Comparative Study of Variation of Mooney-Rivlin Hyperelastic Material Models under Uniaxial Tensile Loading

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ABSTRACT

A Hyperelastic material is type of the ideally elastic material for which elasticity shows non-linear behaviour, because of that the stress strain relationship for them expressed in terms of strain energy density. Hyperelastic models are used to model the mechanical behaviour of rubber-like materials ranging from elastomers, such as natural rubber and silicon, to biologic materials, such as muscles and skin tissue. The presented work is carried out to study the effect of the different Mooney-Rivlin hyperelastic models used in commercial finite element software. All the models studied have under gone same loading and boundary conditions and finally compared with testing. The final conclusion based on their capturing hyperelasticity of material is stated.

Keyword: - *Hyperelastic, Rubber, Mooney-Rivlin, FEM*

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1. INTRODUCTION

In beginning of the study of rubber like materials Mooney[1], presented the general strain energy function for representation of gum stock and tread stock. Mooney first used concept of stretch to define the strain energy function. After rubber material study by Mooney, Treloar[2] had carried out testing of rubber materials. It is most reliable testing data for rubber materials. Later Rivlin[3] studied the formation made by Mooney for further development and suggested new formation based on Mooney function known as Mooney-Rivlin material model. Rivlin stated strain energy as function of the Invariant of deformation matrix.

Hyperelastic materials such as rubber are widely used for diverse structural applications in variety of industries ranging from tire to aerospace. The most attractive property of rubbers is their ability to experience large deformation under small loads and to retain initial configuration without considerable permanent deformation after load is remove. Their stress-strain behaviour is highly non-linear and a simple modulus of elasticity is no longer sufficient. Therefore, characterization of elastic behaviour of highly extensible, nonlinear materials is of great importance.

The constitutive behaviour of hyperelastic material is derived from Strain Energy Function (SEF) 'W' based on three strain invariants I_1, I_2 and I_3. It is the energy stored in material per unit of reference volume (volume in the initial configuration) as a function of strain at that point in material.

$$W=f(I_1,I_2,I_3) \quad \textbf{(1)}$$

where I_1, I_2 and $I_3$are three invariants of Green deformation tensor defined in terms of principal stretch ratios λ_1, λ_2 and λ_3 given by:

$$I_1= \lambda_1^2 + \lambda_2^2+ \lambda_3^2$$
$$I_2= \lambda_1^2*\lambda_2^2+ \lambda_2^2*\lambda_3^2+ \lambda_3^2*\lambda_1^2$$
$$I_3 = \lambda_1^2 * \lambda_2^2* \lambda_3^2 \quad \textbf{(2)}$$

The stretch ratios λi represent the deformation of a differential cubic volume element along the principle axes of a Cartesian coordinate system. They are defined as the ratio of the deformed length li to the undeformed length Li. The stretch ratio equals 1 in undeformed state.

$$\lambda_i = \frac{li}{Li} \quad i \in [1,2,3]$$

Note that given the assumption of incompressibility of the material, the third strain invariant I_3 yields,

$$I_3 = \lambda_1^2 * \lambda_2^2 * \lambda_3^2=1,$$

Hence only two independent strain measures namely I_1 and I_2 remain. This implies that 'W' is a function of I_1 and I_2 only;

$$I_1= \lambda_1^2 + \lambda_2^2+ \lambda_3^2$$
$$I_2= \lambda_1^{-2} + \lambda_2^{-2}+ \lambda_3^{-2}$$
$$W=W(I_1-3,I_2-3) \qquad \textbf{(3)}$$

The selection of suitable SEF depends on its application, corresponding variables and available data for material parameter identification of an efficient hyperelastic material model as follows:

▪. It should have the ability to exactly reproduce the entire 'S' shaped response of rubber

▪. The change of deformation mode should not be problematic, i.e. if the model operates satisfactorily in uniaxial tension, it must also be satisfactory in simple shear or in equibiaxial extension

▪. The number of fitting material parameters should be small, in order to decrease the number of experimental tests needed for their determination

▪. The mathematical formulation should be simple to render possible the numerical implementation of the model.

For a precise prediction of rubber behaviour, used in an assembly (e.g. flexible joint), by finite element simulation, it should be tested under same loading conditions to which original assembly will be subjected. The uniaxial tests are easy to perform and are well understood but uniaxial data alone does not produce reliable set of coefficients for material models, especially if the original assembly experiences complex stress states. Therefore, biaxial, planar (pure shear) and volumetric tests need to be performed along with a uniaxial tension test to incorporate the effects of multiaxial stress states in the model.

For specific applications, the tailoring of rubber mechanical properties is carried out by the addition of various chemicals. Minor changes in chemical composition can alter mechanical properties significantly. Therefore, it is essential to test a particular rubber composition and simulate through FEA to have an opposite SEF. Once determined, this can be used for simulating the assembly in which particular rubber has been used. FE software packages like ANSYS offer a number of SEFs to accommodate the nonlinear behavior of rubber and other hyperelastic materials. This study shows that Mooney-Rivlin model has an advantage over other available material models because of its good match with experimental data over large strain values for given rubber composition.

