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ORIGINAL ARTICLE

Comparative study of chained systems theory and fuzzy logic as a solution for the nonlinear lateral control of a road vehicle

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Abstract This paper presents a comparative study of different lateral controllers applied to the autonomous steering of automobiles. The nonlinear nature of vehicle dynamics makes it a challenging problem in the Intelligent Transportation Systems (ITS) field, as long as a stable, accurate controller is compulsorily needed in order to ensure safety during navigation. The problem has been tackled under two different approaches. The first one is based on chained systems theory, while the second controller relies on fuzzy logic. A comparative analysis has been carried out based on the results achieved in practical trials. Real tests were conducted using a DGPS-driven electric Citroen Berlingo in a private test circuit located at the Industrial Automation Institute of the CSIC (Arganda del Rey, Madrid). The final results and conclusions are presented.

Keywords Nonlinear steering kinematics · Chained systems · Fuzzy controller · Autonomous vehicles

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1 Introduction

Lateral automatic driving has become a challenging topic in the ITS Research field in the last years. Basically, this problem can be stated as that of determining an appropriate control law for commanding the vehicle steering wheel. Several research groups have already demonstrated impressive results on this control task, especially in the ITS field [3, 4, 9, 11, 20]. Many steering control laws are designed in the literature [1, 2, 5, 12, 13, 16], comprising both traditional and artificial intelligence approaches. A comparative study of different lateral control strategies for road vehicles can be found in [14, 19], where a linearized model of the lateral vehicle dynamics is used for controller design based on the fact that it is possible to decouple the longitudinal and lateral dynamics.

The nonlinear nature of vehicles dynamics makes it hard to design an a priori stable controller following the schemes provided by classical control theory. This is particularly critical for the case of automobiles as they run at high speed. Any control algorithm designed to autonomously guide a road vehicle must be stable and highly accurate in order to ensure safety during navigation. Otherwise small instabilities (i.e., lateral error overshoot greater than 1.5 m) would derive in lane departure in real driving conditions. As a means to tackle the challenging problem of autonomous steering of automobiles we undertook the design of the vehicle controller under two different approaches. First, a simplified nonlinear lateral kinematic controller was developed based on chained systems theory. Then, a fuzzy logic based controller was designed and tuned to the same problem. This double strategy aimed at gaining better understanding into the comparative performance of classical and fuzzy theories applied to a real critical problem, namely automobiles lateral steering, where the nonlinear nature of vehicle dynamics and kinematics becomes the main issue.

Both lateral steering strategies were implemented on Babieca, an Electric Citroen Berlingo, using DGPS (Differential GPS) as the main sensor to measure the position of the vehicle in the road. Real tests were carried out on a private circuit emulating an urban quarter, composed of streets, intersections (crossroads), and roundabouts, located at the Industrial Automation Institute (IAI) in Arganda del Rey, Madrid. Two live demonstrations were carried out at International Conferences exhibiting the system capacities on autonomous steering. The first one was performed during the IEEE Intelligent Vehicles Symposium 2002 in a private circuit located at Satory (Versailles), France. The second took place during the ITS World Congress 2003 in Madrid.

The work described in this paper is organised in the following sections: Section 2 contains a description of both the nonlinear lateral controller based on chained systems theory as well as the fuzzy logic based lateral controller. In Section 3 the comparative results of both systems are presented, while a discussion including conclusions and future work is provided in Section 4.

2 Lateral control

Considering the case of an autonomous vehicle driving along some reference trajectory, the main goal of the lateral control module is to ensure proper tracking of the reference trajectory by correctly keeping the vehicle in the center of the lane with the appropriate orientation (parallel to the desired trajectory). This constraint can be generalized as the minimization of the vehicle lateral and orientation errors (d_e, θ_e) with respect to the reference trajectory, at a control point, as illustrated in Fig. 1. To solve this controllability problem and design a stable lateral controller, a model describing the behavior of d_e and θ_e is needed.

2.1 Kinematics model

The kinematics model of the vehicle is the starting point to model the kinematics of the lateral and orientation errors. The vehicle model is approximated by the popular Ackerman (or bicycle) model [1], assuming that the two front wheels turn slightly differentially and thus, the instantaneous rotation center can be purely computed by kinematic means. Let $\kappa(t)$ denote the instantaneous curvature of the trajectory described by the vehicle.

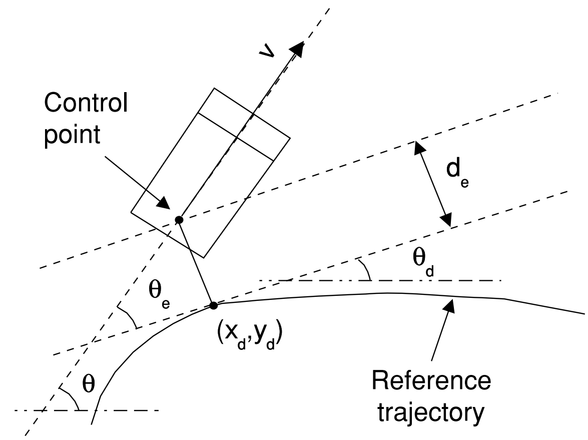

Fig. 1 Lateral and orientation errors at the control point

$$\kappa(t) = \frac{1}{R(t)} = \frac{\tan\phi(t)}{L} = \frac{d\theta(t)}{ds} \tag{1}$$

where R is the radius of curvature, L is the wheelbase, ϕ is the steering angle, and θ stands for the vehicle orientation in a global frame of coordinates. The temporal variation of θ is computed in Equation (2) as a function of vehicle velocity v.

