Comparative analysis of overland flow models using finite volume schemes

P. Costabile, C. Costanzo and F. Macchione

ABSTRACT

In this paper attention is first focused on a comparative analysis of three hydraulic models for overland flow simulations. In particular, the overland flow was considered as a 2D unsteady flow and was mathematically described using three approaches (fully dynamic, diffusive and kinematic waves). Numerical results highlighted that the differences among the simulations were not very important when the simulations referred to commonly used ideal tests found in the literature in which the topography is reduced to plane surface. Significant differences were observed in more complicated tests for which only the fully dynamic model was able to provide a good prediction of the observed discharges and water depths. Then, attention is focused on the fully dynamic model and in particular on the analysis of two numerical schemes (TVD-MacCormack and HLL) and the influence of the grid size. Numerical tests carried out on irregular topography show that, as the grid size decreases, the performance of the HLL scheme becomes closer to that of the TVD-MacCormack scheme in shorter computational times at least for high rainfall intensity.

Key words | diffusive wave, HLL scheme, kinematic wave, overland flow, TVD-MacCormack scheme, 2D shallow water equations

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INTRODUCTION

Flooding events represent the most common natural hazard in the world and may cause enormous economical, social and environmental damage and even loss of lives. Moreover, in recent years the perception exists that extreme climatic and hydrological events have become more frequent, suggesting that this phenomenon may be due to man-induced global warming.

Surface runoff is a dynamic part of the response of watershed from rainfall: it is known to cause surface erosion and it is quite often associated with a sudden rise of the stream hydrograph. In particular, intense localized precipitation may cause flash floods which often occur in small catchments (e.g. those of less than 100–1000 km^2) and primarily in hilly or mountainous areas due to prevailing convective rainfall mechanisms; in general, this type of flood event is short in duration, but is nonetheless frequently connected with severe damage.

In order to obtain a reliable prediction of the hydraulic risk associated with extreme events, the use of numerical simulation models, appropriately validated using both experimental and real event data, seems to be necessary. A remarkable analysis of the sociotechnical forces that have driven the evolution of numerical modelling and more in general of the applications of numerical modelling in hydroinformatics may be found in Abbott & Vojinovic (2009).

The mathematical modelling of overland flow is very complex because it involves the description of the surface and groundwater flow with seepage at the ground surface (Singh & Bhallamudi 1998; Kolditz *et al.* 2008). In particular, the hydraulic description of the overland flow is very important in determining flow depths and velocities and notable efforts have been devoted to the modelling these situations in the literature. As a consequence, several models have been proposed to deal with this issue based on different levels of detail reflecting the simplifications introduced to describe the hydraulic processes.

doi: 10.2166/hydro.2011.077

The 2D fully dynamic shallow water equations (SWE) is the most accurate and exhaustive approach dealing with flow behaviour for locally complex topography. One of the earliest attempts at modelling overland flow using the 2D fully dynamic SWE, solved by a finite difference scheme, was presented by Zhang & Cundy (1989); their results, in particular, showed the importance of soil surface microtopography in overland flow processes since the water depth and velocities simulated on the variable microtopography deviated significantly from those obtained using a plane surface with mean slope gradient. Singh & Bhallamudi (1998) proposed a numerical approach based on a conjunctive surface–subsurface modelling of overland flow: the surface flow is described by the complete 1D Saint-Venant equations while the 2D Richards equation is used to take into account the subsurface flow. Esteves *et al.* (2000) and Fiedler & Ramirez (2000) developed numerical models that couple the surface flow and infiltration processes considering the variations in topographic elevation and in soil hydraulics parameters. Both the aforementioned models consider the Green–Ampt infiltration equation. More recently, Ajayi *et al.* (2008) proposed a numerical model to simulate Hortonian overland flow for tropical humid catchment to include the effects of vegetation in the rainfall interception phenomenon.

Problems of instabilities and convergence due to highly nonlinear nature of the governing equations limited in the past the use of fully dynamic model and, as a consequence, different approximations of unsteady flow equations, as kinematic and diffusive wave models, are also commonly used to simulate the overland flow processes (Tayfur *et al.* 1993, Di Giammarco *et al.* 1996, Feng & Molz 1997, Howes *et al.* 2006, Kazezyilmaz-Alhan & Medina 2007, Gottardi & Venutelli 2008). Several authors have studied the conditions for which those approximations are completely justified (Woolhiser & Liggett 1967; Ponce *et al.* 1978; Moussa & Bacquillon 1996; Moramarco & Singh 2002). A comprehensive review of the applicability criteria may be found in Tsai (2003) where the backwater effects have been also included in the analysis. However, it is important to observe that, as already mentioned, the microtopography may be a dominant factor causing spatial variation in overland flow depth, velocity and directions (Zhang & Cundy 1989; Tayfur *et al.* 1993) while the model performances were often analysed in the literature using a very simplified idealised topography, reducing complex hillslopes to plane surface with constant hydraulic properties.

It is well known that the unsteady flow equations admit analytical or semianalytical solutions only under certain restrictive conditions and, consequently, numerical techniques have to be used for solving the governing equations. Several numerical schemes were proposed in the literature. Explicit and implicit finite-difference methods were intensively used not only in the past (e.g. Liggett & Woolhiser 1967; Chow & Ben-Zvi 1973; Zhang & Cundy 1989) but also in the recent years (Ajayi *et al.* 2008; Tseng 2010) as well as finite-element methods (e.g. Akanbi & Katopodes 1988; Di Giammarco *et al.* 1996; Jaber & Mothar 2003). A very popular approach, especially used for high unsteady computation and dam break problems, is the finite-volume method (e.g. Hirsch 1990; LeVeque 2002) that is a framework for developing numerical schemes conserving mass and momentum. It often considers a Riemann problem which is an initial-value problem in which a discontinuity in the initial condition occurs. In order to solve discontinuities while obtaining at the same time high-order accuracy, a numerical scheme has to ensure the Total Variation Diminishing (TVD) property that the summation of variations between the states of adjacent cells does not increase over time. In this framework, a huge number of finite-volume schemes were developed in the last three decades (for a review see Toro 2001; Toro & García-Navarro 2007). An in-depth comparative analysis on the performances of several first- and second-order upwind and central numerical schemes including HLL, HLLC, Roe scheme, MacCormack-TVD scheme may be found in the literature (Costanzo *et al.* 2002; Macchione & Morelli 2003; Macchione & Viggiani 2004; Costanzo & Macchione 2005). The above analysis was carried out focusing attention on both computational aspects, such as implementation burdensomeness and computational times, and on practical aspects such as the accuracy of the solution in terms of maximum water levels, arrival times and velocities. From the above-mentioned papers, it may be deduced that the simulations carried out by means of the MacCormack-TVD scheme were the most accurate predictions; the HLL scheme also works very well and is very competitive in terms of computational time.

