Comparison Methods for Stochastic Models and Risks

Alfred Müller

University of Karlsruhe, Germany

Dietrich Stoyan

Freiberg University of Mining and Technology, Germany



Contents

Pr	eface	9	ix
1	Uni	variate Stochastic Orders	1
	1.1	Introduction	1
	1.2	Usual Stochastic Order	2
	1.3	Hazard Rate Order	8
	1.4	Likelihood Ratio Order	12
	1.5	Convex Orders	15
		1.5.1 Fundamental Properties	15
		1.5.2 Sufficient Conditions and Strassen's Theorem	23
		1.5.3 Majorization	31
	1.6	Higher Convexity Orders and Laplace Transform Order	37
	1.7	Dispersive Order and Relative Inverse Function Orderings	40
	1.8	Lifetime Distributions and Notions of Aging	45
	1.9	Bivariate Characterizations	51
	1.10	Extremal Elements	55
	1.11	Monotone Approximations	59
	1.12	Relationships and Comparison Criteria for Univariate Stochas-	
		tic Orders	60
_	-		~ -
2		······································	65
	2.1		65
	2.2		67
	2.3	0	69
	2.4		73
	2.5		75
	2.6	Strassen Type Theorems	80
3	Mu	ltivariate Stochastic Orders	85
-	3.1		85
	3.2		89
	3.3		90

	CON	ITE	NTS
--	-----	-----	-----

	3.4	Convex O	orders	98
	3.5		onvex Orders	
	3.6		ntwise Convex Order	
	3.7	-	c Orders Defined by Difference Operators	
	3.8		nce Orders	
	3.9	-	lular Order	
	3.10	-	of Dependence	
		-	ate Likelihood Ratio Orders	
			ally Convex Order	
			c Ordering of Multivariate Normal Distributions 1	
			hips and Comparison Criteria for Multivariate	
	0		c Orders	145
4	Sto	chastic M	Iodels, Comparison and Monotonicity	149
	4.1	General C	Considerations Concerning Stochastic Models	149
	4.2	Monotoni	city and Comparability	154
		4.2.1 M	onotonicity	154
		4.2.2 Co	omparability	154
	4.3	Methods	for Establishing Monotonicity and Comparability	
		Properties	s	155
		4.3.1 Th	ne Functional Method	155
		4.3.2 Th	ne Mapping Method	156
		4.3.3 Th	ne Coupling Method	166
	4.4	Extremal	Problems	171
5		-	y and Comparability of Stochastic Processes 1	
	5.1		ion	
	5.2	-	bility and Monotonicity of Markov Processes 1	
			onotone and Comparable Operators	180
		5.2.2 M	onotonicity and Comparability Conditions for Markov	
			rocesses	
			omogeneous Markov Processes with Discrete State Space	192
			onotonicity Properties of Second Order Characteris-	
			s of Markov Chains	198
		-	pplication of Monotone Markov Chains: Perfect	
			mulation	
			arkov Decision Processes	
	5.3		city and Comparability of Non-Markov Processes 2	
	5.4	Comparis	on of Point Processes	211
e	Ъ.Γ		- Dron ontion and Dron de fan Oaraating Saataan (0177
6			y Properties and Bounds for Queueing Systems 2	
	6.1		ts for $GI/GI/1$ and $G/G/1$	
	6.2		city Properties of $GI/GI/1$ and $G/G/1$ Queues 2	
	6.3	Comparis	on Properties of $GI/GI/1$ and $G/G/1$	221

CONTENTS

	6.4	Bound	Is Obtained from Comparison Properties of $GI/GI/1$	225			
	6.5 Bounds in the Case of Non-renewal Input						
	6.6	Basic Facts for the Multi-server System $GI/GI/s$					
	6.7		conicity Properties of <i>GI/GI/s</i> Queues				
	6.8		arability Properties of $GI/GI/s$				
	6.9		rks on other Queueing Systems				
7	Applications to Various Stochastic Models						
	7.1		conicity Properties and Bounds for the Renewal Function				
	7.2	Reliab	ility Applications	239			
		7.2.1	Coherent Systems				
		7.2.2	Comparison of Maintenance Policies	242			
	7.3	PERT	and Scheduling Problems	244			
	7.4	Compa	arison of Random Sets and Point Processes	247			
		7.4.1	Comparison of Random Closed Sets	247			
		7.4.2	Comparison of Point Processes	251			
	7.5	Monot	conicity and Comparison of Models of Statistical Physics	253			
		7.5.1	Monotonicity and Comparison Properties of the Ising				
			Model	253			
		7.5.2	Comparison of Gibbs Distributions	259			
8	Cor	nnarir	ng Risks	265			
0	8.1		mics of Uncertainty				
	0.1	8.1.1	Basics of Stochastic Dominance				
		8.1.2	First- and Second-Order Stochastic Dominance				
		8.1.3	Stochastic Dominance with DARA Utility Functions				
	8.2						
	0.2	8.2.1	Consistency of Mean-deviation Rules				
		8.2.2	Portfolio Optimization				
	8.3		ing of Actuarial Risks				
	0.0	8.3.1	Bounds for Aggregate Claims of Dependent Risks				
		8.3.2	Some Models for Dependent Risks				
		8.3.3	Indistinguishable Individuals				
		8.3.4	Distinguishable Individuals				
Lis	st of	Symbo	ols	299			
Re	fere	nces .		303			
Inc	dex			325			