$$\dot{\theta} = \frac{d\theta}{dt} = \frac{d\theta}{ds}\cdot\frac{ds}{dt} = \kappa(t)\cdot v(t) = \frac{\tan\phi(t)}{L}\cdot v(t) \tag{2}$$

Let ϕ and v be the variables of the vehicle control input space, representing the steering wheel turning angle and vehicle velocity, respectively. On the other hand, the vehicle configuration space is composed of the global position and orientation variables, described by (x, y, θ), under the flat terrain assumption. Mapping from the control input space to the configuration space can be solved by using the popular Fresnel equations, which are also the so-called dead reckoning equations typically used in inertial navigation. Equation (3) shows the kinematics of (x, y, θ).

$$\dot{x} = \frac{dx}{dt} = v(t)\cos\theta(t)$$
$$\dot{y} = \frac{dy}{dt} = v(t)\sin\theta(t) \tag{3}$$
$$\dot{\theta} = \frac{d\theta}{dt} = v(t)\frac{\tan\phi(t)}{L}$$

where $v(t)$ represents the velocity of the midpoint of the vehicle rear axle, denoted as control point. Global information about the position and orientation of the vehicle (x, y, θ) is then transformed so as to develop a model that describes the open-loop lateral and orientation error kinematics. As observed in Fig. 1, the lateral error d_e is defined as the distance between the vehicle control point and the closest point along the vehicle desired trajectory, described by coordinates (x_d, y_d). This implies that d_e is perpendicular to the tangent to the reference trajectory at (x_d, y_d). The scope of the tangent at (x_d, y_d) is denoted by θ_d, and represents the desired orientation at that point. Based on this, d_e and θ_e suffice to precisely characterize the location error between the vehicle and some given reference trajectory, as described in Equations (4) and (5).

$$d_e = -(x - x_d)\cdot\sin\theta_d + (y - y_d)\cdot\cos\theta_d \tag{4}$$
$$\theta_e = \theta - \theta_d \tag{5}$$

Computing the derivative of d_e with respect to time yields Equation (6), while the time derivative of θ_e is shown in Equation (7). Thus, the complete nonlinear kinematics model for d_e and θ_e is formulated in Equation (8).

$$\begin{aligned}\dot{d}_e &= -\dot{x}\sin\theta_d + \dot{y}\cos\theta_d\\ &= -V\cos\theta\sin\theta_d + V\sin\theta\cos\theta_d\\ &= V\sin(\theta - \theta_d) = V\sin\theta_e\end{aligned} \tag{6}$$
$$\dot{\theta}_e = \frac{d(\theta - \theta_d)}{dt} = \dot{\theta} - \dot{\theta}_d = \dot{\theta} \tag{7}$$
$$\begin{aligned}\dot{d}_e &= V\sin\theta_e\\ \dot{\theta}_e &= \frac{V}{L}\tan\phi\end{aligned} \tag{8}$$

The solution to the lateral control of a road vehicle has been tackled under two different approaches. On the one hand, a nonlinear lateral controller has been designed based on chained systems theory. On the other hand, another controller has been designed based on fuzzy logic. The description of both systems is provided in the following sections.

2.2 Nonlinear control law

The control objective is to ensure that the vehicle will correctly track the reference trajectory. For this purpose, both the lateral error d_e and the orientation error θ_e must be minimized. A fuzzy velocity controller is used in order to keep the vehicle velocity at a given reference value. The velocity profile is a priori designed depending on the mission that the vehicle has to execute. Thus, the reference velocity is selected among a set of constant values depending on the trajectory characteristics. According to the selected profile, and due to the action of the velocity controller, the vehicle velocity v can be assumed to be constant during a given trajectory despite the fact that significant acceleration or deceleration may occur in practice. Additionally, the constant velocity assumption helps simplify the controller design stage. The design of the control law is based on general results in the so-called chained systems theory. An excellent example on this topic can be found in [7]. Nevertheless, these results are extended and generalized in this paper so as to provide a stable nonlinear control law for steering of Ackerman-like vehicles based on local errors. From the control point of view, the use of the popular tangent linearization approach is avoided as it is only locally valid around the configuration chosen to perform the linearization, and thus, the initial conditions may be far away from the reference trajectory. On the contrary, some state and control variables changes are posed in order to convert the nonlinear system described in Equation (8) into a linear one, without any approximation (exact linearization approach). Nevertheless, due to the impossibility of exactly linearizing systems describing mobile robots dynamics, these nonlinear systems can be converted in almost linear ones, termed as chained form. The use of the chained form permits to design a control law using linear systems theory to a high extent. In particular, the nonlinear model for d_e and θ_e (Equation (8)) can be transformed into chained form using the state diffeomorphism and change of control variables, as in

Equation (9).

$$Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \Theta(X) = \begin{bmatrix} d_e \\ \tan\theta_e \end{bmatrix}$$
$$W = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \Upsilon(U) = \begin{bmatrix} v\cos\theta_e \\ \dfrac{v\tan\phi}{L\cos^2\theta_e} \end{bmatrix} \tag{9}$$

