

# COMPARATIVE STUDY OF WING LIFT DISTRIBUTION ANALYSIS USING NUMERICAL METHOD

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## ABSTRACT

This research focuses on calculating the force distribution on the wings of the LSU 05-NG aircraft by several numerical methods. Analysis of the force distribution on the wing is important because the wing has a very important role in producing sufficient lift for the aircraft. The numerical methods used to calculate the lift force distribution on the wings are Computational Flow Dynamics (CFD), Lifting Line Theory, Vortex Lattice Method, and 3D Panel Method. The numerical methods used will be compared with each other to determine the accuracy and time required to calculate wing lift distribution. Because CFDs produce more accurate estimates, CFD is used as the main comparison for the other three numerical methods. Based on calculations performed, the 3D Panel Method has an accuracy that is close to CFD with a shorter time. 3D Panel Method requires 400 while CFD 1210 seconds with results that are not much different. While LLT and VLM have poor accuracy, however, a shorter time is needed. Therefore to analyze the distribution of lift force on the wing it is enough to use the 3D Panel Method due to accurate results and shorter computing time.

**Keywords:** *Wing Lift Distribution, CFD, 3D Panel Methode, LLT, VLM, Numerical Methode.*

## 1 Introduction

Wings have a very important role in producing sufficient lift force for airplanes. The lift force produced by these wings is the main lift force of an aircraft. Failure on the wing will make the aircraft lose lift and crash.

The estimation of the load on the wing will be determined by the wings component. Thus, failure happening on the wing could be rectified. One of the loads work on the wing is aerodynamic load.

The results of the analysis of lift force distribution along the wingspan can be used to predict aerodynamic loads in wing structures (Silitonga & Moelyadi, 2018). The method of estimating lift force distribution was first introduced by Prandtl in 1918. Prandtl introduced the

first practical theory to estimate aerodynamic characteristics of finite wings (Anderson Jr, 2001). This theory is known as Lifting-line Theory (LLT). However, this theory still assumes that the coefficient of lift force linear to changes in the effective angle of attack, whereas after reaching the stall point the curve of lift force curve becomes nonlinear. (Anderson Jr, 2001) introduced an extensive model of the classical LLT developed by Prandtl to accommodate nonlinear areas on the curve of lift force coefficient to the effective angle of attack.

Lifting-line Theory model only applies to elliptic or straight wings, while for wing shapes that have a low aspect ratio, wings with swept angles, and a delta wing, this LLT model cannot be used.

Therefore, the Vortex Lattice Method (VLM) was developed by Margason in 1985. Vortex Lattice Method assumes the wing is a surface consisting of small elements in the form of horseshoe vortex or ring vortex (Bertin & Cummings, 2009).

Lifting-line Theory and VLM models are widely used by researchers to estimate the aerodynamic characteristics of aircraft wings. The basic LLT model developed by Prandtl was modified by H.B. Helmholtz in 1942 to accommodate the LLT model on a straight wing with a low aspect ratio (Anderson Jr, 2001). Meanwhile, (Küchemann, 1953) developed LLT in the case of wings with swept angles. The mathematical model of LLT was also developed (Sivells & Neely, 1947), (Multhopp, 1955), and (Weber, Kirby, & Kettle, 1956). Recent research using the LLT method is related to the optimization of wing or aircraft design, such as research by (Fonseca, Pinheiro, & Arcos, 2018) and (Carvalho & Brito, 2017).

Vortex Lattice Method has been widely implemented in various software tools for conducting aerodynamic analysis in the early stages of aircraft design. The software includes Tornado (Melin, 2000), AVL (Buzdiak, 2015), and XFLR5 (Deperrois, 2013). (Gryte et al., 2018) conducted aerodynamic modeling of unmanned aircraft using wind tunnel data and numerical calculation data using the VLM method. Numerical data is obtained through XFLR5 software. (Loya, Maqsood, & Muhammad Duraid, 2018) analyzed the aerodynamic parameters of unmanned aircraft using XFLR5 with the VLM method whose results were compared with DATCOM and Computational Fluid Dynamics (CFD).

