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# A Comparative Review of Methods for Stability Monitoring in Electrical Power Systems and Vibrating Structures

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## Abstract

This paper provides a review of methods used for stability monitoring in two different fields, electrical power systems and vibration analysis, with the aim of increasing awareness of and highlighting opportunities for cross-fertilisation. The paper discusses the nature of the problems that require stability monitoring in both fields and outlines the similarities between the problems as well as the approaches that have been taken. The review of power systems methods is presented in two parts: methods for ambient or normal operation and methods for transient or post-fault operation. Similarly, the review of methods for vibration analysis is presented in two parts: methods for stationary or linear time-invariant data and methods for non-stationary or non-linear time-variant data. Some observations and comments are made regarding methods that have already been applied in both fields including recommendations for the use of different sets of algorithms that have not been utilised to date. Additionally, methods that have been applied to vibration analysis, and have potential for power systems stability monitoring are discussed and recommended.

*Keywords:* Power Systems; Stability Monitoring; Vibration Analysis

## I. Introduction

Oscillations are inherent in power systems, and as electrical power networks are being operated closer to their limits, it is becoming increasingly important to monitor the stability of the system because poorly damped oscillations can pose various problems such as limiting transfer capacities and in more severe cases can lead to system instability causing a wide-scale blackout. Over the last few years there has been significant deployment of system-wide measurement technology known as Wide Area Monitoring Systems (WAMS) that collect data from GPS-assisted Phasor Measurement Units (PMUs) at different locations. As a result, there is opportunity to obtain a real-time view of power systems and to determine system characteristics from data. However one of the main challenges facing researchers is that the system inputs that cause the random fluctuations in system measurements during operation of power systems cannot be measured. Hence, much research has been geared towards the inference of system stability from only output measurements in both normal and post-fault operation of the system.

A field of research that is tackling a similar problem is vibration analysis. Vibration analysis generally encompasses structural, mechanical and aeronautical engineering. The stability of structures in these fields needs to be monitored because structures deteriorate over time as a result of stress from continuous use as well as forces from external factors such as wind. As structures are exposed to these forces, they can develop fractures which can lead to critical failure over time. However, the structures cannot be excited for two reasons: they are continuously in use, and excitation of the structure could cause catastrophic damage. Thus, stability has to be inferred from measurements of vibrations of the structures hence the term *vibration analysis*. Additionally, similar to electrical power systems, it is hard to measure the input excitation to structures when in use and therefore the majority of research in this field has been on system identification using *output-only measurements*.

An analogy can hence be drawn between the two fields whereby structures are equivalent to power transmission networks, the exciting forces on the structures are equivalent to the excitations/loads applied to power systems and the measured structural vibrations are equivalent to the power system measurements of power flows, voltages, currents and frequencies. The aim of this paper is to show that the research problems in the fields of vibration analysis and electrical power systems can be tackled using similar methodologies hence the potential for cross-fertilization seems great. In order to demonstrate this aim, a review of methods from both fields is carried out to show the similarities in the approaches that have been taken and also to demonstrate some methods that can be applied to the stability monitoring problem in electrical power systems.

The paper is organised as follows: Section II presents a general overview of methods from electrical power systems and discusses the top level hierarchical classification of methods. Section III reviews the methods in literature that have been applied during the normal operation of power systems while section IV reviews those methods that have been applied in post-fault operation of power systems. Sections III and IV contain a greater amount of mathematical detail of methods than other sections because other "top-level" reviews on these topics have been identified to be lacking in technical detail, and also because some of the introduced computations are important for the discussion of some methods in these as well as later sections. Additionally, presenting some of the fundamental details will enable a comparison with methods from the field of vibration analysis. Section V provides an overview of the top-level classification of methods from vibration analysis and also draws a comparison to the methods in electrical power systems. Sections VI and VII review the two types of methods in vibration analysis that have been identified in the classification presented in section V. Some observations and comments regarding the methods and their applicability to power systems are also drawn in these sections. Section VIII presents a brief overview of methods for system identification in which the input is known. This group of methods is reviewed separately because there are some situations in both power systems and vibration analysis where the system inputs are assumed to be known. Various sources in literature give detailed examples of these methods hence they are reviewed in brief. Finally section IX presents a brief summary and discussion of the subject matter of the paper.

## II. ELECTRICAL POWER SYSTEMS - OVERVIEW

A survey of the methods available for analysis of signals from electrical power systems is presented in [1], which provides a classification of the methods available and examples of methods that fall within the respective classifications. The same classification augmented by a tree diagram is used in this paper, including more recent applications. The tree diagram is shown in Figure 1 (at the end of the paper). Greater detail of the methods that fall within the classifications is also presented.

The top-level classification of methods refers to the system response during which they are applicable. A power system response can be described as an ambient response or a transient response.

An *ambient response* describes measurements from the system during ambient operation when the system can be assumed to be reasonably linear (around the operating point) and the excitation (load variation) can be approximated as being random and Gaussian. The outputs from the system are stochastic in nature. A *transient response* describes measurements from the system during transient operation which is initiated after a disturbance has been applied or a fault has occurred. This fault-induced response is usually characterised by a large deviation in system frequency or other system measurements, for example power flow in a transmission line. In [1], the methods applicable to these operation types are referred to as mode-meter and ring-down methods respectively. *Mode-meter methods* are called so because they give mode frequency estimations more readily than damping estimations while *ring-down methods* are called so because they work on signals that characterize the damped oscillatory behaviour of the system. Figure 2 demonstrates an example of a system response under ambient and transient operations. The first of the following sections describes the classification of methods for ambient operation while the second describes the classification of methods for transient operation.

## III. ELECTRICAL POWER SYSTEMS – AMBIENT OPERATION

Signals measured during ambient operation are stochastic in nature and are dominated by broadband noise which originates in the load demand. Additionally, since the load cannot be measured everywhere in the system, the input is assumed to be unknown. The aim of analysis is to determine the damping of the system using measurements. It can be difficult to obtain information about the system damping of modes from outputs only. In order to make such estimations easier, *probing* might be applied. Methods for ambient operation can therefore be sub-divided into methods that require probing and methods that do not require probing.

### A. *Methods that require probing*

*Probing* refers to the injection of an external disturbance into a system (through a large load or an interconnector) and measurement of the response of the system. In this case, the probing signal is taken as a system input and the measurements the system outputs. The system response is then calculated from standard input-output system theory. Such methods are discussed as methods for system identification with known input, for example in [2] which provides an overview. These methods are described in section VIII. All the methods in the subsequent sub-sections under "Ambient operation" do not use probing signals.

### *B. Methods that do not require probing - Parametric and non-parametric methods*

Following [1], the methods for ambient operation which do not use a probing signal can be further divided into non-parametric and parametric methods. *Parametric methods* refer to those approaches that aim to determine transfer functions of systems by first selecting and confining the search to a set of possible models while *non-parametric methods* are those that aim to determine the transfer functions by direct techniques [2]. Non-parametric methods tend to work on data to estimate characteristics of the data itself whereas parametric methods tend to work on data to make inferences about the system generating the data. Non-parametric methods are mainly spectral methods and can used to estimate mode frequencies in data. However, they cannot provide system damping information.

#### *1) Non-parametric methods*

The hierarchical classification tree in Figure 1 shows that the well-known Fast Fourier Transform (FFT) [3] and Welch Periodogram methods fall into the category of non-parametric methods. These methods transform a time-domain signal into a frequency-domain function where the signal is decomposed into a set of oscillatory components with a magnitude and phase. Other non-parametric methods in the same branch of the tree are higher order spectral (HOS) methods and basis function decomposition methods. *HOS* methods such as the bispectrum and trispectrum provide information about the magnitude and phase relationships between frequencies in a signal and can be used for mode detection as well as fault monitoring. An example is [4] where bicoherence (the normalized bispectrum) is used for condition monitoring of turbine blades by making use of the phase-coupling characteristics of fault signals. *Basis function decomposition methods* are multivariate analysis techniques that transform an input matrix of data into a different dimensional space where relationships between the different sets of data are more easily observable. An example of such a method is Principal Component Analysis (PCA) which decomposes a set of inputs into a set of weighted uncorrelated orthonormal functions called principal components (PCs); these PCs describe all the variability in the data [5]. Spectral methods have also been used for mode shape and coherency estimation for example in [6] where the mode shapes are estimated using synchrophasor measurements from cross-spectral densities derived from the FFTs of the signals, and in [7] where PCA is used to obtain coherent groups of generators by clustering the weightings of the PCs obtained using simulated speed measurements at the rotors of generators. The reason for mentioning this application is to briefly highlight this branch of research (mode shapes and coherency) which has also been extensively researched in vibration analysis.