These transformations are invertible whenever the vehicle speed v is different from zero, and the orientation error θ_e is different from $\frac{\pi}{2}$. This implies that the singularities of the transformations can be avoided by asuring that the vehicle moves ($v > 0$) and that its orientation error is maintained under 90 degrees (the vehicle orientation must not be perpendicular to the reference trajectory). These conditions are reasonably simple to meet in practice. From Equation (9) the vehicle model can be rewritten as in Equation (10), considering y_1 and y_2 as the new state variables.

$$\dot{y}_1 = \dot{d}_e = v\sin\theta_e = w_1 y_2$$
$$\dot{y}_2 = \frac{d(\tan\theta_e)}{dt} = \frac{1}{\cos^2\theta_e}\cdot\dot{\theta}_e = \frac{v\tan\phi}{L\cos^2\theta_e} = w_2 \tag{10}$$

In order to get a velocity independent control law, the time derivative is replaced by a derivation with respect to ς, the abscissa along the tangent to the reference trajectory as graphically depicted in Fig. 2. Analytically,

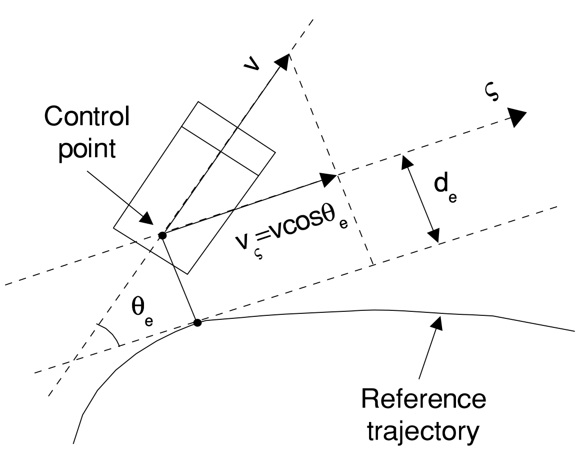

Fig. 2 Graphical description of ς

ς is computed as the integral of velocity v_ς, measured along axis ς.

$$\varsigma = \int v_\varsigma dt = \int v\cos\theta_e dt \Rightarrow \dot{\varsigma} = \frac{d\varsigma}{dt}$$
$$= v\cos\theta_e = w_1 \tag{11}$$

The time derivative of the state variables y_1 and y_2 is expressed as a function of ς in Equation (12).

$$\dot{y}_1 = \frac{dy_1}{dt} = \frac{dy_1}{d\varsigma}\cdot\frac{d\varsigma}{dt} = y_1'\cdot\dot{\varsigma}$$
$$\dot{y}_2 = \frac{dy_2}{dt} = \frac{dy_2}{d\varsigma}\cdot\frac{d\varsigma}{dt} = y_2'\cdot\dot{\varsigma} \tag{12}$$

where y_1' and y_2' stand for the derivative of y_1 and y_2 with respect to ς. Solving for y_1' and y_2' yields Equation (13).

$$y_1' = \frac{\dot{y}_1}{\dot{\varsigma}} = \frac{v\sin\theta_e}{v\cos\theta_e} = \tan\theta_e = y_2$$
$$y_2' = \frac{\dot{y}_2}{\dot{\varsigma}} = \frac{v\tan\phi}{L\cos^2\theta_e v\cos\theta_e} = \frac{\tan\phi}{L\cos^3\theta_e} = \frac{w_2}{w_1} = w_3 \tag{13}$$

As observed in the previous equation, the transformed system is linear and thus, state variables y_1 and y_2 can be regulated to zero (so as to yield $d_e = d_{e,\text{ref}} = 0$ and $\theta_e = \theta_{e,\text{ref}} = 0$) by using the control low proposed in Equation (14).

$$w_3 = -K_d y_2 - K_p y_1 \quad (K_d, K_p) \in \Re^{+2} \tag{14}$$

Using Equations (13) and (14) and solving for variable y_1 yields Equation (15), where the dynamic behavior of y_1 with respect to ς is proved to be linear.

$$y_1'' + K_d y_1' + K_p y_1 = 0 \tag{15}$$

Under the assumption of positive values of constants K_d and K_p, variable y_1 tends to zero as long as variable ς grows. In fact, variable y_1 has a second-order linear dynamic behavior dominated by two poles with negative real components. According to this, y_1 tends to zero as the independent variable (ς in this case, not time) tends to infinite. This statement ensures that d_e tends to zero as ς tends to infinite, as dictated by Equation (9) ($y_1 = d_e$). From Equation (13), if y_1 is zero, then y_2 is also zero ($y_2 = y_1'$). Likewise, if y_2 is zero then θ_e is zero from Equation (9) ($y_2 = \tan\theta_e$). Thus, both variables, y_1 and y_2 tend to zero as variable ς grows. The previous statement is analytically expressed in Equation (16).

$$\lim_{\varsigma\to\infty} d_e = \lim_{\varsigma\to\infty} \theta_e = 0 \tag{16}$$

Accordingly, variable ς must always grow so as to ensure that both d_e and θ_e tend to zero. This condition is met whenever $v > 0$ and $-\pi/2 < \theta_e < \pi/2$. In other words, the vehicle must continuously move forward and the absolute value of its orientation error should be below $\pi/2$ in order to guarantee proper trajectory tracking. Thus, the non linear control law is finally derived from Equations (13) and (14).

