International Journal of Industrial Engineering Computations 4 (2013) 29–50

Contents lists available at GrowingScience

International Journal of Industrial Engineering Computations

homepage: www.GrowingScience.com/ijiec

Comparative performance of an elitist teaching-learning-based optimization algorithm for solving unconstrained optimization problems

R. Venkata Rao* and **Vivek Patel**

Department of Mechanical Engineering, S.V. National Institute of Technology, Ichchanath, Surat, Gujarat – 395 007, India

ARTICLE INFO

Article history:
Received 11 July 2012
Received in revised format
14 August 2012
Accepted August 31 2012
Available online
1 September 2012
Keywords:
Teaching-learning-based optimization; Elitism
Population size
Number of generations
Unconstrained optimization problems

ABSTRACT

Teaching-Learning-based optimization (TLBO) is a recently proposed population based algorithm, which simulates the teaching-learning process of the class room. This algorithm requires only the common control parameters and does not require any algorithm-specific control parameters. In this paper, the effect of elitism on the performance of the TLBO algorithm is investigated while solving unconstrained benchmark problems. The effects of common control parameters such as the population size and the number of generations on the performance of the algorithm are also investigated. The proposed algorithm is tested on 76 unconstrained benchmark functions with different characteristics and the performance of the algorithm is compared with that of other well known optimization algorithms. A statistical test is also performed to investigate the results obtained using different algorithms. The results have proved the effectiveness of the proposed elitist TLBO algorithm.

1. Introduction

Some of the recognized evolutionary algorithms are, Genetic Algorithms (GA), Differential Evolution (DE), Evolution Strategy (ES), Evolution Programming (EP), Artificial Immune Algorithm (AIA), Bacteria Foraging Optimization (BFO), etc. Among all, GA is a widely used algorithm for various applications. GA works on the principle of the Darwinian theory of the survival of the fittest and the theory of evolution of the living beings (Holland 1975). DE is similar to GA with specialized crossover and selection method (Storn & Price 1997, Price et al. 2005). ES is based on the hypothesis that during the biological evolution the laws of heredity have been developed for fastest phylogenetic adaptation (Runarsson & Yao, 2000). ES imitates, in contrast to the GA, the effects of genetic procedures on the phenotype. EP also simulates the phenomenon of natural evolution at phenotype level (Fogel *et al*. 1966). AIA works on the immune system of the human being (Farmer 1986). BFO is inspired by the social foraging behavior of Escherichia coli (Passino, 2002). Some of the well known swarm intelligence based algorithms are, Particle Swarm Optimization (PSO) which works on the principle of foraging behavior of the swarm of birds (Kennedy & Eberhart 1995); Ant Colony Optimization (ACO) which works on the principle of foraging behavior of the ant for the food (Dorigo et al. 1991); Shuffled

* Corresponding author. Tel: 91-9925207027
E-mail: ravipudirao@gmail.com (R. V. Rao)

doi: 10.5267/j.ijiec.2012.09.001

Frog Leaping (SFL) algorithm which works on the principle of communication among the frogs (Eusuff & Lansey, 2003); Artificial Bee Colony (ABC) algorithm which works on the principle of foraging behavior of a honey bee (Karaboga, 2005; Basturk & Karaboga 2006; Karboga & Basturk, 2007-2008; Karaboga & Akay 2009).

There are some other algorithms which work on the principles of different natural phenomena. Some of them are: Harmony Search (HS) algorithm which works on the principle of music improvisation in a music player (Geem et al. 2001); Gravitational Search Algorithm (GSA) which works on the principle of gravitational force acting between the bodies (Rashedi et al. 2009); Biogeography-Based Optimization (BBO) which works on the principle of immigration and emigration of the species from one place to the other (Simon, 2008); Grenade Explosion Method (GEM) which works on the principle of explosion of grenade (Ahrari & Atai, 2010); and League championship algorithm which mimics the sporting competition in a sport league (Kashan, 2011).

All the evolutionary and swarm intelligence based algorithms are probabilistic algorithms and required common controlling parameters like population size and number of generations. Beside the common control parameters, different algorithm requires its own algorithm specific control parameters. For example GA uses mutation rate and crossover rate. Similarly PSO uses inertia weight, social and cognitive parameters. The proper tuning of the algorithm specific parameters is very crucial factor which affect the performance of the above mentioned algorithms. The improper tuning of algorithm-specific parameters either increases the computational effort or yields the local optimal solution. Considering this fact, recently Rao et al. (2011, 2012a, 2012b) and Rao and Patel (2012a) introduced the Teaching-Learning-Based Optimization (TLBO) algorithm which does not require any algorithm-specific parameters. TLBO requires only common controlling parameters like population size and number of generations for its working. Thus, TLBO can be said as an algorithm-specific parameter-less algorithm.

The concept of elitism is utilized in most of the evolutionary and swarm intelligence algorithms where during every generation the worst solutions are replaced by the elite solutions. The number of worst solutions replaced by the elite solutions is depended on the size of elite. Rao and Patel (2012a) described the elitism concept while solving the constrained benchmark problems. The same methodology is extended in the present work and the performance of TLBO algorithm is investigated considering a number of unconstrained benchmark problems. The details of TLBO algorithm along with its computer program are available in Rao and Patel (2012a) and hence those details are not repeated in this paper.

2. Elitist TLBO algorithm

In the TLBO algorithm, after replacing worst solutions with elite solutions at the end of learner phase, if the duplicate solutions exist then it is necessary to modify the duplicate solutions in order to avoid trapping in the local optima. In the present work duplicate solutions are modified by mutation on randomly selected dimensions of the duplicate solutions before executing the next generation as was done in Rao and Patel (2012a). In the TLBO algorithm, the solution is updated in the teacher phase as well as in the learner phase. Also, in the duplicate elimination step, if duplicate solutions are present then they are randomly modified. So the total number of function evaluations in the TLBO algorithm is = {(2 × population size × number of generations) + (function evaluations required for duplicate elimination)}. In the entire experimental work of this paper, the above formula is used to count the number of function evaluations while conducting experiments with TLBO algorithm. Since the function evaluations required for duplication removal are not clearly known, experiments are conducted with different population sizes and based on these experiments it is reasonably concluded that the function evaluations required for the duplication removal are 7500, 15000, 22500 and 30000 for population sizes of 25, 50, 75 and 100 respectively when the maximum function evaluations of the algorithm is 500000. The next section deals with the experimentation of the elitist TLBO algorithm on various unconstrained benchmark functions.

3. Experiments on unconstrained benchmark functions

The considered unconstrained benchmark functions have different characteristics like unimodality/multimodality, separability/non-separability and regularity/non-regularity. In this section three different experiments are conducted to identify the performance of TLBO and compare the performance of TLBO algorithm with other evolutionary and swarm intelligence based algorithms. A common platform is provided by maintaining the identical function evolution for different algorithms considered for the comparison. Thus, the consistency in the comparison is maintained while comparing the performance of TLBO with other optimization algorithms. However, in general, the algorithm which requires less number of function evaluations to get the same best solution can be considered as better as compared to the other algorithms. If an algorithm gives global optimum solution within certain number of function evaluations, then consideration of more number of function evaluations will go on giving the same best result. Rao et al. (2011, 2012a) showed that TLBO requires less number of function evaluations as compared to the other optimization algorithms. However, in this paper, to maintain the consistency in comparison, the number of function evaluations of 500000, 100000 and 5000 is maintained for experiments 1, 2 and 3 respectively for all optimization algorithms including TLBO algorithm.

3.1. Experiment 1

In the first experiment, the TLBO algorithm is implemented on 50 unconstrained benchmark functions taken from the previous work of Karaboga and Akay (2009). The details of the benchmark functions considered in this experiment are shown in Table 1.

Table 1
Benchmark functions considered in experiment 1, D: Dimension, C: Characteristic, U: Unimodal, M: Multimodal, S: Separable, N: Non-separable

No.	Function	Formulation	D	Search range	C
1	Stepint	$F_{\min} = 25 + \sum_{i=1}^{D} \lvert x_i \rvert$	5	[-5.12, 5.12]	US
2	Step	$F_{\min} = \sum_{i=1}^{D} \left[\lvert x_i + 0.5 \rvert \right]^2$	30	[-100, 100]	US
3	Sphere	$F_{\min} = \sum_{i=1}^{D} x_i^2$	30	[-100, 100]	US
4	SumSquares	$F_{\min} = \sum_{i=1}^{D} i x_i^2$	30	[-10, 10]	US
5	Quartic	$F_{\min} = \sum_{i=1}^{D} i x_i^4 + rand\,(0,1)$	30	[-1.28, 1.28]	US
6	Beale	$F_{\min} = \sum_{i=1}^{D} (1.5 - x_1 + x_1 x_2)^2 + \left(2.25 - x_1 + x_1 x_2^2\right)^2 + \left(2.625 - x_1 + x_1 x_2^3\right)^2$	5	[-4.5, 4.5]	UN
7	Easom	$F_{\min} = -\cos(x_1)\cos(x_2)\exp\left(-(x_1 - \pi)^2 - (x_2 - \pi)^2\right)$	2	[-100, 100]	UN
8	Matyas	$F_{\min} = 0.26\left(x_1^2 + x_2^2\right) - 0.48 x_1 x_2$	2	[-10, 10]	UN
9	Colville	$F_{\min} = 100\left(x_1^2 - x_2\right)^2 + (x_1 - 1)^2 + (x_3 - 1)^2 + 90\left(x_3^2 - x_4\right) +$ $10.1\left((x_2 - 1)^2 + (x_4 - 1)^2\right) - 0.48 x_1 x_2 + 19.8(x_2 - 1)(x_4 - 1)$	4	[-10, 10]	UN
10	Trid 6	$F_{\min} = \sum_{i=1}^{D} (x_i - 1)^2 - \sum_{i=2}^{D} x_i x_{i-1}$	6	[-D^2, D^2]	UN
11	Trid 10	$F_{\min} = \sum_{i=1}^{D} (x_i - 1)^2 - \sum_{i=2}^{D} x_i x_{i-1}$	10	[-D^2, D^2]	UN

Table 2
Benchmark functions considered in experiment 1, D: Dimension, C: Characteristic, U: Unimodal, M: Multimodal, S: Separable, N: Non-separable (Cont.)

