

# Comparing Stress ECG Enhancement Algorithms

*With an introduction to a filter bank based approach*

There are two predominant types of noise that contaminate the electrocardiogram (ECG) acquired during a stress test: the baseline wander noise (BW) and electrode motion artifact, and electromyogram-induced noise (EMG) [1]. BW noise is at a lower frequency, caused by respiration and motion of the subject or the leads. The frequency components of BW noise are usually below 0.5 Hz, and extend into the frequency range of the ST segment during a stress test. EMG noise, on the other hand, is predominantly at higher frequencies, caused by increased muscle activity and by mechanical forces acting on the electrodes. The frequency spectrum of the EMG noise overlaps that of the ECG signal and extends even higher in the frequency domain. In this article, we review some of the published ECG enhancing techniques to overcome the noise problems, and compare their performance on stress ECG signals under adverse noise scenarios. We also describe the Filter Bank (FB) based ECG enhancing algorithm [9].

## Overview

Figure 1 shows a noise-free ECG beat with ST-segment depression induced by exercise (top) and various epochs of this ECG with different noise conditions, as during a stress test. It is important to measure the dynamic changes in the morphology of segments of the ECG induced by the exercise, even in the presence of noise.

Many ECG enhancing techniques to address the noise problem have been reported in the literature. In Ref. [2] a combination of mean and median algorithms is used on the filtered ECG. Reference [3] presents a BW noise removal filter which meets specifications in Ref. [4] and a time-varying filter to remove high-frequency EMG noise. Reference [5] provides a technique which subtracts the current heart-beat average to get a 'QRS-free' signal, estimates the BW from the down-sampled QRS-free signal, and then sub-



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Paper  
Award  
Winner**

tracts the estimated BW from the noisy ECG. Reference [6] uses a source consistency filtering technique that seeks to develop a transfer function of the cardiac dipole. In Ref. [7], an adaptive baseline wander filter is designed as a cascade of two adaptive filters. Reference [8] uses a cubic spline technique to estimate and then subtract the BW in the ECG.

The cubic spline method works well

when the 'knots' used to estimate the BW are accurately determined. However, the determination of the knots can be adversely affected by a noisy ECG signal, which compromises the BW estimate. Adaptive filtering of the ECG assumes that either the signal or noise is stationary or nonstationary. These characteristics are not guaranteed in a stress ECG recording.

The various ECG enhancement algorithms addressed in this article compute an enhanced beat from a set of ECG beats or epochs. We refer to the enhanced beat of any enhancing algorithm as the *composite* beat. Other articles may refer to the enhanced beat as the "averaged" beat, but we wish to avoid confusion with the usual sense of the word "averaged," which indicates the arithmetic mean of data.

## Signal Enhancing Algorithms

**1. Mean composite:** The mean composite  $C_{mean}$  is determined by computing the arithmetic mean of a set of noisy beat epochs. The epochs are time aligned using a fiducial point in the heart beat, such as the R wave. The mean composite is given by:

$$C_{mean}(n) = \frac{1}{N} \sum_{k=1}^N x_k(n)$$

$$0 \leq n \leq L-1$$

where  $x_k$ ,  $k = 1, \dots, N$  are the noisy epochs of length  $L$ .

This is the simplest strategy of enhancing the ECG. In an ideal situation, where the noise is uncorrelated with the signal, is stationary, and has a Gaussian distribution, the signal-to-noise-ratio is improved by a factor of  $\sqrt{N}$ .

If one of the epochs has a sudden ECG baseline shift, or is an arrhythmia such as a ventricular ectopic beat, the resulting mean composite will be distorted. A pre-processing step prior to the mean composite algorithm should determine the 'goodness' of the epoch and decide

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whether it should be included in the composite or rejected.

**2. Median composite:** The median composite,  $C_{median}$ , is determined by computing the median of sample values across the set of epochs, for each time instant. The epochs are time aligned by using a fiducial point in the heart beat, such as the R wave.

$$C_{median}(n) = median\{x_1(n), x_2(n), \dots, x_N(n)\} \\ 0 \leq n \leq L-1$$

where  $x_k, k = 1, \dots, N$  are the noisy epochs of length  $L$ .

The median technique removes any outliers in the distribution of data at each instant across the epochs. Thus, baseline shifts in an epoch will be disregarded and not affect the median composite. Bursts of high frequency noise in the ECG will be removed, since these represent outliers in the set of epochs to be composed.

**3. Hybrid composite:** The hybrid composite algorithm combines the benefits of the mean and median composite algorithms [2]. The mean composite algorithm is computationally efficient, and optimal for high-frequency noise reduction, but is susceptible to low-frequency noise. The median composite algorithm is computationally expensive. Their combination is an algorithm that is near optimal for high-frequency noise, good at low-frequency noise reduction, and is computationally efficient.