The more sophisticated technology makes fluid flow problems can be solved by numerical computation. CFD analysis

allows us to get more accurate results but requires a longer computational time than previous methods. Comparisons between methods for estimating aerodynamic parameters have been made by (Loya et al., 2018), (Ugargol & Ugargol, 2017) and (Spall, Phillips, & Pincock, 2012). In addition, several studies have been conducted comparing CFD analysis with wind tunnel tests. (Choi, Yu, & Kwon, 2014) conducted a study on the comparison between CFD and wind tunnel experiments to analyze tall buildings. The results shown from the comparison are not much different although there are several different for certain cases. Fouad et al., (2018) also conducted a study on the comparison of CFD techniques with the results of wind tunnel data. The results obtained by the CFD give very accurate and good results. The resulting Kakatatan depends on the number of grids during the calculation process. These two studies show that CFD can be used as a base for comparing previous methods.

This research will discuss a comparative study of the performance of the LLT, VLM, Panel, and CFD methods in estimating lift force distribution in terms of accuracy and computational time. The purpose of this research is to lift the wing force distribution as a basis for experimental testing of the existing load on the wing. The output data from this study are used as a base for the experimental test

## **2 Methodology**

The phenomenon of flying objects is inseparable from the presence of airflow on the object by to reduce force. This force is called the aerodynamic force. This aerodynamic force basically comes from the pressure and shear stress that occurs on the surface of the object as shown in Figure 2-1.

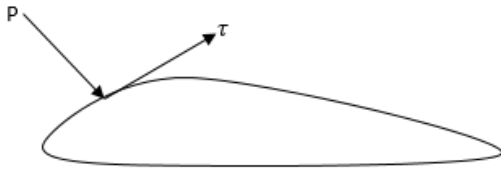


Figure 2-1: Pressure and shear stress

The pressure on each of these small surfaces accumulates resulting in the resultant forces and moments. Two types of force and moment components are determined based on the direction of the force that is the force in the direction of the object and the direction of the wind. This type of force component is illustrated in Figure 2-2.

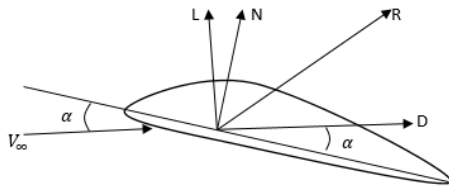


Figure 2-2: Resultant and component force

Where  $L$  is the lift force, that is the force component which is perpendicular to the wind direction ( $V_\infty$ ), while  $D$  is the drag force, that is the force component which is in the direction of  $V_\infty$ .  $N$  is the normal force that is the force component which is perpendicular to  $c$ , while  $A$  is the axial force that is the force component which is in the direction of  $c$ . The angle of attack  $\alpha$  is defined as the angle between the direction of the wind and  $c$  so that  $\alpha$  can also be defined as the angle between  $L$  and  $N$  as well as  $D$  and  $A$ . Therefore, the following equation can be formed:

$$L = N\cos(\alpha) - A\sin(\alpha) \quad (2-1)$$

$$D = N\sin(\alpha) + A\cos(\alpha) \quad (2-2)$$

In the discipline of aerodynamics, some dimensionless forces and moments are often known as aerodynamic coefficients. For example  $\rho_\infty$  and  $V_\infty$  are the density and velocity of air in an open

flow (freestream), then dynamic pressure can be defined as:

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 \quad (2-3)$$

Dynamic pressure has the same units as pressure (Newton and similar units). For example,  $S$  is the area of an object reference and  $l$  is the object reference length, then the aerodynamic coefficient is defined as follows:

$$C_L = \frac{L}{q_\infty S} \quad (2-4)$$

$$C_D = \frac{D}{q_\infty S} \quad (2-5)$$

$$C_M = \frac{M}{q_\infty S l} \quad (2-6)$$

## 2.2 Problem Definition

The process of making an airplane, especially on the wing, needs to be analyzed the distribution of forces that exist along the wing. This is done to make the structure of the aircraft wing so that it is strong with the weight of the aircraft. Because the load received by the wing is large, it is necessary to analyze the force distribution on the wing.

