

COMPARISON OF CONTACT STRESS DISTRIBUTION FOR FOAM SEAT AND SEAT OF AUXETIC SPRING SKELETON

M. JANUS-MICHALSKA* and D. JASIŃSKA

Institute of Structural Mechanics
Cracow University of Technology
ul. Warszawska 24, 31-155 Kraków, POLAND
E-mail: mjm@limba.wil.pk.edu.pl
jasinska@limba.wil.pk.edu.pl

J. SMARDZEWSKI

Department of Furniture Design
Poznań University of Life Science
ul. Wojska Polskiego 28, 60-637 Poznań, POLAND
E-mail: jsmardzewski@up.poznan.pl

The objective of this paper is to present and compare the results of numerical solutions of contact problem for two types of seats subjected to typical sitting loadings. The first seat is made of a typical hyperelastic foam, the other is designed with an auxetic polyamid spring skeleton. Computer simulations of the seat structure under a typical static loading exerted by a human body are performed by means of ABAQUS FEA. The model provides an insight into deformation modes and stress field in relation to geometric and material parameters of the seat structure. The other type of seat, due to the fact of global auxeticity and progressive springs characteristics reduces contact stress concentrations, giving an advantageous distribution of pressure and provides the sensation of physical comfort. The proper seat skeleton shape leads to an improvement of ergonomic quality.

Key words: auxetic structures, seat, utility load, contact problem.

1. Introduction

The contact interaction between a human body and a seat is the main factor of feeling comfort when seated. In the first stages of the design process a seat should be tested for its degree of comfort by computer simulations with models of the human body and the seat. Seating comfort is becoming increasingly important, due to the fact that people spend more and more time in the sitting position. Many research studies indicate that comfort is primarily associated with the physiological and biomechanical factors. Ebe and Griffin (2001), Zhao *et al.* (1994), Park and Kim (1997) report a dependence of subjective personal sensation of comfort and average pressure, maximum pressure, the size and symmetry of the contact area. A correct pressure distribution is crucial to seated comfort. Surface pressure is responsible for constriction of blood vessels in underlying tissues, restricting blood flow, which the sitter experiences as discomfort. Because of the large variance in peak pressure patterns among people of different sizes and shapes, it is difficult to design ideal seat and back contours or cushion compliance levels that would minimize uncomfortable pressure points for all sitters.

Considerations on the subject of contact interaction started with the Hertz solution. Wang and Lakes (2002) report the history of investigations on the contact problems. The classical Hertz theory takes into

* To whom correspondence should be addressed

account a frictionless interface condition between two 3D elastic ellipsoidal deformable bodies in contact. The special case is for contact of an elastic sphere with a halfspace as illustrated in the figure below.

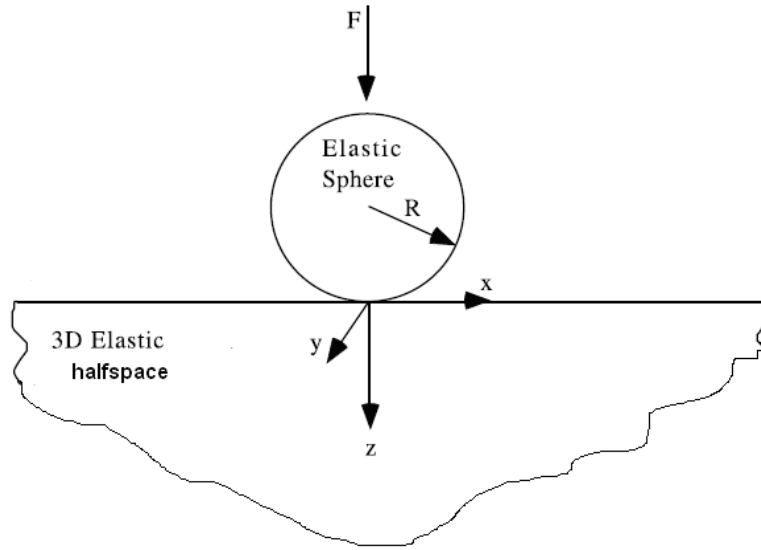


Fig.1. Schematic diagram for Hertz contact model.

The pressure distribution $p(x, y)$ as reported by Wang (2002) is as follows

$$p(x, y) = p_{\max} \sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}} \quad (1.1)$$

where

$$p_{\max} = \frac{3F}{2\pi a b}, \quad a = b = (FDR)^{1/3}, \quad D = \frac{3}{4} \left(\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right)$$

E_1, ν_1 - material Young's modulus and Poisson's ratio for the halfspace,

E_2, ν_2 - material Young's modulus and Poisson's ratio for the sphere.

For the contact of an infinitely rigid sphere ($E_2 = \infty$) with a halfspace the maximum pressure and vertical displacement are as follows

$$p_{\max} = \frac{3F}{2\pi (FDR)^{2/3}}, \quad \delta_{\max} = \left(\frac{D^2 F^2}{R} \right)^{1/3}. \quad (1.2)$$

The classical contact problem between a rigid sphere and a half-space elastic layer with finite thickness is described by the Fredholm integral equation. Due to the complexity of the equation, only asymptotic solutions can be obtained. For thick-layer problems, the solutions can be calculated iteratively, but for thin layer ones, the problem can be solved numerically. A closed-form series solution for the contact problem with a finite thickness elastic layer was proposed by Sakamoto *et al.* (1996).