12	Zakharov	$F_{\min}=\sum_{i=1}^{D}x_i^2+\left(\sum_{i=1}^{D}0.5ix_i\right)^2+\left(\sum_{i=1}^{D}0.5ix_i\right)^4$	10	[-5, 10]	UN
13	Powell	$F_{\min}=\sum_{i=1}^{D/4}\left(x_{4i-3}+10x_{4i-2}\right)^2+5\left(x_{4i-1}-x_{4i}\right)^2+\left(x_{4i-2}+10x_{4i-1}\right)^4+10\left(x_{4i-3}-x_{4i}\right)^4$	24	[-4, 5]	UN
14	Schwefel 2.22	$F_{\min}=\sum_{i=1}^{D}\lvert x_i\rvert+\prod_{i=1}^{D}\lvert x_i\rvert$	30	[-10, 10]	UN
15	Schwefel 1.2	$F_{\min}=\sum_{i=1}^{D}\left(\sum_{j=1}^{i}x_j^2\right)^2$	30	[-100, 100]	UN
16	Rosenbrock	$F_{\min}=\sum_{i=1}^{D}[100(x_i^2-x_{i+1})^2+(1-x_i)^2]$	30	[-30, 30]	UN
17	Dixon-Price	$F_{\min}=\left(x_1-1\right)^2+\sum_{i=2}^{D}i\left(2x_i^2-x_{i-1}\right)^2$	30	[-10, 10]	UN
18	Foxholes	$F_{\min}=\left[\frac{1}{500}+\sum_{j=1}^{25}\frac{1}{j+\sum_{i=1}^{2}\left(x_i-a_{ij}\right)^6}\right]^{-1}$	2	[-65.536, 65.536]	MS
19	Branin	$F_{\min}=\left(x_2-\frac{5.1}{4\pi^2}x_1^2+\frac{5}{\pi}x_1-6\right)^2+10\left(1-\frac{1}{8\pi}\right)\cos x_1+10$	2	[-5, 10] [0, 15]	MS
20	Bohachevsky 1	$F_{\min}=x_1^2+2x_2^2-0.3\cos\left(3\pi x_1\right)-0.4\cos\left(4\pi x_2\right)+0.7$	2	[-100, 100]	MS
21	Booth	$F_{\min}=\left(x_1-2x_2-7\right)^2+\left(2x_1+x_2-5\right)^2$	2	[-10, 10]	MS
22	Rastrigin	$F_{\min}=\sum_{i=1}^{D}\left[x_i^2-10\cos(2\pi x_i)+10\right]$	30	[-5.12, 5.12]	MS
23	Schwefel	$F_{\min}=-\sum_{i=1}^{D}\left(x_i\sin\left(\sqrt{\lvert x_i\rvert}\right)\right)$	30	[-500, 500]	MS
24	Michalewicz 2	$F_{\min}=-\sum_{i=1}^{D}\sin x_1\left(\sin\left(\frac{ix_i^2}{\pi}\right)\right)^{20}$	2	[0, π]	MS
25	Michalewicz 5	$F_{\min}=-\sum_{i=1}^{D}\sin x_1\left(\sin\left(\frac{ix_i^2}{\pi}\right)\right)^{20}$	5	[0, π]	MS
26	Michalewicz 10	$F_{\min}=-\sum_{i=1}^{D}\sin x_1\left(\sin\left(\frac{ix_i^2}{\pi}\right)\right)^{20}$	10	[0, π]	MS
27	Schaffer	$F_{\min}=0.5+\frac{\sin^2\left(\sqrt{x_1^2+x_2^2}\right)-0.5}{\left(1+0.001\left(x_1^2+x_2^2\right)\right)^2}$	2	[-100, 100]	MN
28	6 Hump CamelBack	$F_{\min}=4x_1^2-2.1x_1^4+\frac{1}{3}x_1^6+x_1x_2-4x_2^2+4x_2^4$	2	[-5, 5]	MN
29	Bohachevsky 2	$F_{\min}=x_1^2+2x_2^2-0.3\cos\left(3\pi x_1\right)\left(4\pi x_2\right)+0.3$	2	[-100, 100]	MN
30	Bohachevsky 3	$F_{\min}=x_1^2+2x_2^2-0.3\cos\left(3\pi x_1+4\pi x_2\right)+0.3$	2	[-100, 100]	MN
31	Shubert	$F_{\min}=\left(\sum_{i=1}^{5}i\cos\left(\left(i+1\right)x_1+i\right)\right)\left(\sum_{i=1}^{5}i\cos\left(\left(i+1\right)x_2+i\right)\right)$	2	[-10, 10]	MN
32	GoldStein-Price	$F_{\min}=\left[1+\left(x_1+x_2+1\right)^2\left(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2\right)\right]\left[30+\left(2x_1-3x_2\right)^2\left(18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2\right)\right]$	2	[-2, 2]	MN
33	Kowalik	$F_{\min}=\sum_{i=1}^{11}\left(a_i-\frac{x_1\left(b_i^2+b_ix_2\right)}{b_i^2+b_ix_3+x_4}\right)^2$	4	[-5, 5]	MN
34	Shekel 5	$F_{\min}=-\sum_{i=1}^{5}\left[\left(x-a_i\right)\left(x-a_i\right)^T+c_i\right]^{-1}$	4	[0, 10]	MN
35	Shekel 7	$F_{\min}=-\sum_{i=1}^{7}\left[\left(x-a_i\right)\left(x-a_i\right)^T+c_i\right]^{-1}$	4	[0, 10]	MN

Table 2
Benchmark functions considered in experiment 1, D: Dimension, C: Characteristic, U: Unimodal, M: Multimodal, S: Separable, N: Non-separable (Cont.)

36	Shekel 10	$F_{\min} = -\sum_{i=1}^{10}\left[(x-a_i)(x-a_i)^T + c_i\right]^{-1}$	4	[0, 10]	MN
37	Perm	$F_{\min} = \sum_{k=1}^{D}\left[\sum_{i=1}^{D}(i^k+\beta)\left(\left(x_i/i\right)^k - 1\right)\right]^2$	4	[-D, D]	MN
38	PowerSum	$F_{\min} = \sum_{k=1}^{D}\left[\left(\sum_{i=1}^{D} x_i^k\right) - b_k\right]^2$	4	[0, D]	MN
39	Hartman 3	$F_{\min} = -\sum_{i=1}^{4} c_i \exp\left[-\sum_{j=1}^{3} a_{ij}\left(x_j - p_{ij}\right)^2\right]$	3	[0, 1]	MN
40	Hartman 6	$F_{\min} = -\sum_{i=1}^{4} c_i \exp\left[-\sum_{j=1}^{6} a_{ij}\left(x_j - p_{ij}\right)^2\right]$	6	[0, 1]	MN
41	Griewank	$F_{\min} = \frac{1}{4000}\sum_{i=1}^{D} x_i^2 - \prod_{i=1}^{D}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	30	[-600, 600]	MN
42	Ackley	$F_{\min} = -20\exp\left(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2}\right) - \exp\left(\frac{1}{D}\sum_{i=1}^{D}\cos 2\pi x_i\right) + 20 + e$	30	[-32, 32]	MN
43	Penalized	$F_{\min} = \frac{\pi}{D}\left[10\sin^2(\pi y_1) + \sum_{i=1}^{D-1}(y_i-1)^2\left\{1+10\sin^2(\pi y_{i+1})\right\} + (y_D-1)^2\right]$ $+\sum_{i=1}^{D} u(x_i,10,100,4),\quad u(x_i,a,k,m) = \begin{cases} k(x_i-a)^m & x_i > a, \\ 0, & -a \le x_i \le a, \\ k(-x_i-a)^m, x_i < -a \end{cases}$ $y_i = 1 + 1/4(x_i+1)$	30	[-50, 50]	MN
44	Penalized 2	$F_{\min} = 0.1\left[\sin^2(\pi x_1) + \sum_{i=1}^{D-1} \begin{matrix}(x_i-1)^2\left\{1+\sin^2(3\pi x_{i+1})\right\} + (x_D-1)^2 \\ +\left(1+\sin^2(2\pi x_D)\right)\end{matrix}\right]$ $+\sum_{i=1}^{D} u(x_i,5,100,4),\quad u(x_i,a,k,m) = \begin{cases} k(x_i-a)^m & x_i > a, \\ 0, & -a \le x_i \le a, \\ k(-x_i-a)^m, x_i < -a \end{cases}$	30	[-50, 50]	MN
45	Langerman 2	$F_{\min} = -\sum_{i=1}^{D} c_i \left(\exp\left(-\frac{1}{\pi}\sum_{j=1}^{D}\left(x_j - a_{ij}\right)^2\right)\cos\left(\pi\sum_{j=1}^{D}\left(x_j - a_{ij}\right)^2\right)\right)$	2	[0, 10]	MN
46	Langerman 5	$F_{\min} = -\sum_{i=1}^{D} c_i \left(\exp\left(-\frac{1}{\pi}\sum_{j=1}^{D}\left(x_j - a_{ij}\right)^2\right)\cos\left(\pi\sum_{j=1}^{D}\left(x_j - a_{ij}\right)^2\right)\right)$	5	[0, 10]	MN
47	Langerman 10	$F_{\min} = -\sum_{i=1}^{D} c_i \left(\exp\left(-\frac{1}{\pi}\sum_{j=1}^{D}\left(x_j - a_{ij}\right)^2\right)\cos\left(\pi\sum_{j=1}^{D}\left(x_j - a_{ij}\right)^2\right)\right)$	10	[0, 10]	MN
48	FletcherPowell 2	$F_{\min} = \sum_{i=1}^{D}(A_i - B_i)^2 \quad A_i = \sum_{j=1}^{D}\left(a_{ij}\sin\alpha_j + b_{ij}\cos\alpha_j\right),\quad B_i = \sum_{j=1}^{D}\left(a_{ij}\sin x_j + b_{ij}\cos\right.$	2	[-π, π]	MN
49	FletcherPowell 5	$F_{\min} = \sum_{i=1}^{D}(A_i - B_i)^2 \quad A_i = \sum_{j=1}^{D}\left(a_{ij}\sin\alpha_j + b_{ij}\cos\alpha_j\right),\quad B_i = \sum_{j=1}^{D}\left(a_{ij}\sin x_j + b_{ij}\cos\right.$	5	[-π, π]	MN
50	FletcherPowell 10	$F_{\min} = \sum_{i=1}^{D}(A_i - B_i)^2 \quad A_i = \sum_{j=1}^{D}\left(a_{ij}\sin\alpha_j + b_{ij}\cos\alpha_j\right),\quad B_i = \sum_{j=1}^{D}\left(a_{ij}\sin x_j + b_{ij}\cos\right.$	10	[-π, π]	MN

For the considered test problems, the TLBO algorithm is run for 30 times for each benchmark function. In each run the maximum function evaluations is set as 500000 for all the functions for fair comparison purpose and the results obtained using the TLBO algorithm are compared with the results given by other well known optimization algorithms for the same number of function evaluations. Moreover, in order to identify the effect of population size on the performance of the algorithm, the algorithm is experimented with different population sizes viz. 25, 50, 75 and 100 with number of generations of

9850, 4850, 3183 and 2350 respectively so that the function evaluations in each strategy is 500000. Similarly, to identify the effect of elite size on the performance of the algorithm, the algorithm is experimented with different elite sizes, viz. 0, 4, and 8. Here elite size 0 indicates no elitism consideration. The results of each benchmark function are presented in Table 2 in the form of best solution, worst solution, average solution and standard deviation obtained in 30 independent runs on each benchmark function along with the corresponding strategy (i.e. population size and elite size).