## 2.3 Method

### 2.3.1 Computational Fluid Dynamics

Computational Fluid Dynamics (CFD) is a numerical simulation tool for analyzing and designing fluid flow systems, heat transfer, and other fluid phenomena. In this study, CFD simulations are used to analyze the airflow that occurs in the area of the aircraft with the same actual flying conditions.

The basic concept of CFD is derived from physical phenomena that occur in the fluid. This phenomenon is described in a mathematical model called the governing equation. The general equation

commonly used to represent fluid flow behavior is the Navier-Stokes equation (Panagiotou, Kaparos, Salpingidou, & Yakinthos, 2016). For the assumption of incompressible flow, the Navier-Stokes equation is expressed as

$$\rho \left( \frac{\partial v}{\partial t} + (v \nabla) v \right) = -\nabla p + \mu \nabla^2 v + \rho F \quad (2-7)$$

In general, CFD analysis is carried out in three major stages, namely:

- 1) Pre-processing
- 2) Solving
- 3) Post- Processing

In pre-processing there is a process of determining the turbulent model. This turbulent model is used to model fluid flows that move randomly and are unstable. In this study to solve this problem using the Shear Stress Transport model provided by ANSYS-CFX. The Shear Stress Transport model works by solving the models based on turbulence/frequency ( $k-\omega$ ) on walls and  $k-\epsilon$  in fluid flow (Ansys, 2004). K-epsilon is a turbulent equation model with two equations to solve the turbulent kinetic energy  $k$  and the dissipation power  $\epsilon$ . Whereas  $k-\omega$  is an alternative equation to replace the k-epsilon equation. Similarly, the k-epsilon equation, the  $k-\omega$  equation model is used to solve the turbulent kinetic energy  $k$  and the specific dissipation power  $\omega$ . The advantage of  $k-\omega$  compared to  $k-\epsilon$  is that this equation increases the boundary layer performance due to pressure gradients.

### 2.3.2 Vortex Lattice Method.

Vortex Lattice Method is a numerical method used to analyze fluid dynamics. In principle, this VLM models a surface on an aircraft into an infinite number of vortices to analyze or calculate the pressure distribution on an aircraft, in this case, VLM can also be used to calculate the force distribution on an

aircraft. In this method, it is assumed that fluid flow is incompressible, inviscid and irrotational and the influence of thickness on force is neglected.

Based on the assumptions above, flow that is incompressible and irrotational, according to (Anderson Jr, 2001) can be synthesized by adding a similar elementary flow of energy. The intended elementary flow can be a point or a vortex line.

There are four important theories that are used to illustrate this effect and to model an air flow around the wing (Budziak, 2015). The four theories are

- a. Biot-Savart Law
- b. Kutta-Joukowski theorem
- c. Hermann von Helmholtz theory
- d. Prandtl lifting-line theory

According to Biot-Savart's law the vortex line induces a certain velocity field, at any point P there is a distance  $r$  from a filament causing that speed to be induced by the vortex. This concept is illustrated in the picture

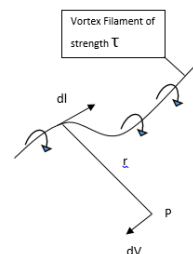


Figure 2-3: Illustration of Biot – Savart Law

Based on the illustration in Figure 2-3, the form of a mathematical model of Biot-Savart law as follows:

$$dV = \frac{\tau}{4\pi} \frac{(dl \times r)}{|r|^3}$$

atau

$$V = \frac{\tau}{4\pi} \int \frac{(dl \times r)}{|r|^3} \quad (2-8)$$

Where

- $dl$  : infinite small filament partitions
- $r$  : distance from point P to the point in the filament

$\tau$  : Vortex strength

$V$  : Induced Speed

According to the Kutta-Joukowski theorem, certain moving vortex with the strength of the speed which is bound in the flow velocity  $V_\infty$  will produce lift force, in other words

$$L = \rho V_\infty \tau \quad (2-9)$$

with

$\rho$  : air density

$V_\infty$  : Freestream speed

$L$  : Lift Force (Lift)

Hermann von Helmholtz's theory illustrates the principle of a vortex filament must be from a closed path (eg vortex ring) and circulation along the vortex filament is constant.

