Comparative Effectiveness of Matching Methods for Causal Inference

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ioint work with

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Talk at Quantitative Issues in Cancer Research Working Seminar, Biostatistics, HSPH, 10/18/10

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- Problem: Many matching methods & specifications
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- Problem: The most commonly used method can increase imbalance!
- Solution: Other methods do not share this problem
- → Lots of insights revealed in the process

Replication: Doyle and Sambanis, APSR 2000

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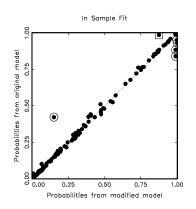
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- Data analysis: Logit model
- The question: How model dependent are the results?

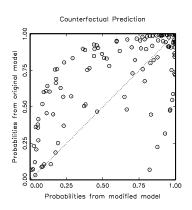
Two Logit Models, Apparently Similar Results

	Original "Interactive" Model			Modified Model		
Variables	Coeff	SE	P-val	Coeff	SE	P-val
Wartype	-1.742	.609	.004	-1.666	.606	.006
Logdead	445	.126	.000	437	.125	.000
Wardur	.006	.006	.258	.006	.006	.342
Factnum	-1.259	.703	.073	-1.045	.899	.245
Factnum2	.062	.065	.346	.032	.104	.756
Trnsfcap	.004	.002	.010	.004	.002	.017
Develop	.001	.000	.065	.001	.000	.068
Exp	-6.016	3.071	.050	-6.215	3.065	.043
Decade	299	.169	.077	-0.284	.169	.093
Treaty	2.124	.821	.010	2.126	.802	.008
UNOP4	3.135	1.091	.004	.262	1.392	.851
Wardur*UNOP4	<u> </u>	_	_	.037	.011	.001
Constant	8.609	2.157	0.000	7.978	2.350	.000
N		122			122	

Log-likelihood -45.649 -44.902 Pseudo R^2 .423 .433

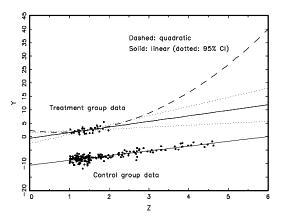
Doyle and Sambanis: Model Dependence



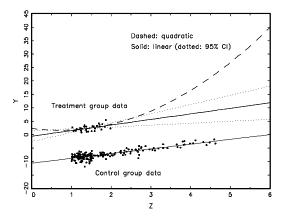


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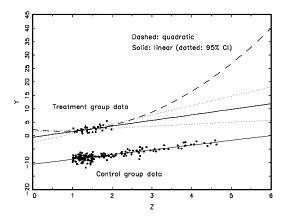


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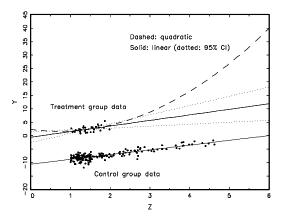
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Preprocess I: Eliminate extrapolation region

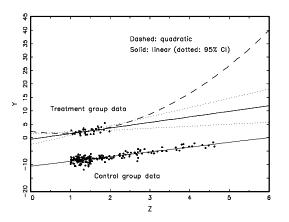
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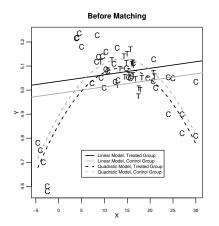


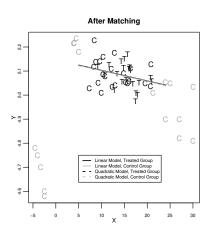
What to do?

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- Preprocess II: Match (prune bad matches) within interpolation region
- Model remaining imbalance

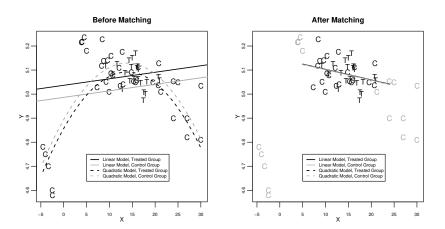
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Matching reduces model dependence, bias, and variance

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 - Can apply other matching methods within CEM strata (inherit CEM's properties)

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The Bias-Variance Trade Off in Matching

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 - Classic measure: Difference of means (for each variable)
 - Better measure (difference of multivariate histograms):

$$\mathcal{L}_1(f,g;H) = rac{1}{2} \sum_{\ell_1 \cdots \ell_k \in H(\mathbf{X})} |f_{\ell_1 \cdots \ell_k} - g_{\ell_1 \cdots \ell_k}|$$