Table 2

Results Obtained by the TLBO algorithm for 50 bench mark functions over 30 independent runs with 500000 function evaluations

No.	Function	Optimum	Best	Worst	Mean	SD	PS	ES
1	Stepint	0	0	0	0	0.00E+00	25, 50, 75,100	0, 4, 8
2	Step	0	0	0	0	0.00E+00	25, 50, 75,100	0
3	Sphere	0	0	0	0	0.00E+00	25, 50, 75,100	0
4	SumSquares	0	0	0	0	0.00E+00	25, 50, 75,100	0
5	Quartic	0	0.001245	0.0074937	0.0043519	1.99E-03	25	8
6	Beale	0	0	0	0	0.00E+00	25, 50, 75,100	0, 4, 8
7	Easom	-1	-1	-1	-1	0.00E+00	25, 50, 75,100	0, 4, 8
8	Matyas	0	0	0	0	0.00E+00	25, 50, 75,100	0, 4, 8
9	Colville	0	0	0	0	0.00E+00	25, 50	0, 4, 8
10	Trid 6	-50	-50	-50	-50	0.00E+00	25, 50, 75,100	0, 4, 8
11	Trid 10	-210	-210	-210	-210	0.00E+00	25, 50, 75,100	0, 4, 8
12	Zakharov	0	0	0	0	0.00E+00	25, 50, 75	0
13	Powell	0	3.96E-11	1.92E-07	5.86E-08	8.13E-08	25	0
14	Schwefel 2.22	0	0	0	0	0.00E+00	25, 50	0
15	Schwefel 1.2	0	3.97E-197	2.60E-177	2.60E-178	7.86E-183	25	0
16	Rosenbrock	0	2.76E-07	1.17E-04	1.62E-05	3.64E-05	50	0
17	Dixon-Price	0	0.6666667	0.6666667	0.6666667	0.00E+00	25, 50, 75,100	0, 4, 8
18	Foxholes	0.998	0.9980039	0.9980039	0.9980039	0.00E+00	25, 50, 75,100	0, 4, 8
19	Branin	0.398	0.3978874	0.3978874	0.3978874	0.00E+00	25, 50, 75,100	0, 4, 8
20	Bohachevsky 1	0	0	0	0	0.00E+00	25, 50, 75,100	0, 4, 8
21	Booth	0	0	0	0	0.00E+00	25, 50, 75,100	0, 4, 8
22	Rastrigin	0	1.78E-15	3.73E-14	1.47E-14	1.16E-14	75	4
23	Schwefel	-12569.5	-12569.49	-12173.15	-12414.884	1.18E+02	50	8
24	Michalewicz 2	-1.8013	-1.801303	-1.801303	-1.801303	0.00E+00	25, 50, 75,100	0, 4, 8
25	Michalewicz 5	-4.6877	-4.687658	-4.537656	-4.672658	4.74E-02	100	8
26	Michalewicz 10	-9.6602	-9.66015	-9.51962	-9.6172	4.52E-02	100	4
27	Schaffer	0	0	0	0	0.00E+00	25, 50, 75,100	0, 4, 8
28	6 Hump Camel Back	-1.03163	-1.03163	-1.03163	-1.03163	0.00E+00	25, 50, 75,100	0, 4, 8
29	Bohachevsky 2	0	0	0	0	0.00E+00	25, 50, 75,100	0, 4, 8
30	Bohachevsky 3	0	0	0	0	0.00E+00	25, 50, 75,100	0, 4, 8
31	Shubert	-186.73	-186.731	-186.731	-186.731	0.00E+00	25, 50, 75,100	0, 4, 8
32	GoldStein-Price	3	3	3	3	0.00E+00	25, 50, 75,100	0, 4, 8
33	Kowalik	0.00031	0.0003076	0.0003076	0.0003076	0.00E+00	25, 50, 75,100	0, 4, 8
34	Shekel 5	-10.15	-10.1532	-10.1532	-10.1532	0.00E+00	25, 50, 75,100	0, 4, 8
35	Shekel 7	-10.4	-10.4029	-10.4029	-10.4029	0.00E+00	25, 50, 75,100	0, 4, 8
36	Shekel 10	-10.53	-10.5364	-10.5364	-10.5364	1.87E-15	25, 50, 75,100	0, 4, 8
37	Perm	0	1.27E-07	1.97E-03	6.77E-04	7.45E-04	75	0
38	PowerSum	0	3.78E-13	3.52E-04	7.43E-05	1.11E-04	25	0
39	Hartman 3	-3.86	-3.862782	-3.862782	-3.862782	0.00E+00	25, 50, 75,100	0, 4, 8
40	Hartman 6	-3.32	-3.322368	-3.322368	-3.322368	0.00E+00	25, 50, 75,100	0
41	Griewank	0	0	0	0	0.00E+00	25, 50, 75,100	0
42	Ackley	0	0	0	0	0.00E+00	25, 50, 75,100	4
43	Penalized	0	2.67E-08	2.67E-08	2.67E-08	6.98E-24	25, 50, 75,100	0, 4, 8
44	Penalized 2	0	2.34E-08	2.34E-08	2.34E-08	0.00E+00	25, 50, 75,100	4, 8
45	Langerman 2	-1.08	-1.080938	-1.080938	-1.080938	2.34E-16	25, 50, 75,100	0, 4, 8
46	Langerman 5	-1.5	-0.939706	-0.939646	-0.939702	1.55E-05	100	4
47	Langerman 10	NA	-0.806	-0.428355	-0.64906	1.73E-01	100	4
48	FletcherPowell 2	0	0	0	0	0.00E+00	25, 50, 75,100	0
49	FletcherPowell 5	0	0	10.66247	2.2038134	4.39E+00	50	0
50	FletcherPowell 10	0	1.042963	224.8249	35.971004	7.13E+01	75	4

PS = Population size, ES = Elite size, SD = Standard deviation

It is observed from Table 2 that for functions 5, 13, 15, and 38, strategy with population size of 25 and number of generations of 9850 produced the best results than the other strategies. For functions 16, 23 and 49, strategy with population size of 50 and number of generations of 4850 gave the best results. For functions 22, 37 and 50, strategy with population size of 75 and number of generations of 3183 and for functions 25, 26, 46 and 47 strategy with population size of 100 and number of generations of 2350 produced the best results. For function 12, strategy with population size 25, 50 and 75 while for function 9 and 14, strategy with population size 25 and 50 produced the identical results. For rest of the functions all the strategies produced the same results and hence there is no effect of population size on these functions to achieve their respective global optimum values with same number of function evaluations.

Similarly, it is observed from Table 2 that for functions 2-4, 12-16, 37, 38, 40, 41, 48 and 49, strategy with elite size 0, i.e. no elitism produced best results than the other strategies having different elite sizes. For functions 22, 26, 42, 46, 47 and 50, strategy with elite size of 4 produced the best results. For functions 5, 23, and 25, strategy with elite size of 8 produced the best results. For function 44, strategy with elite size 4 and 8 produced the same results. For rest of the functions all the strategies (i.e. strategy without elitism consideration as well as strategies with different elite sizes consideration) produced the same results.

The performance of TLBO algorithm is compared with the other well known optimization algorithms such as GA, PSO, DE and ABC. The results of GA, PSO, DE and ABC are taken from the previous work of Karaboga and Akay (2009) where the authors had experimented benchmark functions each with 500000 function evaluations with best setting of algorithm specific parameters. Table 3 shows the comparative results of the considered algorithm in the form of mean solution (M), standard deviation (SD) and standard error of mean (SEM). In order to maintain the consistency in the comparison the values bellow 10^{-12} are assumed to be 0 as considered in the previous work of Karaboga and Akay (2009). It is observed from Table 3 that TLBO algorithm outperforms the GA, PSO, DE and ABC algorithms for Powell, Rosenbrock, Kowalik, Perm, and Power sum functions in every aspect of comparison criteria. For Rastrigin, Hartman 6, and Griewank functions, performance of the TLBO and ABC algorithms are alike and outperforms the GA, PSO and DE algorithms. For Shekel 5, Shekel 7, Shekel 10, Hartman 3, and Ackley functions, performance of the TLBO, DE and ABC algorithms are alike and outperforms the GA and PSO algorithms. For Colville function, performance of PSO and TLBO while for Zakharov function, performance of TLBO, DE and PSO are same and produce better results.

For Stepint, Step, Sphere, Sum squares, Schwefel 2.22, Schwefel 1.2, Schaffer, Bohachevsky 2 and GoldStein-Price functions, performance of TLBO, ABC, DE and PSO are identical and produced better results than GA. For Michalewicz 2 and Langerman 2 functions, performances of TLBO, ABC, DE and GA are same and better results are produced than PSO algorithm. For Dixon-Price, Schwefel, Michalewicz 5, Michalewicz 10, FletcherPowell 5, FletcherPowell 10, Penalized and Penalized 2 functions, the results obtained using ABC algorithm are better than the rest of the considered algorithms. For Langerman 5 and Langerman 10 functions, the results obtained using DE are better than other algorithms though the results of TLBO are better than GA, PSO and ABC. Similarly for Quartic function, the PSO algorithm produced better results than rest of the algorithms though the results of TLBO are better than GA, DE and ABC. To investigate the results obtained using different algorithms more deeply, a statistical test is performed in the present work. t-test is performed on the pair of the algorithms to identify the significance difference between the results of the algorithms. In the present work Modified Bonferroni Correction is adopted while performing the t-test. For t-test, first the p-value is calculated for each function and then the p-values are ranked in ascending order. The inverse rank is obtained and then the significance level (α) is found out by dividing 0.05 level by inverse rank. For any function if obtained p value is less than the significance level then there is a significance difference between pair of the algorithms on that function. Tables 4-7 show the results of the statistical test.