Prandtl lifting-line theory describes vortex rings as horseshoe vortex. This is because the vortex ring can be changed into four vortex filaments that are definitely closed.

### 2.3.3 The 3D Panel Methode

The 3D Panel method in XFLR5 software is applied to analyze in 3D form by considering wing thickness. Different from VLM that only considers the mean camber line. Moreover 3D Panel method is also used to analyze the pressure distribution on the upper and lower surface of the wing. In principle, this method models the existing disturbance on the wing surface with a number of doublets and sources which is distributed above and below the wing surface (Deperrois, 2013). This method uses doublets and sources on a flat and linear panel. In detail, mathematical model for the 3D Panel method there is in reference.

### 2.4 VLM Model and 3D Panel Model

Numerical simulation using VLM and Panel Method to calculate the distribution of lift force on the wing as an

initial prediction for the process of calculating the load analysis on the structure. In this calculation used XFLR5 software. XFLR5 is software used to analyze aerodynamic parameters in airfoil, wings and fuselage, also can be used to analyze full configuration.

Modeling using the first XFLR5 software requires airfoil coordinates which is the basis for making wings. Then with that input, wing model with XFLR5 was made and determine the number of elements on the wing. Figure 2-4 below is the result of wing modeling with XFLR5 software using VLM and Horseshoe Vortex analysis

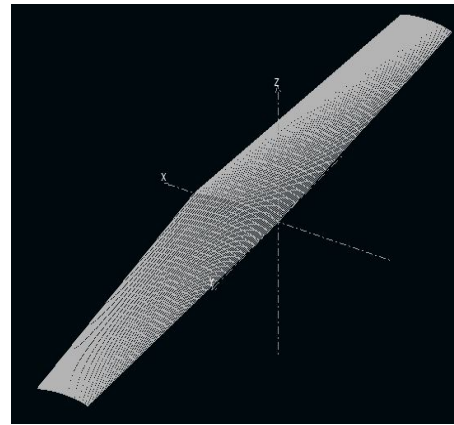


Figure 2-4: Image of wings with VLM and Horseshoe Vortex with XFLR5

While for the wing image using the 3D Panel Method on XFLR5 the image is obtained as follows

