Comparative Analysis of MIT Rule and Lyapunov Rule in Model Reference Adaptive Control Scheme

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Abstract

Adaptive control involves modifying the control law used by the controller to cope with the fact that the parameters of the system being controlled change drastically due to change in environmental conditions or change in system itself. This technique is based on the fundamental characteristic of adaptation of living organism. The adaptive control process is one that continuously and automatically measures the dynamic behavior of plant, compares it with the desired output and uses the difference to vary adjustable system parameters or to generate an actuating signal in such a way so that optimal performance can be maintained regardless of system changes. This paper deals with application of model reference adaptive control scheme and the system performance is compared with Lyapunov rule and MIT rule. The plant which is taken for the controlling purpose is the first order system for simplicity. The comparison is done for different values of adaptation gain between MIT rule and Lyapunov rule. Simulation is done in MATLAB and simulink and the results are compared for varying adaptation mechanism due to variation in adaptation gain.

Keywords: Adaptation gain, MIT rule, Model reference adaptive control, Lyapunov theory.

1. Introduction

A control system is in the broadest sense, an interconnection of the physical components to provide a desired function, involving some type of control action with it. The requirement of high performance control system for industrial applications has produced great research efforts for the application of modern control theory and, in particular, adaptive control. As compared to fixed gain PID controllers Adaptive Controllers are very effective to handle the situations where the parameter variations and environmental changes are frequent. The controller parameters are adjusted to give a desired closed-loop performance. The adaptive controller maintains constant dynamic performance in the presence of unpredictable and immeasurable variations. Adaptive control changes the control algorithm coefficients in real time to compensate for variations in the environment or in the system itself. It also varies the system transfer function according to situation.

Adaptive control is most recent class of control techniques though research in adaptive control has a long and vigorous history. In 1950s, it was motivated by the problem of designing autopilots for aircraft operating at wide range of speeds and altitudes. Consequently, gain scheduling based on some auxiliary measurements of air speed was adapted. Kalman developed the concept of a general self tuning regulator with explicit identification of the parameters of linear, single-input, single-output system and he used these parameter estimates to update an optimal linear quadratic controller. In 1960s Lyapunov's stability theory was established as tool for proving convergence in adaptive control scheme. Further, Parks found a way of redesigning the update laws proposed in 1950s for model reference adaptive control. Nowadays the adaptive control schemes are making their place where the conventional control system is not able to cope-up with the situation, like

- Loads, inertias and other forces acting on system change drastically.
- Possibility of unpredictable and sudden faults.
- Possibility of frequent or unanticipated disturbances.

The conventional PID controllers with fixed gain are unable to cop up with the problems discussed above. Though recently advanced fuzzy PID controllers have been developed to deal with such problems for electrical and mechanical systems. Hwang (2000) and Tang (2001) gave the concept of fuzzy PID controller. Rubaai (2008) developed the DSP-Based Laboratory model of Hybrid Fuzzy-PID Controller Using Genetic optimization for high-performance Motor Drives. Later on the concept of neural network is applied to develop the PID controllers to enhance the dynamic characteristics of controller. Sun (2006) and Yao (2008) developed such PID controllers based on the artificial Neural Network technique. Still to obtain the complete adaptive nature, specific adaptive control techniques are needed. Out of many adaptive control schemes, this paper mainly deals with the model reference adaptive control (MRAC) approach. In MRAC, the output response is forced to track the response of a reference model irrespective of plant parameter variations. The controller parameters are adjusted to give a desired closed-loop performance. In practical cases, a MRAC system is generally best implemented with a digital computer, owing to the complexity of the controller. Koo (2001) and Tsai (2004) developed the concept of MRAC by using Fuzzy logic. The model reference adaptive controllers are mainly designed by using different approaches like MIT rule, Lyapunov theory and theory of augmented error. Ehsani (2007) used the MIT rule to control the DC servo motor. Swarnkar (2010, 2011) applied this rule to systems of different order and compare the results with the conventional techniques for the same systems. Wu (2004) used the concept of Lyapunov theory to develop the MRAC system of Linear motor drive. Shyu (2008) and Stefanello (2008) used the Lyapunov theory for improving the performance of shunt active power filter.