Table 3
Comparative results of TLBO with other evolutionary algorithms over 30 independent runs

Function		GA	PSO	DE	ABC	TLBO	Function		GA	PSO	DE	ABC	TLBO
Stepint	M	0	0	0	0	0	Step	M	1.17E+03	0	0	0	0
	SD	0.00E+00	0	0	0	0		SD	76.56145	0	0	0	0
	SEM	0.00E+00	0	0	0	0		SEM	13.978144	0	0	0	0
Sphere	M	1.11E+03	0	0	0	0	Sum Squares	M	1.48E+02	0	0	0	0
	SD	74.214474	0	0	0	0		SD	12.409289	0	0	0	0
	SEM	13.549647	0	0	0	0		SEM	2.265616	0	0	0	0
Quartic	M	0.1807	0.001156	0.001363	0.030016	0.004351	Beale	M	0	0	0	0	0
	SD	0.027116	0.000276	0.000417	0.004866	1.99E-03		SD	0	0	0	0	0
	SEM	0.004951	5.04E–05	7.61E–05	0.000888	3.64E-04		SEM	0	0	0	0	0
Easom	M	-1	-1	-1	-1	-1	Matyas	M	0	0	0	0	0
	SD	0	0	0	0	0		SD	0	0	0	0	0
	SEM	0	0	0	0	0		SEM	0	0	0	0	0
Colville	M	0.014938	0	0.0409122	0.0929674	0	Trid 6	M	-49.9999	-50	-50	-50	-50
	SD	0.007364	0	0.081979	0.066277	0		SD	2.25E–5	0	0	0	0
	SEM	0.001344	0	0.014967	0.0121	0		SEM	4.11E–06	0	0	0	0
Trid 10	M	-209.476	-210	-210	-210	-210	Zakharov	M	0.013355	0	0	0.0002476	0
	SD	0.193417	0	0	0	0		SD	0.004532	0	0	0.000183	0
	SEM	0.035313	0	0	0	0		SEM	0.000827	0	0	3.34E–05	0
Powell	M	9.703771	0.00011	2.17E–07	0.0031344	5.86E-08	Schwefel 2.22	M	11.0214	0	0	0	0
	SD	1.547983	0.00016	1.36E–7	0.000503	8.13E-08		SD	1.386856	0	0	0	0
	SEM	0.282622	2.92E–05	2.48E–08	9.18E–05	1.48E-08		SEM	0.253204	0	0	0	0
Schwefel 1.2	M	7.40E+03	0	0	0	0	Rosenbrock	M	1.96E+05	15.088617	18.203938	0.0887707	1.62E-05
	SD	1.14E+03	0	0	0	0		SD	3.85E+04	24.170196	5.036187	0.07739	3.64E-05
	SEM	208.1346	0	0	0	0		SEM	7029.1062	4.412854	0.033333	0.014129	6.65E-06

Table 3
Comparative results of TLBO with other evolutionary algorithms over 30 independent runs (Cont.)

Function		GA	PSO	DE	ABC	TLBO	Function		GA	PSO	DE	ABC	TLBO
Dixon-Price	M	1.22E+03	0.6666667	0.6666667	0	0.6666667	Foxholes	M	0.998004	0.9980039	0.9980039	0.9980039	0.9980039
	SD	2.66E+02	E–8	E–9	0	0		SD	0	0	0	0	0
	SEM	48.564733	1.82E–09	1.82E–10	0	0		SEM	0	0	0	0	0
Branin	M	0.397887	0.3978874	0.3978874	0.3978874	0.3978874	Bohachevsky 1	M	0	0	0	0	0
	SD	0	0	0	0	0		SD	0	0	0	0	0
	SEM	0	0	0	0	0		SEM	0	0	0	0	0
Booth	M	0	0	0	0	0	Rastrigin	M	52.92259	43.977137	11.716728	0	0
	SD	0	0	0	0	0		SD	4.56486	11.728676	2.538172	0	0
	SEM	0	0	0	0	0		SEM	0.833426	2.141353	0.463405	0	0
Schwefel	M	-11593.4	-6909.1359	-10266	-12569.487	-12414.884	Michalewicz 2	M	-1.8013	-1.5728692	-1.801303	-1.8013034	-1.801303
	SD	93.25424	457.95778	521.84929	0.00E+00	1.18E+02		SD	0.00E+00	0.11986	0	0	0
	SEM	17.025816	83.611269	95.28	0.00E+00	2.15E+01		SEM	0.00E+00	0.021883	0	0	0
Michalewicz 5	M	-4.64483	-2.4908728	-4.683482	-4.6876582	-4.6726578	Michalewicz 10	M	-9.49683	-4.0071803	-9.591151	-9.6601517	-9.6172
	SD	0.09785	0.256952	0.012529	0.00E+00	4.74E-02		SD	0.141116	0.502628	0.064205	0	4.52E-02
	SEM	0.017865	0.046913	0	0	8.66E-03		SEM	0.025764	0.091767	0.011722	0	8.24E-03
Schaffer	M	0.004239	0	0	0	0	Six Hump CamelBack	M	-1.03163	-1.032	-1.032	-1.032	-1.03163
	SD	0.004763	0	0	0	0		SD	0	0	0	0	0
	SEM	0.00087	0	0	0	0		SEM	0	0	0	0	0
Bohachevsky 2	M	0.06829	0.00	0.00	0.00	0.00	Bohachevsky 3	M	0.00	0.00	0.00	0.00	0.00
	SD	0.078216	0.00	0.00	0.00	0.00		SD	0.00	0.00	0.00	0.00	0.00
	SEM	0.01428	0.00	0.00	0.00	0.00		SEM	0.00	0.00	0.00	0.00	0.00
Shubert	M	-186.731	-186.73091	-186.7309	-186.73091	-186.7309	GoldStein-Price	M	5.250611	3	3	3	3
	SD	0	0	0	0	0		SD	5.870093	0	0	0	0
	SEM	0	0	0	0	0		SEM	1.071727	0	0	0	0

Table 3
Comparative results of TLBO with other evolutionary algorithms over 30 independent runs (Cont.)

Function		GA	PSO	DE	ABC	TLBO	Function		GA	PSO	DE	ABC	TLBO
Kowalik	M	0.005615	0.0004906	0.0004266	0.0004266	0.0003076	Shekel 5	M	-5.66052	-2.0870079	-10.1532	-10.1532	-10.1532
	SD	0.008171	0.000366	0.000273	6.04E–5	0		SD	3.866737	1.17846	0	0	0
	SEM	0.001492	6.68E–05	4.98E–05	1.10E–05	0		SEM	0.705966	0.215156	0	0	0
Shekel 7	M	-5.34409	-1.9898713	-10.40294	-10.402941	-10.4029	Shekel 10	M	-3.82984	-1.88	-10.54	-10.54	-10.5364
	SD	3.517134	1.420602	0	0	0		SD	2.451956	0.432476	0	0	0
	SEM	0.642138	0.259365	0	0	0		SEM	0.447664	0.078959	0	0	0
Perm	M	0.302671	0.0360516	0.0240069	0.0411052	0.0006766	PowerSum	M	0.010405	11.390448	0.0001425	0.0029468	0.0000743
	SD	0.193254	0.048927	0.046032	0.023056	0.0007452		SD	0.009077	7.3558	0.000145	0.002289	0.0001105
	SEM	0.035283	0.008933	0.008404	0.004209	0.000136		SEM	0.001657	1.342979	2.65E–05	0.000418	2.02E-05
Hartman 3	M	-3.86278	-3.6333523	-3.862782	-3.8627821	-3.862782	Hartman 6	M	-3.29822	-1.8591298	-3.226881	-3.3219952	-3.322368
	SD	0.00E+00	0.116937	0	0	0		SD	0.05013	0.439958	0.047557	0	0
	SEM	0.00E+00	0.02135	0	0	0		SEM	0.009152	0.080325	0.008683	0	0
Griewank	M	10.63346	0.0173912	0.0014792	0	0	Ackley	M	14.67178	0.1646224	0	0	0
	SD	1.161455	0.020808	0.002958	0	0		SD	0.178141	0.493867	0	0	0
	SEM	0.212052	0.003799	0.00054	0	0		SEM	0.032524	0.090167	0	0	0
Penalized	M	13.3772	0.0207338	0	0	2.67E-08	Penalized 2	M	125.0613	0.0076754	0.0021975	0	2.34E-08
	SD	1.448726	0.041468	0	0	0		SD	12.0012	0.016288	0.004395	0	0
	SEM	0.2645	0.007571	0	0	0		SEM	2.19111	0.002974	0.000802	0	0
Langerman 2	M	-1.08094	-0.679268	-1.080938	-1.0809384	-1.080938	Langerman 5	M	-0.96842	-0.5048579	-1.499999	-0.93815	-0.939702
	SD	0	0.274621	0	0	0		SD	0.287548	0.213626	0	0.000208	1.55E-05
	SEM	0	0.050139	0	0	0		SEM	0.052499	0.039003	0	3.80E–05	2.83E-06
Langerman 10	M	-0.63644	-0.0025656	-1.0528	-0.4460925	-0.64906	Fletcher Powell 2	M	0	0	0	0	0
	SD	0.374682	0.003523	0.302257	0.133958	0.1728623		SD	0	0	0	0	0
	SEM	0.068407	0.000643	0.055184	0.024457	0.03156		SEM	0	0	0	0	0
Fletcher Powell 5	M	0.004303	1457.8834	5.988783	0.1735495	2.2038134	Fletcher Powell 10	M	29.57348	1364.4556	781.55028	8.2334401	35.971004
	SD	0.009469	1269.3624	7.334731	0.068175	4.3863209		SD	16.02108	1325.3797	1048.8135	8.092742	71.284369
	SEM	0.001729	231.75281	1.34	0.012447	0.8059744		SEM	2.925035	1325.3797	241.98	1.48	13.014686