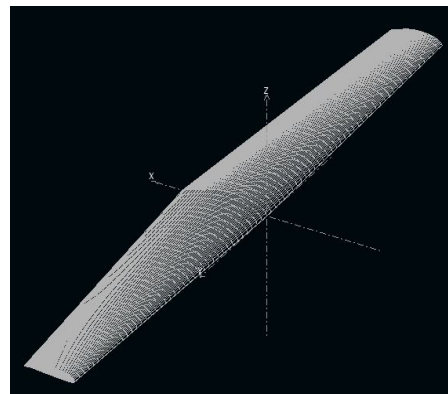


Figure 2-5: Image of wings with the 3D Panel Method with XFLR5

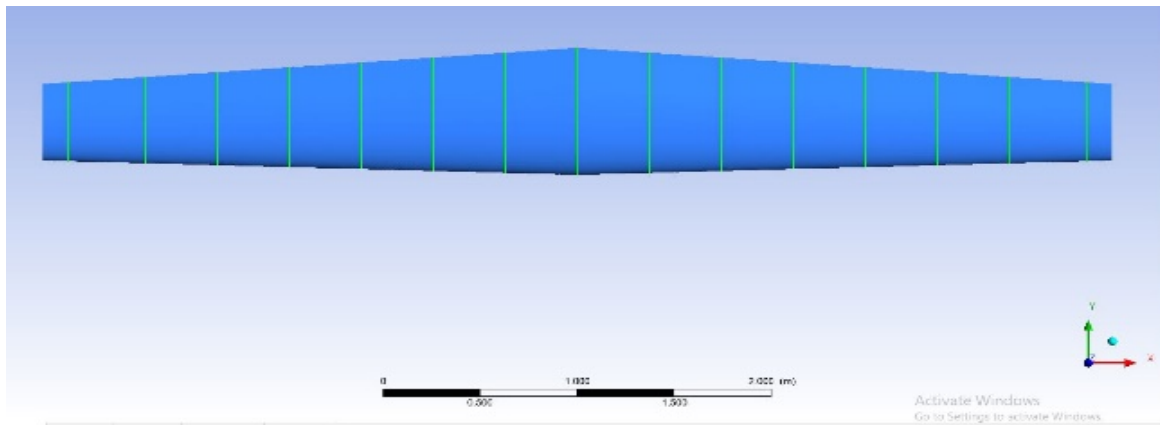


Figure 2-6: The Position of observation of Wing Lift Distribution on the wing

Then the resulting images were analyzed by VLM, Horseshoe Vortex and 3D Panel methods.

### 2.5 CFD Model

Lift force distribution calculations are performed on the cruise configuration. There are a total of 15 points to be observed based on the need for wiffletree testing. These points can be seen in Table 2-1.

Table 2-1: The position of observation of lift force distribution

No	Position observation
1.	-2.62
2.	-2.22
3.	-1.85
4.	-1.48
5.	-1.11
6.	-0.74
7.	-0.37
8.	0.00
9.	0.37
10.	0.74
11.	1.11
12.	1.48
13.	1.85
14.	2.22
15.	2.62

The position of observation can also be seen in Figure 2-6, where the numbering sequence starts sequentially from left to right. The yellow line in Figure 2-6 is the airfoil-shaped wing intersection where the lift value is calculated

### 3. Result and Analysis

This simulation is done on cruise configuration. The parameters used in both configurations can be seen in following.

Table 3-1: Parameters Input

Parameter	Symbol	Value
Configuration	-	Cruise
Velocity	$v$	30 m/s
Air density	$\rho$	1.09224 kg/m <sup>3</sup>
Wing area	$S$	3.2175 m <sup>2</sup>
MAC	-	0.597 m
Span	$b$	5.5 m

The simulation process using CFD on the cruise configuration the number of mesh used was 4099054 elements. Aerodynamic force analysis which includes lift coefficient, drag coefficient, CL vs CD coefficient and efficiency (L / D) using CFD is shown in Figure 3-1.

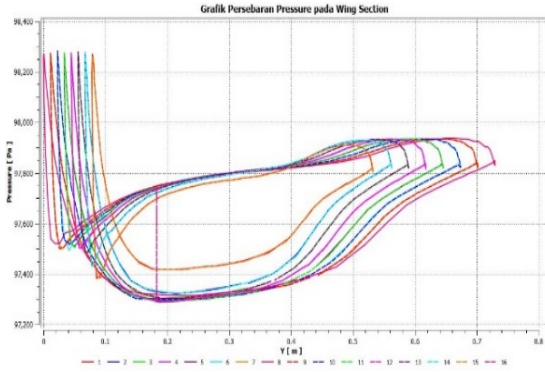


Figure 3-1: Pressure graph vs Y axis in 15 different positions

Can be seen in Figure 3-1 that there are only 8 different graphs. This means that there are two graphs that have the same pressure distribution. The method used in the interp1 command in Matlab is the Spline method. A partition of  $n = 100$  is used. From the calculation results, the lift distribution value obtained at 15 points as in Table 3-2 follows.

Table 3-2: Value of lift respect to position of wing section

Position X (m)	L' (N/m)
-2.62	88.5899
-2.22	139.9946
-1.85	163.1239
-1.48	180.4520
-1.11	195.0827
-0.74	206.7499
-0.37	217.7947
0.00	221.4290
0.37	216.2650
0.74	207.1259
1.11	195.1621
1.48	180.7269
1.85	163.5302
2.22	140.1262
2.62	88.8286

Figure 3-2 bellow displays a lift graph to the position of the wing section.

