Comparison of Compression Ratios for ECG Signals by Using Three Time-Frequency Transformations

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Abstract: In this paper are presented compression results of ECG signal by using three time-frequency transformations: Discrete Wavelet Transform, Wavelet Packets and Modified Cosine Transform. By using transforms mentioned, samples of signals are transformed to appropriate groups of transformation coefficients. Almost all coefficients below the determined threshold are rounded to zero values and by inverse transform the similar signal to original one is created. By using run-length coder, consecutive zero value coefficients can be replaced by single value that shows how many consecutive coefficients with zero value exists. In this way small number of coefficients is stored, and compression is obtained.

Depending on transform used, different number of coefficients is rounded to zero in different positions, hence the reconstructed signal is more or less similar to the original one. In general there exists measures that show how much reconstructed signal is similar to the original one, and the most used is Percentage Root mean square Difference (PRD). Comparison of compression is performed in obtaining the larger compression ratio for the smaller PRD.

Keywords: ECG, compression, Discrete Wavelet Transform, Wavelet Packets, Cosine transform, Cosine-modulated filter bank.

1 Introduction

To record ECG signal waveform, a large amount of data should be saved. To reduce the space for data storage, some compression must be used, but only if the difference between decompressed - reconstructed signal and the original one is minimal, i.e. if reconstructed signal is not distorted and if cardiologist can obtain the same diagnosis from reconstructed signal as if he would obtain it from original signal.

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There are several ways to obtain compression of non - stationary signals [1] and almost all of them use transform coding like in [2–4]. In this paper, compression of the signal is obtained by wavelet transforms - Discrete Wavelet Transform (DWT) and Wavelet Packets (WP) and by cosine-modulated filter bank.

It is important to find the best compression method for all different shapes of ECG signal regardless of heart disease of patient. The compression ratio also depends on the parameters how ECG signal is digitized (sampling frequency and number of bits per sample) and the level of decomposition. In the paper [5], authors obtained the compression results for ECG signal sampled at 360 Hz with 16 bits per sample using the wavelet packets at 3rd level. In this paper, compression of ECG signal from the device described at [6] is analyzed. The signal is quantized with 10 bits per sample and by sampling frequency of 960 Hz.

2   Decomposition and Reconstruction of Signals Using the Discrete Wavelet Transform

Discrete Wavelet Transform (DWT) enables decomposition of the input signal into two signals - Approximation *A* and Details *D*. Details are obtained when the signal is passed through the half band, high-pass filter (HPF), which impulse response represents the wavelet function ($\psi(t)$). Approximation is obtained if the signal is passed through the half band, low-pass filter (LPF) which is the quadrature mirror (QMF) of the previous one [7]. The impulse response of the LPF is the scaling function ($\phi(t)$).

At the output of both QMF filter pair, number of coefficients is two times larger compared to the number of samples at their input. This is a reason why downsampling is then performed by factor 2 on both output signals. Only one coefficient out of two is kept.

The decomposition process can be iterated, with successive approximations being decomposed in turn, so that one signal is decomposed into many lower resolution components. This is called the wavelet decomposition tree. The number of iterations represents level (l) of decomposition. The initial signal is transformed in a set of coefficients which are grouped into the lowest approximation and l details. The total number of coefficients (in the lowest approximation level and in l details) is the same as the number of the samples in original signal. Without any further processing it is irrelevant which set of data will be stored in a file. In the Figure 1(a) is presented decomposition process. Coefficient groups which represents DWT of the signal are bordered with dot line.

Original signal can be reconstructed using the modified wavelet ($\psi'(t)$) function and modified scaling ($\phi'(t)$) function. The coefficients of the modified func-

tions have the same values as the original ones but reversed ordered. To reconstruct Approximation at the higher level (A_j), Approximation and Detail at the lower level (A_{j+1} and D_{j+1}) are used. Coefficients at lower level must be up-sampled and passed through the modified half band Low and High Pass Filters. Up-sampling is performed by inserting zeros between two consecutive coefficients. Reconstruction of Approximation and Details at the higher levels is shown in the Figure 1(b).
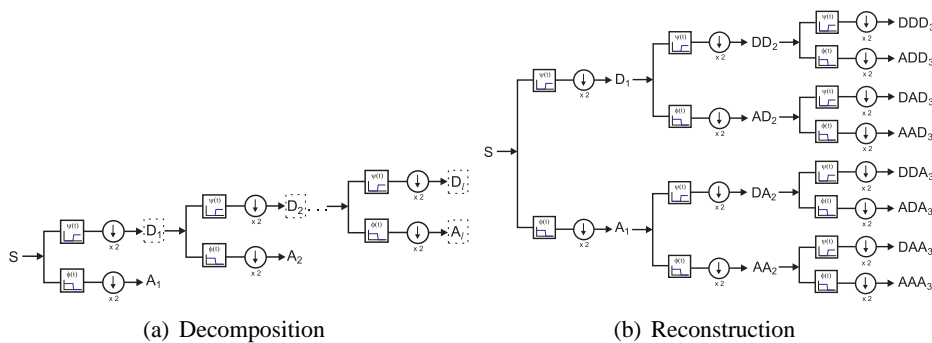