Indeed a number of numerical problems exist in the use of the 2D unsteady flow modeling for the propagation of a surface runoff in complex topography, even if they are not

explicitly considered herein. For example, Unami *et al.* (2009) used the 2D complete unsteady flow equations, solved with a finite volume method, to study the runoff processes in Ghanaian inland valleys during flood events. In this model, particular attention was paid to achieve a stable computation in complex topographies. Heng *et al.* (2009) proposed a numerical model to describe the overland flow and the associated soil erosion phenomena. The author's numerical scheme, based on a MUSCL-Hancock method, minimized the spurious oscillation that may arise from both the numerical imbalance between source terms and flux gradient and the treatment of wet–dry fronts with very shallow flows. Costabile *et al.* (2010) highlighted the importance of both a robust wet–dry procedure and a suitable numerical treatment of friction slope to improve the stability of the computations using the MacCormack-TVD scheme.

It should be borne in mind that the choice of the numerical solver is a significant source of uncertainty in the fields of flood modelling and computational fluids dynamics that did not received much attention in the past (Claeys *et al.* 2010) unlike the friction coefficient (e.g. Aronica *et al.* 2002; Bates 2004; Pappenberger *et al.* 2005), the grid cell size (e.g. Werner 2001; Fewtrell *et al.* 2008), the structure of flood inundation model (Horritt & Bates 2002), the boundary conditions (e.g. Pappenberger *et al.* 2006), the topography (e.g. Bates *et al.* 1997; Sanders 2007). The estimation of model uncertainty is a very important issue (Pappenberger & Beven 2006) but is beyond the scope of this paper. Recent reviews on this topic can be found in Montanari (2007) and Solomatine & Shrestha (2009).

The analysis of the hydraulic processes associated to overland flow starts from the choice of the most suitable method able to describe the main features of propagation dynamic. Then, in practical studies, it is important to find numerical integration schemes able to provide reliable results in short computational times especially for the analyses at a basin scale in which the accuracy of a numerical scheme should be weighted with the burdensomeness of the computations. These aspects represent the context in which the paper aims to give its contribution.

On one hand, the paper will provide an in-depth comparative analysis of the performances of overland flow models. In particular, models based on fully dynamic, diffusive and kinematic wave properties have been first developed and validated with numerical tests commonly used in the literature and then compared with reference to experimental tests. More in detail, the paper aims to highlight those situations in which the use of a simplified modelling can induce poor predictions respect to a more detailed approach. For that reason, the attention will be also focused on the analysis of benchmark tests characterized by more complicated hydraulic conditions than those traditionally used in the literature in which a complex hillslope topography is dramatically simplified as plane surfaces.

On the other hand, the performances on two numerical schemes will be compared not only using the numerical tests proposed in the literature but also focusing the attention on a real topography. In particular, for the reasons explained above, the numerical integration was carried out using both a first-order upwind (HLL scheme) and a second-order central (TVD-MacCormack) scheme. The influence of the grid size on the numerical results obtained by the two schemes was also analysed.

MATHEMATICAL MODEL

The implemented codes are based on the fully conservative shallow water equations:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{S} \tag{1}$$

where

$$\mathbf{U} = \begin{pmatrix} h \\ hu \\ hv \end{pmatrix}; \quad \mathbf{F} = \begin{pmatrix} hu \\ hu^2 + gh^2/2 \\ huv \end{pmatrix}; \quad \mathbf{G} = \begin{pmatrix} hv \\ huv \\ hv^2 + gh^2/2 \end{pmatrix};$$
$$\mathbf{S} = \begin{pmatrix} r - f \\ gh(S_{0x} - S_{fx}) \\ gh(S_{0y} - S_{fy}) \end{pmatrix} \tag{2)–(5)}$$

in which t is time; x, y are the horizontal coordinates; h is the water depth; u, v are the depth-averaged flow velocity in x- and y-directions; g is the gravitational acceleration; S_{0x}, S_{0y} are the bed slopes in x- and y-directions; S_{fx}, S_{fy} are the friction slopes in x- and y-directions, which can be calculated from Strickler's formula; r is the rain intensity and f are the infiltration losses.

By neglecting the local and convective acceleration in the momentum conservation equations, it is possible to obtain the following diffusive model:

$$\frac{\partial \mathbf{U_d}}{\partial t}+\frac{\partial \mathbf{F_d}}{\partial x}+\frac{\partial \mathbf{G_d}}{\partial y}=\mathbf{S_d} \tag{6}$$

with

$$\mathbf{U}_\mathrm{d}=\begin{pmatrix} h \\ 0 \\ 0 \end{pmatrix};\quad \mathbf{F}_\mathrm{d}=\begin{pmatrix} hu \\ gh^2/2 \\ 0 \end{pmatrix};\quad \mathbf{G}_\mathrm{d}=\begin{pmatrix} hv \\ 0 \\ gh^2/2 \end{pmatrix};$$
$$\mathbf{S}_\mathrm{d}=\begin{pmatrix} r-f \\ gh(S_{0x}-S_{fx}) \\ gh(S_{0y}-S_{fy}) \end{pmatrix} \tag{7)–(10}$$

and ignoring also the depth gradient terms one may obtain the following kinematic model:

$$\frac{\partial \mathbf{U_k}}{\partial t}+\frac{\partial \mathbf{F_k}}{\partial x}+\frac{\partial \mathbf{G_k}}{\partial y}=\mathbf{S_k} \tag{11}$$

with

$$\mathbf{U}_\mathrm{k}=\begin{pmatrix} h \\ 0 \\ 0 \end{pmatrix};\quad \mathbf{F}_\mathrm{k}=\begin{pmatrix} hu \\ 0 \\ 0 \end{pmatrix};\quad \mathbf{G}_\mathrm{k}=\begin{pmatrix} hv \\ 0 \\ 0 \end{pmatrix};$$
$$\mathbf{S}_\mathrm{k}=\begin{pmatrix} r-f \\ gh(S_{0x}-S_{fx}) \\ gh(S_{0y}-S_{fy}) \end{pmatrix} \tag{12)–(15}$$