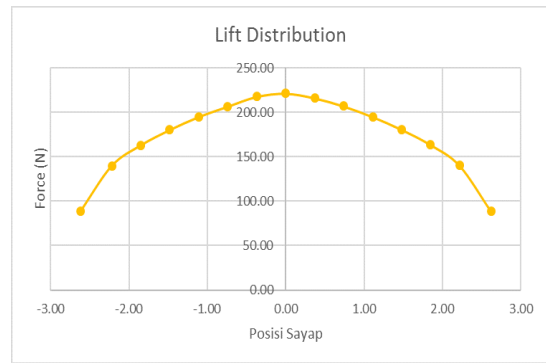


Figure 3-2: Lift value graph in Wing Position

Based on research conducted by (Choi et al., 2014) and (Fouad, Mahmoud, & Nasr, 2018) the results generated from CFD simulations for the analysis of lift distribution are accurate results. So that it can be used as a reference for comparing LLT, 3D Panel, Horseshoe Vortex and VLM methods

Analysis of aerodynamic forces using VLM and 3D Panels was carried out on 6000 and 12150 panels, respectively. The large number of panels shows increasingly similar to the original wing shape. On the other hand, the large number of panels affects the simulation time.

There are differences of amount in analysis with VLM and 3D Panel. This difference can be seen in Figure 3-1 and 3-2. This is because VLM only analyzes the mean camber line while 3D Panel analyzes many three-dimensional panels on the wing.

Analysis of force distribution with VLM and 3D Panel on XFLR5 software only produces raw data. Then the data is processed into data that represents the distribution of forces along the wing. The results of the data processing are made in graphical form as follows. This shows that VLM is the result of the development of the Horseshoe Vortex method.

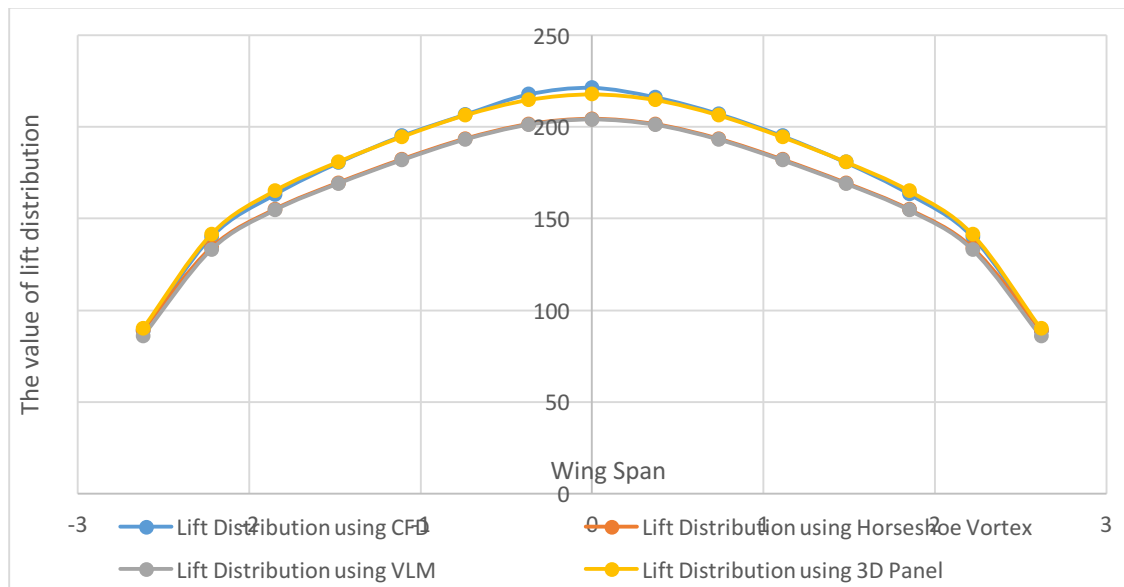


Figure 3-3 : Lift distribution with 4 different methods

However, the comparison of results between VLM and 3D Panel is maximum of 13 N, a large enough difference.