Analytical investigations on the contact problems between two homogeneous and isotropic elastic bodies were performed by Wang and Lakes (2002). They also simulated the contact of human buttocks and seat cushions. The cushion materials' Poisson's ratio were allowed to be negative. The human buttocks were modeled as an ideal elastic sphere. The peak contact pressure was reduced by adjusting Poisson's ratio and contour curvature of cushions according to the Hertz theory. An analysis by both the Hertz model and a finite thickness 3D elasticity model showed that using negative Poisson's ratio cushions could further reduce the pressure. Negative Poisson's ratio cushions may be beneficial in the prevention of pressure sores or ulcers in the sick and in reduction of pressure-induced discomfort in seated people. Wang and Lakes investigated the influence of Poisson's ratios, in the full range including negative values on the peak pressure between two contacting elastic bodies and on their deformation.

Generally, a soft cushion helps reduce pressure, but a cushion which is too compliant will bottom out, resulting in increased pressure. The 3D elasticity finite thickness solution incorporates some aspects of bottoming. However a full understanding of bottoming is limited by the complexity of the asymptotic solutions and by the fact that the assumption of linearity excludes the material nonlinearity of the foam and human tissue at high strain. If the cushion's Young's modulus is held constant, the optimal Poisson's ratio is zero. A contoured cushion reduces peak pressure. Combining the concepts of contoured shape and negative Poisson's ratio materials offers superior cushion performance. An analysis which incorporates more of the complexities of the problem such as material nonlinearity should incorporate the finite element method treatment.

2. Formulation of the contact problem

For the unilateral static contact problem of a linear elastic body with a stiff body the following system of equations must be fulfilled (Kikuchi and Oden, 1988)

$$\sigma_{ij,j} + f_i = 0, \quad \sigma_{ij} = S_{ijkl}\varepsilon_{kl}, \quad \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad \text{in } \Omega, \quad (2.1)$$

completed with boundary conditions

$$u_i = \hat{u}_i \quad \text{on } \Gamma_D, \quad \sigma_{ij} \cdot n_j = t_i \quad \text{on } \Gamma_F, \quad (2.2)$$

contact conditions on Γ_C

$$\sigma_n \cdot (u_n - g) = 0, \quad \sigma_n \leq 0, \quad u_n - g \leq 0, \quad (2.3)$$

and friction conditions on Γ_C

$$|\sigma_T| < \mu |\sigma_n| \Rightarrow \Delta \mathbf{u}_T = 0, \quad |\sigma_T| = \mu |\sigma_n| \Rightarrow \exists \lambda > 0; \quad \Delta \mathbf{u}_T = -\lambda \sigma_T \quad (2.4)$$

where: σ_{ij} - Cauchy stress tensor, ε_{ij} -small strain tensor, S_{ijkl} - elastic stiffness matrix, u_i - displacement vector, f_i -body forces, \hat{u}_i - prescribed displacements on Γ_D , t_i - forces acting on Γ_F , n_i -unit normal vector, $\Gamma_D \cup \Gamma_F \cup \Gamma_C$ - boundary of the domain Ω , g - initial gap, $\sigma_n = \sigma_{ij}n_i n_j$ - contact pressure, $u_n = \mathbf{u} \circ \mathbf{n}$ - displacement normal to the boundary, $\sigma_{Ti} = \sigma_{ij} \cdot n_j - \sigma_n \cdot n_i$ - tangential contact force, and $\Delta \mathbf{u}_T = \Delta(\mathbf{u} - u_n \cdot \mathbf{n})$ -increment of tangential displacement.

To solve the boundary value problem formulated above (nonlinear due to conditions (2.3) and (2.4)) the FEM approach is used.

3. The main idea of seat design

Negative Poisson's ratio materials are called auxetics. They are classified as smart materials having a significant technological potential. An example of auxetics are some cellular materials. Cellulars with various microstructures constitute a special kind of elastic media. The auxecity in the selected directions and required magnitude of Poisson's ratio may be tailored according to the designer demands by a proper choice of microstructural geometrical parameters (Janus-Michalska, 2009). The theoretical problem of material design for anisotropic elastic cellular bodies with respect to contact problems is presented by Jasińska and Janus-Michalska (2008; 2010). Special attention is paid to materials having reentrant structure, which yield negative Poisson's ratio. These properties and the orientation of material symmetry axes with respect to load direction significantly influence contact stress distribution and may play an important role in reducing the peak contact pressure.

This work discusses the application of auxetic skeleton structures in seats. The seat skeleton is composed of 'springs', which are based on auxetic frame structures designed similarly to the microstructure of auxetic cellular materials as described by Jasińska and Janus-Michalska (2012).

Seat design is an example of application of the features mentioned above. A new design can be tested by computer simulations using models of the human body and the seat.