*2) Parametric methods*

The other branch of the hierarchical tree under the probing signal classification contains parametric methods. Parametric methods usually make assumptions of the nature of the system inputs that result in the measured outputs by assuming a functional form for the probability distribution functions of the observations [8]. If the nature of the input-generating mechanism is unknown, it is usually assumed to be a random and Gaussian varying-load process hence methods based on this assumption are applicable to estimate the system characteristics that produce the observed output measurements.

Parametric methods can be further classified as recursive or non-recursive methods. This classification refers to the nature of the estimation involved. *Recursive methods* are those that converge to a solution for the model parameters with respect to time where new data is used to update a previously calculated solution, whereas *non-recursive methods* are those that re-calculate a new solution for every set of new data. Therefore, there is a further branching in the tree distinguishing between recursive and non-recursive methods.

*a) Recursive Methods*

The most used recursive methods are adaptive filter techniques of which least squares (LS) algorithms are the ones that have been applied to power systems [9],[10]. LS algorithms estimate a model for a set of data by assuming a functional form of the input probability density function and minimizing a penalty or cost function which is the sum of the square of the differences between the observed values and the ones obtained from the model. They can be described as optimization algorithms in this sense. The general adaptive linear filter algorithm is of the form:

$$y_{filt}(k) = X^{\mathrm{T}}(k)W(k)$$

$$e(k) = y_{obs}(k) - y_{filt}(k)$$

$$\Delta W(k) = \mathrm{f}[e(k)]$$

The modelled uncorrelated system outputs $y_{filt}$ are a weighted function ($W$) of the system inputs $X$ (white noise), where $e$ is a matrix of errors between the modelled outputs and the measured outputs $y_{obs}$ at instant $k$. The weighting function ($W$) is then adjusted (by $\Delta W$) as a function of the cost function (error). The adjustment to the weights is carried out using an optimization algorithm to iteratively search the space of the previous solution, for example using the method of steepest descent, Newton's method or the Gauss-Newton Method. The Least Means Squares (LMS) method uses the Gauss-Newton method and an error-squared cost function in the estimation of the filter weights [11]. The error $e$ associated with the previous estimation can also be used as an estimate of the unknown system input. The LMS algorithm is usually stable and simple to code, but is slow to converge. An evolution of the LMS algorithm is the Recursive Regularized Least Squares (R3LS) algorithm in which an initial state of the filter coefficients is specified and included in the cost function such that the deviation from the initial state is also minimized.

$$\Delta W(k) = \mathrm{q}[W(k) - \hat{\mathrm{W}}] + \lambda \mathrm{f}[e(k)]$$

$q$ is a function representing the confidence in the deviation from the initial guess of filter estimates $\hat{\mathrm{W}}$. Additionally, a forgetting factor, $\lambda$, which reduces the influence of large prediction errors is used to weight the error cost function [9]. The reason for including a forgetting factor is to make the filter less sensitive to small departures from the assumed functional form of the input probability distribution. This algorithm can therefore be applied to a collection of other similar distributions [8]. Such departures can be caused by missing or erroneous data. However, there can be large estimation errors if the initial filter states are wrongly specified. These described techniques are generally well known from system identification [2]. The algorithms are used to estimate autoregressive (AR) model paameters in [9] and [10]. The structure of these models is introduced in the following sub-section including the procedure of obtaining the system mode frequencies and damping values.

*b) Non-Recursive Methods*

Non-recursive methods, as previously defined, calculate a new estimate for every new set of data and discard the previous estimate entirely. The hierarchical tree in Figure 1 shows that these methods can be further sub-divided into *time-domain* methods which use the measurement time series directly and *frequency-domain* methods which use the spectra of the measurements. The hierarchical tree shows that there are two main types of time domain non-recursive methods implemented in literature for the analysis of ambient measurements from power systems: methods based on the Yule Walker (YW) algorithms and Subspace Identification (SSI) methods.

*(i) Time domain methods – Yule Walker*

It is reported in [1] that the earliest implementation of a non-recursive method is the YW implementation of an

Auto-Regressive (AR) model in [12]. The YW algorithm is further extended to estimate an Auto-Regressive Moving Average (ARMA) model in [13]. These models represent the most common implementations of the YW algorithm in system identification literature. The AR and ARMA models are linear input-output models used to describe stationary data. An ARMA model is formed by combining an AR model and a Moving Average (MA) model; it represents a generic form of representing an input-output relationship. An AR model represents the present output measurement as a weighted sum of previous outputs and an uncorrelated noise term, whereas a MA model represents the present output measurement as a weighted sum of present and previous inputs. The inputs are usually assumed to be white noise sequences. The following equation represents an ARMA model [18]:

$$y(k) = -\sum_{i=1}^{m} a_i y(k-i) + \sum_{i=1}^{n} b_i x(k-i) + \varepsilon(k)$$

$a_1$, $a_2$, …, $a_m$ are the coefficients of the $m$-order AR-part of the model, $b_0$, $b_1$, …$b_n$ are the coefficients of the $n$-order MA-part of the model, $x(k)$ are the elements of the input, $y(k)$ are the elements of the observed output and the $\varepsilon(k)$ represents the uncorrelated noise in the output. The AR model is therefore an ARMA model where all the coefficients of the MA model are zero. The AR model is also referred to as an all-pole or infinite impulse response filter. The system poles are obtained by solving the $z$-polynomial characteristic equation comprising the $a$ coefficients of the model.

$$1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_m z^{-m} = 0$$

These poles correspond to the eigenvalues of the power system; hence provide the system mode frequencies and damping values. The coefficients of the model are not easily obtained by least squares estimation; the YW algorithm enables us to obtain these coefficients by expressing the estimation problem as a matrix equation using the estimated autocorrelations of the signals which can easily be solved. However, a main drawback of these methods is that the order of the models ($m$ and $n$) need to be chosen. These values need to be chosen to be high enough to capture all the dynamics of interest as well as noise [13], while ensuring that they are not too large to be computationally inefficient.

A similar YW method is reported in [14], where the authors use ambient measurements from a power system to estimate the autocorrelation functions (ACFs) of the measured system outputs and use an AR algorithm to estimate the system modes. They observed that there is an error introduced from the use of a standard one step predictor model in the presence of signal noise, and therefore extend the analysis to a multi-step predictor solution which they called the interleaved Prony (IP) method. In [15], an AR algorithm is implemented in conjunction with a Kalman filter (KF) to recursively determine the AR model parameters (AR + KF method in the hierarchical tree). The

Kalman filter is a recursive linear filter that can be used to estimate the state of a linear process in the presence of noise [16]. It is however shown in [17] that the estimation of modal damping by this method is only reliable when an excitation is applied to the power system.

*(ii) Time domain methods – SSI*

The second set of commonly used methods shown in the hierarchical tree in Figure 1 consists of methods for Subspace Identification (SSI). SSI methods make use of linear subspaces which are ways of representing data in different planes of reduced dimensionality. It is reported in [19] that in these subspaces, the eigenvectors resulting from noise added to a signal are separated from those of the signals themselves hence making its resolution theoretically independent of the signal-to-noise ratio (SNR). SSI is a method used to estimate the state-space model of a system. A discrete state-space model of a system is of the form:

$$\begin{aligned}\mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{w}(k)\\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k) + \mathbf{v}(k)\end{aligned}$$

At time $k$, $\mathbf{x}(k)$ represents the system states, $\mathbf{y}(k)$ the measurable system outputs, $\mathbf{u}(k)$ the system inputs, $\mathbf{w}(k)$ the process noise, $\mathbf{v}(k)$ the measurement noise, **A** the state transition matrix, **B** the input matrix, **C** the output matrix, and **D** the feed-through matrix. The system dynamics are contained in the state transition matrix **A**, hence the eigenvalues of the system are obtained by eigenvalue analysis of **A**. SSI aims to estimate the **A**, **B**, **C** and **D** matrices by estimating the state vector $\mathbf{x}(k)$ first, using regression analysis to determine the state-space model and finally determining the transfer matrix. This contrasts to the classical methods of system identification that use regression analysis to estimate the transfer matrix, realizing it as a state-space model and finally calculating or predicting the state-vector. The state vector is estimated using reliable numerical algorithms such as Rayleigh Quotient (QR) methods and Singular Value Decomposition (SVD) [20]. It is however assumed that the eigenvalues of the **A** matrix lie within the unit circle, and that the measurement noise, $\mathbf{v}(k)$ is stationary, zero-mean white noise [21]. Some of the SSI algorithms that have been applied to the problem of mode identification under ambient conditions are Canonical Variate Analysis (CVA) [22], the Numerical Algorithm for Subspace State-Space System Identification (N4SID) [23] and the Multivariate Output Error State Space (MOESP) algorithm [24].