(a) Decomposition   (b) Reconstruction

Fig. 1. DWT decomposition and reconstruction

3   Decomposition and Reconstruction of Signals Using the Wavelet Packets

In the similar way, decomposition of the input signal into Approximations *A* and Details *D* using the Wavelet Packets can be performed. By using DWT, only input signal and Approximations can be decomposed into Approximations and Details on the lower level. There is no decomposition of Details. By using WP, Details can be also decomposed, and the tree of nodes is created with input signal at the top of the tree. Coefficients of each node can be decomposed into two subsets (nodes), regardless the originating node represents Details or Approximation. The decomposition tree of WP up to the third level is presented at the Figure 2

Coefficients which belong to the end nodes (the rightmost at Figure 2) need not to be those from which the input signal is going to be reconstructed. Sometimes it is better not to decompose coefficients of some node into Approximation and Details on the lower level. For each node is calculated entropy, based on the values of coefficients belong to that node. If the entropy of the originating node is less than the sum of entropies of successor nodes, decomposition of that node is not performed. By pruning the tree with respect to the entropy criteria, the best tree is obtained with the least cumulative entropy [8].

There exists several ways of calculating entropy [9]. The simplest way which
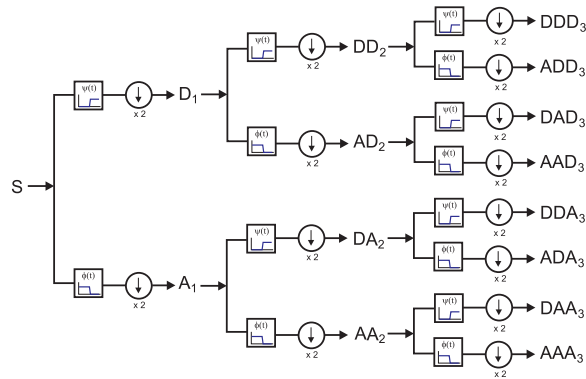
Fig. 2. WP Decomposition

obtains very good results is to select threshold entropy according to formula:
$E(s) = \#\{\text{i such as } |s_i| > \varepsilon\}$ where ε is selected threshold value.

4 Decomposition and Reconstruction of Signals Using the M-channel Cosine-Modulated Filter Bank

By using M-channel cosine-modulated filter bank, input signal is passed through
the bank of M QMF filters ($H_k(Z)$) and on their outputs, decimation by factor M is
performed, i.e. each M-th sample is kept. In this way input signal is decomposed
on M groups of coefficients [10–15]. Reconstruction of signal is performed if M
consecutive zero value coefficients are added after each coefficient obtained in de-
composition process and passed through modified filter bank ($F_k(Z)$). In contrast
to wavelet transformations which have Perfect Reconstruction (PR), i.e. original
and reconstructed signal are the same, cosine-modulated transformation does not
have perfect reconstruction. Because the relative difference between original and
reconstructed signal is very low, this transformation has, as is used to say, nearly
PR. The impulse responses of these filters are given by equations:

$$h_k[n] = 2 \cdot p[n] \cdot \cos\left((2k+1) \cdot \frac{\pi}{2M} \cdot \left(n - \frac{N-1}{2}\right) + (-1)^k \cdot \frac{\pi}{4}\right) \qquad (1)$$

and

$$f_k[n] = 2 \cdot p[n] \cdot \cos\left((2k+1) \cdot \frac{\pi}{2M} \cdot \left(n - \frac{N-1}{2}\right) - (-1)^k \cdot \frac{\pi}{4}\right) \qquad (2)$$

where $p[n]$ is impulse response of so-called prototype filter given by equation:

$$p[n] = \left(\frac{\sin\left((n - \frac{N}{2}) \cdot \omega_{c,6dB} \right)}{\pi \cdot (n - \frac{N}{2})} \right) \cdot w[n] \tag{3}$$

and $w[n]$ is windowing function. In this case Blackman windowing function is selected. Decomposition and reconstruction process by using M-channel cosine-modulated filter bank is shown at the Figure 3
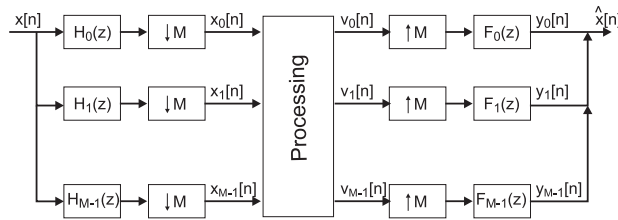