This paper compares the MIT rule and Lyapunov rule for developing the adaptation mechanism on the basis of different time response specifications. Simulation is done in MATLAB and results are shown for the first order system. The results show that the nature of adaptation depends on a constant known as the adaptation gain. The parameter convergence rate is greatly affected by the correct choice of this adaptation gain. It is thus important to know the reasonable value of this parameter.

2. Model Reference Adaptive Control

This technique of adaptive control comes under the category of Non-dual adaptive control. A reference model describes system performance. The adaptive controller is then designed to force the system or plant to behave like the reference model. Model output is compared to the actual output, and the difference is used to adjust feedback controller parameters.

MRAS has two loops: an inner loop or regulator loop that is an ordinary control loop consisting of the plant and regulator, and an outer or adaptation loop that adjusts the parameters of the regulator in such a way as to drive the error between the model output and plant output to zero.

2.1 Components of Model Reference Adaptive Controller

Reference Model: It is used to specify the ideal response of the adaptive control system to external command. It should reflect the performance specifications in control tasks. The ideal behavior specified by the reference model should be achievable for the adaptive control system.

Controller: It is usually parameterized by a number of adjustable parameters. In this paper two parameters θ_1 and θ_2 are used to define the controller law. The control law is linear in terms of the adjustable parameters (linear parameterization). Adaptive controller design normally requires linear parameterization in order to obtain adaptation mechanism with guaranteed stability and tracking convergence. The values of these control parameters are mainly dependent on adaptation gain which in turn changes the control algorithm of adaptation mechanism.

Adaptation Mechanism: It is used to adjust the parameters in the control law. Adaptation law searches for the parameters such that the response of the plant which should be same as the reference model. It is designed to guarantee the stability of the control system as well as conversance of tracking error to zero. Mathematical techniques like MIT rule, Lyapunov theory and theory of augmented error can be used to develop the adaptation mechanism. In this paper both MIT rule and Lyapunov rule are used for this purpose.

3. The MIT Rule

This rule is developed in Massachusetts Institute of technology and is used to apply the MRAC approach to any practical system. In this rule the cost function or loss function is defined as

$$F(\theta) = e^2 / 2 \qquad (1)$$

Where, e is the output error and is the difference of the output of the reference model and the actual model, while θ is the adjustable parameter.

In this rule the parameter θ is adjusted in such a way so that the loss function is minimized. For this it is reasonable to change the parameter in the direction of the negative gradient of *F*, that is,

$$d\theta/dt = -\gamma\, \partial F / \partial\theta \qquad (2)$$

$$= -\gamma\, e\, \partial e / \partial\theta \qquad (3)$$

The partial derivative term $\partial e/\partial\theta$, is called the sensitivity derivative of the system. This shows how the error is dependent on the adjustable parameter, θ. There are many alternatives to choose the loss function *F*, like it can be taken as mode of error also. Similarly dθ/dt can also have different relations for different applications.

Sign-sign algorithm:

$$d\theta/dt = -\gamma\, \text{sign}(\partial e / \partial\theta)\, \text{sign}\, e \qquad (4)$$

Or it may be chosen as

$$d\theta/dt = -\gamma\, (\partial e / \partial\theta)\, \text{sign}\, e \qquad (5)$$

Where sign e = 1 for e › 0

= 0 for e = 0

= -1 for e ‹ 0

In some industrial applications it is found that the choice of adaptation gain is critical and its value depends on the signal levels. So MIT rule has to be modified as follows:

$$d\theta/dt = -\gamma\, \zeta\, e \qquad (6)$$

Where $\zeta = \partial e / \partial\theta$

Also $$d\theta/dt = -\gamma\, \zeta\, e / (\beta + \zeta^T \zeta) \qquad (7)$$

Where β › 0 is introduce to avoid the zero division when $\zeta^T \zeta$ is small.

In this paper Model Reference Adaptive Control Scheme is applied to first order system using MIT rule. The MIT rule which is used for simulation is described in equation (1).

Let the first order system is described by

$$dy/dt = -a\, y + b\, u \qquad (8)$$

Let a=3 and b=2

where u is the controller output or manipulated variable. The transfer function can be written as

$$Y(s)/U(s) = 2/(s+3) \quad (9)$$

Similarly the reference model is described by

$$dy_m/dt = -a_m\, y_m + b_m\, r \quad (10)$$

Take a_m =4 and b_m=4

where r is the reference input.