*(iii) Frequency-Domain Methods - SOFR*

The right-hand branch of non-recursive methods in the hierarchical tree in Figure 1 shows frequency-domain methods. The first method under this group is Second Order Frequency Regression (SOFR) that was applied in [25]. A least-squares algorithm based on the Newton-Raphson method is applied to estimate the mode frequency and damping utilizing the frequency spectrum of the derivative of the phase angle difference between two phasor measurements. The authors propose that the derivative of the angle difference between the measurements ($y$) during ambient operation can be modelled using a second order equation which is equivalent the representation of the signal computed via a finite Fourier series.

$$\frac{d^2y}{dt^2} + 2\zeta\omega_n \frac{dy}{dt} + \omega_n{}^2 y = \sum_{i=0}^{m/2} A_i \cos\left(\omega_i t + \phi_i\right)$$

$\zeta$ is the damping ratio and $\omega_n$ the undamped natural angular frequency of the interarea mode. The second order decomposition can be compared to the frequency spectrum of the signal via the magnitude $A_i$ and phase $\phi_i$ of the $i$th mode obtained from the FFT. By establishing the relationships between the known values from the FFT, hence finite Fourier series, and the unknown parameters $\zeta$ and $\omega_n$ from the second order model, a least-squares optimization can be carried out. The demonstrated method however requires a large amount of data (spanning almost an hour with an inter-sampling time of 0.1 s) and has a large variance when implemented with actual measured data. The authors propose that this variation can be removed by averaging the estimates obtained over a long time scale (for example 3 hours).

*(iv) Frequency-Domain Methods – YWS and FDD*

Two other recent frequency-domain methods shown in the hierarchical tree in Figure 1 that have been applied for mode identification without probing under ambient conditions are reported in [1]: the Yule-Walker with Spectral Analysis (YWS) algorithm and Frequency Domain Decomposition (FDD). The YWS method is introduced in [26], and it is very similar to the YW method described in the previous section from [12], the only difference being that the autocorrelation functions of the signals are estimated from the spectra of the signals whereby the autocorrelation functions (at lag $\lambda$), $R(\lambda)$, are the inverse Fourier transforms, $\Im^{-1}\{\}$, of the power spectra, $P(\omega)$, which are the squares of the frequency spectra, $X(\omega)$.

$$R(\lambda) = \Im^{-1}\{ P(\omega)\}$$

$$P(\omega) = X(\omega)X^*(\omega)$$

The drawback of the YWS algorithm is the same as that of the YW algorithm: the order of the model needs to be pre-determined.

Unlike YWS, FDD is a purely frequency domain method which involves SVD of the matrix of spectra of the output measurements. SVD is a way of factorizing a matrix into a set of linear approximations which expose the underlying structure of the matrix [27]. FDD is described as the process of decomposing a matrix of spectra of outputs into a function of the spectra of the unknown inputs in [28].

$$G_{yy}(j\omega) = H^*(j\omega)G_{xx}(j\omega)H(j\omega)^T$$

$G_{xx}(j\omega)$ is the estimated power spectral density (PSD) matrix of the $r$ unknown inputs given the PSD matrix

$G_{yy}(j\omega)$ of the $m$ responses where $H(j\omega)$ is the $m\times r$ matrix of frequency response functions (FRFs) which map the estimated inputs to the measured outputs. $H(j\omega)^T$ and $H^*(j\omega)$ represent the transpose and complex conjugate of $H(j\omega)$ respectively. The FRFs consist of singular vectors whose corresponding singular values are the power spectral densities of the equivalent single degree of freedom systems. The power spectral densities are then taken back to the time domain via an inverse Fourier transform which are analyzed using the logarithmic decrement method to obtain the mode natural frequencies and damping values [29]. This method is applied to ambient measurements from a power system in [30]. The FDD is an efficient and reliable technique. However, the number of singular values formed during SVD can be large and a threshold is required to determine the dominant values. This can be difficult to automate. Additionally, there are some errors introduced by (i) truncation of the Fourier series and (ii) the phenomenon of spectral leakage due to the use of finite data lengths: these cause the damping to be over-estimated and under-estimated respectively [29].

### *C. Observations and Comments*

The above overview of the methods that have currently been applied to the problem of system identification without probing shows that parametric methods are dominant. The key steps in applying parametric methods are:

- Estimation of the probability density function (PDF) or nature of the input excitation, which is assumed to be random and Gaussian.
- Choice of a system model that can be represented as a stochastic process.
- Estimation of system model parameters.
- Inferring the system eigenvalues hence mode frequencies and damping.

Of the methods described in this section, the time-domain parametric methods have the advantage that they are able to use models that relate more directly to the underlying system structure. However, an important observation to note in all the described implementations of parametric methods is that the underpinning assumption is that the system is driven by white noise. However, there is no concrete practical evidence in the literature to establish this theoretical assumption.

## IV. Electrical Power Systems – Transient Operation

Transient operation as previously defined refers to the response of a power system after a fault has occurred or a large disturbance has been initiated. This operation is characterised by large deviations in measurable system parameters such as frequency. It is assumed that the post-fault/post-disturbance transient represents the true impulse response of the systems hence the aim of transient analysis is to measure the stability of the system by determining the oscillatory frequency and damping of the transient. Methods in the literature are sub-divided according to the assumption of linearity of the system during such excursions. The hierarchical tree in Figure 1 therefore branches into *linear* and *non-linear* methods.

### *A. Linear Methods*

Linear methods assume that the system maintains linearity after the fault or disturbance and they aim to fit a model of a sum of decaying sinusoids to the transient data. The measured output $y(t)$ is composed of a weighted sum of $n$ decaying sinusoids $\lambda_i$ (with weights $B_i$) where $\lambda_i$ can be decomposed into a frequency component $\omega_i$ and a exponential decay component $\alpha_i$.

$$y(t) = \sum_{i=1}^{n} B_i e^{\lambda_i t}$$

The differences between the methods result from the approach taken to fit this model. The hierarchical tree in Figure 1 is further subdivided into *time-domain* methods and *frequency-domain* methods.

#### *1) Time-Domain Methods*

These methods fit a linear model to data by analysing the time-series of the data. The hierarchical tree shows four different time-domain methods: Prony methods, the Matrix-Pencil (MP) method, the Eigenvalue Realisation Algorithm (ERA) and the Hankel Total Least Squares (HTLS) method. The methods described below are univariate and several of them make use of a Hankel matrix, a square matrix which includes the signal and time-shifted versions of the signal giving the form below where $k$ is the sampling time and $\ell$ is a delay.

$$\begin{pmatrix} y(k) & y(k+\ell) & y(k+2\ell) & y(k+3\ell) \\ y(k+\ell) & y(k+2\ell) & y(k+3\ell) & y(k+4\ell) \\ y(k+2\ell) & y(k+3\ell) & y(k+4\ell) & y(k+5\ell) \\ y(k+3\ell) & y(k+4\ell) & y(k+5\ell) & y(k+6\ell) \end{pmatrix}$$

##### *a) Prony Methods*

Prony methods are those that utilise Prony analysis to determine the linear model described previously. Prony analysis is a two-step process that involves creating a linear prediction model (LPM) of the data and having obtained the coefficients of the model, determining the eigenvalues of the system by evaluating the roots of the least-squares polynomial created by the LPM. This is quite similar to the AR model introduced in the previous section for ambient methods. Prony methods suffer from problems evaluating damping of closely-spaced modes. Despite this, they are widely used for transient analysis in power systems. Examples are available in [31]-[34].

*b)* *MP*

The Matrix-Pencil method makes use of functions known as pencil functions. Pencil functions represent the system matrix as a function of two known matrices ($\mathbf{Y}_1$ and $\mathbf{Y}_2$) such that the system eigenvalues $\lambda$ are the same as the eigenvalues of the function $\mathbf{Y}_2 - \lambda\mathbf{Y}_1$. It is introduced in [36] as a one-step process to obtain system modes and damping and as an alternative to Prony analysis. The MP method involves the SVD of a matrix of time-shifted output measurements where the length of the rows and columns of the matrix depends on the length of the time-shift which is itself dependent on the number of dominant modes to be estimated and a parameter termed the pencil parameter. After SVD, the resulting singular matrices are truncated to the number of dominant modes required, and by carrying out two different inverse SVD operations on these truncated matrices, the pencil functions $\mathbf{Y}_1$ and $\mathbf{Y}_2$ can be obtained, and hence the system eigenvalues. The MP method has the advantage of being more robust to noise than Prony methods and is demonstrated in [37].