Table 4
Significance test for GA and TLBO

No.	Function	t	SED	p	R	IR	new α	Sign
42	Ackley	451.107	0.033	0	1	50	0.001	TLBO
2	Step	83.7021	13.978	0	2	49	0.0010204	TLBO
3	Sphere	81.921	13.55	0	3	48	0.0010417	TLBO
4	SumSquares	65.3244	2.266	0	4	47	0.0010638	TLBO
22	Rastrigin	63.5001	0.833	0	5	46	0.001087	TLBO
44	Penalized 2	57.0767	2.191	0	6	45	0.0011111	TLBO
43	Penalized	50.5754	0.264	0	7	44	0.0011364	TLBO
41	Griewank	50.1456	0.212	0	8	43	0.0011628	TLBO
14	Schwefel 2.22	43.5277	0.253	0	9	42	0.0011905	TLBO
15	Schwefel 1.2	35.5539	208.135	0	10	41	0.0012195	TLBO
49	FletcherPowell 5	34.6409	0.806	0	11	40	0.00125	GA
13	Powell	34.3348	0.283	0	12	39	0.0012821	TLBO
23	Schwefel	29.9167	27.459	0	13	38	0.0013158	TLBO
16	Rosenbrock	27.8841	7029.106	0	14	37	0.0013514	TLBO
5	Quartic	26.7317	0.001	0	15	36	0.0013889	TLBO
17	Dixon-Price	25.1074	48.565	0	16	35	0.0014286	TLBO
10	Trid 6	24.3432	0	0	17	34	0.0014706	TLBO
12	Zakharov	16.1404	0.001	0	18	33	0.0015152	TLBO
36	Shekel 10	14.9813	0.448	0	19	32	0.0015625	TLBO
11	Trid 10	14.8387	0.035	0	20	31	0.0016129	TLBO
9	Colville	11.1106	0.001	0	21	30	0.0016667	TLBO
37	Perm	8.5591	0.035	0	22	29	0.0017241	TLBO
35	Shekel 7	7.8781	0.642	0	23	28	0.0017857	TLBO
34	Shekel 5	6.3639	0.706	0	24	27	0.0018519	TLBO
38	PowerSum	6.2333	0.002	6E-08	25	26	0.0019231	TLBO
27	Schaffer	5.1869	0.008	0.000003	26	25	0.002	TLBO
29	Bohachevsky 2	4.7821	0.014	0.000001	27	24	0.0020833	TLBO
26	Michalewicz 10	4.4496	0.027	3.959E-05	28	23	0.0021739	TLBO
33	Kowalik	3.5561	0.001	0.0007573	29	22	0.0022727	TLBO
50	FletcherPowell 10	3.4796	13.339	0.0016333	30	21	0.002381	GA

t: t-value of student t-test, SED: standard error of difference, p: p-value calculated for t-value, R: rank of p-value, IR: Inverse rank of p-value, Sign: Significance

Table 5
Significance test for PSO and TLBO

No.	Function	t	SED	p	R	IR	new α	Sign
23	Schwefel	63.7666	86.342	0	1	50	0.001	TLBO
26	Michalewicz 10	60.8881	0.092	0	2	49	0.00102	TLBO
25	Michalewicz 5	45.7345	0.048	0	3	48	0.001042	TLBO
34	Shekel 5	37.4899	0.215	0	4	47	0.001064	TLBO
36	Shekel 10	33.3766	0.259	0	5	46	0.001087	TLBO
35	Shekel 7	32.4372	0.259	0	6	45	0.001111	TLBO
22	Rastrigin	20.5371	2.141	0	7	44	0.001136	TLBO
40	Hartman 6	18.2165	0.08	0	8	43	0.001163	TLBO
47	Langerman 10	16.6788	0.032	0	9	42	0.00119	TLBO
46	Langerman 5	11.1491	0.039	0	10	41	0.00122	TLBO
39	Hartman 3	10.7463	0.021	0	11	40	0.00125	TLBO
24	Michalewicz 2	10.4387	0.022	0	12	39	0.001282	TLBO
5	Quartic	8.6952	0	0	13	38	0.001316	PSO
38	PowerSum	8.4814	1.343	0	14	37	0.001351	TLBO
45	Langerman 2	8.0112	0.05	0	15	36	0.001389	TLBO
49	FletcherPowell 5	6.2814	231.754	5E-08	16	35	0.001429	TLBO
50	FletcherPowell 10	5.4821	242.33	9.6E-07	17	34	0.001471	TLBO
41	Griewank	4.5778	0.004	0.000025	18	33	0.001515	TLBO
37	Perm	3.9597	0.009	0.0002076	19	32	0.001563	TLBO
13	Powell	3.765	0	0.0003909	20	31	0.001613	TLBO
16	Rosenbrock	3.4192	4.413	0.0011554	21	30	0.001667	TLBO

t: t-value of student t-test, SED: standard error of difference, p: p-value calculated for t-value, R: rank of p-value, IR: Inverse rank of p-value, Sign: Significance

Table 6
Significance test for DE and TLBO

No.	Function	t	SED	p	R	IR	new α	Sign
43	Penalized	1.46E+06	0	0	1	1	0.001	DE
46	Langerman 5	1.98E+05	0	0	2	2	0.0010204	DE
22	Rastrigin	25.284	0.463	0	3	3	0.0010417	TLBO
23	Schwefel	21.9989	97.682	0	4	4	0.0010638	TLBO
16	Rosenbrock	19.7981	0.919	0	5	5	0.001087	TLBO
40	Hartman 6	10.9974	0.009	0	6	6	0.0011111	TLBO
47	Langerman 10	8.2386	0.064	0	7	7	0.0011364	DE
5	Quartic	8.0364	0	0	8	8	0.0011628	DE
13	Powell	5.446	0	1.09E-06	9	9	0.0011905	TLBO
50	FletcherPowell 10	3.8847	191.928	0.0002654	10	10	0.0012195	TLBO
37	Perm	2.7756	0.008	0.0007405	11	11	0.00125	TLBO

t: t-value of student t-test, SED: standard error of difference, p: p-value calculated for t-value, R: rank of p-value, IR: Inverse rank of p-value, Sign: Significance

Table 7
Significance test for ABC and TLBO

No.	Function	t	SED	p	R	IR	new α	Sign
17	Dixon-Price	3.13E+24	0	0	1	50	0.001	ABC
46	Langerman 5	40.7424	0	0	4	47	0.00102	TLBO
13	Powell	34.1302	0	0	5	46	0.001042	TLBO
5	Quartic	26.7317	0.001	0	6	45	0.001064	TLBO
33	Kowalik	10.5736	0	0	7	44	0.001087	TLBO
37	Perm	9.5993	0.004	0	8	43	0.001111	TLBO
9	Colville	7.683	0.012	0	9	42	0.001136	TLBO
12	Zakharov	7.4107	0	0	10	41	0.001163	TLBO
23	Schwefel	7.1763	21.543	0	11	40	0.00119	ABC
38	PowerSum	6.8655	0	0	12	39	0.00122	TLBO
16	Rosenbrock	6.2815	0.014	5E-08	13	38	0.00125	TLBO
26	Michalewicz 10	5.2101	0.008	2.61E-06	14	37	0.001282	ABC
49	FletcherPowell 5	2.5349	0.801	0.0013078	15	36	0.001316	ABC
50	FletcherPowell 10	2.1176	13.098	0.0013285	16	35	0.001351	ABC

t: t-value of student t-test, SED: standard error of difference, p: p-value calculated for t-value, R: rank of p-value, IR: Inverse rank of p-value, Sign: Significance

It is observed from Table 4 that for 28 functions TLBO is better than GA and on two functions GA is better than TLBO while for remaining 20 functions both the algorithms showed the equal performance. From Table 5, on 29 functions there is no significance difference between PSO and TLBO but on 20 functions TLBO is better than PSO while on one function PSO is better than TLBO. From Table 6, on 7 functions TLBO performed better than DE while on 4 functions DE is better than TLBO. On remaining 39 functions there is no significance difference between DE and TLBO. From Table 7, on 34 functions TLBO and ABC showed equal performance. On 11 functions, TLBO performed better than ABC while ABC performs better than TLBO on 5 functions.

3.2. Experiment 2

In this section, the performance of TLBO is compared with the different evolutionary algorithms like Canonical evolution strategies (CES), Fast evolution strategies (FES), Covariance matrix adaptation evolution strategies (CMA-ES) and Evolution strategies learned with automatic termination (ESLAT) along with the swarm intelligence based algorithm ABC. In this experiment the TLBO algorithm is implemented on 23 unconstrained benchmark functions taken from the previous work of Karaboga and Akay (2009). The details of the benchmark functions considered in this experiment are shown in Table 8. For the considered test problems, the TLBO algorithm is run for 50 times for each benchmark function. To maintain the consistency in the comparison between TLBO and other algorithms, in each run the algorithm is terminated when it has completed 100000 function evaluations or when it reached the global minima within the gap of 10^{-3}. The results obtained using the TLBO algorithm are compared with the results obtained by other well known optimization algorithms for the same termination criteria. Here also the TLBO algorithm is implemented with different combinations of population size, number of generation and elite size. After conducting experiments with different population sizes the function evaluations required for duplication removal considered are 2500, 5000, 7500 and 10000 for population sizes of 25, 50, 75 and 100 respectively when the maximum function evaluations of the algorithm is 100000.

Table 8
Benchmark functions considered in experiment 2 D: Dimension, C: Characteristic, U: Unimodal, M: Multimodal, S: Separable, N: Non-separable

No.	Function	D	Search range	C	No.	Function	D	Search range	C
1	Sphere	30	[-100, 100]	US	13	Penalized 2	30	[-50, 50]	MN
2	Schwefel 2.22	30	[-10, 10]	UN	14	Fox holes	2	[-65.536, 65.536]	MS
3	Schwefel 1.2	30	[-100, 100]	UN	15	Kowalik	4	[-5, 5]	MN
4	Schwefel 2.21	30	[-100, 100]	UN	16	6 Hump camel back	2	[-5, 5]	MN
5	Rosenbrock	30	[-30, 30]	UN	17	Branin	2	[-5, 0] × [10, 15]	MS
6	Step	30	[-100, 100]	US	18	Goldstein-Price	2	[-2, 2]	MN
7	Quartic	30	[-1.28, 1.28]	US	19	Hartman 3	3	[0, 1]	MN
8	Schwefel	30	[-500, 500]	MS	20	Hartman 6	6	[0, 1]	MN
9	Rastrigin	30	[-5.12, 5.12]	MS	21	Shekel 5	4	[0, 10]	MN
10	Ackley	30	[-32, 32]	MN	22	Shekel 7	4	[0, 10]	MN
11	Griewank	30	[-600, 600]	MN	23	Shekel 10	4	[0, 10]	MN
12	Penalized	30	[-50, 50]	MN					

Table 9 shows the best results obtained using the TLBO algorithm along with its corresponding strategy.