### 3.1. Comparison Results of VLM, Horseshoe Vortex, 3D Panel and CFD

The analysis results obtained by using CFD, VLM, 3D Panel and Horseshoe Vortex made a comparison of the values of the force distribution at 15 observation points. Comparison of the results of the four methods is presented in Figure 3-3. The distribution of lift values at the observation position at the point mentioned in Table 3-4 shows the difference in lift values at each point not too far away. Based on Figure 3-3 to approach the results of the lift force distribution calculation with CFD is the 3D Panel method. The difference in distribution values at each observation point is very small. As for the VLM and Horseshoe Vortex methods, the difference in lift distribution at each observation point is relatively close to CFD. The biggest difference between CFD and VLM and Horseshoe Vortex lies in the root chord wing. To find out the value at each

point of observation can be seen in Table 3-4.

However, in terms of time to do one simulation with variance of the angle of attack each method is presented in Table 3-3.

Table 3-3: Time required for each method

Method	Time
3D Panel	400 second
VLM	65 second
Horseshoe Vortex	66 second
CFD	1210 second

As already mentioned that the simulation is done on cruise conditions means that it is only simulated when the angle of attack is zero. Based on Table 3-3 it appears that the VLM method is a fast method to do simulations with the time required is 65 seconds. While CFD requires quite a lot of time which is 1210 seconds. For the VLM method, Horseshoe Vortex and 3D Panel can simulate for several angles of attack with a fairly fast time, not much different from simulations for one angle of attack.

Table 3-4: Lift Force Distribution Value with Four Methods at 15 observation points



Observation position	METHOD			
	3D Panel	VLM	Horseshoe Vortex	CFD
-2.62	89.92180075	85.73947843	89.05205992	88.59
-2.22	141.5271871	133.0338307	134.0970227	139.99
-1.85	165.2175825	154.5349692	155.0616294	163.12
-1.48	181.0207952	169.0349681	169.5274925	180.45
-1.11	194.5916861	181.7632156	1822917797	195.08
-0.74	206.4318603	193.0378102	193.5467664	206.75
-0.37	214.812132	201.077463	201.5339771	217.79
0	217.86	204.01	204.44	221.43
0.37	214.8121321	201.077463	201.5339771	217.79
0.74	206.4318604	193.0378102	193.5467665	206.75
1.11	194.5916863	181.7632157	182.2917798	195.08
1.48	181.0207954	169.0349682	169.5274926	180.45
1.85	165.2175828	154.5349694	155.0616295	163.12
2.22	141.5271874	133.033831	134.0970229	139.99
2.62	89.92180115	85.73947906	89.05206017	88.59

Whereas CFD within 1210 seconds is only for one angle of attack. Difference of 810 seconds with the 3D Panel method. The difference in the results of the 3D Panel method analysis with VLM and Horseshoe Vortex is due to the geometry formed using the 3D Panel method which is very similar to the CFD. Therefore the 3D Panel method can analyze the distribution of pressure on the upper and lower surface of the wing.

Referring to Table 3-4, it appears that the difference between the results of the CFD and 3D Panel is not too far away. Only in the root chord the difference is only 4N adrift. With a small enough difference, for the analysis of the lift force distribution on the wing for prediction of static wing testing.

Based on research conducted by (Choi et al., 2014) and (Fouad et al., 2018), that CFD can approach experimental results. Therefore, with a shorter and faster time, to do the analysis of the force distribution on the the aircraft wings is enough to do with the 3D Panel method.

#### 4 Conclusions

In this research, the lift force distribution analysis on the wings of LSU 05-NG has been carried out with four methods namely CFD, VLM, Horseshoe Vortex and 3D Panel. This analysis process begins with making wing geometry with the FX 76-MP 160 airfoil. Then the analysis is carried out based on the results and time required for analysis. Based on the results of research, it was found that VLM as an efficient method in terms of time. However, VLM provides inaccurate results to approach the CFD. The 3D Panel method takes 400 seconds, giving very good results to approach the analysis results of the lift force distribution with CFD. With a relatively fast time, 3D Panel is an effective and efficient method to approach the analysis results with CFD.

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### Contributorship Statements

AS analyzed the result and developed the VLM, Horseshoe Vortex and 3D Panel Method simulations; KH designed CFD simulations method, AR and YGW prepared the manuscript

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