In the hybrid algorithm, a sequence of incoming beats is partitioned into three groups by using one of three strategies. A random grouping strategy partitions the incoming beats into the three groups randomly. A block grouping strategy partitions the first one-third of the epochs into the first group, the second one-third into the second group, and the final one-third into the last group. A sequential grouping strategy partitions sequential beats into different groups. The arithmetic mean composite is computed for each of the three groups, regardless of the strategy used. The baseline level estimated from the PQ segment is then removed from each of the three mean composites. For each group, a low-pass filter with a corner frequency of approximately 15 Hz is used to remove the correlation between the high-frequency noise and the low-frequency noise.

The hybrid composite algorithm is then obtained by summing the median of the low-frequency signals with the me-

dian of the high-frequency signals. The algorithm is represented as follows:

$$E = \{x_1, x_2, \dots, x_N\} \\ E_k = Group(E) \quad k = 1, 2, 3 \\ y_k = mean\_composite(E_k) \quad k = 1, 2, 3 \\ z_k = baseline\_correction(y_k) \quad k = 1, 2, 3 \\ LP_k(n) = \\ \frac{1}{J} (z_k(n - \frac{J}{2}) + z_k(n - \frac{J}{2} + 1) + \dots + z_k(n + \frac{J}{2})) \quad k = 1, 2, 3$$

$$HP_k(n) = z_k(n) - LP_k(n) \quad k = 1, 2, 3$$

$$C_{hyb} = median\_composite\{LP_1, LP_2, LP_3\} + median\_composite\{HP_1, HP_2, HP_3\}$$

where  $x_1, x_2, \dots, x_N$ , are the noisy epochs,  $N$  is the number of noisy epochs each of length  $L$ , and  $J$  is chosen based on the sampling frequency to get a cutoff frequency of approximately 15 Hz.  $LP_k$  and  $HP_k$  refer to the low-passed and high-passed signals respectively.

**4. Trimmed mean composite:** A trimmed composite  $C_{Trim}$  of a set of time-aligned epochs is computed by first sorting the values at each time instant through the epochs. The bottom 20% and top 20% of the 'sorted' epochs are discarded. For example, if there are 10 epochs in an episode, after sorting and discarding there will be  $10 - 2 - 2 = 6$  "sorted" epochs left. The arithmetic mean of the remaining epochs is then computed to obtain the trimmed composite:

$$E = \{x_1, x_2, \dots, x_N\}$$

$$E' = sort(E) = \{x'_1, x'_2, \dots, x'_N\}$$

where

$$x'_1(n) \leq x'_2(n) \leq \dots \leq x'_N(n) \quad 0 \leq n \leq L-1$$

$$E'' = \{x'_{i+1}, \dots, x'_{N-i}\} \quad i = round(0.2 \times N)$$

$$C_{Trim} = mean\_composite(E'')$$

and where  $x_1, x_2, \dots, x_N$ , are the noisy epochs,  $x'_1, x'_2, \dots, x'_N$ , are the "sorted" noisy epochs, and  $N$  is the number of noisy epochs each of length  $L$ .  $E'$  is the set of "sorted" noisy epochs, and  $E''$  the trimmed set of "sorted" noisy epochs.

The trimmed mean algorithm removes outlier data values in the set of epochs to be composed before computing the mean composite of the residual data. It thus

incorporates a feature of the median algorithm, in that extreme data values, or outliers, do not influence the resultant composite. In addition, the use of the mean algorithm to determine the final composite ensures optimal noise reduction in the case of Gaussian distributed, uncorrelated, and stationary noise.

**5. Incremental composite:** The fixed incremental composite,  $C_{IncrF}$ , is computed by increasing or decreasing each sample value in the current composite beat by a fixed amount [10]. The direction of the change in value of each sample depends on the corresponding sample in the next epoch. If the sample in the noise epoch is greater than the corresponding one in the current (running) incremental beat, the latter point is increased by a fixed amount. The increment parameter in the algorithm is specified as corresponding to a specific *mm* distance on a strip chart recording of the ECG with a 1 mV/cm vertical axis sensitivity. This choice provides a balance between immunity to noise and dynamic response of the composite.

The algorithm for the fixed incremental composite is given as:

$$E = \{x_1, x_2, \dots, x_N\}$$

$$C_{IncrF} = x_1$$

for  $k = 2$  to  $N$  {

$$\Delta = C_{IncrF} - x_k$$

$$C_{IncrF} = C_{IncrF} - P_{Fixed} \times sign(\Delta)$$

}

where  $x_1, x_2, \dots, x_N$ , are the noisy epochs,  $sign(\Delta)$  operates on each element of the vector  $\Delta$ , and is 1 if  $\Delta(n) > 0$ , -1 if  $\Delta(n) < 0$ , or 0 otherwise,  $N$  is the number of noisy epochs each of length  $L$ , and  $P_{Fixed}$  is the fixed incremental parameter.