Table 9
Results Obtained by the TLBO algorithm for 23 bench mark functions over 50 independent runs

No.	Function	Best	Worst	Mean	SD	PS	NOG	ES
1	Sphere	2.09E-05	7.27E-04	1.15E-04	6.21E-05	25	2000	0
2	Schwefel 2.22	3.53E-05	9.58E-04	2.38E-04	3.26E-05	25	2000	0
3	Schwefel 1.2	3.87E-05	8.17E-04	8.90E-05	2.57E-05	25	2000	0
4	Schwefel 2.21	6.69E-05	7.27E-04	3.32E-04	7.14E-04	25	2000	0
5	Rosenbrock	13.88407	19.08793	16.3213	1.3564	75	666	0
6	Step	1.81E-05	8.06E-04	3.57E-04	9.54E-05	25	2000	0
7	Quartic	0.001253	0.014182	6.25E-03	3.86E-03	50	1000	4
8	Schwefel	-12569.49	-12158.04	-12409.752	149.1062	75	666	4
9	Rastrigin	1.78E-04	8.67E-04	7.38E-04	1.18E-04	25	2000	4
10	Ackley	1.89E-04	6.21E-04	4.81E-04	1.37E-04	25	2000	0
11	Griewank	8.93E-05	5.72E-04	2.83E-04	2.69E-04	25	2000	0
12	Penalized	2.67E-04	8.27E-04	6.02E-04	1.09E-04	75	666	0
13	Penalized 2	2.37E-08	6.77E-04	3.68E-04	1.16E-04	75	666	4
14	Fox holes	0.998	0.998004	0.998	3.45E-06	25	2000	0
15	Kowalik	0.000308	0.000309	3.08E-04	3.16E-05	75	666	4
16	6 Hump camel back	-1.031628	-1.031628	-1.031628	2.34E-04	25	2000	0
17	Branin	0.3978	0.3984	0.398	5.85E-07	25	2000	0
18	Goldstein-Price	3	3	3	2.05E-07	25	2000	0
19	Hartman 3	-3.8628	-3.8624	-3.8628	2.91E-04	25	2000	0
20	Hartman 6	-3.3224	-3.3223	-3.3224	3.16E-05	75	666	0
21	Shekel 5	-10.152	-10.151	-10.151	2.35E-02	75	666	0
22	Shekel 7	-10.402	-10.402	-10.402	2.87E-04	25	2000	0
23	Shekel 10	-10.53641	-10.5334	-10.534	1.87E-03	25	2000	0

Comparative results of all the considered algorithms in the form of mean solution and standard deviation are shown in Table 10. Except TLBO, results of other algorithms are taken from the previous work of Karaboga and Akay (2009), Yao and Liu (1997) and Hedar and Fukushima (2006). The computational effort of all the considered algorithms in the form of mean number of function evaluations is shown in Table 11.

Table 10
Comparative results of TLBO with other evolutionary algorithms over 50 independent runs

NO.	Function	CES		FES		ESLAT		CMA_ES		ABC		TLBO	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
1	Sphere	1.7E–26	1.1E–25	2.5E–04	6.8E–05	2.0E–17	2.9E–17	9.7E–23	3.8E–23	7.57E–04	2.48E–04	1.15E-04	6.21E-05
2	Schwefel 2.22	8.1E–20	3.6E–19	6.0E–02	9.6E–02	3.8E–05	1.6E–05	4.2E–11	7.1E–23	8.95E–04	1.27E–04	2.38E-04	3.26E-05
3	Schwefel 1.2	337.62	117.14	1.4E–03	5.3E–04	6.1E–06	7.5E–06	7.1E–23	2.9E–23	7.01E–04	2.78E–04	8.90E-05	2.57E-05
4	Schwefel 2.21	2.41	2.15	5.5E–03	6.5E–04	0.78	1.64	5.4E–12	1.5E–12	2.72	1.18	3.32E-04	7.14E-04
5	Rosenbrock	27.65	0.51	33.28	43.13	1.93	3.35	0.4	1.2	0.936	1.76	16.3213	1.3564
6	Step	0	0	0	0	2.0E–02	0.14	1.44	1.77	0	0	3.57E-04	9.54E-05
7	Quartic	4.7E–02	0.12	1.2E–02	5.8E–03	0.39	0.22	0.23	8.7E–02	9.06E–02	1.89E–02	6.25E-03	3.86E-03
8	Schwefel	-8.00E+93	4.90E+94	-1.26E+04	3.25E+01	2.30E+15	5.70E+15	-7637.14	895.6	-12563.673	23.6	-12409.752	149.1062
9	Rastrigin	13.38	43.15	0.16	0.33	4.65	5.67	51.78	13.56	4.66E–04	3.44E–04	7.38E-04	1.18E-04
10	Ackley	6.0E–13	1.7E–12	1.2E–02	1.8E–03	1.8E–08	5.4E–09	6.9E–12	1.3E–12	7.81E–04	1.83E–04	4.81E-04	1.37E-04
11	Griewank	6.0E–14	4.2E–13	3.7E–02	5.0E–02	1.4E–03	4.7E–03	7.4E–04	2.7E–03	8.37E–04	1.38E–03	2.83E-04	2.69E-04
12	Penalized	1.46	3.17	2.8E–06	8.10E-07	1.5E–12	2.0E–12	1.2E–04	3.40E-02	6.98E–04	2.78E–04	6.02E-04	1.09E-04
13	Penalized 2	2.4	0.13	4.7E–05	1.5E–05	6.4E–03	8.9E–03	1.7E–03	4.5E–03	7.98E–04	2.13E–04	3.68E-04	1.16E-04
14	Fox holes	2.2	2.43	1.2	0.63	1.77	1.37	10.44	6.87	0.998	3.21E-04	0.998	3.45E-06
15	Kowalik	1.3E–03	6.3E–04	9.7E–04	4.22E–04	8.1E–04	4.1E–04	1.5E–03	4.2E–03	1.18E–03	1.45E-04	3.08E-04	3.16E-05
16	6 Hump camel back	-1.031	1.2E–03	-1.0316	6.00E-07	-1.0316	9.7E–14	-1.0316	7.70E-16	-1.031	3.04E-04	-1.031628	2.34E-04
17	Branin	0.401	3.6E–3	0.398	6.00E-08	0.398	1.0E–13	0.398	1.40E-15	0.3985	3.27E-04	0.398	5.85E-07
18	Goldstein-Price	3.007	1.2E–02	3	0	3	5.8E–14	14.34	25.05	3	3.09E-04	3	2.05E-07
19	Hartman 3	-3.8613	1.2E–03	-3.86	4.00E-03	-3.8628	2.9E–13	-3.8628	4.80E-16	-3.862	2.77E-04	-3.8628	2.91E-04
20	Hartman 6	-3.24	5.8E–2	-3.23	0.12	-3.31	3.3E–2	-3.28	5.8E–02	-3.322	1.35E-04	-3.3224	3.16E-05
21	Shekel 5	-5.72	2.62	-5.54	1.82	-8.49	2.76	-5.86	3.6	-10.151	1.17E-02	-10.151	2.35E-02
22	Shekel 7	-6.09	2.63	-6.76	3.01	-8.79	2.64	-6.58	3.74	-10.402	3.11E–04	-10.402	2.87E-04
23	Shekel 10	-6.42	2.67	-7.63	3.27	-9.65	2.06	-7.03	3.74	-10.535	2.02E-03	-10.534	1.87E-03

Table 11
Mean number of function evaluation (Mean FE) required by ESLAT, CMA-ES, ABC and TLBO algorithms for the benchmark functions considered in experiment 2

No.	Function	CES	FES	ESLAT	CMA-ES	ABC		TLBO	
		Mean FE	Mean FE	Mean FE	Mean FE	Mean FE	SD of FE	Mean FE	SD of FE
1	Sphere	69724	150,000	69724	10721	9264	1481	4648	148
2	Schwefel 2.22	60859	200000	60859	12145	12991	673	7395	163
3	Schwefel 1.2	72141	500000	72141	21248	12255	1390	12218	1305
4	Schwefel 2.21	69821	500000	69821	20813	100000	0	9563	715
5	Rosenbrock	66609	1500000	66609	55821	100000	0	100000	0
6	Step	57064	150000	57064	2184	4853	1044	13778	1491
7	Quartic	50962	300000	50962	667131	100000	0	100000	0
8	Schwefel	61704	900000	61704	6621	64632	23897	100000	0
9	Rastrigin	53880	500000	53880	10079	26731	9311	34317	13866
10	Ackley	58909	150000	58909	10654	16616	1201	3868	2634
11	Griewank	71044	200000	71044	10522	36151	17128	10090	16237
12	Penalized	63030	150000	63030	13981	73440	2020	10815	1430
13	Penalized 2	65655	150000	65655	13756	8454	1719	30985	12937
14	Fox holes	1305	10000	1305	540	1046	637	524	150
15	Kowalik	2869	400000	2869	13434	6120	4564	2488	2700
16	6 Hump	1306	10000	1306	619	342	109	447	175
17	Branin	1257	10000	1257	594	530	284	362	88
18	Goldstein-Price	1201	10000	1201	2052	15186	13500	452	244
19	Hartman 3	1734	10000	1734	996	4747	16011	547	135
20	Hartman 6	3816	20000	3816	2293	1583	457	24847	29465
21	Shekel 5	2338	10000	2338	1246	6069	13477	1245	114
22	Shekel 7	2468	10000	2468	1267	7173	9022	1272	99
23	Shekel 10	2410	10000	2410	1275	15392	24413	1270	135

Here the mean number of function evaluation indicates the function evaluations required to obtain global best solution within the gap of 10^{-3} averaged over 30 independent runs.