Fig. 3. Decomposition and reconstruction of signal by using M-channel cosine-modulated filter bank

5   Compression Method and Material

When the original signal is decomposed by time-frequency transformation into groups of coefficients, each group represents signal in appropriate bandwidth. Statistically, coefficients which belong to lower frequency bandwidths have higher amplitudes, and vice versa - coefficients which belong to higher frequency bandwidths have lower amplitudes. Hence, if some number of coefficient which absolute amplitudes are close to zero are neglected, the compression is obtained, with no big impact on shape of the signal after reconstruction.

The method of setting the value of coefficient to zero if the absolute value of a coefficient is below the threshold defined, is calling "thresholding" [16]. It is very important to select appropriate value for threshold(s). Large threshold values lead to very good compression but distortion might appear in reconstruction. Small threshold values lead to low compression but reconstructed signal is very similar to the original one.

There exists several measures for "quality" of reconstructed signal after compression. The most popular is Percentage Root mean square Difference (PRD) which is defined by equation:

$$PRD = \sqrt{\dfrac{\sum\limits_{n=1}^{N}(x(n)-\tilde{x}(n))^2}{\sum\limits_{n=1}^{N}(x(n)-\bar{x}(n))^2}} \qquad (4)$$

where $\bar{x}(n)$ represents signal's mean value.

Unfortunately, PRD does not show exactly how much is signal distorted in different time position, but shows only cumulative distortion. Very often Maximal Absolute Difference measure for the difference between reconstructed and original signal is used. Original and reconstructed signal without and with distortion are shown at the Figure 4.
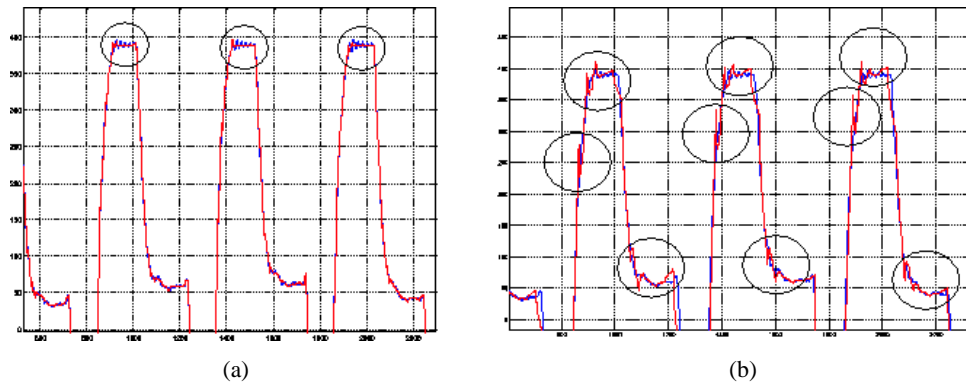

(a)   (b)

Fig. 4. Reconstruction of ECG signal after thresholding method: (a) correct reconstruction and (b)reconstruction with distortion

By replacing the sequence of consecutive zero value coefficients with a number that shows how many consecutive zeros are there in appropriate group of coefficients, the lower amount of data is going to be stored. The Number of Zeros (NZ) is the measure that shows what is the ratio between number of zero valued coefficients and total number of coefficients in the transform of a signal. This measure will be used as the compression ratio in this paper.

The comparison of compression ratios and quality of reconstructed signal is done by changing the following parameters: Level of decomposition and wavelet used, for DWT and WP and number of filters and appropriate number of filter's coefficients.

Several wavelet functions are used in this study: Daubechies wavelets: 2, 4, 6, 8 and 10, and Biorthogonal 4.4 wavelet, 3 levels of decomposition: 5, 6 and 7 and two different orders of Modified Cosine transform: 161 and 321 with 16 and

32 filters respectively. Samples of ECG waveforms of 10 patients who belong to 3 groups of diseases: Acute myocardial infarction (AIM), absoluta and left bundle branch block are being compressed.

6 Results

For each patient, 12 leads (channels) with 6 seconds of ECG signal per lead are recorded in a database. Every signal is then decomposed using parameters mentioned and thresholding is performed. For selected parameters, after reconstruction of signal in each lead, compression ratio, maximal absolute difference and PRD is calculated, as it is shown in the Table 6 for DWT with decomposition on 4th level using Daubechies 2 (db2) wavelet function.