NUMERICAL MODELS

The finite-volume method, widely adopted in the literature, has been used to discretize the previous equations. It considers the integral form of the shallow water equations which facilitate the implementation of shock capturing schemes on different mesh types. The system of equations is integrated over an arbitrary control volume $\Omega_{i,j}$ and, in order to obtain surface integrals, the Green theorem has been applied to each component of the flux vectors (for example **F** and **G**) leading to

$$\frac{\partial}{\partial t}\int_{\Omega_{i,j}}\mathbf{U}d\Omega+\oint_{\partial\Omega_{i,j}}[\mathbf{F},\mathbf{G}]\cdot\mathbf{n}\,dL=\int_{\Omega_{i,j}}\mathbf{S}d\Omega \tag{16}$$

where $\partial\Omega_{i,j}$ beings to the boundary enclosing $\Omega_{i,j}$, $\mathbf{n}$ is the unit vector normal and dL is the length of each boundary. Denoting by $\mathbf{U}_{i,j}$ the average value of the flow variables over the control volume $\Omega_{i,j}$ at a given time, Equation (16) may be discretized as

$$\mathbf{U}_{i,j}^{n+1}=\mathbf{U}_{i,j}^{n}-\frac{\Delta t}{\Omega_{i,j}}\sum_{r=1}^{4}[\mathbf{F},\mathbf{G}]_r\cdot\mathbf{n}_r\,\Delta L_r+\Delta t\mathbf{S}_{i,j}^{n}. \tag{17}$$

The finite-volume method, as represented by Equation (17), allows the decomposition of a two-dimensional problem into a series of local one-dimensional problems to evaluate normal flux through every side of a cell.

Generally, the most popular finite-volume schemes are *upwind* schemes and *central* schemes. In the former schemes the computational cells are selected according to the propagation of the perturbations while the latter are characterized by a central discretization of the flux vectors through a side of the cell.

Herein, in the analysis presented, firstly the HLL first-order upwind scheme has been implemented and used for integrating the complete and kinematic model. The HLL scheme only considers the left and right wave characteristics as representative of the minimum and the maximum speed of the perturbation. That scheme, applied to the two-dimensional equations, gives the following expression for the numerical flux across the edge of the computational cell Ω_L on the left and Ω_R on the right:

$$[\mathbf{f},\mathbf{g}]_r\cdot\mathbf{n}_r=\begin{cases}[\mathbf{f},\mathbf{g}]_L\cdot\mathbf{n}_r & \text{if } s_L\geq 0\\ \dfrac{s_R([\mathbf{f},\mathbf{g}])_L\cdot\mathbf{n}_r-s_L([\mathbf{f},\mathbf{g}])_R\cdot\mathbf{n}_r+s_Ls_R(\mathbf{U}_R-\mathbf{U}_L)}{s_R-s_L} & \text{if } s_L\leq 0\leq s_R\\ [\mathbf{f},\mathbf{g}]_R\cdot\mathbf{n}_r & \text{if } s_R\leq 0.\end{cases} \tag{18}$$

For the expressions of the wave celerities S_L and S_R one may refer to Toro (2001). In the case of the discretization of the kinematic model, Equation (18) was only applied to the mass conservation equation while the momentum equations, along the two directions x and y, were simply resolved computing the velocities through the kinematic equations using Gauckler–Strickler's formula.

As regards the use of a second-order scheme, it is well known that the upwind schemes present drawbacks similar to those of the central discretization schemes: they generate numerical oscillations around discontinuities. Therefore, through the TVD theory, nonlinear limiters were introduced in second-order upwind schemes in order to prevent these drawbacks. It is interesting to recall that the TVD approach has shed new light on second-order central schemes belonging to the Lax–Wendroff family (Macchione & Morelli 2003). Indeed, thanks to TDV theory, schemes similar to those of Lax–Wendroff with an artificial viscosity term were obtained through the introduction of particular limiters in second-order upwind schemes, with the advantage that the above term can be formulated without calibrating empirical constants case by case. Now it is well known that the MacCormack scheme belongs to the Lax–Wendroff family. In this paper its version with TVD artificial viscosity has been applied to the complete, diffusive and kinematic models.

Several authors have used the MacCormack scheme to simulate the propagation of overland flow processes (see for instance Esteves *et al.* 2000; Fiedler & Ramirez 2000; Gandolfi & Savi 2000; Kazezyilmaz-Alhan & Medina 2007). The numerical integration of the system was performed in the form

$$\mathbf{U}_{i,j}^{p} = \mathbf{U}_{i,j}^{n} - \frac{\Delta t}{\Omega_{i,j}} \sum_{r=1}^{4} [\mathbf{F}, \mathbf{G}]_{r}^{n} \cdot \mathbf{n}_r \, \Delta L_r + \Delta t \mathbf{S}_{i,j}^{n} \tag{19}$$

$$\mathbf{U}_{i,j}^{c} = \mathbf{U}_{i,j}^{n} - \frac{\Delta t}{\Omega_{i,j}} \sum_{r=1}^{4} [\mathbf{F}, \mathbf{G}]_{r}^{p} \cdot \mathbf{n}_r \, \Delta L_r + \Delta t \mathbf{S}_{i,j}^{p} \tag{20}$$

$$\mathbf{U}_{i,j}^{n+1} = \frac{1}{2}\left(\mathbf{U}_{i,j}^{p} + \mathbf{U}_{i,j}^{c}\right) \tag{21}$$

where p and c stand for predictor and corrector values. For each side ($r = 1, \ldots, 4$), $\mathbf{F}_r$ and $\mathbf{G}_r$ are obtained referring to upstream and downstream volumes alternately.

In order to obtain a high resolution extension of MacCormack's scheme, the term $\mathbf{U}_{i,j}^{n+1}$ is corrected according to TVD theory. The added normal flux is expressed as

$$\mathbf{D}_{x_r}^{n} = \frac{1}{2}\sum_{k=1}^{3} \tilde{\alpha}^k \Psi\left(\tilde{a}^k\right)\left[1 - \lambda\left|\tilde{a}^k\right|\right]\left[1 - \phi\left(\rho^k\right)\right]\tilde{\mathbf{e}}^k \tag{22}$$

where $\tilde{\alpha}$ is the characteristic variable; $\tilde{a}$ and $\tilde{\mathbf{e}}$ are the eigenvalues and eigenvectors of approximate Jacobian matrix; λ is equal to $\Delta t/d$, where d is the distance between neighbouring centroids; $\Psi(\tilde{a})$ is the entropy correction to the modulus of $\tilde{a}$, thereby avoiding the appearance of non-physical solutions and $\varphi = \varphi(\rho)$ represents the limiter which allows the TVD condition to be fulfilled. In this work the *minmod* limiter is used (Hirsch 1990).