A second version of this algorithm computes a percentage incremental composite  $C_{IncrP}$ , where the incremental parameter is a percentage of the difference between the running incremental beat and the incoming epoch.

$$E = \{x_1, x_2, \dots, x_N\}$$

$$C_{IncrP} = x_1$$

for  $k = 2$  to  $N$  {

$$\Delta = C_{IncrP} - x_k$$

$$C_{IncrP} = C_{IncrP} - P_{Per} \times \Delta$$

}

where  $x_1, x_2, \dots, x_N$ , are the noisy epochs,  $N$  is the number of noisy epochs each of

length  $L$ , and  $P_{Per}$  is the percentage incremental parameter.

### Filter Bank Based Composite

The filter bank-based strategy relies on a filter bank (FB) containing a set of analysis filters  $H_l(z)$  and synthesis filters  $F_l(z)$  (Fig. 2). The analysis filters decompose the input signal,  $x(n)$ , into  $M$  frequency bands (subbands) and subsample by a factor of  $M$ . Processing can be performed on each subband independently. The synthesis filters combine the processed subbands to reconstruct the input signal. Thus, a FB-based algorithm involves decomposing a signal into frequency subbands, processing these subbands according to the application, and reconstructing the processed subbands. The design and use of FBs is widely reported in the literature [11-13].

Many signals contain specific energy distributions in the frequency domain. For example, a significant proportion of the energy from the QRS complex in the ECG extends to a frequency of 40 Hz, and even more if the Q, R, and S waves have a very sharp morphology. The P and T waves, in general, have a significant proportion of energy only up to 10 Hz. Thus, there is a benefit in using a FB-based algorithm, where time and frequency dependent processing can be performed.

**FB block diagram:** Assume that the FB contains  $M$  analysis and synthesis filters, each of length  $L$ . The analysis filters  $H_l(z)$ ,  $l = 0, 1, \dots, M-1$ , bandpass the input signal  $X(z)$  to produce the subband signals  $U_l(z)$ :

$$U_l(z) = H_l(z)X(z) \quad l = 1, 2, \dots, M-1$$

Since the effective bandwidth of  $U_l(z)$  is  $\frac{2\pi}{M}$ , it can be downsampled to reduce

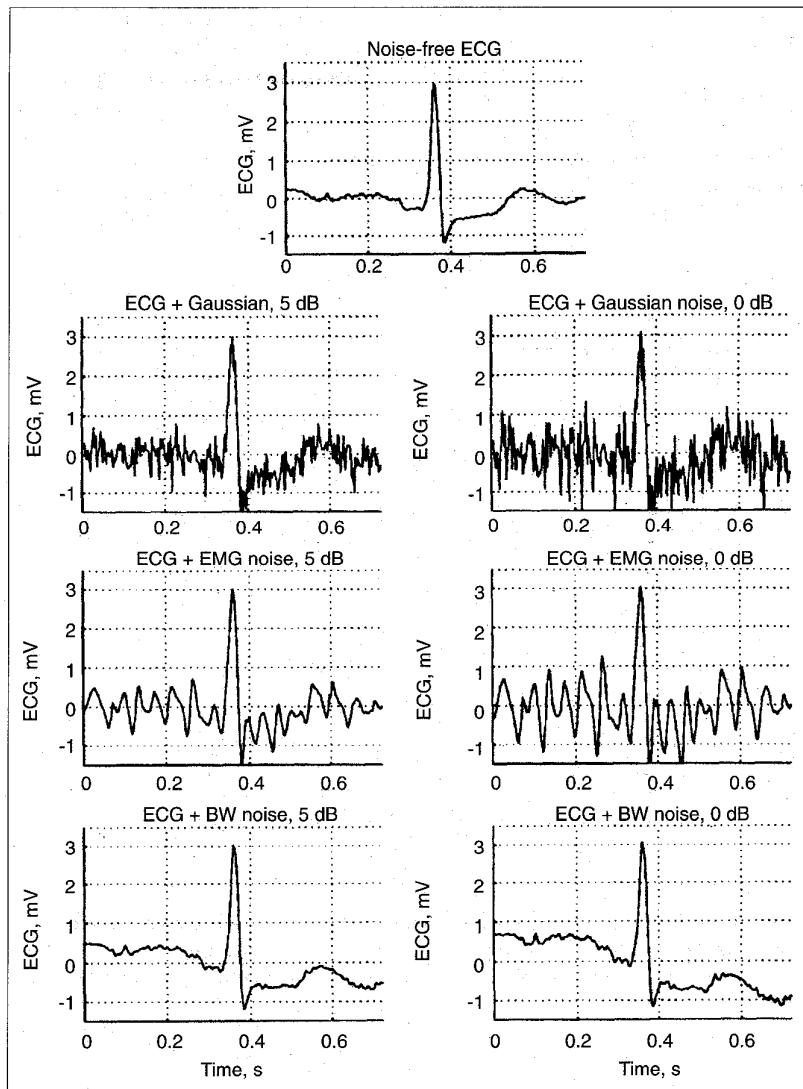
the total rate. The downsampling process keeps one sample out of  $M$  samples. The downsampled signal  $W_l(z)$  is:

$$W_l(z) = \frac{1}{M} \sum_{k=0}^{M-1} U_l(z^{1/M} e^{-jk2\pi/M})$$

for  $l = 0, 1, \dots, M-1$

Taking advantage of the downsampling, we can efficiently apply the filtering process at  $\frac{1}{M}$  the input rate. This implementation is referred to as polyphase implementation and contributes to the computational efficiency of the FB-based algorithm [11].

Time and frequency dependent processing can now be performed on some or all of the subband signals to result in a



1. Examples of a noise-free ECG beat contaminated with various types of noise. EMG is electromyogram noise and BW is baseline wander noise. The SNR level is low enough so as to distort the underlying ECG signal.

processed and downsampled subband signal,  $W_{pl}(z)$ . The nature of the processing involved is application dependent, and stress ECG enhancement is explained later.

The reconstruction is achieved by upsampling and interpolating the subband signals using a set of bandpass filters,  $F_l(z)$ . Similar to the analysis bank, the filtering process in the synthesis bank can be efficiently implemented by taking advantage of the  $M-1$  zeros in the upsampled sequence,  $V_l(z)$ . This processing contributes to the overall computational efficiency of the FB-based algorithm.

The processed subband signals,  $O_l(n)$ , can then be added algebraically point-by-

point to result in a time and frequency dependent processed version,  $Y(z)$ , of the input signal,  $X(z)$ . The analysis and synthesis filters must be designed to incorporate useful properties for the application at hand.

### Filter Bank Properties

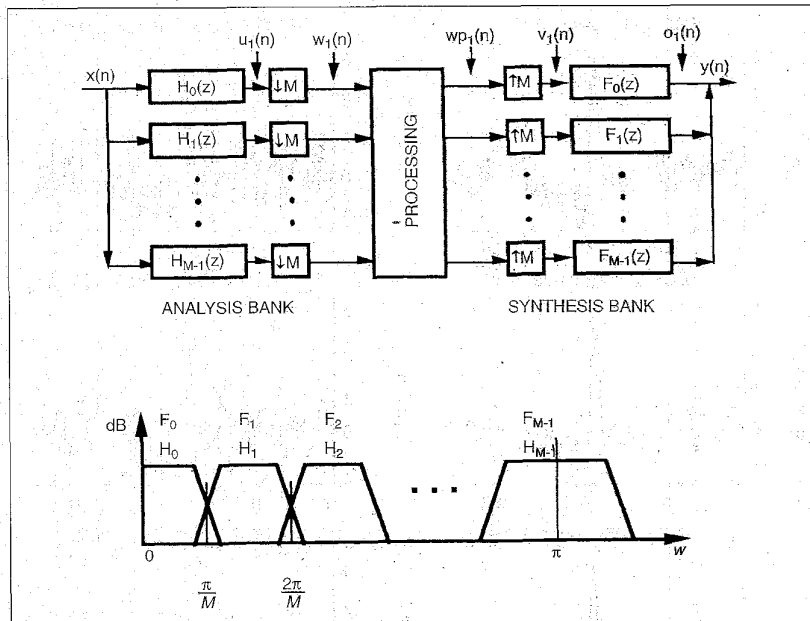
The analysis filters decompose the ECG into uniform frequency subbands. For stress ECG enhancement, and many other ECG processing tasks, it is useful while processing each subband to have a fixed or deterministic relationship between data in the processed subbands and data at the input. This requirement implies that the analysis filters have *linear phase* frequency characteristics, or *constant*

group delay for all frequencies in the pass-band.

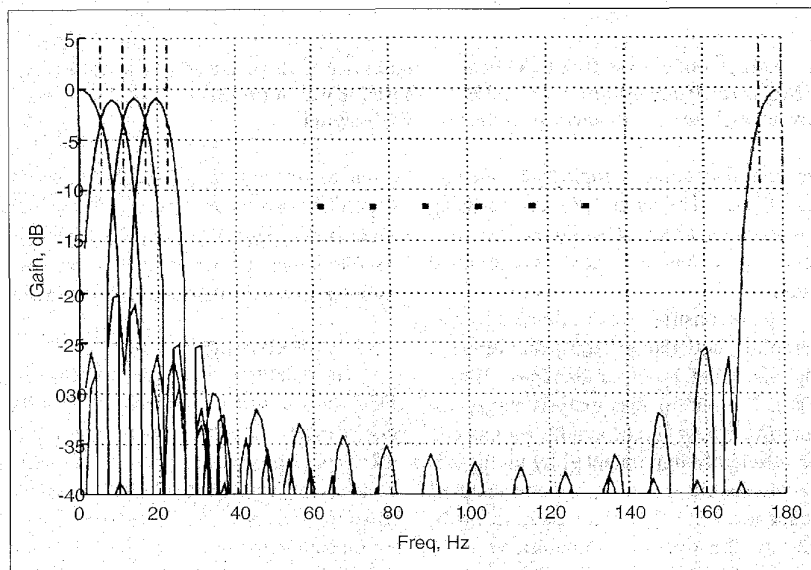
The linear phase requirement ensures that all frequencies at the input will have the same *sample delay* through the analysis filters. It is then possible, for example, to determine the exact location of the R wave and other fiducial points in the sub-