*c)* *ERA*

The Eigenvalue Realisation Algorithm was introduced in [38] and involves the singular value decomposition (SVD) of the Hankel matrix of measured outputs to obtain the reduced state-space model of the system. The state-transition matrix is then analysed to obtain the system modes and damping. Variants of this method are applied to power systems in [39]-[41]. In [40], the numerical characteristics of the method are improved by using the Hankel matrix and its transpose to obtain a symmetrical matrix which is then analysed by partial SVD.

*d)* *HTLS*

The Hankel Total Least Squares method begins with SVD of the Hankel matrix of outputs just as the ERA method but the rest of the method is different. In the HTLS method, the eigenvalues of the system are taken to be the same of the eigenvalues of a matrix $\mathbf{Q}$ obtained by the least-squares solution of the equation:

$$\mathbf{U}_1\mathbf{Q} = \mathbf{U}_2$$

where $\mathbf{U}_1$ and $\mathbf{U}_2$ are matrices formed by omitting the first and last rows respectively of the unitary matrix obtained from the SVD step [42]. This method is applied to power systems in [41].

*2)* *Frequency Domain Methods*

The second group of linear methods for transient analysis presented in the hierarchical tree is frequency-domain methods. These methods fit a linear model to data by analysing the frequency spectra of the data. There are two methods presented under this heading in the hierarchical tree: frequency domain identification (fID) and z-transform identification (zID). fID is introduced in [43] and uses the FFT of a sliding window of data to obtain the relative amplitude of modes in the signal. This relative amplitude corresponds to the damping ratio of the mode. zID on the other hand performs the search for system poles in the z-domain using estimations obtained from the FFT of the signal. This is carried out by searching special z-plane contours for positions where the evaluated frequency spectrum from the estimations match that obtained from the measurements via conditions of magnitude maximality and phase reversal. This method is due to Corinthios [44] and is applied to power systems in [45].

### *B. Non-linear Methods*

The second group of transient methods in the hierarchical tree in Figure 1 comprises non-linear methods. Non-linear methods recognise that power systems are inherently non-linear to an extent and assume that the post-fault or post-disturbance response is mainly non-linear. In this case, non-linearity refers to interactions between the frequency components in the transient response. This interaction can be observed using Higher Order Spectral (HOS) analysis, for example in [35]. Therefore, non-linear methods aim to track the evolution of frequency and damping over short intervals of time. The non-linear methods that have been reviewed in the literature can be classified further into parametric and non-parametric methods. In this context, parametric methods provide specific values for the damping of modes while non-parametric methods do not.

#### *1) Non-parametric methods*

The methods presented under non-parametric methods in the hierarchical tree in Figure 1 are frequency domain pattern recognition (FDPR) and the Short Time Fourier Transform (STFT). The FDPR and STFT methods are based on the observation that undamped oscillations are related to excess kinetic energy in the power system which in turn can be related to the peaks in the power spectral densities (PSD) obtained from the output measurements. The peak in the PSD shifts towards 0 Hz as the oscillation becomes less damped. These methods are applied to power systems in [46] and [47] respectively. The FDPR method in [46] used Artificial Neural Networks (ANNs) to track the movement of the peaks in the frequency spectra. In the STFT method in [47], the peaks are tracked by measuring the rate of change of the maximum peak over time and by using a threshold to determine whether the level of damping has gone below a critical value. However, in both these methods, an exact value of damping is not obtained.

#### *2) Parametric methods*

The final group of methods in this part of the hierarchical tree is parametric methods. These methods determine instantaneous values of frequency and damping within the analyzed window of data. Two main types of such methods have been implemented in power systems: wavelets and Hilbert Analysis. These methods are reviewed and compared in [48].

*a) Wavelets*

Wavelet methods decompose signals as a function of a mother wavelet which is chosen prior to application of the method, and can provide good time-frequency resolution. The mother wavelet can be described as an oscillating sinusoid that exists in a small finite time period. This instantaneous time-localization property implies that it can be used to represent non-stationary and nonlinear signals. Wavelet analysis works by comparing a wavelet with the signal being analyzed and then defining a coefficient which is high if the wavelet looks like the signal or low otherwise. By comparing the signal with various wavelets over different time intervals, the signal can be decomposed as a weighted sum of the damped wavelets over different time periods. This leads to the concept of instantaneous frequency and damping.

*b) Hilbert Analysis*

Hilbert Analysis or the Hilbert Huang Transform (HHT) is a two-step procedure for the evaluation of the instantaneous time-frequency characteristics of signals. In the first step, the measured signal is decomposed into *intrinsic mode functions* (IMFs) by a process known as empirical mode decomposition (EMD). Each IMF represents a simple oscillatory source that is both amplitude and frequency modulated. The second step involves determining the analytic form of each IMF via the Hilbert transform. The Hilbert transform describes the extracted IMFs in terms of instantaneous amplitude, frequency and phase functions [48]. There have been various improvements made on the HHT, mainly to improve the IMF extraction via EMD and applications of these variants to power systems are described in [49]-[52].

### *C. Observations and Comments*

Methods for transient analysis are applied to analyse the post-fault or post-disturbance response of a system. The steps involved in the application of these methods are:

- Assumption that the post-fault or post-disturbance response represents the true system frequency and damping.
- Choice of a linear or non-linear method for analysis.
- Estimation of frequency and damping.

## V. VIBRATION ANALYSIS – OVERVIEW

There are many methods in structural, mechanical and aeronautical engineering which have similar approaches to those being used for power system analysis described in sections III and IV. These methods fall under a general category labelled as vibration analysis and they are listed in the classification tree of Figure 3 with relevant references supplied in the diagram.

The main analogies between vibration analysis and power systems analysis are:

- Vibrating structures are continuously excited by random inputs just as power networks are continuously loaded.
- The input signal cannot be measured in most cases.
- Model structures chosen for identification are similar.
- Damping information is very important in order to predict failure.

There are also some fundamental differences:

- The concept of transient operation is usually not defined because excitations to structures rarely take the form of large disturbances.
- Instead, a distinction is made between stationary and non-stationary methods.

Figure 4 demonstrates an example of a system response under stationary and non-stationary vibration analysis. The figure shows measurements at the onset of an earthquake where the response is initially stationary but becomes non-stationary. *Stationary* methods are those that assume that the system is linear and time-invariant whereas *non-stationary* methods are those that assume that the system in time-variant as a result of non-linearity. Sections VI and VII describe a brief selection of the methods from vibration analysis that may have potential for being applied in power systems analysis. They describe the methods, speculate about the problems which they may be able to solve in electrical power systems and explain why they might be useful. The first of the sections describes the classification of stationary methods while the second describes the classification of methods for non-stationary methods.

## VI. Vibration Analysis – Stationary methods

Stationary methods for vibration analysis can be broadly split into methods that use both excitations and responses and those that use responses only [53].

### A. *Excitation and Response Methods*

These methods are similar to the methods discussed in the section for ambient operation of power systems that require probing. However in practice, it can be difficult to measure the excitations on the structure unless the excitation is known and quantifiable, for example tidal impact on the bases of bridges that can be measured using sensors. For such cases, standard input-output methods of system identification can be applied. These methods are available in [2] and are reviewed in section VIII as methods for system identification with known input.

### *B. Response-Only Methods*

A review of response-only methods that have been applied to system identification of vibrating structures is presented in [54]. In [54], the initial classification of methods is between time-domain and frequency domain methods; we will however make an initial classification similar to the one presented for methods that do not require probing under ambient operations in electrical power systems. The hierarchical classification tree shown in Figure 3is hence split into parametric and non-parametric methods.

#### *1) Non-parametric methods*

The main non-parametric method that has been applied for mode frequency estimation is the Peak Picking (PP) method. In the PP method, the eigenfrequencies of the system are obtained by identifying the peaks in the spectra of the measured outputs [55]. This is very similar to the Welch Periodogram method since the main algorithm is the FFT. It is reported in [54] that the half-power bandwidth method is commonly used to obtain the mode damping in the PP method. However, it is recognised that this extension of the PP method into a parametric approach does not yield reliable estimates. The PP method also has limitations that the system modes should be lightly damped and well separated in the spectral plots. Its accuracy is also diminished in the presence of noise. Variants of the PP method are also reported such as methods that use coherence functions. Such methods perform better under low SNRs and are similar to higher order spectral methods presented in the electrical power systems classification tree.