The transfer function can be written as

$$Y_m(s)/R(s) = 4/(s+4) \quad (11)$$

Here the object is to compare the actual output (y) and the reference output (y_m) and by applying Model Reference Adaptive Control Scheme the overall output will be improved.

$$u(t)=\theta_1\, r(t)- \theta_2\, y(t) \quad (12)$$

The controller parameters are chosen as

$$\theta_1=b_m/ b \text{ and } \theta_2 = (a_m- a) / b \quad (13)$$

The update rule for the controller parameters using MIT rule is described by

$$d\theta_1/dt = - \gamma\, e\, \partial e / \partial \theta_1$$

$$=- \gamma\, e\, [b\, r / (p + a_m)]$$

$$=- \alpha\, e\, [a_m\, r / (p + a_m)]$$

and $d\theta_2/dt =- \alpha\, e\, [a_m\, y / (p+ a_m)]$

where $\alpha = \gamma\, b/a_m$ is the adaptation gain.

From fig 5 it can be observed that controller parameter 1 (θ_1) converges to 1.46 and controller parameter 2 (θ_2) converges to -0.04. Now by using equation (11) the estimated plant parameters come out to be

$b^{'} = b_m / \theta_1 = 2.739$ and $a^{'} = a_m - b^{'} \theta_2 = 4.109$

4. The Lyapunov Rule

The Lyapunov stability theory can be used to describe the algorithms for adjusting parameters in Model Reference Adaptive control system. For the above mention system the controller law is defined by equation (10). The error is given by

$$e(t)= y - y_m \quad (14)$$

Now the change in error with respect to time can be written as

$$de/dt = - a_m\, e – (b\, \theta_2 + a – a_m)y + (b\, \theta_1 - b_m)\, u_c \quad (15)$$

The Lyapunov function is described $V(e, \theta_1, \theta_2)$.

This function should be positive semi definite and is zero when error is zero. For stability according to Lyapunov theorem the derivative dV/dt must be negative. The derivative dV/dt requires the values of $d\theta_1/dt$ and $d\theta_2/dt$. If the parameters are updated then

$d\theta_1/dt = -\alpha re$, $d\theta_2/dt= \alpha ye$, $dV/dt= - a_m e^2$

So dV/dt is negative semi definite. This shows that

$V(t) \Leftarrow V(0)$ and e, θ_1, θ_2 must be bounded.

5. Simulation and results

The Model reference Adaptive Control Scheme is applied to first order system by using MIT rule and Lyapunov rule. The models are simulated in MATLAB which are shown in fig 2 and fig 3. Fig 4 compares the time responses of same plant controlled by MIT and Lyapunov rules for α = 1. The characteristics show that there is very less difference in responses for both the models, though the complicacy is reduced to large extent in Lyapunov rule.

Fig 5 shows the variation of θ_1 and θ_2 with respect to time for MIT rule. It can be observed that controller parameter 1 (θ_1) converges to 1.46 and controller parameter 2 (θ_2) converges to -0.04. Similarly fig 6 shows the variation of θ_1 and θ_2 with respect to time for Lyapunov rule. Here θ_1 converges to 1.375 and θ_2 converges to -0.125.

Fig 7 and fig 8 show the effect of adaptation gain on time response curves for MIT rule and Lyapunov rule respectively. There is improvement in the performance of the system with the increment in adaptation gain. Every system gives its best for the limited range of the adaptation gain. In this paper the range of adaptation gain is chosen from 0.1 to 10 for the system under consideration. Beyond this range the system performance is not up to the mark. It has been seen that the response is very slow with the smaller value of adaptation gain but there are no oscillations in the response. With the increment in α, the maximum overshoot is increased but at the same time speed becomes faster (the settling time is reduced). It is noticed that the improvement is faster in the system performance for Lyapunove rule with the increment in adaptation gain. The tracking error ($y-y_m$) is shown in fig 9 for both the rules. There is not much difference in error convergence, though it is faster with Lyapunov rule.