The MacCormack scheme was applied to the diffusive model (Equation (6)) discretizing the mass conservation equation as in Equations (19)–(21). For the diffusive model the flow equations were considered in the following form:

$$\frac{\partial H}{\partial x} = \frac{u\sqrt{u^2+v^2}}{K_s^2 h^{4/3}}; \quad \frac{\partial H}{\partial y} = \frac{v\sqrt{u^2+v^2}}{K_s^2 h^{4/3}} \tag{23), (24}$$

where $H = z + h$ is water elevation and z is bed elevation. In the kinematic wave model, the momentum equations were reduced to the uniform law equations from which the values of the velocities were computed.

APPLICATIONS

This section is divided into two parts. In the first one the attention is focused on the comparison of the modelling approach while the latter is devoted to the analysis of the numerical schemes performances considered in the paper.

As stated before, one of the main purpose of the paper is to evaluate the effects of the simplifications of the governing equations especially in those situations in which the hydraulic phenomenon is more complicated than that occurring over a plane. Several numerical tests concerning overland flow are available in the literature and some of them were already reported by the authors (Costabile *et al.* 2009) for model validation purposes.

The discussion that follows focuses first on simple cases in which the performances of the models are quite similar, at least for the diffusive and fully dynamic model. These tests (test 1 and 2) were also used for the validation of the implemented numerical codes comparing their results with both analytical solutions or the simulations carried out by other authors. Then the simulation of a more complicated test is presented (test 3).

In the second part of this section, a comparative analysis of the two numerical schemes considered is presented using

both experimental tests and a numerical test with irregular topography.

All the simulations were performed using a structured Cartesian grid.

COMPARISON AMONG OVERLAND FLOW MODELS

Test 1: Time-varying rainfall intensity over a plane

These tests, proposed in Govindaraju *et al.* (1988) and Gottardi & Venutelli (2008), consist in a time variable and constant spatial rainfall intensity over a plane, 22 m long, with constant slope and Chézy coefficient $\chi = 1.336\ \mathrm{m}^{1/2}/\mathrm{s}$. Two different slopes were considered: 0.001 and 0.04. The numerical results, obtained by using the MacCormack scheme, are shown in Figures 1(a) and (b) respectively. In these tests the numerical runoff computed by simplified models are compared with the solutions obtained by the complete models. The computational domain for both tests has been divided in the cells of dimensions 0.1×0.1 m while the Courant number was set to 0.1. It should be born in mind that it is difficult to achieve stable computations in overland flow simulations due to both the very shallow water depth values and the high shear stress values induced by bed roughness. So in these simulations the Courant number value is smaller than that commonly used for flood propagation analysis (see for instance Esteves *et al.* 2000; Gottardi & Venutelli 2008).

It is interesting to observe that when using a slope equal to 0.001 the simulations are quite different and in particular the kinematic approximation provides poor prediction because, in this case, the depth gradient contribution was not negligible in comparison to the bottom slope (Figure 1(a)). Moreover, it should be noted that the kinematic model cannot consider downstream boundary conditions and this fact may represent another important reason of the different results since subcritical flow occur in this test. However the solutions of the models are very similar when using a slope equal to 0.04 (Figure 1(b)). In both cases, the numerical results were in a good agreement with those presented by other authors.

Test 2: Constant rainfall intensity over an ideal basin

In this test (Stephenson & Meadows 1986; Di Giammarco *et al.* 1996) an ideal basin, composed of two constant slope hillsides at whose bottom a constant slope channel is located, was considered. This is one of the few available literature test in which 2D features clearly occur in the pattern flow. A constant rainfall intensity (10.8 mm/h) falls over two planes 800×1000 m, having Manning coefficient $n = 0.015\ \mathrm{s/m}^{1/3}$, transversal slope 0.05 and no longitudinal slope, whose discharges flow into a constant slope (0.02) channel with Manning coefficient $n = 0.15\ \mathrm{s/m}^{1/3}$.

Figure 2 shows the results obtained by the different models compared with the analytical solution in terms of both the outflow discharge coming down each hillside and the discharge at the channel outlet. In both figures the numerical results obtained by the implemented models agreed with the analytical solution. No significant differences appear among the results obtained with the three models. This fact can be explained by the bottom slopes

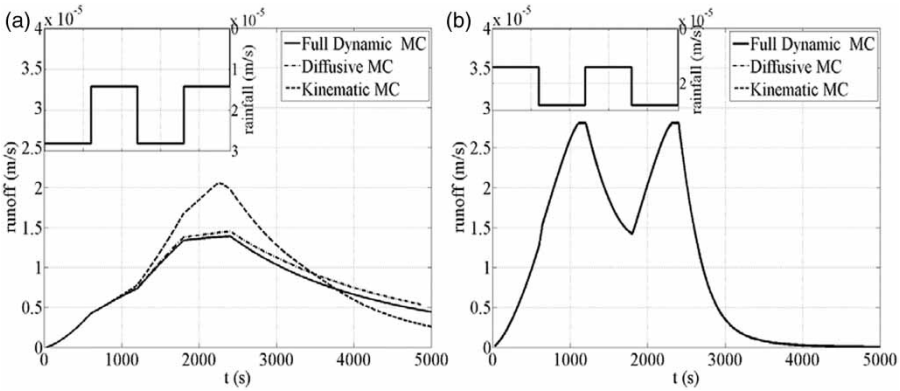

Figure 1 | Test 1: Comparison of the simulated runoff hydrographs at the channel outlet. (a) Slope 0.001, (b) slope 0.04.

