cdot \frac{\partial^2 w}{\partial t^2} = 0. \tag{6}$$

As a solution of (6), it can be used a separation of variables solution for harmonic free vibration:

$$w(x,t) = e^{i\omega_n t} W(x), \tag{7}$$

where ω_n is the frequency and $W(x)$ is the mode shape function of the lateral deflection. Substitution of this solution into (6) eliminates the time dependency and yields the following characteristic value problem:

$$\frac{d^4 W(x)}{dx^4} - \zeta^2 W(x) = 0, \tag{8}$$

where λ is the dimensionless frequency of the beam vibrations given by

$$\zeta = \sqrt{\frac{\omega_n^2 \rho_m A}{E_{\text{ef}} I_{yy}}}. \tag{9}$$

For a cantilever composite laminated beam shown in Figure 1, the boundary conditions at the two ends are

$$W = \frac{dW(x)}{dx} = 0 \quad \text{at } x = 0, \tag{10}$$

due to the deflection and rotation both being zero at the clamped end, and

$$\frac{d^2 W(x)}{dx^2} = 0 \quad \text{at } x = L, \tag{11}$$

due to the bending moment and shear force both vanishing at the free end.

The analytical solution of (8) subjected to (10) and (11) yields the frequency equation:

$$\cos(\beta L)\cosh(\beta L) + 1 = 0, \qquad \beta = \zeta^4, \tag{12}$$

which may be found in the relevant literature [13].

3. DQM Solution

DQM method is carried out for the approximate solution of the characteristic value problem in (8) with the boundary conditions given by (10) and (11) by first discretizing the interval $[0, L]$ such that $0 = x_1 < x_2 < \cdots < x_N = L$, where N is the number of grid points. Application of the DQM to discrete the derivative in (8) leads to

$$\sum_{j=1}^{N} A_{ij}^{(4)} W_j - \zeta^2 W_i = 0, \qquad i = 3, 4, \ldots, (N-2), \tag{13}$$

where $A_{ij}^{(4)}$ are the weighting coefficients of the fourth-order derivative which can be calculated using the explicit relations given by Shu [14]. Note that we have two boundary conditions specified at both ends. These two conditions at the same point provoke a great challenge for the DQM, because we have only one quadrature equation at one point in the DQM, which prevents implementing the two boundary conditions. We use δ-point technique to eliminate the difficulties in implementing two conditions at a single boundary point (Figure 2). Following the same approach presented in [15], the boundary conditions at $x = 0$ can be discretized as

$$W_1 = 0, \qquad \sum_{j=1}^{N} A_{2j}^{(1)} W_j = 0. \tag{14}$$

Similarly, the boundary conditions at $x = L$ can be discretized as

$$\sum_{j=1}^{N} A_{(N-1)j}^{(2)} W_j = 0, \tag{15}$$

$$\sum_{j=1}^{N} A_{N\,j}^{(3)} W_j = 0. \tag{16}$$

The assembly of (13) through (15) yields the following set [14] of linear equations:

$$\begin{bmatrix} [S_{bb}] & [S_{bd}] \\ [S_{db}] & [S_{dd}] \end{bmatrix} \begin{Bmatrix} \{W_b\} \\ \{W_d\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \zeta\{W_d\} \end{Bmatrix}, \tag{17}$$

where the subscripts b and d indicate the grid points used for writing the quadrature analog of boundary conditions and the governing differential equation, respectively. By matrix substructuring of (17), one has the following two equations:

$$\begin{aligned} [S_{bb}]\{W_b\} + [S_{bd}]\{W_d\} &= \{0\}, \\ [S_{db}]\{W_b\} + [S_{dd}]\{W_d\} &= \zeta^2\{W_d\}. \end{aligned} \tag{18}$$

From the first part of (18), one obtains

$$\{W_b\} = -[S_{bd}]^{-1}[S_{bd}]\{W_d\}. \tag{19}$$

Back-substituting (19) into the second part of (18), one gets

$$[S]\{W_d\} - \lambda^2[I]\{W_d\} = \{0\}, \tag{20}$$

where $[S]$ is of order $(N-4) \times (N-4)$ and given by

$$[S] = -[S_{db}]\ [S_{bb}]^{-1}[S_{bd}] + [S_{dd}]. \tag{21}$$

Both the eigenvalues being the frequency squared values and the eigenvectors $\{W_d\}$ describing the mode shapes of the freely vibrating beam may be obtained simultaneously from the $[S]$ matrix.

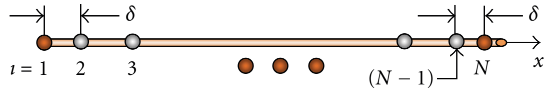

Figure 2: A one-dimensional quadrature grid with adjacent δ-points.

4. BPES Solution

The BPES [16–23] is applied to (8) through setting the expression

$$W(x) = \frac{1}{2N_0} \sum_{k=1}^{N_0} \lambda_k \times B_{4k}\left(x \times \frac{r_k}{L}\right), \tag{22}$$

where B_{4k} are the $4k$-order Boubaker polynomials, $x \in [0, L]$ is the normalized time, r_k are B_{4k} minimal positive roots, N_0 is a prefixed integer, and $\lambda_k|_{k=1,\ldots,N_0}$ are unknown pondering real coefficients.