1.1 Mooney-Rivlin Material Model:

In continuum mechanics Mooney–Rivlin solid is a hyperelastic material model where the strain energy density function W is a linear combination of two invariants of the left Cauchy–Green deformation tensor. The model was proposed by Melvin Mooney in 1940 and expressed in terms of invariants by Ronald Rivlin in 1948.

There are so many hyperelastic materials model exist in commercial FEA softwares. Mooney-Rivlin material model is one of the few famous materials models. It has very good advantage when behaviour of hyperelastic material is unknown. There are four different variations present. Each model is differentiated by its use of the number of the independent constants.

The Mooney–Rivlin model is a special case of the generalized Rivlin model which has the form [3],

$$W = \sum_{i+j=1}^{N} C_{ij}(\overline{I_1} - 3)^i(\overline{I_2} - 3)^j + \sum_{k=1}^{N} \frac{1}{d_k}(J_{el}-1)^{2k} \qquad \textbf{(4)}$$

with $C_{00} = 0$ where C_{ij} are material constants related to the distortional response and $1/d_k$ are material constants related to the volumetric response. Assumption of the incompressibility makes last term volumetric response zero. So for different formulation are with incompressibility as follows,

- Two Term MR Model:
  $W= C_{10}(I_1 - 3) + C_{01}(I_2 - 3)$
- Three term M-R Model:
  $W=C_{10}(I_1 - 3)+C_{01}(I_2 - 3) +C_{11}(I_1 - 3)(I_2 - 3)$
- Five Term M-R Model:
  $W=C_{10}(I_1 - 3) +C_{01}(I_2 - 3)+ C_{20}(I_1 - 3)^2 + C_{11}(I_1 - 3)(I_2 - 3)$
- Nine Term M-R Model:
  $W=C_{10}(I_1 - 3) +C_{01}(I_2 - 3)+ C_{20}(I_1 - 3)^2 + C_{30}(I_1 - 3)^3 + C_{11}(I_1 - 3)(I_2 - 3)$ **(5)**

As a result, the incompressible polynomial model is expressed in terms of the first and second strain invariant only.

The volume is subject to the uniaxial tensile stress σ. Axes 1, 2 and 3 denote coordinate axes, which are parallel to the principal axes of the cube. The three principal stretches with regard to the coordinate axes are λ1, λ2 and λ3. If λ is the stretch parallel to the tensile stress σ, deformations in 2 and 3 are equal [1].The corresponding mathematical expressions are

$$\lambda_1 = \lambda \text{ and } \lambda_2 = \lambda_3$$

Since the material is considered incompressible,

$$\lambda_2 = \lambda_3 = \lambda^{1/2}$$

Resulting from this, the two strain invariants for an incompressible material in tension or compression are

$$I_1 = \lambda^2 + 2\lambda^{-1} \text{ and}$$
$$I_2 = \lambda^{-2} + 2\lambda$$

The actual relation between engineering stress and stretch for an incompressible material under tension/compression is,

$$\sigma_e = 2 * (\lambda - \lambda^{-2}) * \left(\frac{dW}{dI1} + \frac{1}{\lambda} * \frac{dW}{dI2}\right) \quad \textbf{(6)}$$

Where:

σ_e: Engineering stress λ: Stretch, parallel to σ_e

W: Strain energy I_1, I_2: Strain invariants

By using above equation the stress values for different materials are evaluated in further chapter. The strain energy density function for different material is considered from equation....

2. EXPERIMENTAL DETAILS:

For the comparison purpose of rubber materials, test data has prime importance among all, so reliable test data should be used for study of hyperelastic materials. For this study uniaxial test data is taken from the reference [5]. The details regarding test data given below,

2.1. Material:

Natural rubber with reinforced carbon-black was used for testing. Chemical composition is given in Table 1.

Table 1. Chemical Composition of Rubber:

Natural Rubber (gms)	**Carbon-Black (gms)**	**Stearic Acid (gms)**	**Zinc Oxide (gms)**	**MBT (gms)**	**Sulphur-80 (gms)**
100	40	6	3	1	1

2.2 Mechanical Testing:

This section describes the standard test performed by reference [5]. The test data of stress- strain is required to evaluate material constants for the different variation of Mooney-Rivlin Model. For the study estimated uniaxial test data consider. The test was performed on standard dumbbell shaped specimen according to Type IV ASTM D638.

3. EXPERIMENTAL RESULTS AND CURVE FITTING FOR MOONEY-RIVLIN MODELS SELECTION:

The different material constant for the Mooney-Rivlin material model are calculated from test data used. Constant are obtained with the help of ANSYS software. Materials constant evaluated for calculating the stress and strain property of different Mooney-Rivlin Model.