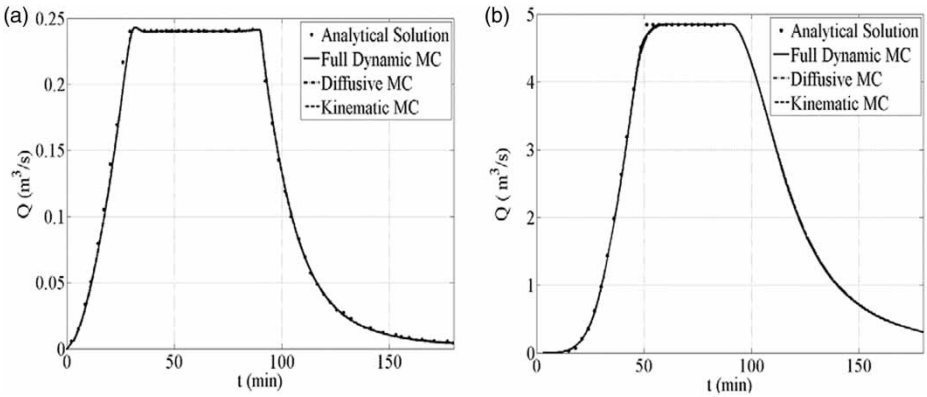

Figure 2 | Test 2: Comparison of numerical and analytical flood wave (a) at the bottom of the hillside, (b) at the channel outlet.

whose values dominated over the other terms in the momentum equations. Moreover it should be noted that a step between the valley sides and the bottom of the channel avoided backwater effects on the valley sides.

Test 3: Space-varying and time-constant rainfall intensity over a plane

From the above results, it seems that no significant differences appear between the complete model and its simplifications, at least for the diffusive approximation. Indeed, they refer to very idealised situations characterized by simple topographies and hydraulic phenomena very far from those occurring during flash floods real events. So there is the need to focus the models comparison on more complex tests such as those carried out by Iwagaki (1955) and used as validation test in Feng & Molz (1997) and Fiedler & Ramirez (2000).

These experiments consist in varying space but leaving constant in time the rainfall intensity over a cascade of three planes. Each plane section was 8 m long, with slopes of 0.02, 0.015 and 0.01 in the downstream directions; each section received a constant rainfall input of 389, 230 and 288 cm h^{-1}, respectively. Discharge and water depth hydrographs are available with reference to three rainfall durations ($t = 10$ s, $t = 20$ s, $t = 30$ s). For each test, the computational domain was obtained using a structured mesh with a cell size equal to 0.1 m; the Manning coefficient was set equal to 0.01 s/$m^{1/3}$. In Figures 3 and 4, a comparison of the numerical results and the experimental data, relative to the shortest and to the longest rainfall duration, is shown. In particular, for each test, the water depth profiles refer to the time instant in which the rain ends (30 s, 10 s). Numerical results are in a quite good agreement with the experimental data. In particular, as shown in Figure 3(a), all the numerical hydrographs gave a good prediction of

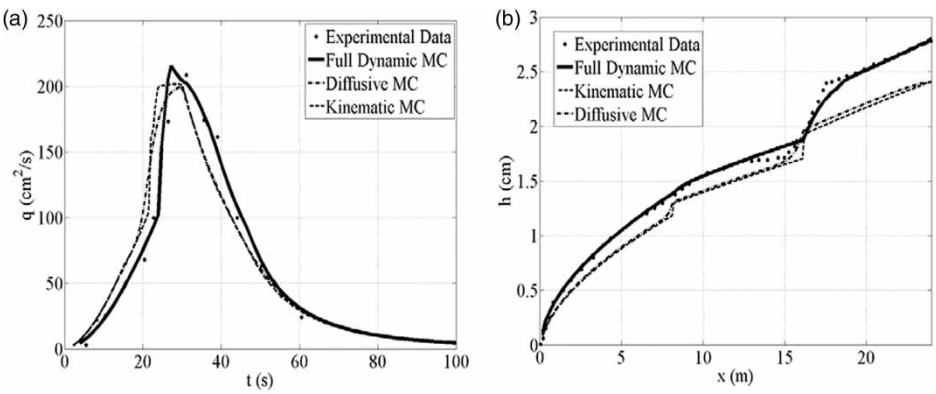

Figure 3 | Test 3: Comparison of experimental data and numerical results (rain duration $t = 30$ s): (a) flood wave at the channel outlet, (b) longitudinal water depths profile at the end of the rainfall input.

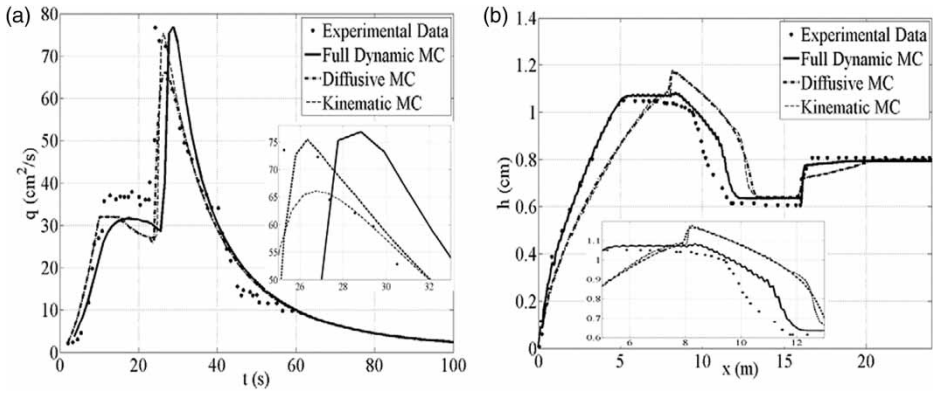

Figure 4 | Test 3: Comparison of experimental data and numerical results (rain duration $t = 10$ s): (a) flood wave at the channel outlet, (b) longitudinal water depths profile at the end of the rainfall input.

the peak value. The fully dynamic wave model provides the better overall solution with reference to both the rising and recession limbs of the hydrograph and to the water depth profile at the end of the rainfall duration (Figure 3(b)). In particular, the water depth values predicted by the simplified models underestimated the experimental data especially in the last plane. The most difficult simulation refers to the situation in which a rainfall duration equal to 10 s occurs. In this experiment a shock wave, which arrives at the downstream end at approximately 25 s, is produced (Fiedler & Ramirez 2000).

For this test, the numerical simulations gave different predictions of the flood wave at the end of the last plane. In particular, the fully dynamic and kinematic model make good predictions of the observed peak discharge value while the diffusive model provides a significant underestimation (Figure 4(a)).