#### *2) Parametric methods*

The other branch in the hierarchical tree (Figure 3) under response-only methods contains parametric methods. *Parametric methods* aim to fit a model onto measurements of outputs by making assumptions about the inputs of the models. It is difficult to estimate the inputs or excitations applied to a structure and hence it is usually assumed that the excitations during normal use of a structure are random and realisations of a stochastic process. Based on this assumption, a system model can be identified. Parametric methods can be classified with reference to the type of model that is chosen to represent the system. The hierarchical tree shows that there is further branching into time-domain models, state-space models and frequency-domain models. The classification refers to the domain in which the system parameters are identified. *Time-domain models* represent the system as a realisation of present and past inputs, and past outputs. *Frequency-domain models* identify system parameters from the spectral representation of the system outputs. Finally, *state-space models* identify the system parameters by computing the state-space representation of an equivalent system with the measured outputs as its outputs.

*a) Time-Domain Methods*

Three methods are presented in the hierarchical tree in Figure 3 under time-domain models. These methods are the Autoregressive (AR), Autoregressive Moving Average (ARMA) and Random Decrement (RD) methods.

*(i) AR and ARMA models*

AR and ARMA models have already been described as Yule-Walker (YW) methods for time-domain non-recursive implementations of parametric methods that do not require probing in ambient analysis of electrical power systems. The AR and ARMA model parameters can similarly be estimated using Prediction Error Methods (PEMs). The AR model is easily solved using such methods and has been applied in [57]. However, in the case of ARMA models, this procedure leads to a non-linear optimisation problem due to the MA parameters. The method is additionally not robust when using real-life data. Despite these drawbacks, ARMA methods using PEMs have found extensive application in civil engineering, for example in [58] and [59]. ARMA models have alternatively been solved for vibrating structures using a group of methods known as Instrumental Variable (IV) methods. A general explanation of IV methods is presented in section VIII as a parametric correlation approach; the basic principle is that the model parameters are obtained by minimizing the correlation between past data and the errors between the model and measured outputs; this follows from the hypothesis that the errors should be completely uncorrelated with past data if the model parameters perfectly describe the system. Imposing such conditions effectively reduces the ARMA model into an AR model. Examples of IV methods that have been applied to vibration analysis are the Ibrahim Time Domain (ITD) and the Least Squares Complex Exponential (LSCE) methods [57].

*(ii) Random Decrement*

The final method presented in this part of the hierarchical tree is the Random Decrement (RD) method. The RD method assumes that the system input can be decomposed into a series of steps, impulses and a random component. The measured system output is therefore composed of the responses due to each of the components. By averaging the mean-centred measured outputs subject to a threshold condition, the responses due to the random and step components average to zero leaving an estimate of the impulse response [60]. One drawback of this method is that this decomposition is only strictly valid in the case of a stochastic input, however mathematical proof in [61] shows that the RD can be used to estimate correlation functions when the input is not strictly stochastic.

*b) Frequency-Domain Methods*

The middle branch in the hierarchical classification tree in Figure 3 shows two parametric response-only frequency-domain methods. The first of these methods is the Complex Mode Identification Function (CMIF). The CMIF is a parametric extension of the non-parametric PP method. This is achieved by diagonalisation of the spectral density matrix from the PP method by SVD [62]. In [54], this decomposition is interpreted as separation of the system response into equivalent single-degree-of-freedom (SDOF) systems which can be analysed using modal techniques to yield the system eigenfrequencies and corresponding damping. In [28], the method was reintroduced as the Frequency-Domain Decomposition (FDD) method. This FDD method has recently been applied to electrical power systems in [30]. The advantages and disadvantages of the method have also been discussed in the section describing the FDD approach that was applied to electrical power systems.

The second frequency-domain method in the hierarchical tree is the Maximum Likelihood (ML) method. The ML method is an optimization technique that aims to optimize the parameters of a model by minimizing the error norm between the output of the model and measurements made from the system. It is therefore a realization of a Prediction Error Method (PEM). It has been applied in structural engineering in the frequency domain in [63] in order to make it applicable to the problem of output-only identification. In [63], the method is used to identify the parameters of a model that is a function of the ratio between the output and unknown input spectra. Similar to the ARMA time-domain model method, the ML method leads to a set of nonlinear equations which require an iterative solution and can hence be computationally demanding. It however has been shown to be a robust method for identification of modal parameters from large noisy data sets, unlike the ARMA model.

*c) State-Space Models*

The final branch of parametric methods in the hierarchical tree shows state-space models. State-space models have been introduced in the review of methods for ambient operation in electrical power systems. One technique that can be used to obtain state-space models is Subspace Identification (SSI). Publications in vibration analysis have used singular value decomposition (SVD) of Hankel or Toeplitz matrices of outputs followed by truncation to a defined number of modes. Using the SVD representation, the parameters of the state-space model are estimated [54]. In vibration analysis, SSI methods can be further divided based on the nature of the approach taken to solve the parameters of the model. The hierarchical tree in Figure 3 hence branches into covariance-driven and data-driven approaches. D*ata-driven approaches* work directly on output data to estimate the state-space model while *covariance-driven approaches* first estimate the output covariances before the derivation of the state-space model. The data-driven SSI-DAT method uses the measured outputs in the matrices while the covariance-driven SSI-COV uses estimations of the output covariances in the matrices. An alternative name for SSI-COV presented in literature is the Matrix Block Hankel Stochastic Realization Algorithm (MBHSRA) [64]. There are also variants of the SSI-DAT and SSI-COV methods which are realized by weighting the matrices prior to SVD. Examples of such variants are Canonical Variate Analysis (CVA), Principal Components (PC) and Balanced Realization (BR) [65]. These methods are applicable to both the data- and covariance- driven approaches and are hence placed directly under methods for state-space models in the hierarchical tree in Figure 3. The main shortcoming of SSI methods is that it is assumed that the outputs are realizations of a stochastic process and hence if this is not truly the case, an error is introduced in the estimation of the state-space model. The methods of SSI that have been used for ambient analysis of power systems are all data-driven, providing future opportunities for the application of some covariance-driven SSI methods.

### *C. Observations and Comments*

The key steps in applying stationary response-only methods for system identification are:

- Assumption of stochastic excitation of a linear time-invariant system.
- Choice of a suitable input-output model.
- Finding a stochastic output-only realization of model.
- Estimation of system model parameters.
- Inferring the system eigenvalues hence mode frequencies and damping.

It can be noted that the key steps taken in the application of response-only stationary methods are similar to those presented for the application of parametric methods of analysis of power systems under ambient conditions without probing. In fact, many of the methods that have been reviewed in this section have previously been reviewed in section III. However, this review highlights some promising approaches that have not yet been applied to power systems including ARMA models solved using IV methods, the RD method and variants of the SSI method using Principal Components (PCs). They are simple and robust approaches which can be applied in real-time: ARMA models solved using IV methods lead to a linear solution compared to those solved using PEMs, the RD method is very easy to apply because it only requires the operation of averaging, and SSI based on PCs can help in screening measured signals to determine the best measurements for model identification.

## VII. Vibration Analysis – Non-Stationary methods

Non-stationary methods are those that assume that the system is time-variant as a result of non-linearity. This can be the case in rotating machinery or during earthquake vibration analysis, as well as non-linearity due to non-stationary stochastic excitation on structures. Hence, the statistical properties of the measured signal, for example mean and variance, vary with time. Thus, these methods aim to identify time-varying parameters of models. In literature, non-stationary methods are generally divided into parametric and non-parametric methods [66], whose definitions are similar to the ones described previously. The hierarchical tree in Figure 3 therefore branches into these two groups of methods.

### *A. Non-Parametric Methods*

Non-stationary non-parametric methods decompose a measured response into a representation localized in frequency and/or time. A review of these methods is presented in great detail in [67] therefore only a brief introduction is presented in this section. Following [67], the branch of non-parametric methods in the hierarchical tree can be further divided into linear and quadratic forms. This classification refers to the nature of the decomposition of the signal.