$$\phi = \arctan[-L\cos^3\theta_e \cdot (K_d \tan\theta_e + K_p d_e)] \tag{17}$$

The control law is then modified by a sigmoidal function as shown in Equation (18), to account for physical limitations in the vehicle wheels turning angle and prevent from actuator saturation. On the other hand, the use of sigmoidal functions preserves the system stability [10].

$$\phi = \arctan\left[-KL\cos^3\theta_e \cdot \frac{1-\exp^{-K(K_d\tan\theta_e+K_p d_e)}}{1+\exp^{-K(K_d\tan\theta_e+K_p d_e)}}\right] \tag{18}$$

The control law is saturated to $\phi_{\max}$ by properly tuning parameter K. The stability of the saturated system is ensured as demonstrated in [15, 18]. Thus, the maximum value of Equation (18) is $\phi_{\max} = \pm\arctan(-KL)$. Therefore, K is chosen to ensure that $\phi_{\max} = \pm\frac{\pi}{6}$ rad (physical limitation of the vehicle), given the wheelbase $L = 2.69$ m, yielding a practical value $K = 0.2146$.

$$K = \frac{\tan\frac{\pi}{6}}{L} \tag{19}$$

From observation of Equation (15), the dynamic response of variable y_1 can be considered to be a second-order linear one. In practice, it is not indeed linear due to the sigmoidal function used to saturate the control law, although it can be reasonably approximated as such. Thus, an analogy between constants K_d, K_p, and the parameters of a second-order linear system ξ (damping coefficient) and ω_n (natural frequency) can be established, yielding Equation (20).

$$\omega_n = \sqrt{K_p} \qquad \xi = \frac{K_d}{2\sqrt{K_p}} \tag{20}$$

Likewise, system overshoot M_p and settling distance d_s (given that the system error dynamics is described as a function of space variable ς, not time) can be obtained from Equation (21).

$$M_p = \exp^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} \qquad d_{s|2\%} = \frac{4}{\xi\omega_n} \tag{21}$$

The design of constants K_d and K_p is undertaken considering that the system overshoot must not exceed 10% of the step input, and that the settling distance should be below some given threshold. Thus, for a typical settling time $t_s = 20$ s, and given a vehicle velocity v, the proper settling distance can be computed as in Equation (22).

$$d_s = t_s \cdot v = 20v \tag{22}$$

The value of K_d is derived from Equations (20) and (21) yielding the velocity dependant expression in Equation (23).

$$K_d = \frac{8}{d_s} = \frac{0.4}{v} \tag{23}$$

Likewise, dumping coefficient ξ is derived from Equations (20) and (21), as shown in Equation (24).

$$\xi = \sqrt{\frac{1}{\left[\frac{\pi}{\ln 0.1}\right]^2 + 1}} = \frac{K_d}{2\sqrt{K_p}} = \frac{4}{d_s\sqrt{K_p}} \tag{24}$$

Finally, K_p is deduced from the previous equation, yielding Equation (25).

$$K_p = \left[\frac{6.766}{d_s}\right]^2 = \left[\frac{0.3383}{v}\right]^2 \tag{25}$$

The dependence of K_p and K_d on vehicle velocity v permits to ensure proper dynamic response. In particular, vehicle turning angle will be smooth at high speeds, therefore avoiding possible oscillations due to physical constraints in steering dynamics.

2.3 Fuzzy control law

Due to the impossibility of exactly linearizing systems that describe mobile robots dynamics, in this case, a car, two solutions appear in order to solve the control law design problem. The first of all is the transformation of the nonlinear system in a set of linear systems and deal with them, as described in Section 2.2. The second solution is to use Artificial Intelligence control methods that overcome the nonlinearity limitations of the classical control methods and base their performance in the human knowledge and experience. The latter is the solution we are presenting here. In our case, we have added to the system an alternative AI-based lateral controller, using fuzzy logic, which is a widely studied subject with a well-known use as control method, since the experiments of Sugeno at late 80s [21]. The main feature of this kind of control is that it does not try to model the system, the steering of a vehicle, rather, it tries to model the management of the system, this is, the human driver actions. This makes this method very useful for treating uncertainty. In a fuzzy system, we do not work with the crisp values of the elements to control, instead we use fuzzy or linguistic values. These values indicate the degree of truth that a variable of the process has in any moment and depending on it, some control actions will be taken. These control actions, are caused by the inference of a set of linguistic fuzzy rules with the pattern antecedent implies consequent, where the antecedents are the linguistic variables for the input values and the consequents are the control actions to be taken in order to control the system [22]. For example, in the case of a speed regulator, the variable to be controlled may be the speed error from a reference speed, and some simple rules as follows.

if *speederror* negative then *throttle* stepon
if *speederror* positive then *throttle* stepoff

where the control action is to stepon or stepoff the throttle pedal depending on the car going slower or faster than the reference [23]. It does not mean that there is a rule simplification. On the contrary, this rule implies that the output fuzzy variable *throttle* has two associated linguistic labels called *stepon* and *stepoff* with their membership functions represented as singletons. The output of the inference of the rules represents the resulting degree of truth of the consequent of each rule depending on corresponding the antecedent. Then each rule will generate a degree of truth for the output fuzzy variable *throttle*. Once finished the inference of all rules, the defuzzyfication operation is executed. In our case, we use the center of mass method to obtain the crisp value of the fuzzy inference. The activation weight of a rule represents its contribution to the global control action (calculated as the minimal degree of current crisp input membership value of its respective fuzzy partitions), which relates the different values of the inference of each rule. Accordingly, it can be stated that there are not only two different throttle positions, but the *throttle* output fuzzy variable has been modeled with two linguistic labels.