The prediction of the water depth profiles provided by the simplified models is poor. In the first plane, a systematic underestimation of the water level is simulated. Moreover, the numerical results give a sudden rise of the water level, not observed in the experiment, at the beginning of the second plane along which the water depth is clearly overestimated. The hydraulic jump, that occurs at the beginning of the third plane, cannot be simulated by the simplified models due to the absence of the convective inertial terms. The inertial terms are very important in this test due to the impulsive behaviour of the flood wave.

As regards the prediction of water level profile and discharge hydrograph, Figure 4 highlights a very good agreement between model results and experimental data.

Another aspect related to the consequences associated with the use of simplified models may be represented by the influence of the computational cell size on the results. On the other hand, the study of overland flow processes in a real situation involves the analysis of the phenomenon in large areas. As a consequence, in order to avoid a significant increase in term of both computational time and memory storage, the computational domain may be obtained using very coarse cells. Therefore an analysis of the accuracy of the numerical solutions in relationship to the size of computational cell was performed.

In Figure 5 the comparisons of the discharge hydrographs obtained using the fully dynamic model with different cell sizes ($\Delta x = 0.1$ m, $\Delta x = 0.5$m, $\Delta x = 1$ m) are shown. It is interesting to observe that, as the phenomenon becomes more impulsive (Figure 5(b)), the increase in cell size induces poorer results. This behaviour is confirmed by the diffusive and kinematic models as well (Figure 6). Figure 6(b) highlights that the peak discharge value reduction was similar to that of the complete model, while in the Figure 6(a) it is possible to observe that, in this case, the diffusive model is more sensible to the mesh size variation leading to a very poor prediction of the peak value. In particular, it may be noted that the discharge peak value obtained using the diffusive scheme with $\Delta x = 0.1$ m is equal to the corresponding value obtained using the fully dynamic model with $\Delta x = 1$ m.

COMPARISON BETWEEN NUMERICAL SCHEMES

The analysis of the experimental tests of Iwagaki (1955) suggests that both the modelling approach and the

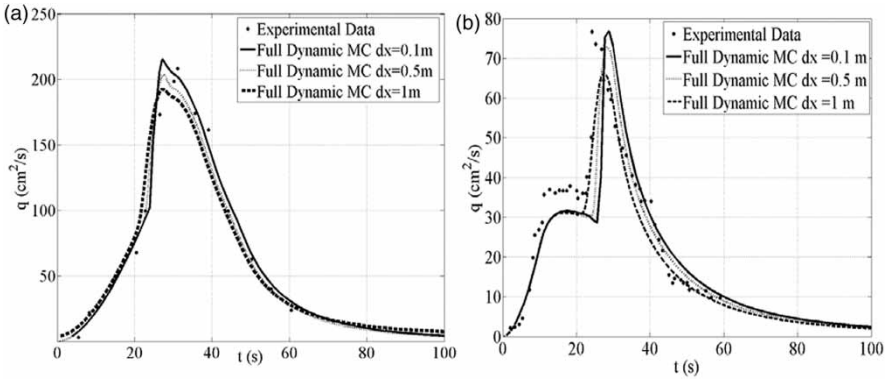

Figure 5 | Test 3: Flood wave at the channel outlet, influence of mesh size on the computed hydrographs using the complete MacCormack scheme: rain duration (a) $t = 30$ s, (b) $t = 10$ s.

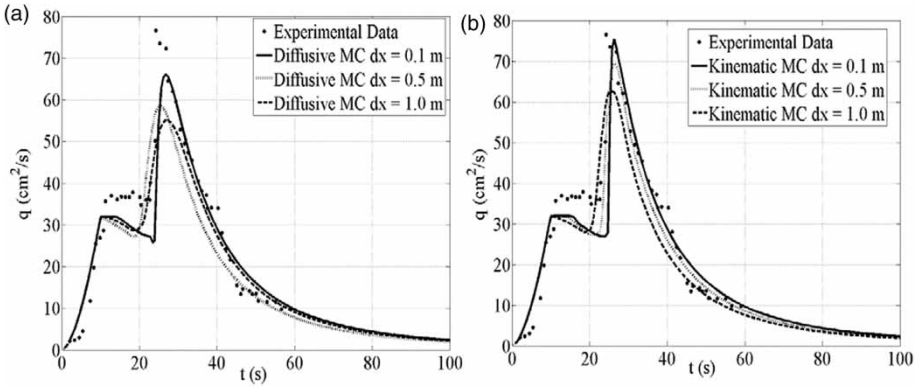

Figure 6 | Test 3: Flood wave at the channel outlet (rain duration $t = 10$ s), influence of mesh size on the computed hydrographs using the MacCormack scheme: (a) diffusive, (b) kinematic approximations.

computational grid have to be chosen carefully especially in the simulation of impulsive hydraulic phenomena on irregular topography. In these situations, the use of simplified models may prevent a suitable description of the flow behaviour and the fully dynamic modelling is thus recommended.

In this section some practical aspects related to the use of the fully unsteady 2D overland flow modelling in real topographies (such as the choice of the most suitable numerical model able to provide reliable results in short computational times and the influence of the grid size on the results) are analysed.

In particular, the performances of the implemented numerical schemes, the second-order central TVD-MacCormak's scheme and the first-order upwind HLL scheme were investigated simulating both the above-discussed tests and an overland flow on an irregular topography.

Analysis of literature tests

With reference to test 1, the HLL scheme shows a small diffusion with a decrease of the outflow discharge using a slope equal to 0.001 due to the first order of accuracy (Figure 7(a)) while the results are similar to those obtained with the second-order MacCormack's scheme when the plane's slope is equal to 0.04. A small increase of the outflow discharge is obtained, using HLL scheme, at the channel outlet in test 2 (Figure 7(b)).