#### *1) Linear Forms*

*Linear forms* decompose a signal into a series of components that can be added up to yield the original form. Examples of such methods are Evolutionary Spectra (ES) [68],[69], the Short Time Fourier Transform (STFT) [70], the Gabor Transform (GT) [71],[72] and the Wavelet Transform (WT) [70],[73]. The STFT is a time-varying periodogram obtained by sliding a window across a time record and performing a Fourier transform where the magnitude of the periodogram is plotted as a function of frequency and time. ES and the GT extend this idea to time-varying processes: ES decompose a signal as a function of time-modulated sines and cosines, while the GT decomposes a signal as a function of Gaussian pulse modulated sines and cosines [67]. However these representations do not provide good localizing properties in the time and frequency domain; this is instead provided by the WT [73]. Wavelet methods have been previously described in the section of parametric non-linear methods for transient analysis of power systems.

#### *2) Quadratic Forms*

*Quadratic forms* decompose the energy function of the signal rather than the signal itself. The decomposition involves a time-dependent spectral density related to the weighted local autocorrelation function of the signal. Examples of the weighting functions are Wigner-Ville [74],[75] and Cohen distributions [76],[77]. The details of these methods are presented in greater detail in [67].

### *B. Parametric Methods*

Non-stationary non-parametric methods do not lead directly to the model parameters required for stability inference, such as system damping; non-stationary parametric methods aim to identify the models of systems whose outputs have been measured, but unlike the stationary parametric models, the parameters of the identified model are time-varying. The methods that have therefore been applied in vibration analysis are time-varying extensions of stationary time-domain and state-space parametric models. The hierarchical tree in Figure 3 is therefore divided into

Time-varying Autoregressive Moving Average (TARMA) and Time-varying State Space (TSS) models.

*1) Time-varying Autoregressive Moving Average (TARMA) models*

TARMA models generalize all time-varying time-domain models including AR, ARMA and ARMAX models. They are reviewed extensively in [78] hence only a brief summary is provided here. Following [78], they can be further divided with reference to the structure of evolution of the time-varying model parameters. The hierarchical tree in Figure 3 therefore has three further branches, as outlined in the next three sub-sub-sections.

*a) Unstructured Parameter Evolution Methods*

These methods do not impose a particular structure upon the evolution of the time-varying model parameters [78]. They therefore can only track slow dynamics and have the highest complexity of the three groups of methods. Examples of these methods are the Short-Time ARMA (ST-ARMA) [79],[80] and recursive methods such as Recursive Maximum Likelihood TARMA (RML-TARMA) [81],[82].

*b) Stochastic Parameter Evolution Methods*

These methods impose a stochastic structure upon the evolution of the time-varying model parameters via stochastic smoothness constraints and are also referred to as Smoothness Priors TARMA (SP-TARMA) models [78]. These methods have been mainly reported for the modelling of earthquake ground motions [83],[84]. They can track slow to medium dynamics but are similarly as complex as unstructured parameter evolution methods.

*c) Deterministic Parameter Evolution Methods*

This is the final group of TARMA methods; they impose a deterministic structure upon the evolution of the time-varying model parameters. Examples of these methods are Functional Series TAR (FS-TAR) and TARMA (FS-TARMA) models. These methods have been applied in various fields including modelling of bridge-like structures [85] and earthquake ground motions [86]. They can be solved via algorithms such as two-stage Least Squares (2SLS) [87], Polynomial-Algebraic (PA) [88] and Recursive Extended Least Squares (RELS) [89]. These methods are the least complex of the TARMA models and can track slow, medium and fast dynamics of the system.

*2) Time-varying State Space (TSS) models*

The left-hand branch of non-stationary parametric methods for vibration analysis consists of time-varying State-Space models. As in the case of TARMA models, they are the counterparts of the stationary case parametric state-space models with time-varying parameters. TSS models can be further divided into the groups of methods reviewed in TARMA models; however, the research literature available for output-only methods is confined to unstructured parameter evolution methods such as Short-Time Time-varying Subspace (ST-TSS) [90] and Recursive Time-varying Subspace (R-TSS) methods [91].

### C. Observations and Comments

Non-stationary methods for vibration analysis present a possible direction for the application to power systems. They work on the principle of estimation of time-varying parameters. The non-parametric methods decompose signals into representations localized in frequency and/or time. This approach is similar to the ones for transient operation in power systems such as the Hilbert Huang Transform (HHT) that decomposes a signal into an instantaneous amplitude and phase which can be translated into an instantaneous frequency and damping. However, these methods are only capable of providing system damping in the case when the measured output contains a transient. On the other hand, methods that can obtain system damping in non-linear situations where operation is still defined as ambient would be beneficial. Non-stationary parametric methods for vibration analysis may be able to fulfil this purpose. The main difference between these methods and the methods applied for ambient conditions in power systems is the assumption of linear operation in the latter group: non-stationary methods would be more robust owing to the time-dependent parameters of models; however, care needs to be taken in ensuring that real-time identification can be achieved.

## VIII. Methods For System Identification With Known Inputs

The final review section presented in this paper contains methods for system identification with known inputs. These methods have been identified as methods for mode estimation in power systems under ambient operation when the system has been probed, and also as stationary methods for vibration analysis when both the excitation and response have been measured. These methods are standard methods for system identification and are reviewed in depth in [2]. The hierarchical tree in Figure 5 classifies these methods. The methods are again initially divided into *parametric* and *non-parametric* methods. The definitions of the two are the same as those discussed in previous sections of this paper.

### a) Non-parametric methods

Non-parametric methods are further divided into *time-domain methods* and *frequency-domain methods*, whose definitions have been described in an earlier section. Of the time-domain methods described in [2], the only one applicable to ambient measurements (without any impulse or step excitation) is Correlation Analysis: the system output is obtained by the convolution of the measured system input and system impulse response. Hence, the cross-correlation of the system input and output $R_{yu}(\tau)$ is equivalent to the convolution of the estimate of the system impulse response $g(k)$ and the input autocorrelation function $R_u(\tau)$. The system frequency and damping of a mode can then be obtained by analyzing the estimated impulse response.

$$\hat{R}_{yu}^{N}(\tau) = \sum_{k=1}^{M} \hat{g}(k)\hat{R}_{u}^{N}(k-\tau)$$

If the input were a noise sequence, an estimation of the output autocorrelation would hence be an estimate of the system impulse response. This is the assumption used to obtain the system response prior to the application of the parametric method in [14].

The other applicable methods shown in Figure 5 are frequency domain methods: Empirical Transfer Function Estimation (ETFE) and Spectral Analysis. ETFE makes use of the relationship between the system input and output in a frequency transfer function. Since convolution corresponds to multiplication in the frequency domain, the system output spectrum $Y(j\omega)$ is simply the product of the spectrum of the system response $G(j\omega)$ and that of the input $U(j\omega)$. Hence, using this relationship, the system frequency response can be estimated.

$$\hat{G}(j\omega) = \frac{Y(j\omega)}{U(j\omega)}$$

However, [2] remarks that the estimates of the system response estimated in this way are very crude and that the variance of the estimate does not decrease with the length of the sampled signals. This is because the method stipulates that the estimates at different frequencies are uncorrelated. In order to improve the estimates, it is assumed that the values of the true frequency response function at various frequencies are related. This is the basis of spectral analysis methods which aim to smooth the ETFE estimate using a weighting function or window in the vicinity of the system frequencies, for example the Blackman-Tukey periodogram.

*b) Parametric methods*

The second branch of methods that require probing in the hierarchical tree are parametric methods. Parametric approaches are divided into Prediction-Error Identification Methods (PEMs) and Correlation Approaches (CAs).

*(i) Prediction-Error Identification Methods (PEMs)*

PEMs aim to minimize the prediction error: a function of the error between the outputs of the constructed model and the measured system outputs In PEMs, the prediction error is usually represented as a function of the actual error, in most cases a quadratic norm which is the sum of the square of the errors. A norm is a function that assigns a positive value to a measure. Two examples of such methods are Least Square Error (LSE) methods and Likelihood Estimators for example the Maximum Likelihood (ML) method. LSE models try to minimize the error obtained by applying a linear regression to data. A linear regression model employs a linear predictor 0which is of the form:

$$\hat{y}(t, \theta) = \varphi^T(t)\theta + \mu(t)$$

$\hat{y}(t, \theta)$ is the modelled output which is a function of the vector of linear regressors $\varphi(t)$, the parameter vector $\theta$ and noise $\mu(t)$ at time $t$. The prediction error is therefore:

$$\varepsilon(t, \theta) = y(t) - \varphi^T(t)\theta$$

The aim is then to minimize the quadratic norm of this error. Examples of variants of these methods are LS, RLS, Robust LS, R3LS, AR and ARMA methods which have been described in the section of parametric methods that do not require probing. Another PEM is the ML estimator which is a statistical approach that aims to recreate the probability density function (PDF) of the observations. It however requires the user to know the PDF of the observations. The maximum entropy approach is a method that searches for a PDF model that minimizes the information distance to the true system. Details of these methods are available in [2].