The definition of the fuzzy variables includes associated membership functions $\hat{A}$ that contain the representation of the degree of truth for the determined variable range. The fuzzy control works executing fuzzy control iterations sequentially. It is initially necessary to transform the crisp values of the control variables (given by the process sensors) into fuzzy linguistic ones. This is called fuzzyfication. Once the input values are fuzzyfied, the inference rules are executed, obtaining the degree of truth to be applied to the control action. Then, this degree must be defuzzyfied in order to be transformed again in crisp values that can be applied to the actuators to execute the control. In order to design the fuzzy control system for automatically control the steering wheel of a vehicle, we are going to use the same parameters defined in the classical control described in 2.2, but we will manage them in a different way. The input variables to be considered will be the lateral error (d_e) and the orientation error (θ_e) that must be minimized too. The output variable will be the steering turning signal (ϕ) that must be sent to the steering wheel controlling motor in order to minimize the input errors. We will also consider two fuzzy contexts that will execute depending on the driving circumstances: curve driving mode and straight driving mode. As in humans drivers, the driving error appreciation slightly varies depending on the road situation. This will be reflected in the fuzzy variable design. The control actions will also be the same (e.g. turn the steering wheel to the right when there is a trajectory deviation to the left) in any context. Only varies the magnitude of the actions, not the actions themselves.ext, we are going to define the three stages of the steering fuzzy controller.

2.3.1 Fuzzyfication

The fuzzyfication stage consists on the transformation of the input crisp variables into linguistic variables that will be used as input data for a fuzzy controller. A linguistic label is composed by a set of linguistic labels in which the variable range is divided. Each label has associated a membership function that represents the degree of truth of this label along the range of the variable. In our case, the automatic steering control, we use two fuzzy variables, the lateral and the angular errors. When a human drives, the steering management basically consists on correcting the errors on the trajectory, when the car deviates to the left or to the right. If we take the human experience as reference, we can define two linguistic labels for each input variable: left and right, depending on the trajectory deviation occurring to the left or to the right of the reference route. Each one of this linguistic labels has an associated a membership function, as shown in Fig. 3. In this figure we can appreciate the two driving contexts defined, for curve and straight line driving. (a) and (b) represent the typical driver appreciation when we drive along a straight road. The gradients of the shape of d_e and θ_e are very high. This means that a little trajectory deviation will be considered as important error, similarly as humans do, because we should circulate at high speeds and any deviation may cause a lane depart. The definition of the d_e and θ_e membership functions for the curve-driving context is shown in (c) and (d). In this case, the gradient of the shape is lower than in straight mode. The reason of this is that, similarly to human driving, in curved roads the speed is lower and there are inherent errors due the curvature of the path so, it is not necessary to consider little deviations as important errors. Once defined the labels and membership functions for the fuzzy variables, we can transform the crisp variables into fuzzy ones, comparing it to the corresponding membership functions and obtaining their membership degree.

This definition of the lateral error membership function depends on the combined tuning of the membership functions of all the variables. Lateral error defined in a range of $(-\infty, -10, 0)$ means that the full membership degree is obtained when the lateral error is lower than -10. This means that only in the case of an extreme trajectory deviation, the lateral error may infer to the system the maximum weight. In the cases with a higher error, the system output is a combination of both lateral and angular errors which increases the performance and sensitivity of the fuzzy controller.

2.3.2 Fuzzy rules

The fuzzy rules relates the input variables with the output ones in order to reflect the human behavior in driving actions. In this case, only four rules are necessary for modeling this behavior:

R1: IF θ_e Left THEN ϕ Right
R2: IF θ_e Right THEN ϕ Left

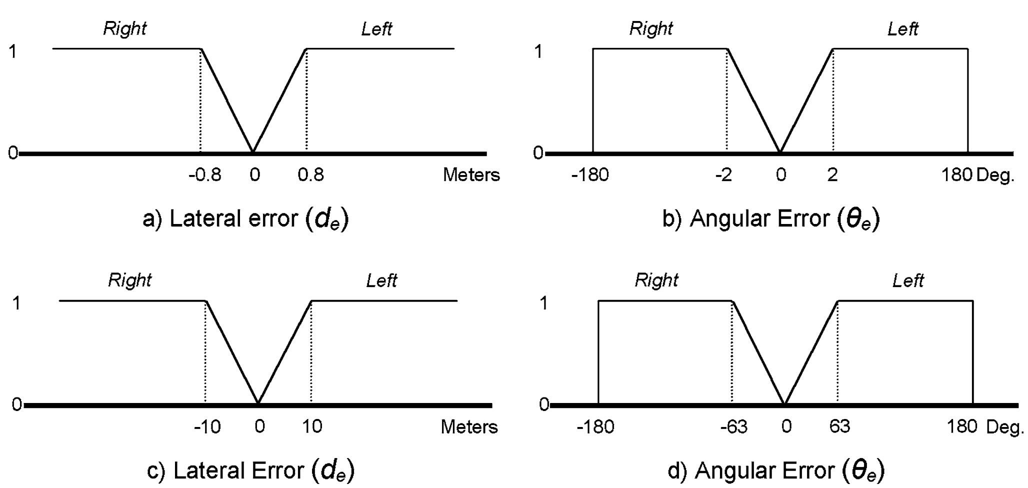

Fig. 3 Membership function definition for the fuzzy input variables

R3: IF d_e Left THEN ϕ Right
R4: IF d_e Right THEN ϕ Left

Although the presented fuzzy rules set has only four rules, in this paper we have tried to show the power of the fuzzy controllers that, by using an extremely simple set of rules and variables, can generate similar results to those obtained by a nonlinear classical controller. However, it is truth that as we increase the variable and rule number softer behavior might appear. Concerning the rapid changing orientation errors it can be stated that they are caused by the low precision of the orientation calculation with GPS at low speeds (under 5 km/h). This is other strong point of the developed fuzzy controllers, as long as they may deal with imprecise data generating good control actions.

The philosophy of these rules (Mamdani type [23]) is to correct the trajectory errors, lateral as well as angular, when they appear, moving the steering wheel in the opposite sense of the error. It is really a simple behavior as humans do. When a human drives, there are some control layers, depending on the abstraction and conscience level [25]. The travel planning and route following are parts of the high level control layer whose subject is out of the focus of this paper. Low level control consists on maintaining the vehicle always into the corresponding road lane. Fuzzy controller automates this low level human driving task using the same information and reasoning as humans do. The aim of the presented controllers is to maintain the vehicle always with the minimum error from the reference trajectory. In a normal road driving operation, several situations may appear. This way, we have divided the automatic fuzzy driving controllers in some independent fuzzy contexts that fit to any of these situations. Straight driving, curve driving or overtaking are the three fuzzy contexts we have defined. It is truth that other rule and membership function configurations may be applied but we have chosen this one in order to prove this three behaviors independently as well as the relationships among them. Any other configuration might be applied but the results will be similar, as shown in the experiments section. No different context appears in this case, because the methods for correcting the errors are always the same. It only varies the amount of the correction that will directly depend on the input variables.

The fuzzy qualifiers number is a tricky topic in fuzzy systems practical deployment as well as from the theoretical point of view. It is also a research topic from the psychological science: Determining the minimum qualifier number to define the possible states of a variable is not a trivial issue. The most common method is to try to find the minimum possible number of qualifiers that allow the control. In our case, the division in fuzzy contexts allows to have very few qualifiers. If we use only one context, we would augment the number of contexts and rules, increasing the complexity of the fuzzy controller. Then, the selection and definition of a multicontext controller is a design decision in order to simplify the system appearance. However, one-context controllers have also been proved obtaining similar results but, this is out of the scope of this paper.

2.3.3 Defuzzyfication

Finally, the defuzzyfication process consists on transforming the results of the inference of the rules into crisp values that permit to be applied directly to the mechanical elements of the car. In our case, we have also defined the output variable (ϕ) as a singleton, very useful in control tasks [24]. We use singletons because they are a good solution for control systems since Sugeno [24] demonstrated the equivalence between this singleton-type-II controllers and type-I, whose output is defined with trapezoidal membership functions.

3 Implementation and results

Both control laws for autonomous steering described in this paper were tested on the so-called Rocinante prototype vehicle (an electric Citroen Berlingo). The vehicle was modified to allow for automatic velocity and steering control at a maximum speed of 90 km/h. Babieca is equipped with a DGPS receiver to provide lateral and orientation position of the ego-vehicle with regard to the center of the lane, a Pentium PC, and a set of electronic devices to provide actuation over the accelerator and steering wheel, as well as to encode the vehicle velocity and steering angle. The DGPS receiver is a Z-12 Real Time model by Ashtech that implements the RTCM SC 104 V2.2 standard. Practical experiments were conducted on a private circuit located at the Industrial Automation Institute in Arganda del Rey (Madrid). The circuit is composed of several streets, intersections, and roundabout points, trying to emulate an urban quarter. The control objective is to achieve the reference error vector $d_{e,\mathrm{ref}} = 0, \theta_{e,\mathrm{ref}} = 0$.

This objective implies proper tracking of the road curvature. Various practical trials were conducted so as to test the validity of the two control laws (nonlinear and fuzzy) for different initial conditions in real circumstances. During the tests, the reference vehicle velocity is assumed to be kept constant by a velocity controller developed in [8]. Constants K_d and K_p were calculated as a function of v using Equations (21) and (23). In the experiments, a quasi-straight reference path was used to autonomously guide the vehicle based on DGPS. Figures 4 and 5 show the transient response of the vehicle lateral and orientation errors for reference velocities in the range 7–25 and 7–40 km/h, respectively. In both cases, the vehicle starts the run at an initial lateral error of about 0.3 and 1 m, respectively, and an initial orientation error in the range $\pm 5^\circ$. As can be clearly appreciated, the steady-state response of the system is satisfactory for both experiments. Thus, the lateral error is bound to ± 20 cm at low speeds and ± 25 cm at $v = 40$ km/h, while the absolute orientation error in steady state remains below 1° in all cases. Just to give an example on how the practical results conform to the expected values as derived from the theoretical development, let us consider the transient response of the vehicle depicted in Fig. 5 for $v = 7$–40 km/h. Assuming a theoretical maximum overshoot of $M_p = 10\%$ and a settling time of $t_s = 20$ s, the controller coefficients are tuned to $K_d = 0.072$ and $K_p = 0.0037$, according to Equations (21) and (23). Nonetheless, from observation of Fig. 4 the maximum overshoot obtained in practice is almost 100% for the lateral error, while the settling time takes some 20s. This is mainly due to the existence of nonlinear actuator dynamics and latencies, not considered in the model. In spite of these slight differences with regard to the theoretical expected values, the practical results exhibited in this section demonstrate that