Table 12
Success rate of ESLAT, CMA-ES, ABC and TLBO algorithms for the benchmark functions considered in experiment 2

No.	Function	ESLAT	CMA-ES	ABC	TLBO
1	Sphere	100	100	100	100
2	Schwefel 2.22	100	100	100	100
3	Schwefel 1.2	100	100	100	100
4	Schwefel 2.21	0	100	0	100
5	Rosenbrock	70	90	0	0
6	Step	98	36	100	100
7	Quartic	0	0	0	0
8	Schwefel	0	0	86	40
9	Rastrigin	40	0	100	100
10	Ackley	100	100	100	100
11	Griewank	90	92	96	100
12	Penalized	100	88	100	100
13	Penalized 2	60	86	100	100
14	Fox holes	60	0	100	100
15	Kowalik	94	88	100	100
16	6 Hump camel back	100	100	100	100
17	Branin	100	100	100	100
18	Goldstein-Price	100	78	100	100
19	Hartman 3	100	100	100	100
20	Hartman 6	94	48	100	96
21	Shekel 5	72	40	98	100
22	Shekel 7	72	48	100	100
23	Shekel 10	84	52	96	100
	Total	9	9	19	19

If for any function, the global best solution is not obtained in this precision then solution obtained in the last cycle is recorded. It is observed from the results that on 14 functions the computational effort of TLBO is less than the rest of the considered algorithms i.e. the convergence of TLBO is faster than rest of the algorithms. On 3 functions, ABC required minimum computational effort than the other algorithms. On 5 functions, CMA-ES and on 1 function ESLAT required less number of function evaluations to achieve the global best solution than rest of the considered algorithms. The success rate of all the algorithms for the considered benchmark functions are shown in Table 12.

It is observed from the results that ESLAT and CMA-ES achieved the best success rate on 9 functions while ABC and TLBO algorithms achieved the best success rate on 19 functions. On Schwefel and Hartman 6 functions ABC achieved higher success rate than TLBO while on Schwefel 2.21, Griewank, Shekel 5 and Shekel 10 functions success rate of TLBO is better than ABC. On Rosenbrock function success rate of CMA-ES is better than rest of the considered algorithms.

3.3. Experiment 3

In this section, the computational effort and consistency of the TLBO algorithm is compared with Self-organizing maps evolution strategy (SOM-ES), Neural gas networks evolution strategy (NG-ES), CMA-ES and ABC algorithms. In this experiment the TLBO algorithm is implemented on 3 unconstrained benchmark functions taken from the previous work of Karaboga and Akay (2009). The details of the benchmark functions considered in this experiment are shown in Table 13.

Table 13
Benchmark functions considered in experiment 3

No.	Function	Formulation	D	Search range	C
1	Modified Rosenbrock	$F_{\min} = 74 + 100\left(x_2 - x_1^2\right)^2 + \left(1 - x_1\right)^2 - 400\exp^{-\left((x_1+1)^2+(x_2+1)^2/0.1\right)}$	2	[-2, 2]	UN
2	Modified Griewank	$F_{\min} = 1 + \frac{1}{200}\left(x_1^2 + x_2^2\right) - \cos\left(x\right)\cos\left(\frac{y}{\sqrt{2}}\right)$	2	[-100, 100]	MN
3	Rastrigin	$F_{\min} = \sum_{i=1}^{D}\left[x_i^2 - 10\cos(2\pi x_i) + 10\right]$	2	[-5.12, 5.12]	MS

For the considered test problems, the TLBO algorithm is run for 10000 times for each benchmark function. In each run the maximum function evaluation is set as 5000 per test function. To maintain the consistency in the comparison, the limiting value of the satisfactory convergence is set as 40, 0.001 and 0.001 for functions 1, 2 and 3 respectively (Karaboga and Akay 2009, Milano *et al*. 2004). Here also the TLBO algorithm is implemented with different combinations of population size, number of generation and elite size and the strategy which produced the best results is considered for the comparison.

Comparative results of all the considered algorithms in the form of mean and standard deviation and success rate are shown in Table 14. Except TLBO, results of other algorithms are taken from the previous work of Karaboga and Akay (2009). It is observed from the results that SOM-ES produced better convergence rate and success rate on modified Rosenbrock function than the other algorithms. On Griewank function, the TLBO produced better success rate though the convergence of the TLBO is slower than the other considered algorithm. On Rastrigin function, the success rate of ABC and TLBO are equally good but the convergence of both the algorithms is slower than the other algorithms.

Table 14
Comparative results of different algorithms for the benchmark functions considered in experiment 3

Algorithm	Modified Rosenbrock		Griewank		Rastrigin	
	Mean FE	% Success	Mean FE	% Success	Mean FE	% Success
5 × 5 SOM-ES	1600 ± 200	70 ± 8	130 ± 40	90 ± 7	180 ± 50	90 ± 7
7 × 7 SOM-ES	750 ± 90	90 ± 5	100 ± 40	90 ± 8	200 ± 50	90 ± 8
NG-ES (m=10)	1700 ± 200	90 ± 7	180 ± 50	90 ± 10	210 ± 50	90 ± 8
NG-ES (m=20)	780 ± 80	90 ± 9	150 ± 40	90 ± 8	180 ± 40	90 ± 7
CMA-ES	70 ± 40	30 ± 10	210 ± 50	70 ± 10	100 ± 40	80 ± 9
ABC	1371 ± 2678	52 ± 5	1124 ± 960	99 ± 1	1169 ± 446	100 ± 0
TLBO	1277 ± 942	62 ± 6	1164 ± 456	100 ± 0	1637 ± 596	100 ± 0

The TLBO algorithm has already been successfully applied by various researchers for solving complex benchmark functions and difficult engineering problems (Azizipanah-Abarghooee *et al*. 2012, Hosseinpour *et al*. 2011, Krishnanand *et al*. 2011, Nayak *et al*. 2011, Niknam *et al*. 2012a, 2012b, 2012c, Rao and Kalyankar 2012a, 2012b, 2012c, Rao and Savsani 2012, Rao and Patel 2012a; 2012b; 2012c; 2012d, Satapathy and Naik 2011, Satapathy *et al*. 2012, Toğan 2012). Contrary to the opinion expressed by Črepinšek *et al*. (2012) that TLBO is not a parameter-less algorithm, this paper has clearly explained that TLBO is an algorithm-specific parameter-less algorithm and this was already stated by Rao and Patel (2012a). Common control parameters are common to run any of the optimization algorithms and algorithm-specific parameters are specific to the algorithm and different algorithms have different specific parameters to control. The TLBO algorithm does not have any algorithm-specific parameters to control and it requires only the control of the common control parameters like population size, number of generations and elite sizes. In fact, many of the comments made by Črepinšek *et al*. (2012) about the TLBO algorithm were already addressed by Rao and Patel (2012a).

4. Conclusion

The tuning of the common controlling parameters such as population size and number of generations is one of the important factors in any probabilistic algorithm. In addition to this, evolutionary and swarm intelligence based algorithms require proper tuning of algorithm-specific parameters. A change in the tuning of the algorithm-specific parameters influences the effectiveness of the algorithm. The recently proposed TLBO algorithm does not require any algorithm-specific parameters. It only requires the tuning of the common controlling parameters of the algorithm for its working.

In the present work, the concept of elitism is introduced in the TLBO algorithm and its effect on the performance of the algorithm for the unconstrained optimization problems is investigated. Furthermore, the effect of common controlling parameters on the performance of TLBO algorithm is also investigated by considering different combinations of common controlling parameters. The proposed algorithm is implemented on 76 unconstrained optimization problems having different characteristics to identify the effect of elitism and common controlling parameters. The results have shown that for some functions the strategy with elitism consideration produced better results than that without elitism consideration. The results obtained by using TLBO algorithm are compared with the other optimization algorithms available in the literature for the considered benchmark problems. Results have shown the satisfactory performance of TLBO algorithm for the unconstrained optimization problems.

References

Ahrari, A. & Atai A. A. (2010). Grenade explosion method - A novel tool for optimization of multimodal functions. *Applied Soft Computing*, 10, 1132-1140.

Azizipanah-Abarghooee, R., Niknam, T., Roosta, A., Malekpour, A.R. & Zare, M. (2012). Probabilistic multiobjective wind-thermal economic emission dispatch based on point estimated method, *Energy*, 37, 322-335.

Basturk, B & Karaboga, D. (2006). An artificial bee colony (ABC) algorithm for numeric function optimization, in: *IEEE Swarm Intelligence Symposium*, Indianapolis, Indiana, USA.

Črepinšek, M., Liu, S-H & Mernik, L. (2012). A note on teaching-learning-based optimization algorithm, *Information Sciences*, 212, 79-93.

Dorigo, M., Maniezzo V. & Colorni A. (1991). Positive feedback as a search strategy, *Technical Report 91-016*. Politecnico di Milano, Italy.

Eusuff, M. & Lansey, E. (2003). Optimization of water distribution network design using the shuffled frog leaping algorithm. *Journal of Water Resources Planning and Management, 2*9, 210-225.

Farmer, J. D., Packard, N. & Perelson, A. (1986).The immune system, adaptation and machine learning, *Physica D,* 22,187-204.

Fogel, L. J, Owens, A. J. & Walsh, M.J. (1966). *Artificial intelligence through simulated evolution*. John Wiley, New York.

Geem, Z. W., Kim, J.H. & Loganathan G.V. (2001). A new heuristic optimization algorithm: harmony search. *Simulation*, 76, 60-70.

Hedar, A. & Fukushima, M. (2006). Evolution strategies learned with automatic termination criteria. *Proceedings of SCIS-ISIS* 2006, Tokyo, Japan.

Holland, J. (1975). *Adaptation in natural and artificial systems*. University of Michigan Press, Ann Arbor.

Hosseinpour, H., Niknam, T. & Taheri, S.I. (2011). A modified TLBO algorithm for placement of AVRs considering DGs, *26th International Power System Conference,* 31st October – 2nd November 2011, Tehran, Iran.

Karaboga, D. (2005). An idea based on honey bee swarm for numerical optimization, *Technical Report-TR06*, Computer Engineering Department. Erciyes University, Turkey.

Karaboga, D. & Akay, B. (2009). A comparative study of Artificial Bee Colony algorithm. *Applied Mathematics and Computation,* 214(1) 108-132.

Karaboga, D. & Basturk, B. (2007). A powerful and efficient algorithm for numerical function optimization: artificial bee colony (ABC) algorithm. *Journal of Global Optimization*, 39 (3), 459–471.