Consequently, it comes for (8) that

$$\begin{aligned} &\frac{1}{2N_0}\left(\frac{r_k}{L}\right)^4 \sum_{k=1}^{N_0} \lambda_k \times \frac{dB_{4k}}{dx^4}\left(x \times \frac{r_k}{L}\right) \\ &\quad - \zeta^2 \frac{1}{2N_0}\left(\frac{r_k}{L}\right)^4 \sum_{k=1}^{N_0} \lambda_k \times B_{4k}\left(x \times \frac{r_k}{L}\right) = 0. \end{aligned} \tag{23}$$

The related boundary conditions expressed through (10) and (12). The BPES protocol ensures their validity regardless main equation features. In fact, thanks to Boubaker polynomials first derivatives properties are

$$\begin{aligned} &\sum_{q=1}^{N} B_{4q}(x)\Big|_{x=0} = -2N \neq 0, \\ &\sum_{q=1}^{N} B_{4q}(x)\Big|_{x=r_q} = 0, \\ &\sum_{q=1}^{N} \frac{dB_{4q}(x)}{dx}\Big|_{x=0} = 0, \\ &\sum_{q=1}^{N} \frac{dB_{4q}(x)}{dx}\Big|_{x=r_q} = \sum_{q=1}^{N} H_q \\ &\text{with } H_n = B'_{4n}(r_n) = \left(\frac{4r_n[2 - r_n^2] \times \sum_{q=1}^{n} B_{4q}^2(r_n)}{B_{4(n+1)}(r_n)} + 4r_n^3\right). \end{aligned} \tag{24}$$

Boundary conditions are inherently verified:

$$\begin{aligned} \frac{dW(x)}{dx}\Big|_{x=0} &= \frac{1}{2N_0} \sum_{k=1}^{N_0} \lambda_k \times \frac{dB_{4k}(x)}{dx}\Big|_{x=0} = 0, \\ \frac{dW(x)}{dx}\Big|_{x=L} &= \frac{1}{2N_0} \sum_{k=1}^{N_0} \lambda_k \times \frac{dB_{4k}(x)}{dx}\Big|_{x=r_k} = 0, \\ &\sum_{k=1}^{N_0} \lambda_k \times H_n = 0. \end{aligned} \tag{25}$$

Table 1: Geometry and material properties of the composite materials.

Descriptions	Present	Unit
L (mm)	320	mm
b (mm)	36	mm
h (mm)	8	mm
E_1 (MPa)	26950	MPa
E_2 (MPa)	21800	MPa
G_{12} (MPa)	7540	MPa
ν_{12}	0.15	
ρ (kg/m^3)	**2030**	kg/m^3

The BPES solution is obtained through five steps:

(i) Integrating, for a given value of N_0, the whole expression given by (23) along the interval $[0, L]$.

(ii) Determining the set of coefficients where $\tilde{\lambda}_k|_{k=1,\ldots,N_0}$ that minimizes the absolute difference D_{N_0}:

$$D_{N_0} = \left| \left(\frac{1}{2N_0} \sum_{k=1}^{N_0} \tilde{\lambda}_k \times \Lambda_k \right) - \zeta \left(\frac{1}{2N_0} \sum_{k=1}^{N_0} \tilde{\lambda}_k \times \Lambda'_k \right) \right|$$

with

$$\Lambda_k = \left(\frac{r_k}{L} \right)^4 \int_0^L \frac{dB_{4k}}{dx^4} \left(x \times \frac{r_k}{L} \right) dx,$$
$$\Lambda'_k = \int_0^L B_{4k} \left(x \times \frac{r_k}{L} \right) dx. \tag{26}$$

(iii) Deducing the corresponding frequency using (9).

(iv) Incrementing N_0.

(v) Ranging the obtained frequencies.

5. Results and Discussion

Natural frequencies of the symmetric laminated composite cantilever beam have been estimated using the Boubaker Polynomials Expansion Scheme (PBES) and the Differential Quadrature Method (DQM), and for parameters values indicated in Table 1. Figure 3 presents the obtained values. The results have been evaluated as quite close to each other.

The natural frequency alteration as a direct result of the change in the stacking sequence causes resonance if the changed frequency becomes closer to the working frequency. Hence, selection of the stacking sequences in the laminated composite beams has to be outlined.

6. Conclusion

This work deals with two protocols for the calculation of natural frequency of the symmetric laminated composite cantilever beam. Calculations performed by means of Boubaker Polynomials Expansion Scheme PBES and Differential Quadrature Method DQM yielded coherent and similar results.

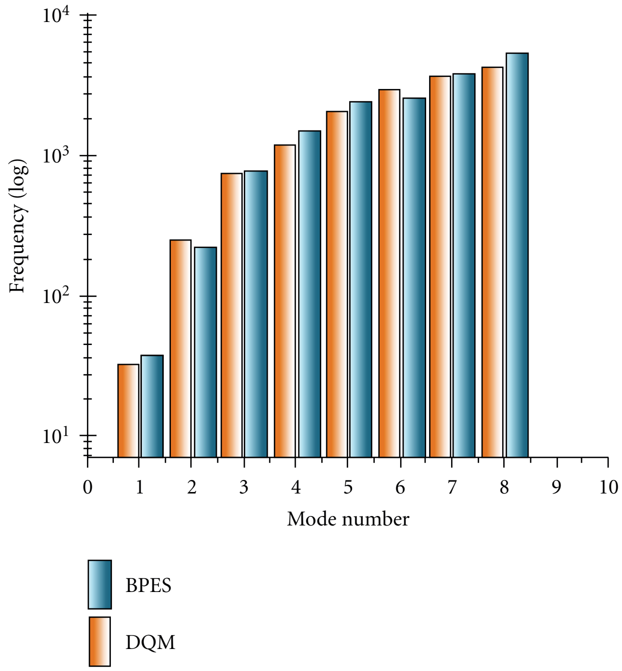

Figure 3: Comparison of BPES and DQM frequencies for cross-ply laminated beam.

All considered results have been seen to be in accordance with each other. Changes in the stacking sequence, which likely allow tailoring of the material to achieve desired natural frequencies and respective mode shapes without changing its geometry, are the subject of following studies.

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