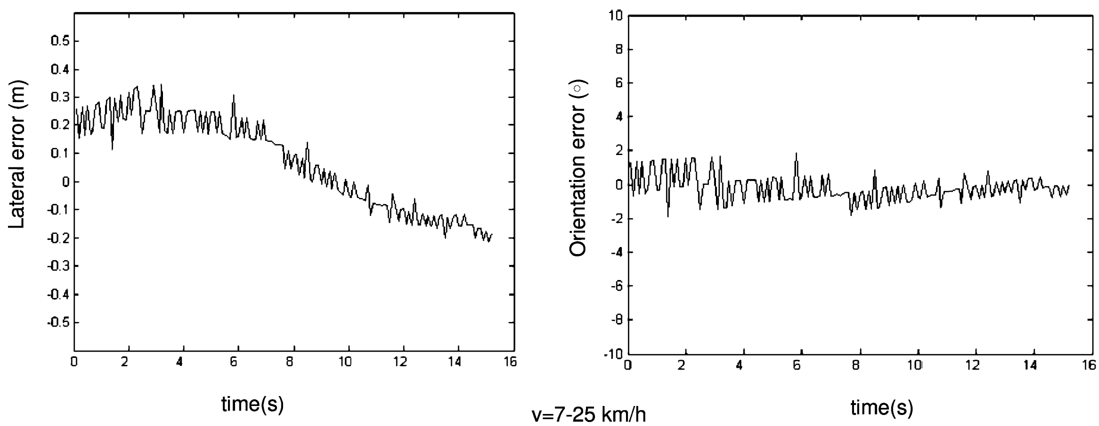

Fig. 4 Transient response of the lateral and orientation error for $v = 7$–25 km/h using the nonlinear controller

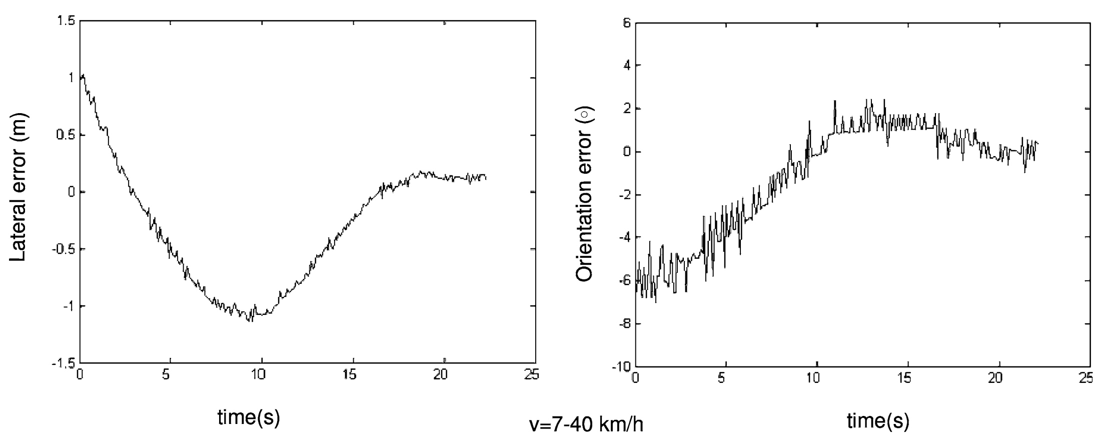

Fig. 5 Transient response of the lateral and orientation error for $v = 7$–40 km/h using the nonlinear controller

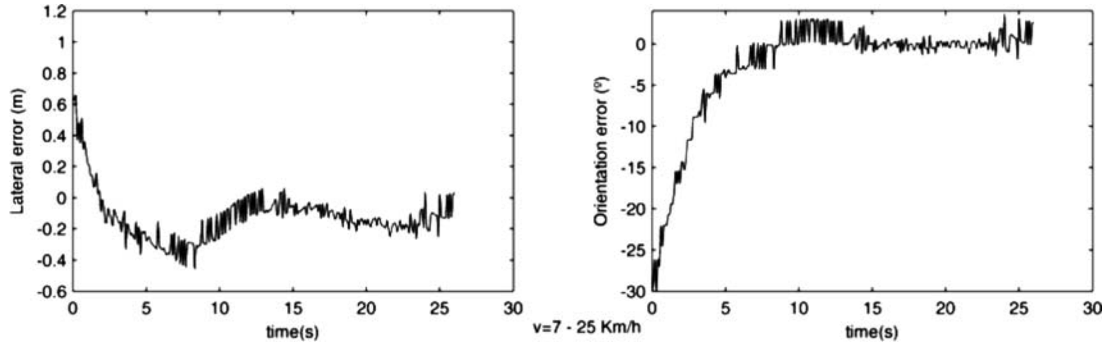

Fig. 6 Transient response of the lateral and orientation error for $v = 7$–25 km/h using the fuzzy controller

the nonlinear lateral control law developed in this work still permits to safely steer the vehicle at operational velocities. Next, the same experiment was repeated using the fuzzy controller for a commanded velocity ranging from $v = 7$–25 km/h. The result is depicted in Fig. 6. As can be observed, the overshoot and settling time are smaller than in the case of the nonlinear controller.