6. Conclusion

A detailed comparison is done between two methods of model reference adaptive control system. The complicacy is reduced in configuration of MRAC with Lyapunov rule as compared to MIT rule. So the physical realization for the system under consideration is comparatively more feasible with Lyapunov theory. But the mathematical modeling of system is simpler for MIT rule. Table I compares the results of two schemes for different values of adaptation gain. In this paper the range of adaptation gain is selected between 0.1 and 10. It can be observed easily that the performance of system for both the methods is improving with the increment in adaptation gain. But the rate of improvement is higher for Lyapunov theory. The system response is not oscillatory for $\alpha = 0.1$, but the response is very sluggish. Now if the adaptation gain is increased slightly, response becomes oscillatory with somewhat reduction in settling time. Now if the adaptation gain is further increased then the maximum overshoot is reduced. The reduction in maximum overshoot is remarkable for Lyapunov rule as compared to MIT rule. So Lyapunov rule confirms the stability of the system. Settling time is also less in case of Lyapunov rule in Model reference controlled system, which improves the speed of the system. It can be concluded that system performance is best for adaptation gain $\alpha = 10$. For this value of adaptation gain maximum overshoot is only 5% with settling time 2.5 seconds for system controlled by MRAC scheme with Lyapunov rule.

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	Adaptation Gain (α)	Peak Time (second)	Maximum Overshoot (%)	Settling Time (second)	θ_1	θ_2
MIT Rule	0.1	-	-	8.3	1.134	-0.367
	1	1.5	15	3.74	1.46	-0.04
	5	0.756	18.5	3	1.765	0.265
	10	0.6	12.5	3.2	1.845	0.345
Lyapunov Rule	0.1	-	-	8.25	1.105	-0.395
	1	1.3	18	4.2	1.375	-0.125
	5	0.6	14	2.85	1.77	0.271
	10	1	5	2.5	1.93	0.43