The influence of the cell size on the numerical results obtained using the HLL for the simulation of the test 3 was also performed (Figure 8). The comparison between Figures 5 and 8 highlights that an increase of the cell size in the MacCormack scheme did not excessively alter the accuracy of the solution (Figure 5), while the results obtained using the HLL scheme were quite sensitive to the

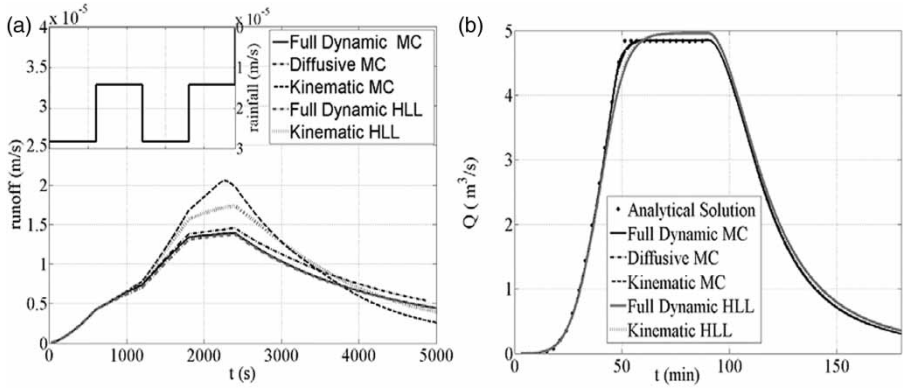

Figure 7 | Comparison of the simulated runoff hydrographs using MacCormack's scheme and the HLL scheme: (a) Test 1, (b) Test 2.

cell size and, in particular, become less accurate as the cell size increased (Figure 8). This is clearly due to the fact that the MacCormack's scheme is a second-order accurate scheme, while the HLL scheme is a first-order accurate scheme. However the difference between the two schemes significantly reduces as cell size becomes lower. At the same time, the computational time associated with the use of HLL scheme is very much lower than that of MacCormack (up to 30% for the simulations considered here). So the HLL scheme may be very useful when using high-resolution meshes.

Simulation of the surface runoff over an irregular topography

The applications of the aforementioned tests refer to ideal situations in which the topography is dramatically simplified. Though it is important to check the performances of overland flow models in those situations, at least for validation purposes, a more reasonable evaluation of the suitability of an overland flow model should be performed by analysing the results deriving from their application in real topographies. This aspect is quite often neglected in the literature. Indeed a number of numerical problems exist in the use of the 2D unsteady flow modelling for the propagation of a surface runoff in complex topography. However, they are beyond the scope of the paper and thus only the question relating to the cell size influence on the numerical results is addressed here.

This application regarded the propagation of the surface runoff due to a rainfall intensity which is constant in time and space (100 mm/h and 10 mm/h) over an irregular topography. The domain is 950 m $\times$ 1100 m. Figure 9 shows the surface elevation of the basin. The domain was subdivided according to a structured grid with different cell sizes (5, 20, and 40 m). Strickler's coefficient was assumed constant

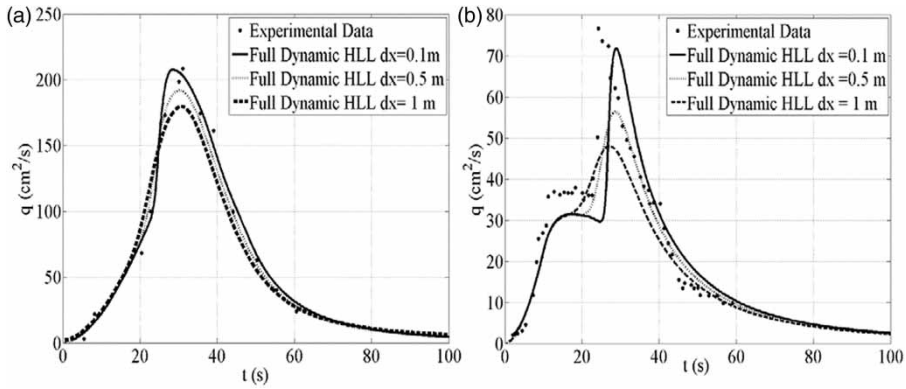

Figure 8 | Test 3: Flood wave at the channel outlet, influence of mesh size on the computed hydrographs using the HLL scheme: rain duration (a) $t = 30$ s, (b) $t = 10$ s.

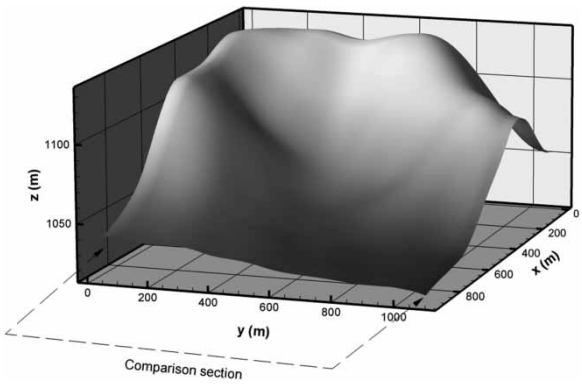

Figure 9 | Surface elevation of the basin.

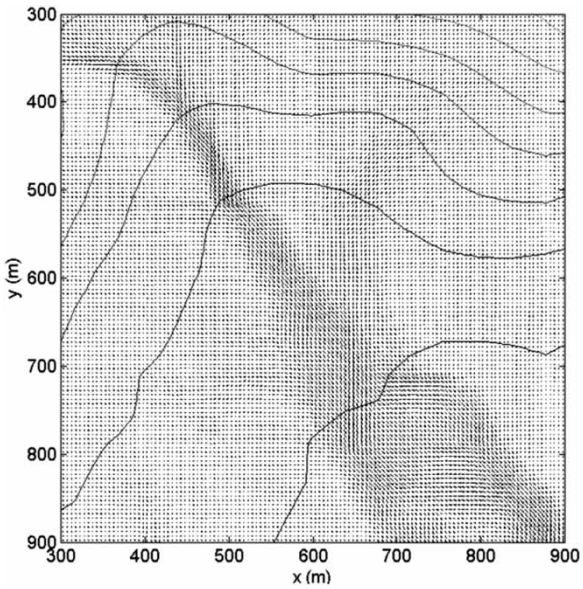

Figure 10 | Flow path at $t = 45$ min.

in all the domain (8 $m^{1/3}/s$) and the infiltration rate was set to zero. In Figure 10, for example, flow vectors at time 45 min, for a 100 mm/h rainfall intensity, are depicted.

In Figure 11, the mesh size influence on the simulated discharge hydrographs, relative to a 100 mm/h rainfall intensity, at the domain outlet is shown. In particular, the flood wave computed by the MacCormack and the HLL schemes are depicted respectively in Figure 11(a) and (b). For both schemes, it is possible to observe that the mesh size mainly influenced the peak discharge while less variation may be noted in the time to peak values.