*(ii) Correlation Approaches (CAs)*

CAs aim to minimize the correlation between the prediction error and past data because an "ideal" model should have an error completely independent of past data [2]. The best known CAs are Instrumental Variable (IV) methods. IV methods try to estimate the parameters of a model that minimize the correlation between the prediction errors and a finite-dimensional vector sequence derived from the past data whose elements are termed instrumental variables. In IV methods, a linear regression is performed on data with the conditions that the IVs $\zeta(t)$ are correlated to the regression variables $\varphi(t)$ but uncorrelated with the noise (hence the residuals) $v_0(t)$. These conditions can be expressed as expectation equations, $E\{\}$.

$$E\{\zeta(t)\, \varphi^T(t)\} \text{ is non-singular}$$

$$E\{\zeta(t)\, v_0(t)\} = 0$$

The IVs are usually then obtained from the past inputs by applying linear filtering; criteria for selection of the IVs are presented in detail in [2].

## IX. Summary And Discussion

The main challenge facing researchers in electrical power systems is the problem of system identification in the presence of unknown inputs. This is especially a challenge in monitoring the normal operation of power networks. As power systems are pushed towards their limits, oscillations are typically lightly damped and can become unstable, hence they need to be monitored in real-time. However during normal operation, load changes cause random fluctuations in the power transfers and other system measurement variables. These load fluctuations cannot be measured from all over the system hence the use of standard input-output models is limited. This review has shown that a similar problem is faced in the fields of vibration analysis where the monitoring of structures that are continuously in use is necessary to detect failure. An example is in structural engineering: structures deteriorate over time and are exposed to damage from external forces such as wind, hence need to be continuously monitored. However, manual testing of structures via excitation-response measurements is not practical lest they suffer more damage, and since structures are continuously in use, they need to be assessed from measurements of the vibrations only. The nature of the research problem in power systems monitoring and these fields is therefore similar.

Research in all these fields has shown that there are stochastic output-only equivalents for most of the standard input-output methods. These methods have been developed by making assumptions of the nature of the unknown system inputs. The implementations of the methods are referred to as stochastic realizations of input-output methods in vibration analysis whereas in electrical engineering, most methods assume that the system input consists of noise that is random and Gaussian. Furthermore, the nature of responses in power systems and vibration analysis are similar: there is more than one frequency present for several cycles of the system oscillation making the subject of multi-swing stability analysis equally applicable. The mathematical fundamentals involved are similar since they can both be modeled as systems with multiple degrees of freedom. The research approaches are therefore also similar. The reviews of methods for analysis under ambient conditions from electrical power systems and stationary methods of vibration analysis show that the approaches that have been implemented are similar, for example AR and ARMA time-domain models, state-space models and frequency-domain models. The methodologies for solving the parameters of the models are also similar for example Least Squares (LS) methods. The review has also shown that some methods for transient analysis in power systems are similar to non-parametric non-stationary methods for vibration analysis. Methods from vibration analysis are promising to become important algorithms for electrical power applications of the future having already found application in the detection and quantification of wide area modes: an example is the Frequency Domain Decomposition (FDD) method which was initially applied in vibration analysis [28] and has also been applied in power systems [30].

However, research on vibration analysis has been established in structural engineering for decades and hence there is a wide pool of methods available that can be applied to the problem of power system monitoring in the electrical engineering field, providing more opportunities for cross-fertilisation. This paper has made some suggestions of promising methods that can be applied to power systems: stationary methods such as Autoregressive Moving Average (ARMA) models solved using Instrumental Variable (IV) methods, and Subspace Identification (SSI) aided by Principal Components (PCs), as well as non-stationary parametric methods like deterministic parameter evolution Time Varying ARMA (TARMA) models.

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**List of Figures:**

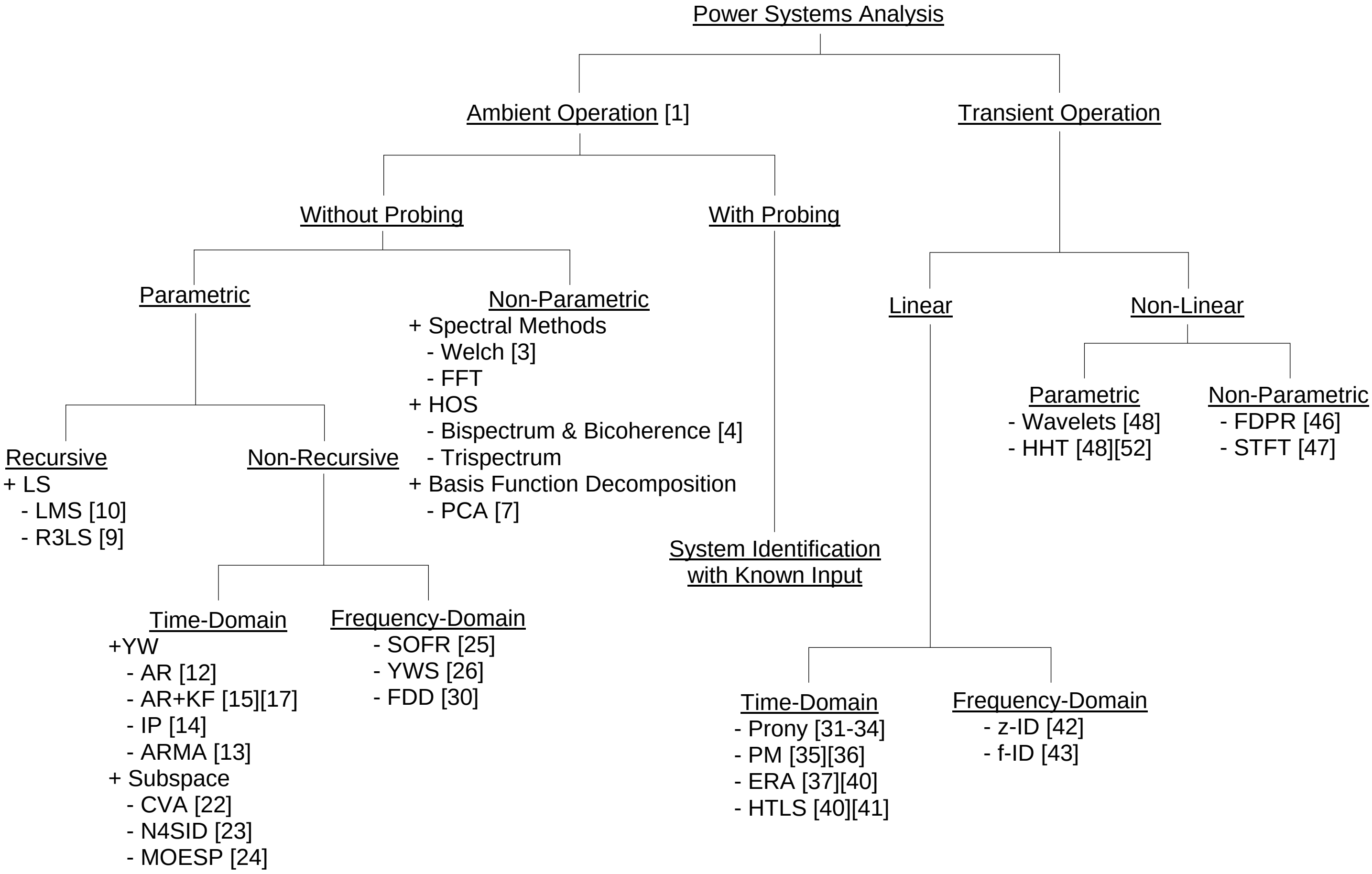


Figure 1: Hierarchical tree showing classification of methods for power systems engineering.