Table 1. Comparison between MIT rule and Lyapunov rule

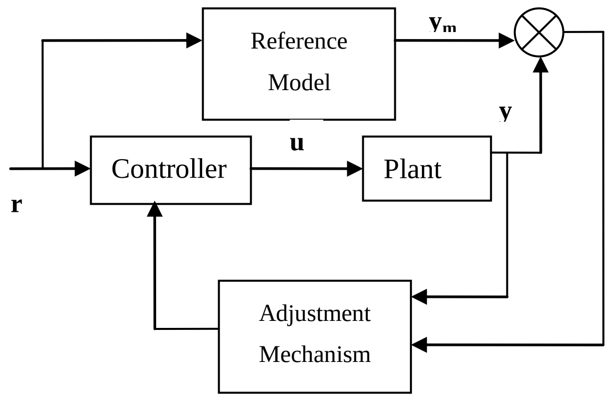

Figure 1. Model Reference Adaptive Controller

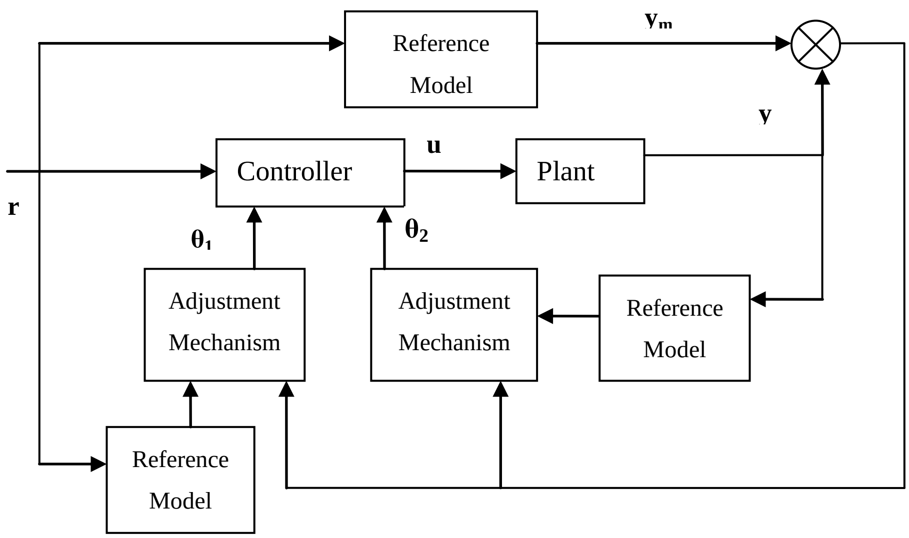

Figure 2. Model for MRAC using MIT rule

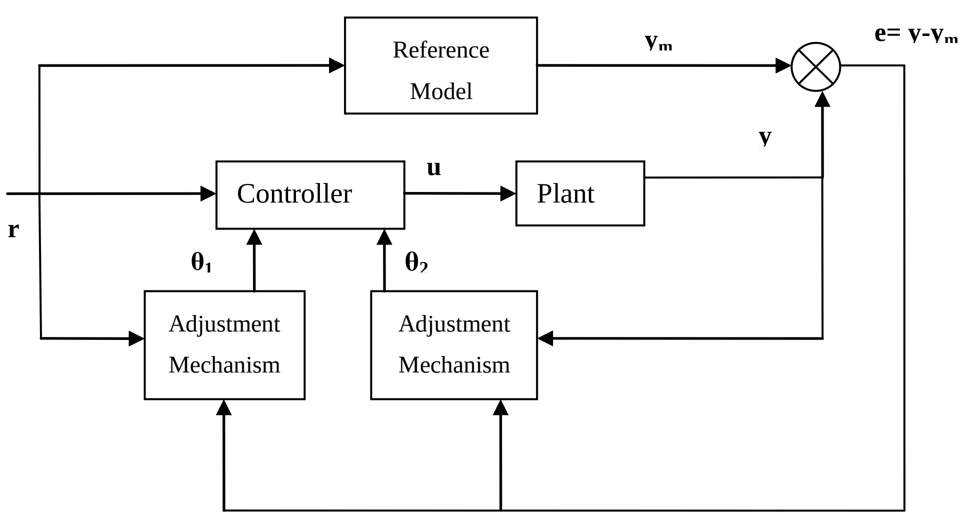

Figure 3. Model for MRAC using Lyapunov rule

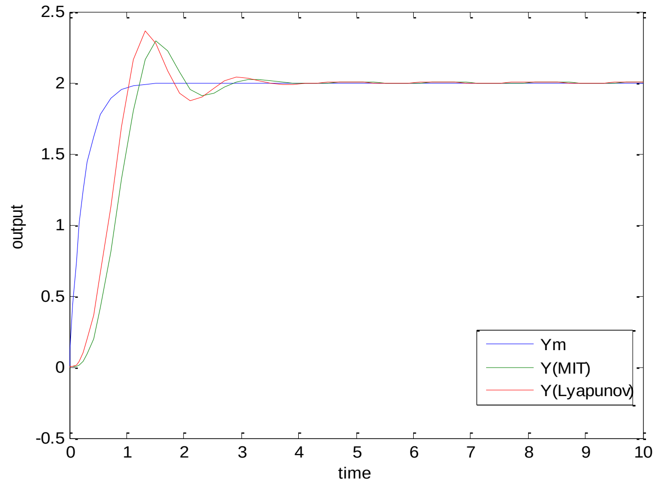

Figure 4. Time response curve for $\alpha = 1$

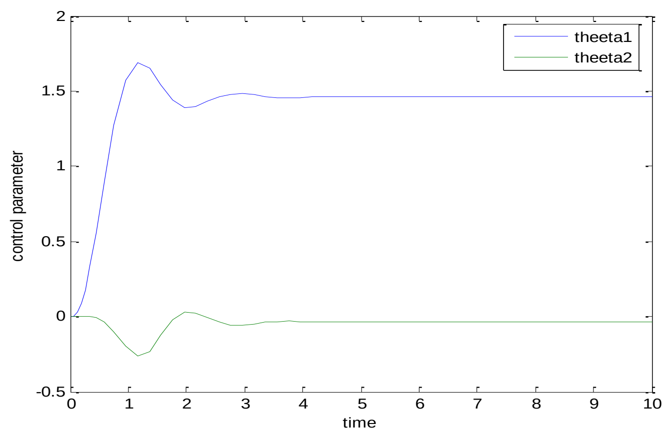

Figure 5. Variation of control parameter θ_1 and θ_2 with time for MIT rule (α=1)