bands. Moreover, this linear phase requirement on each filter in the FB should be distinguished from the linear phase property of the whole FB system. If there were no processing block in Fig. 2, the output,  $y(n)$ , could have three types of distortion from the input,  $x(n)$ . The first distortion is the aliasing introduced by the

downsampling, due to the nonideal nature of the analysis and synthesis filters. Secondly, the output,  $y(n)$ , could have a magnitude distortion compared to the input,  $x(n)$ , if the analysis and synthesis filters were not designed to cancel effects due to the nonideal magnitude response of the filters. Finally, if the output cannot be represented in terms of the input by a simple sample delay it could have a phase distortion compared to the input. Thus, it is possible for the FB system output to have no phase distortion with respect to the input (i.e., a *linear phase FB*) even if each of the analysis and synthesis filters have *nonlinear* phase frequency responses. In ECG processing, however, it is important that each of the analysis and synthesis filters have a linear phase frequency response.



2. A filter bank-based algorithm enables time and frequency dependent processing with a reduced number of computations per second. Ideal magnitude responses of the filters are shown. One set of filters has the potential to accomplish multiple ECG processing tasks.



3. The magnitude responses of the 32 channel filter bank. These filters have linear phase, are orthogonal, and have the perfect reconstruction property.

### Perfect Reconstruction

The output of the FB can differ from the input due to the various reasons stated above. The processing block incorporated into the FB system will itself cause the output to differ from the input. The FB used in this study is a perfect reconstruction (PR) system that reproduces the input signal at the output when there is no processing. The ideal relationship between the input and output in Fig. 2 is:

$$y(n) = cx(n - k)$$

where  $k$  is the system delay, and  $c$  is a constant gain factor.

Aliasing and imaging distortions are the results of nonideal magnitude responses of the filters. The decimated signal in Fig. 2,  $w_1(z)$ , has aliased frequencies of the input. These errors can be cancelled if one designs the analysis and synthesis filters appropriately. This is well explained in various sources in the literature [11-13].

In an alias-free FB,  $y(z) = T(z)X(z)$ , where  $T(z)$  is the overall distortion function. Clearly, there is no distortion if  $T(z) = z^{-k}$ . The system has no magnitude distortion if  $T(z)$  is an all-pass function; and there is no phase distortion if  $T(z)$  is a linear phase function. PR requires that the overall transfer function of the FB have an all-pass magnitude response ( $c$  is a constant), and a linear phase response ( $k$  is a fixed delay).

The reason for using the PR property of filter banks is that an overall goal is to develop one set of filters which will work well for various ECG processing tasks. Neither ECG beat detection nor classifica-

tion requires reconstruction at the output of the filter bank (see Fig. 2). For these applications, only decomposition of the input into frequency subbands is of interest. Stress ECG enhancement, however, requires reconstruction of the processed subbands. This task requires a PR filter bank with no phase or magnitude distortion.

The 32-channel FB was designed, based on an algorithm from Ref. [14]. The analysis and synthesis filters have linear phase responses and their magnitude responses are as shown in Fig. 3.