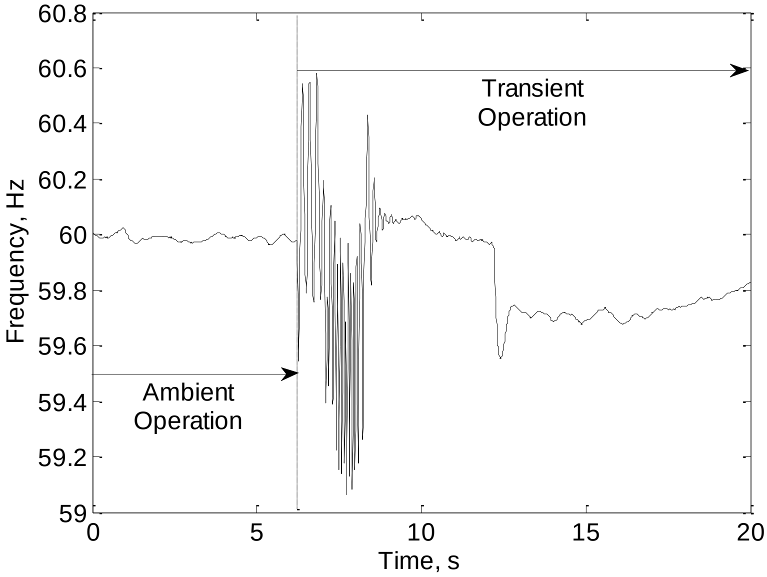

Figure 2: Ambient and transient operation in power systems.

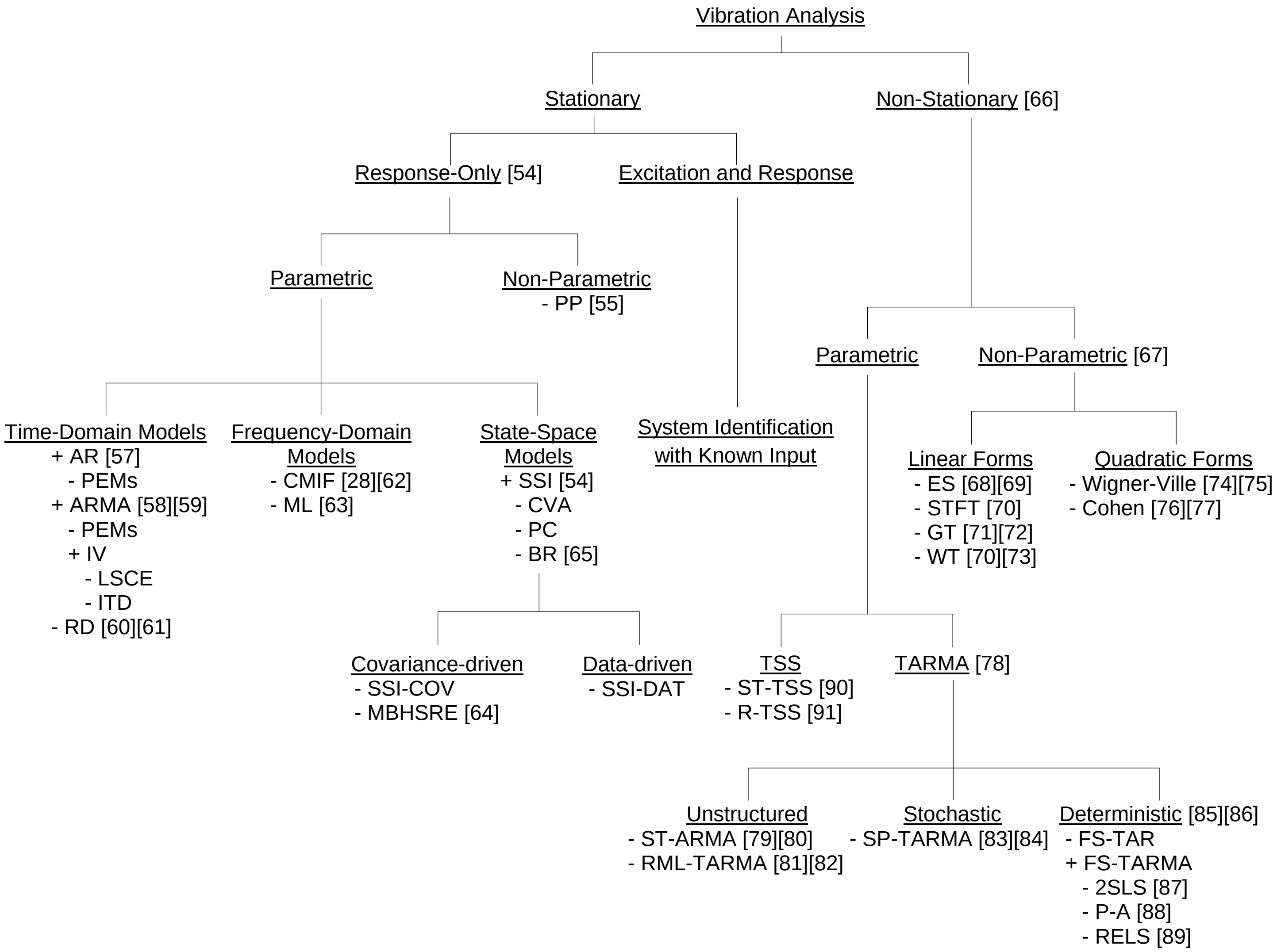


Figure 3: Hierarchical tree showing classification of methods for vibration analysis.

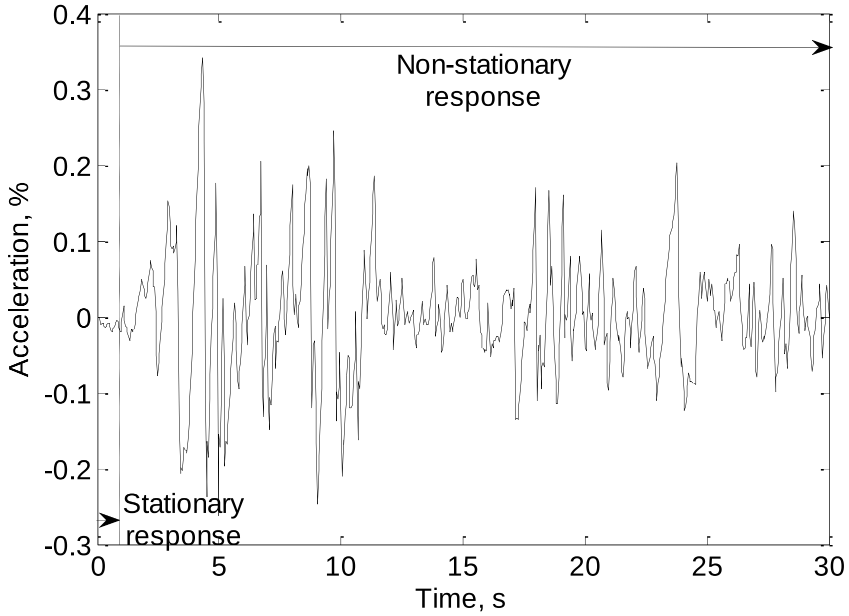

Figure 4: Stationary and non-stationary response in vibration analysis. After

http://www.owlnet.rice.edu/~elec532/PROJECTS00/earthquake/earthquakes.htm, accessed December 2009.

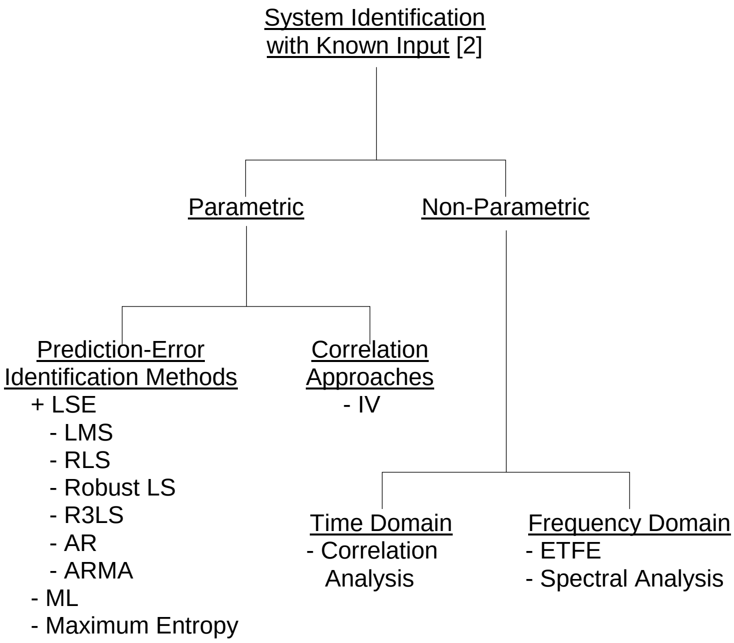


Figure 5: Hierarchical tree showing classification of methods for system identification with known input.