Table-2: Coefficient of Two Term Mooney-Rivlin Material model

C_{10} (MPa)	C_{01} (MPa)	D_1	Residual
0.287606	-0.25942	0	1.968622

Table-3: Coefficient of Three Term Mooney-Rivlin Material model

C_{10} (MPa)	C_{01} (MPa)	C_{11} (MPa)	D_1	Residual
0.030747	0.042667	0.02711	0	0.527063

Table-4: Coefficient of Five Term Mooney-Rivlin Material model

C_{10} (MPa)	C_{01} (MPa)	C_{20} (MPa)	C_{02} (MPa)	C_{11} (MPa)	D_1	Residual
0.169548	-0.11276	0.015961	0.046217	-0.05047	0	0.307677

Table-5: Coefficient of Nine Term Mooney-Rivlin Material model

C_{10} (MPa)	C_{01} (MPa)	C_{20} (MPa)	C_{02} (MPa)	C_{30} (MPa)	C_{03} (MPa)	C_{11} (MPa)	C_{12} (MPa)	C_{21} (MPa)	D_1	Residual
2.996089	-3.0466	-135.056	-152.12	-0.01018	-18.8031	283.4141	33.14071	0.128574	0	0.304562

The behaviour of the different material models considered is having different approach for capturing the rubber material behaviour. According to increasing number of parameter from strain energy density equation, accuracy of material model increases. But there is always trade off for selection of appropriate material model. Curve fit for different material models considered are given below,

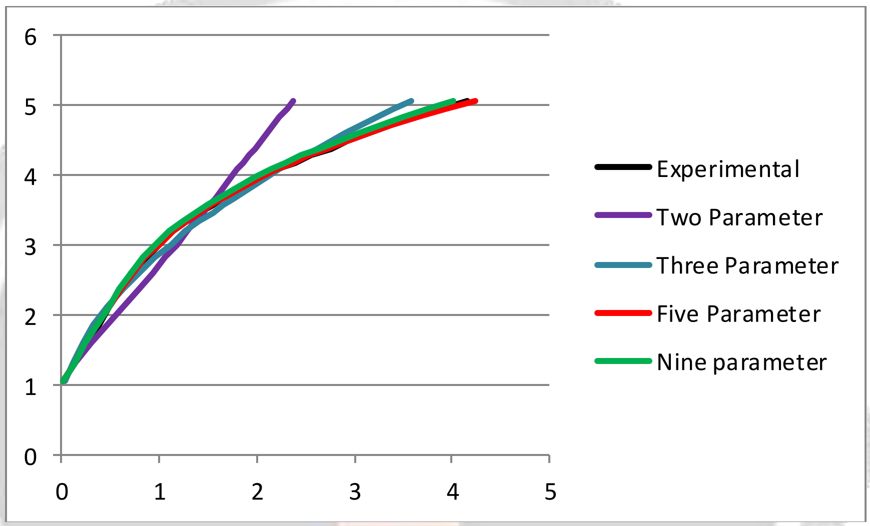

Chart-1: Comparative representation of Mooney-Rivlin Two,Three,Five,Nine parameter model with experimental uniaxial test

Also the error variation of the four models compared to make trade off for selected rubber material. The error variation shows,

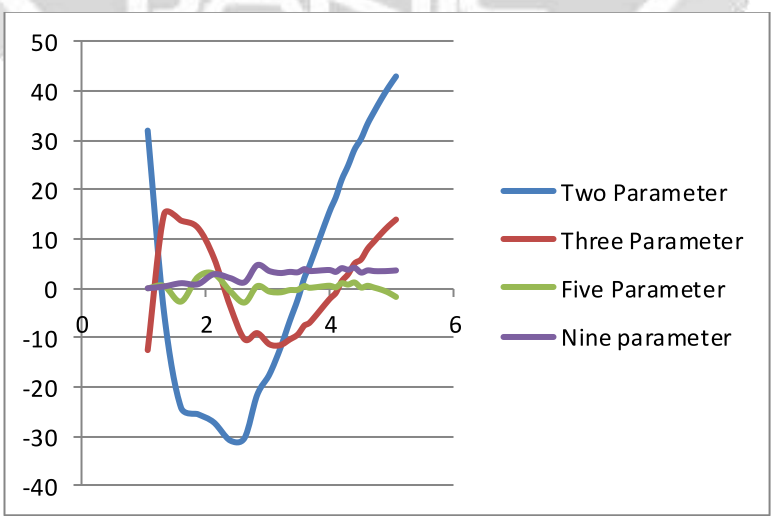

Chart-2: Comparative error variation for Mooney-Rivlin Two, Three, Five, Nine Parameter

4. CONCLUSION:

- The general conclusion based on study is that with increases in number of parameter used in Mooney-Rivlin material model, accuracy of material model increases.
- As in selected material case the error values and residual for Five parameter and nine parameter is nearly same but differ in number of parameter required to evaluate. So comparatively five parameter MR model better for selected material. The nine parameter model is good when two inflection points required in stress-strain curve.
- Two parameter model failed to capture behaviour of the rubber material, where three parameter moderately gives results with marginable error. Both perform better at lower stretch values.
- One can observed from error plot, that for more the error value lesser is the error oscillation is observed. So five parameter and nine parameter model gives oscillating small error, which shows good capturing of material property.

5. REFERENCE:

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