Karaboga, D. & Basturk, B. (2008). On the performance of artificial bee colony (ABC) algorithm. *Applied Soft Computing*, 8 (1), 687–697.

Kashan, A.H. (2011). An efficient algorithm for constrained global optimization and application to mechanical engineering design: League championship algorithm (LCA). *Computer-Aided Design*, 43, 1769-1792.

Kennedy, J. & Eberhart, R. C. (1995). Particle swarm optimization. *Proceedings of IEEE International Conference on Neural Networks,* IEEE Press, Piscataway, 1942-1948.

Krishnanand, K.R., Panigrahi, B.K., Rout, P.K. & Mohapatra, A. (2011). Application of multi-objective teaching-learning-based algorithm to an economic load dispatch problem with incommensurable objectives. *Swarm, Evolutionary, and Memetic Computing, Lecture Notes in Computer Science 7076*, 697-705, Springer-Verlag, Berlin.

Milano, M., Koumoutsakos, P. & Schmidhuber, J. (2004). Self-organizing nets for optimization. *IEEE Transactions on Neural Networks*, 2004, 15(3), 758-765.

Nayak, N., Routray, S.K. & Rout, P.K. (2011). A robust control strategies to improve transient stability in VSC- HVDC based interconnected power systems. *Proc. of IEEE Conference on Energy, Automation, and Signal (ICEAS*), PAS-102, 1-8.

Niknam, T., Fard, A.K. & Baziar, A. (2012a). Multi-objective stochastic distribution feeder reconfiguration problem considering hydrogen and thermal energy production by fuel cell power plants, *Energy*, 42, 563-573.

Niknam, T., Golestaneh, F., & Sadeghi, M.S. (2012b). θ-multiobjective teaching–learning-based optimization for dynamic economic emission dispatch. *IEEE Systems Journal*, 6, 341-352.

Niknam, T., Azizipanah-Abarghooee, R. & Narimani, M.R. (2012c). A new multi objective optimization approach based on TLBO for location of automatic voltage regulators in distribution systems. *Engineering Applications of Artificial Intelligence*, http://dx.doi.org/10.1016/j.engappai.2012.07.004.

Passino, K.M. (2002). Biomimicry of bacterial foraging for distributed optimization and control. *IEEE Control Systems Magazine,* 22, 52–67.

Price K., Storn, R, & Lampinen, A. (2005). Differential evolution - a practical approach to global optimization, *Springer Natural Computing Series*.

Rao, R.V. & Kalyankar, V.D. (2012a). Parameter optimization of modern machining processes using teaching–learning-based optimization algorithm. *Engineering Applications of Artificial Intelligence*, http://dx.doi.org/10.1016/j.engappai.2012.06.007.

Rao, R.V. & Kalyankar, V.D. (2012b). Multi-objective multi-parameter optimization of the industrial LBW process using a new optimization algorithm. *Journal of Engineering Manufacture*, DOI: 10.1177/0954405411435865

Rao, R.V. & Kalyankar, V.D. (2012c). Parameter optimization of machining processes using a new optimization algorithm. *Materials and Manufacturing Processes,* DOI: 10.1080/10426914.2011.602792

Rao, R.V. & Patel, V. (2012a). An elitist teaching-learning-based optimization algorithm for solving complex constrained optimization problems. *International Journal of Industrial Engineering Computations*, 3(4), 535-560.

Rao, R.V. & Patel, V. (2012b). Multi-objective optimization of combined Brayton and inverse Brayton cycle using advanced optimization algorithms, *Engineering Optimization*, doi: 10.1080/0305215X.2011.624183.

Rao, R.V. & Patel, V. (2012c). Multi-objective optimization of heat exchangers using a modified teaching-learning-based-optimization algorithm, *Applied Mathematical Modeling*, doi:10.1016/j.apm.2012.03.043.

Rao, R.V. & Patel, V. (2012d). Multi-objective optimization of two stage thermoelectric cooler using a modified teaching-learning-based-optimization algorithm. *Engineering Applications of Artificial Intelligence*, doi:10.1016/j.engappai.2012.02.016

Rao, R.V. & Savsani, V.J. (2012). *Mechanical design optimization using advanced optimization techniques*. Springer-Verlag, London.

Rao, R.V., Savsani, V.J & Balic, J. (2012b). Teaching-learning-based optimization algorithm for unconstrained and constrained real-parameter optimization problems. *Engineering Optimization*, http://dx.doi.org/10.1080/0305215X.2011.652103

Rao, R.V., Savsani, V.J. & Vakharia, D.P. (2011). Teaching-learning-based optimization: A novel method for constrained mechanical design optimization problems. *Computer-Aided Design,* 43 (3), 303-315.

Rao, R.V., Savsani, V.J. & Vakharia, D.P. (2012a). Teaching-learning-based optimization: A novel optimization method for continuous non-linear large scale problems. *Information Science*s, 183 (1), 1-15.

Rashedi, E., Nezamabadi-pour, H. & Saryazdi, S. (2009). GSA: A gravitational search algorithm, *Information Sciences*, 179, 2232-2248.

Runarsson, T.P. &Yao X. (2000) Stochastic ranking for constrained evolutionary optimization. *IEEE Transactions on Evolutionary Computation,* 4 (3), 284-294.

Satapathy, S.C. & Naik, A. (2011). Data clustering based on teaching-learning-based optimization. *Swarm, Evolutionary, and Memetic Computing, Lecture Notes in Computer Science 7077*, 148-156, Springer-Verlag, Berlin.

Satapathy, S.C., Naik, A. & Parvathi, K. (2012). High dimensional real parameter optimization with teaching learning based optimization. *International Journal of Industrial Engineering Computations*, doi: 10.5267/j.ijiec.2012.06.001.

Simon, D. (2008). Biogeography-based optimization. *IEEE Transactions on Evolutionary Computation,* 12, 702–713.

Storn, R. & Price, K. (1997). Differential evolution - A simple and efficient heuristic for global optimization over continuous spaces. *Journal of Global Optimization,* 11, 341-359.

Toğan, V. (2012). Design of planar steel frames using teaching–learning based optimization, *Engineering Structures*, 34, 225–232.

Yao, X & Liu, Y. (1997). Fast evolution strategies. *Control and Cybernetics,* 26(3), 467- 496.

Appendix A: Code of Elitist TLBO algorithm for unconstrained problems

The code is similar to that given in Rao and Patel (2012a) for the constrained optimization problems. The files of TLBO, OUTPUT, AVG_RESULT, REMOVE DUPLICATE and RUNTLBO remain the same. However, the INITIALIZATION, IMPLEMENT and OBJECTIVE files of Rao and Patel (2012a) are to be replaced by the following files. To run the TLBO code, user has to create separate MATLAB files for each function (i.e. separate .m file for INITIALIZATION, IMPLEMENTATION, OBJECTIVE, etc.) and then the RUNTLBO file is to be executed.

```
%%%%%%%%%%%%%%%%%%%%%%%%%% INITIALIZATION %%%%%%%%%%%%%%%%%%%%%%%
function [Students, select, upper_limit, lower_limit, ini_fun, min_result, avg_result, result_fun, opti_fun, result_fun_new,
opti_fun_new] = Initialize(note1, obj_fun, RandSeed)
format long;
select.classsize =25;
select.var_num = 10;
select.itration =100;
if ~exist('RandSeed', 'var')
    rand_gen = round(sum(100*clock));
end
rand('state', rand_gen);
[ini_fun, result_fun, result_fun_new, opti_fun, opti_fun_new,] = obj_fun();
[upper_limit, lower_limit, Students, select] = ini_fun(select);
Students = remove_duplicate(Students, upper_limit, lower_limit);
Students = result_fun(select, Students);
Students = sortstudents(Students);
average_result = result_avg(Students);
min_result = [Students(1).result];
avg_result = [average_result];
return;
%%%%%%%%%%%%%%%%%%%%%%%%%% %%% IMPLEMENT %%%%%%%%%%%%%%%%%%%%%%%
function [ini_fun, result_fun, result_fun_new, opti_fun, opti_fun_new] = implement
format long;
 ini_fun = @implementInitialize ;
result_fun = @implementresult;
result_fun_new = @implementresult_new;
opti_fun = @implementopti;
opti_fun_new = @implementopti_new;
return;

function [upper_limit, lower_limit, Students, select] = implementInitialize(select)
global lower_limit upper_limit ll ul
Granularity = 1;
lower_limit = ll;
upper_limit = ul;
ll =[-100 -100 -100 -100 -100 -100 -100 -100 -100 -100];
ul =[100 100 100 100 100 100 100 100 100 100];
upper_limit = ul;
for popindex = 1 : select.classsize
    for k = 1 : select.var_num
        mark(k) =(ll(k))+ ((ul(k) - ll(k)) * rand);
     end
        Students(popindex).mark = mark;
end
select.OrderDependent = true;
return;
function [Students] = implementresult(select, Students)
global lower_limit upper_limit
classsize = select.classsize;
for popindex = 1 : classsize
    for k = 1 : select.var_num
        x(k) = Students(popindex).mark(k);
    end
    Students(popindex).result = objective(x);
```

```
end
return
function [Studentss] = implementresult_new(select, Students)
global lower_limit upper_limit
classsize = select.classsize;
for popindex = 1 : size(Students,1)
    for k = 1 : select.var_num
        x(k) = Students(popindex,k);
       end
    Studentss(popindex) = objective(x);
end
return
function [Students] = implementopti(select, Students)
global lower_limit upper_limit ll ul
for i = 1 : select.classsize
    for k = 1 : select.var_num
        Students(i).mark(k) = max(Students(i).mark(k), ll(k));
        Students(i).mark(k) = min(Students(i).mark(k), upper_limit(k));
    end
end
return;
function [Students] = implementopti_new(select, Students)
global lower_limit upper_limit ll ul
for i = 1 : size(Students,1)
    for k = 1 : select.var_num
        Students(i,k)= max(Students(i,k), ll(k));
        Students(i,k) = min(Students(i,k), upper_limit(k));
    end
end
return;
%%%%%%%%%%%%%%%%%%%%%%%%%% OBJECTIVE%%%%%%%%%%%%%%%%%%%%%%%%%
function yy=objective(x)
format long;
for ikl=1 : 10
    p(ikl)=x(ikl);
end
sum1=0;
for ikl=1 : 10
    z1=(p(ikl))^2;
sum1=sum1+z1;
end
yy=(sum1);
```