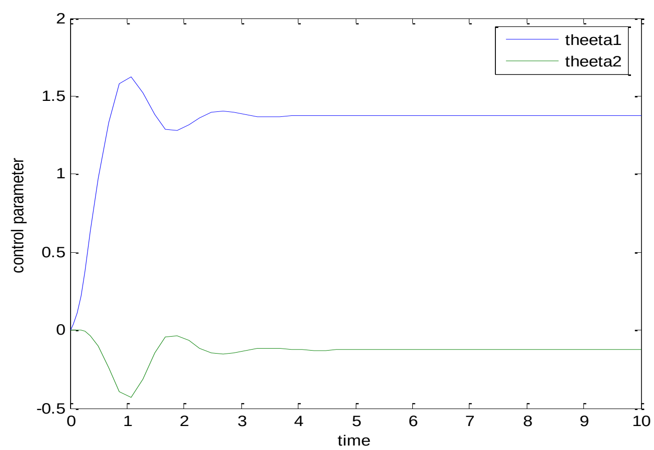

Figure 6. Variation of control parameter θ_1 and θ_2 with time for Lyapunov rule (α=1)

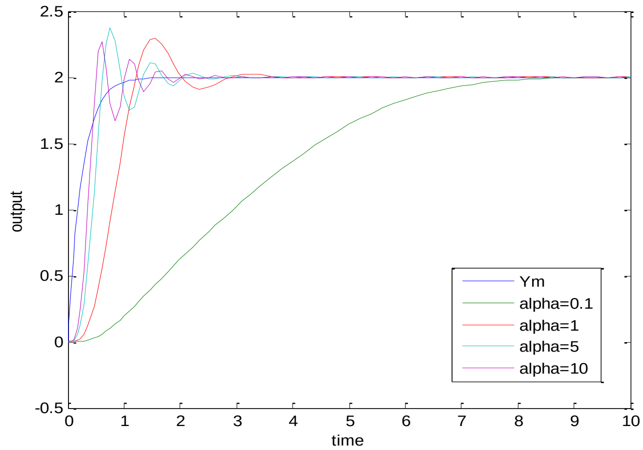

Figure 7. Effect of α on time response curve for MIT rule

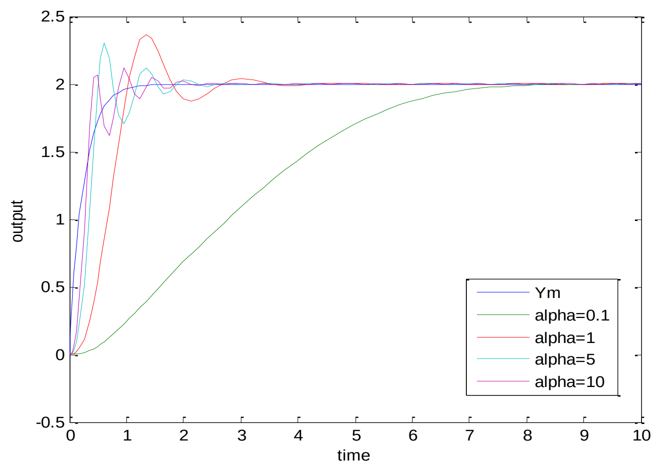

Figure 8. Effect of α on time response curve for Lyapunov rule

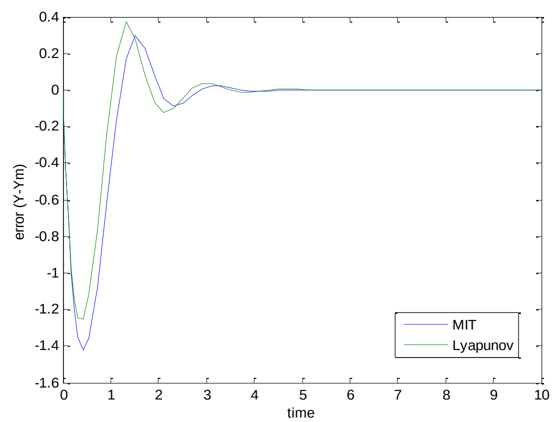

Figure 9. Output error ($y-y_m$) for MIT and Lyapunov rule