An analysis of Figure 11 highlights that the mesh size influence was quite limited when using the MacCormack scheme while it became more significant within the HLL model simulations. The maximum difference in terms of the peak discharge values, using the MacCormack scheme, was less than 5%, while for the HLL scheme it increases to 20%. These results are not surprising since the MacCormack scheme is of second order of accuracy in both time and space while HLL is a first-order scheme. However the differences, as expected, seemed to significantly decrease as the mesh size decreased.

A similar analysis was performed to simulate the surface runoff due to a 10 mm/h rainfall intensity. The simulation of this situation was the most difficult due to the presence, for the entire time period, of shallow water depths that induce numerical instabilities. More in general, it is well known in the literature that small depths over complex topography and wet–dry interfaces may lead to several numerical problems. In overland flow simulations these problems clearly are amplified by the presence of a great number of computational dry cells that become wet because of the rainfall input and subsequently dry out due to high bed slopes. Therefore a robust wet–dry procedure were implemented. For further details one may refer to Costabile *et al.* (2010).

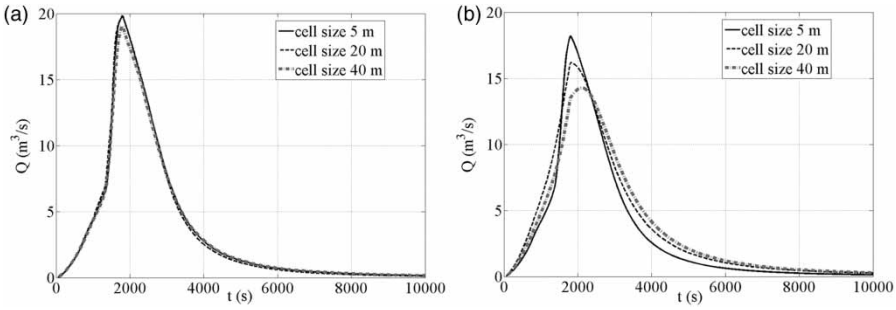

Figure 11 | Discharge hydrographs at the basin outlet computed using different mesh size with 100 mm/h rainfall intensity: (a) MacCormack, (b) HLL results.

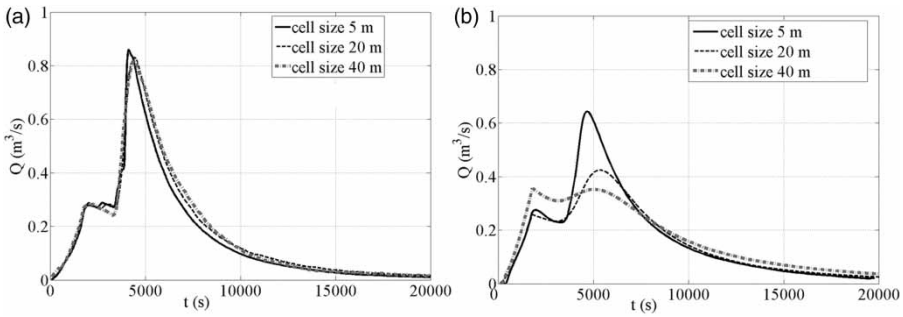

Figure 12 | Discharge hydrographs at the basin outlet computed using different mesh size with 10 mm/h rainfall intensity: (a) MacCormack, (b) HLL results.

The discharge hydrographs obtained are shown in Figure 12. In Figure 12(a) it is possible to observe that the MacCormack results still continued to be similar except for slight differences observed during the rising and recession limbs of the hydrographs. Different conclusions came from the analysis of Figure 12(b) in which one may observe the numerical diffusion induced by the HLL scheme as the mesh size increases. In this case the differences in the two schemes, though decreasing as the grid size decreases, still continued to be significant, up to 20%, despite using a cell size of 5 m.

In all simulations the mass conservation property is reasonably ensured.

CONCLUSIONS

A comparative analysis of different overland flow models based on the shallow water equations and relative approximations are presented in this paper.

Several numerical and experimental tests were used in order to highlight those situations in which the use of a simplified modelling can induce poor predictions respect to a more detailed approach. Numerical simulations showed that the models performances are similar in very simplified tests where the topography is reduced to a plane surface. In particular, the results obtained using the diffusive and the fully dynamic models are in a good agreement in every case, while the kinematic model shows significant overestimation of the peak discharges values when a milder slope was used. So the analysis of the above-mentioned tests seems to suggest that, for overland flow simulations, the use of the diffusive model is completely justified and it provides a very good approximation of the fully dynamic model. It should be borne in mind that they refer to very idealised situations characterized by simple topographies and hydraulic phenomena very far from those occurring during flash floods real events. Indeed, the results coming from the numerical simulation of the experimental test (test 3), in which the generation of a shock wave occurs, lead to mitigate that conclusion. In particular, the diffusive wave model produced a clear underestimation of the flood peak at the outlet of the last plane; this behaviour was not observed in the kinematic model which gave results similar to those obtained with the fully dynamic approach. It is important to observe that the simplified models gave poor results in terms of water depth profiles. This test suggests that the use of simplified models in situations characterized by impulsive phenomena over complex topographies may lead to important errors.

From a numeric point of view, the overall results obtained by using the MacCormack and the HLL scheme are quite good even if the last scheme showed a little diffusion in the tests. No problems of numerical instability were observed despite the small values of the simulated water depths. The numerical results also showed that an increase of cell size causes more important negative effects on the HLL scheme than in the MacCormack scheme; this result was expected since the MacCormack scheme has a second order of accuracy in both time and space. However, in those situations in which high resolution grid should be used, the HLL scheme may be very useful since it may give numerical results more similar to those of high-order schemes in shorter computational times. The latter consideration was confirmed by the analysis of two numerical tests on an irregular topography. The numerical

results showed that the mesh size influence on the MacCormack scheme is quite limited, while it may be significant when using the HLL scheme. As the grid size decreases, the difference between the two schemes seems to decrease, at least for high rainfall intensity situations.

ACKNOWLEDGEMENTS

This work was supported by CMCC – Centro Euro Mediterraneo per i Cambiamenti Climatici (Euro – Mediterranean Centre for Climate Change) in the funding project 'Modellistica idraulica delle alluvioni conseguenti ad eventi meteorologici intensi' (Numerical modelling of floods due to heavy rainfalls).

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First received 27 May 2010; accepted in revised form 2 November 2010. Available online 12 April 2011