In a final trial, the same test was carried out under the same conditions using both controllers in an attempt to establish a comparison of performance. For that purpose, the vehicle was started in the same position (a marked one) in either case, with an initial lateral error of 1 m and an initial orientation error of approximately 0 degrees. In both cases, the vehicle was provided with a commanded velocity $v = 20$ km/h, leaving the control of the accelerator to the velocity controller in order to mantain the reference velocity. The comparison is graphically depicted in Fig. 7.

On the one hand, one can observe how the nonlinear controller provides a response with a great overshoot in the lateral error, while the orientation error response is more stable, with almost no overshoot. Once the nonlinear controller succeeds in reaching null errors the system remains stable, keeping the absolute values of the lateral and orientation errors bounded to 10 cm and 2 degrees, respectively. On the other hand, the fuzzy controller provides faster lateral response, exhibiting minor overshoot and settling time. Nonetheless, there is a second overshoot that does not occur in the case of the nonlinear controller. Likewise, the fuzzy controller provides less stable response concerning the orientation error. Indeed, a transient overshoot takes place with a maximum amplitude of 5 degrees. As a general comment, it can be stated that the nonlinear controller provides a slow but stable response in the lateral and orientation error, while the fuzzy controller provides fast response at the expense of a bit of oscillations in steady state. In any case, the oscillations produced by the fuzzy controller keep the vehicle within a limit that makes autonomous guidance a tractable issue in practice. Conversely, the slow transient response produced by the nonlinear controller puts the issue at risk when

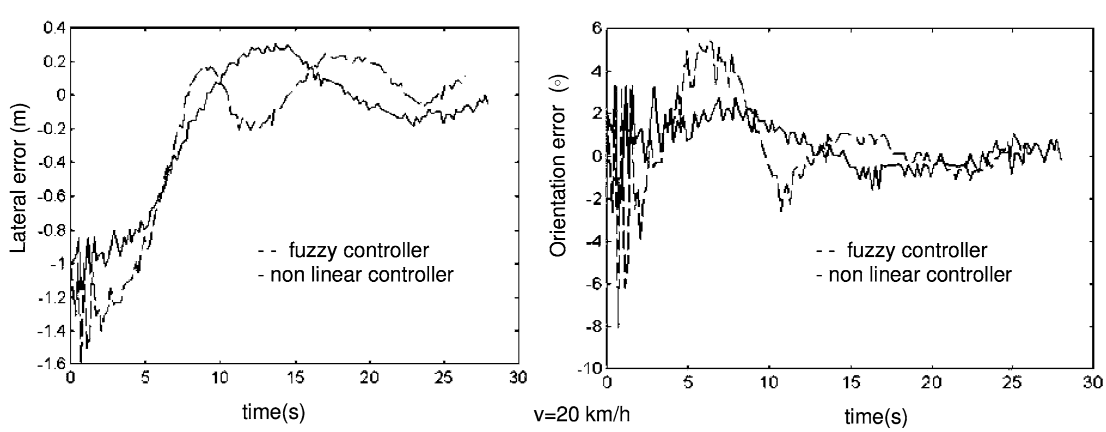

Fig. 7 Comparison between nonlinear controller and fuzzy controller at $v = 20$ km/h

it comes to high speed autonomous navigation, while it remains stable and safe at low speed (25–40 mk/h).

Rocinante has run over hundreds of kilometers in lots of successful autonomous missions carried out along the test circuit located at the IAI, using both the fuzzy and nonlinear control laws described in this paper. Moreover, live demonstrations exhibiting the system capacities on autonomous driving were carried out during the IEEE Conference on Intelligent Vehicles 2002, in a private circuit located at Satory (Versailles), France, as well as in the World ITS Congress that was held in Madrid in 2003.

4 Conclusions

To conclude, the next key points should be remarked.

- First of all, the nonlinear control law described in this work has proved its analytical and empirical stability for lateral driving of Automobiles. In fact, it has been implemented on a real commercial vehicle slightly modified so as to allow for autonomous operation, and tested on two different private circuits.
- The fuzzy controller has been proved to provide stability from the empirical point of view in hundred of experiments.
- Vehicle commanded actuation is taken into account by considering the current velocity in the design of the nonlinear controller coefficients. This permits to provide the system with adaptive capability.
- The fuzzy controller provides faster response than the nonlinear controller.
- The fuzzy controller is more appropriate for high speed navigation, although it suffers from the lack of adaptive capability.
- A more ellaborated fuzzy rules set is considered as a future work in order to implement nonlinear fuzzy rules that provide even softer actuation at all speeds.

A combination of the positive features of both controllers, i.e. fast response, stability, and adaptability, should be highly desirable for achieving a really robust and safe controller of a road vehicle. Indeed, our current work focuses on the development of a combination of nonlinear adaptive theory and fuzzy systems in an attempt to increase stability and comfortability when driving at high speed, while avoiding ad-hoc non-adaptive designs.

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