### Subband Processing

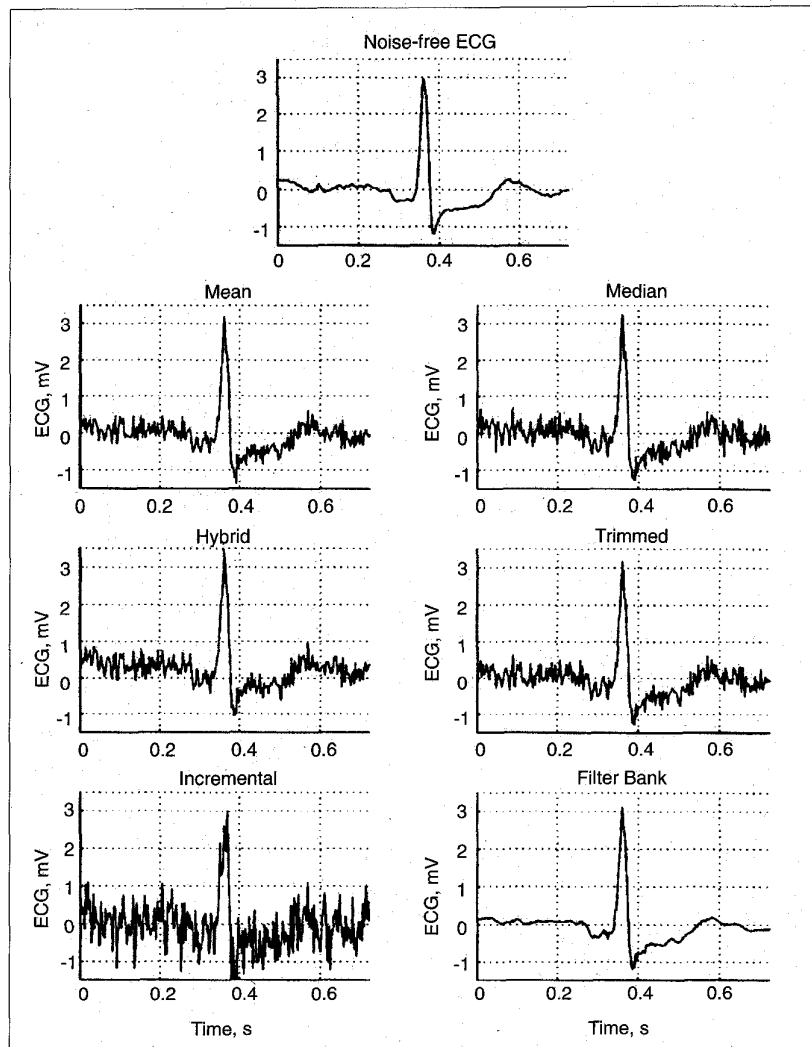
The FB structure enables processing of a signal in a specific time period and a specific frequency region. The FB-based algorithm decomposes the ECG into 32 uniform subband frequencies of the signal (see Fig. 3). The 0 to 180 Hz frequency bandwidth of the input signal is decomposed into 32 uniform frequency subbands, {[0 to 5.625], [5.625 to 11.25], ..., [174.375 to 180]} Hz.

The subband in the [0 to 5.625] Hz range, which contains most of the energy of the P and T waves, is not processed in any way. In the remaining subbands, the signal components are attenuated to various levels in time periods that correspond to the non-QRS region. Since the P and T waves do not have energy at higher frequencies, we attenuate the signal components in these time periods and frequencies. The QRS region of the ECG is not modified in any of the subbands. The processed subband signals are then reconstructed by the synthesis filters to result in a time and frequency dependent processed version of the input signal, in which noise has been reduced without distorting ECG components of interest.

The Q and S fiducial points to mark the QRS complex are determined from the input noisy ECG using the algorithm described in Ref. [15]. Since each of the analysis filters has linear phase, the Q and S waves can be located in each subband signal after accounting for filter delays.

### FB-based Composite

An FB based composite is obtained by performing the above operations on the input ECG. To compute a FB based composite from a set of more than one noisy epoch, the FB composite is computed for each epoch, and then the mean of the resulting composites computed.



4. Composites from various enhancing algorithms using nine noisy epochs of the noise-free ECG. Under a Gaussian noise scenario, (SNR = 0 dB), the FB-based composite has the closest resemblance to the noise-free ECG than the other composites.

## Methods

### Stress ECG data

A noise-free ECG beat cycle with an exercise-induced ST segment depression was obtained from the MIT/BIH database [16]. This beat was used as a template of a noise-free ECG epoch. Noisy beat epochs were constructed by adding pure-noise segments to the template. Noisy data were either generated artificially or obtained from the MIT/BIH database. Artificial noisy data were generated with an independent Gaussian distribution. BW and EMG noise were read from the MIT/BIH database [1]. The noisy beat epochs could be accurately aligned, since the location of the R wave was known from the underlying noise-free ECG beat.

**Signal-to-noise-ratio:** We used the SNR parameter to quantify and compare the performance of the algorithms, and also to determine the noise level in an enhanced ECG beat. The SNR was defined as [17]:

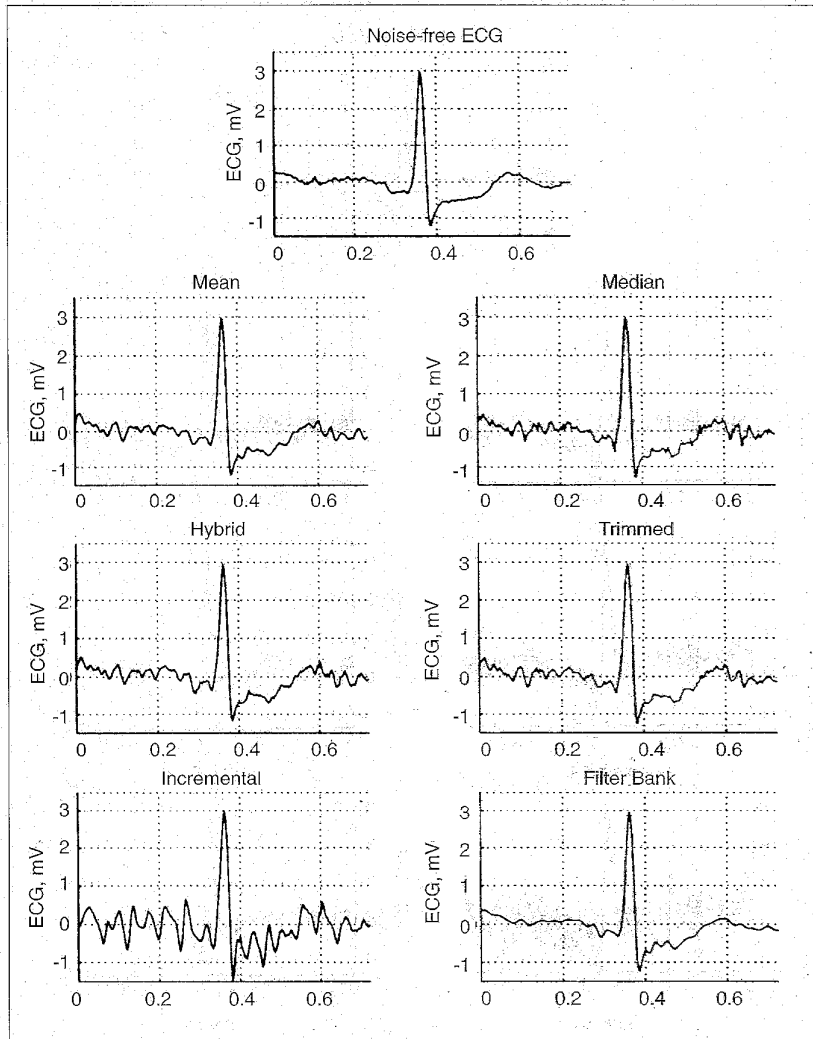
$$SNR = 10 \times \log_{10} \frac{S_{\sigma}}{N_{\sigma}}$$

where  $S$  was a noise-free ECG template of length  $L$ ,  $N$  was a noise vector of length  $L$ , and  $X_{\sigma}$  was defined as:

$$X_{\sigma} = \sum_{l=0}^{L-1} (X(l) - \mu_x)^2$$

where  $\mu_x$  was the mean of the signal  $X$ .

To generate a noisy epoch of an ECG beat, the sample values of the noise vector



5. Composites from various enhancing algorithms using nine noisy epochs of the noise-free ECG. Under a EMG noise scenario, (SNR = 5 dB), the FB-based composite has a good resemblance to the noise-free ECG compared to the other composites.

were multiplied by a gain factor corresponding to the required SNR and added to the noise-free ECG beat. To compute the SNR of an ECG composite beat, the known noise-free ECG beat was subtracted to determine the residual noise, and then the above SNR formula was used.

#### ECG Composites

Figure 1 shows sample noisy stress ECG epochs under various noise scenarios. We computed the composite of each of the algorithms when input with nine noisy ECG epochs. Each of the nine epochs had the same underlying noise-free ECG beat. The Gaussian (0 dB), and EMG (5 dB) noise scenarios were tested. Based

on initial experiments, we decided to use the fixed increment method.

#### SNR Improvement

For each algorithm, we computed the SNR of the composite beat from 1, 2, 3, ..., 9 epochs. Based on this experiment, a comparison can be made among algorithms to determine which results in an increased SNR with fewer epochs. Since the hybrid algorithm partitions the set of noisy epochs into three groups, the incremental numbers of epochs used for this particular algorithm were three, six, and nine. The trimmed method works on a reduced set of the input epochs. Thus, in this experiment, this method only begins to show SNR improvement after about five epochs.

## Results

### Stress ECG data

Figure 1 shows samples of noisy ECGs. Each of the noisy epochs used is very distorted as compared to the noise-free beat, so as to test the robustness of the algorithms.

### ECG Composites

Figure 4 shows the output of each algorithm using nine epochs with Gaussian distributed noise and a SNR of 0 dB. Figure 5 shows the output of each algorithm using nine epochs with EMG distributed noise and a SNR of 5 dB. Under both noise scenarios, the FB-based composite shows best resemblance to the noise-free beat. The P and T waves are closest in morphology to the noise-free beat in the case of the FB composite. The mean algorithm is a favorable performance under EMG noise. The other composites do show a reduction in noise level as compared to their original epochs (see Fig. 1).