Table 1. PRD, Maximal absolute difference (MaxD) and Number of Zeroes (NZ) for a patient with AIM by using DWT with db2 wavelet and decomposition up to 4th level

ECG lead	PRD (%)	MaxD	NZ(%)
D1	1.55	5	91.61
D2	1.80	4	91.60
D3	1.74	6	91.60
aVR	3.64	3	91.62
aVL	2.25	5	91.60
aVF	1.65	4	91.60
V1	2.16	3	91.60
V2	1.31	6	91.61
V3	1.81	10	91.60
V4	3.37	8	91.61
V5	1.40	10	91.60
V6	2.78	6	91.60
Statistics	**3.64**	**10**	**91.60**

In the last row in the Table 6 are shown maximal values of PRD and MaxD for that patient as the worst cases and mean compression ratio, because that is the compression which will be obtained if compressed coefficients of all leads will be stored in a same file. These values will be called statistical values of compression for selected parameters.

Statistical values of obtained compression and reconstruction results are entered in a new table with four additional columns: heart disease id, patient id, wavelet function and level of decomposition. For each combination of patient, wavelet function and level of decomposition the new row is created. There was 10 (patients) $\times$ 3 (levels of decomposition) $\times$ 6 (wavelet functions) = 180 rows in a table which represents compression results for 12 (leads) $\times$ 180 = 2160 ECG

signals. Two tables of described format are created: one for DWT and another for WP. From these huge tables, regardless of patient, the average compression ratios per combination of wavelet function and level of decomposition are calculated and shown in the Table 2 and appropriate maximal PRD in the Table 3. It is very important to say that original and reconstructed signal for each lead (ECG signal) had to be visually checked for distortion.

Table 2. Mean compression results of ECG signal obtained by using DWT and WP by wavelet function and by level of decomposition

method	level	db2	db4	db6	db8	db10	bior4.4
DWT	5	91.87	91.76	91.53	91.57	91.45	91.78
WP	5	* 90.18	91.93	92.17	92.21	92.10	92.46
WP	6	* 90.51	92.28	92.48	92.56	92.41	* 92.81
WP	7	* 90.68	92.44	92.65	92.71	92.57	* 92.96

Table 3. Maximal PRD values of ECG signal obtained by using DWT and WP by wavelet function and by level of decomposition

method	level	db2	db4	db6	db8	db10	bior4.4
DWT	5	9.90	9.71	9.74	9.70	9.69	9.70
WP	5	* 9.27	8.76	8.65	8.73	8.65	8.79
WP	6	* 9.38	8.91	8.68	8.69	8.65	* 8.95
WP	7	* 9.39	8.94	8.68	8.63	8.65	* 9.02

As can be seen from the tables 2 and 3, there are missing two rows - for DWT decompositions for levels 6 and 7. Only ECG signals of couple out of 10 patients could be compressed without a distortion for the levels mentioned. Stars in table show the opposite case - only ECG signals of couple patients couldn't be compressed in these levels without distortion.

For compression of ECG signals using M-channel cosine-modulated filter bank average statistical values are shown in table 4

Table 4. Mean compression results AND mean PRD of ECG signals obtained by using M-channel cosine-modulated filter bank by number of filters and number of filter coefficients

M	Number of coefficients in filter	PRD	NZ
16	161	5.25	91.19
32	321	8.09	94.08

The average compression ratio of 94.08% shows that the best method of compression is obtained using 32-channel cosine-modulated filter bank. It matches with results obtained in [17].

7   Discussion and Conclusion

Intuitively, the larger is the distance between original and reconstructed signal, the compression ratio should be better. But as can be seen from the tables 2 and 3 compression using DWT has the largest PRD but the worst compression ratios. The best compression ratio, by using DWT is obtained by using db2 wavelet.

By performing compression using WP, compression ratio is increasing by increasing decomposition level with increasing PRD, except in the case of wavelet function db8. The best compression ratio is obtained with WP using biorthogonal 4.4 wavelet function in all levels of decomposition compared to ratios obtained by using other wavelet functions. Generally, by using WP, better compression is achieved compared to the DWT.

As can be seen from the table 4, compression ratio by using cosine-modulated filter bank with M=16 filters has average which is the smallest comparing to all DWT and WP compression ratios. But, by using cosine-modulated filter bank with M = 32 filters, the best compression ratio is achieved with the smallest distance between original and reconstructed signal. It is very important result considering that cosine-modulated filter bank is not the tool for obtaining perfect reconstruction.

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