• MDM & PSM: Choose matched n, match, check imbalance

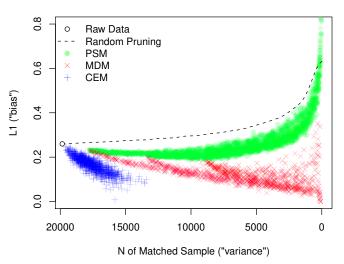
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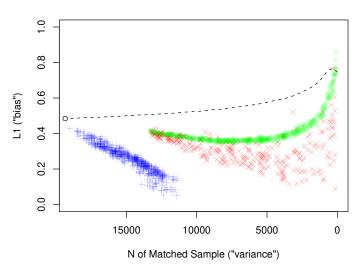
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- Our idea: Compute lots of matching solutions, identify the frontier of lowest imbalance for each given n, and choose a matching solution

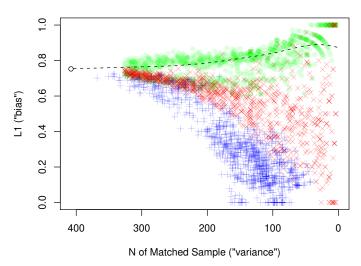
Healthways Data



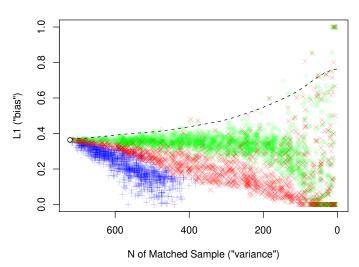
Called/Not Called Data



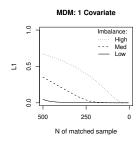
FDA Data

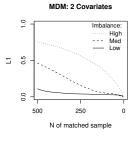


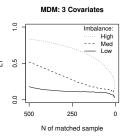
Lalonde Data Subset



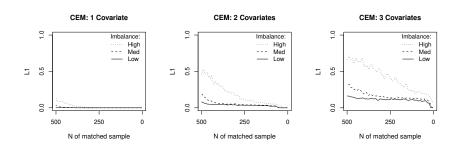
A Space Graph: Simulated Data — Mahalanobis



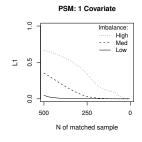


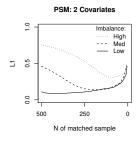


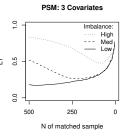
A Space Graph: Simulated Data — CEM



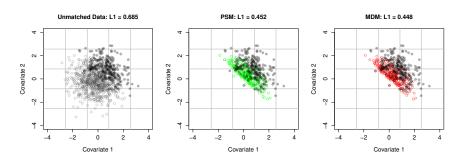
A Space Graph: Simulated Data — Propensity Score



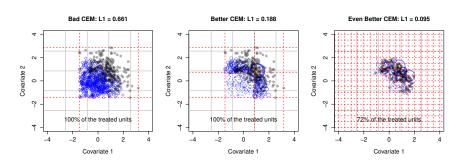




Data where PSM Works Reasonably Well — PSM & MDM



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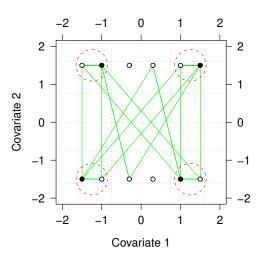
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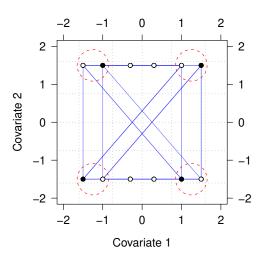
- Gives a better pscore than PSM
- Doesn't match based on crippled information

PSM Approximates Random Matching in Balanced Data



PSM MatchesCEM and MDM Matches

Destroying CEM with PSM's Two Step Approach



CEM MatchesCEM-generated PSM Matches

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- In four data sets and many simulations:
 - CEM > Mahalanobis > Propensity Score

- Propensity score matching:
 - The problem:
 - Imbalance can be worse than original data
 - Can increase imbalance when removing the worst matches
 - Approximates random matching in well-balanced data (Random matching increases imbalance)
 - The Cause: unnecessary 1st stage dimension reduction
 - Implications:
 - Balance checking required
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- You can easily check with the Space Graph

For papers, software (for R and Stata), tutorials, etc.

http://GKing.Harvard.edu/cem