### SNR Improvement

Figure 6 shows the SNR improvement as a function of the number of epochs included in the composite. The FB based method shows the best SNR improvement under Gaussian and EMG noise scenarios. The mean, median, hybrid, and trimmed methods have similar performance. The incremental algorithm does not show as remarkable a SNR improvement as do the other algorithms.

The FB based composite has a higher SNR improvement using a fewer number of epochs than the other techniques. For example, in the EMG noise scenario (5 dB), the FB composite of four epochs has a SNR improvement of about 15 dB, as compared to the mean algorithm which attains 15 dB only after nine epochs (see Fig. 6).

### Discussion and Summary

We used stress ECG data that is extremely contaminated with noise because it was important to test and present results of stress ECG enhancing algorithms operating under adverse noise scenarios.

In a stress ECG system, the enhanced beat is further processed to measure parameters from the ST segment and the overall ECG morphology. It is thus important that the enhanced beat be as noise free as possible, since noise hampers further processing. For example, computing the ST segment depression or elevation re-

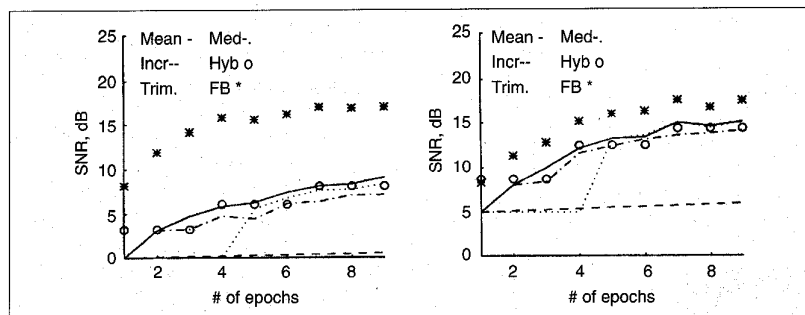
quires determining the isoelectric level as well as the detection of the J point and the onset of the T wave. These processing tasks are compromised by too high a noise level in the enhanced ECG.

The mean algorithm operates best in a stationary and uncorrelated noise scenario. These ideal scenarios do not usually exist in stress ECG records. The median algorithm works well at removing extreme data values in a distribution of ECG epochs. However, a usual preprocessing step for an ECG enhancing algorithm determines the 'goodness' of an ECG epoch before it is included in the set of epochs to be composed. Thus, this primary benefit of the median algorithm normally occurs of in this preprocessing step.

The hybrid algorithm removes noise fairly well as compared to the mean composite, but in addition better handles baseline shifts present in epochs than does the mean algorithm [2]. The trimmed mean method inherently combines mean and median techniques. The incremental algorithm does not directly exploit the nature of the noise. Thus, its enhancing performance is minimal. However, in a noise-free scenario, the incremental technique is potentially useful to track quickly and accurately dynamic changes in the ST segment as compared to the mean composite algorithm, which averages dynamic ST segment changes. Its usefulness in this area remains to be studied.

The FB composite enhances the ECG best under both Gaussian and EMG noise scenarios, and is also computationally inexpensive. It operates on the noise characteristics in the time and frequency domains, independently. The FB also potentially offers a way to perform other ECG processing tasks, such as R wave detection, enhancement and beat classification. These tasks can be carried out because the analysis filters decompose the ECG into various subbands, which can be used for further analysis. Thus, with one set of filters, various tasks could be performed in parallel, improving the overall computational efficiency of existing ECG processing systems.

As for improving the SNR, during a stress test it is important to measure changes in the ST segment as quickly as possible after they occur. Most enhancing algorithms require a set of ECG epochs from which one enhanced beat is computed. Various features of the morphology (such as ST segment depression) are then computed from this enhanced beat by us-



6. SNR improvement over an incremental number of epochs under Gaussian (left) and EMG noise scenarios. The FB composite shows better SNR improvement with a fewer number of epochs

ing other algorithms. Using a large number of epochs in the set will result in 'time-averaged' features. It is thus important to get an enhanced ECG using as few epochs as possible.

The FB-based composite provides the best SNR in the fewest number of beats. However, studies still remain to be performed on the accuracy of ST measurements under various noise scenarios. For example, in a noise-free scenario it is likely that the incremental algorithm will track changes in the ST segment more quickly and accurately than the mean algorithm.

We also compared the performance of the algorithms on ECGs contaminated with Gaussian and EMG noise only. A high-pass filter such as the one described in Ref. [3] or any other that meets the specifications given in Ref. [4] can be used to preprocess the ECG before using any of the above algorithms. In fact, the high-pass filter can be incorporated into the lowest subband of the FB algorithm. This filter would then operate at a lower rate since it is filtering the downsampled 0 to 5.625 Hz subband. The resulting FB algorithm would then filter BW and EMG noise.

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