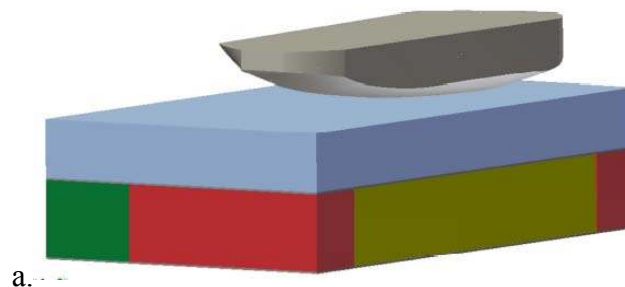
Overall seat comfort is also influenced by seat characteristics (e.g., seat stiffness) as reported by Ebe and Griffin (2000). Progressive characteristics of an elastic medium give an advantageous distribution of pressure. Progressive means that the seat is compliant for a smaller load and becomes stiffer for increasing loads. The skeleton structure is designed to work in two consecutive stages. At first, as an elastic response at small deformations, the network deforms fairly uniformly. Then, under increasing loading, the local elastic collapse occurs, leading to an increased bending in the network. As the structural configuration evolves, new weak points are created and high degrees of bending propagate throughout the structure which results in large deflections. Finally, the network collapses onto itself and self-contact between frame elements results in the stiffening of the structure.

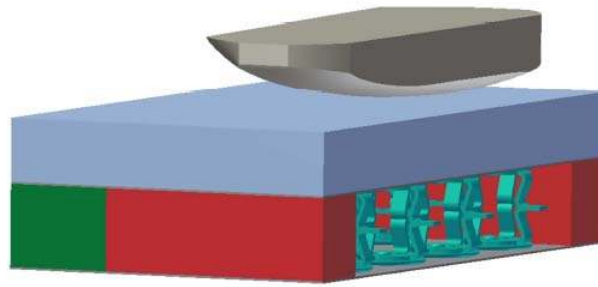
A three-dimensional real contact and stiffness problem of small size auxetic springs constituting a part of an elastic seat cushion is considered. The analysis is performed using FEM. Computer simulations of the seat structure under a typical static loading exerted by a human body were performed by means of ABAQUS FEA (calculations were performed at ACK CYFRONET AGH). Extensive numerical simulations were carried out using nonlinear analysis to calculate the stresses and deflections.

The finite element model of a seat, for both seats could provide an insight into changes in contact interaction between the human and the seat due to variations of geometric and material parameters.

4. Seat structural elements

Two types of seats as illustrated in Fig.1 are considered. In each case a half of the seat structure with appropriate load and symmetry boundary conditions is analysed. A foam seat is composed of various foam elements: seat filling, bodno and upper seat layer. In an auxetic seat the seat filling is replaced by an auxetic skeleton whereas the remaining parts remain unchanged.





b.

Fig.1. Structures of half seat with interlayer. a. foam seat b. seat with auxetic spring skeleton.

The spring skeleton consists of silicon springs of the shape illustrated in the figure below. Special rubber discs are applied to the bottom and top surfaces of the spring skeleton elements to ensure a proper interaction with the aligning felt layer (Fig.2).

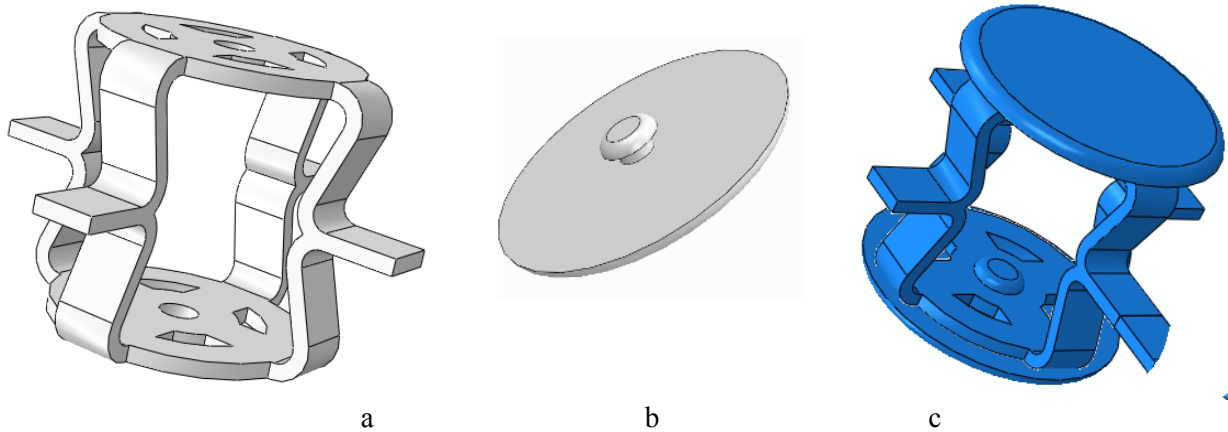


Fig.2. Skeleton elements, a. auxetic silicon spring, b. rubber disc, c. spring with two discs.

5. Material data

The specification of materials for structural elements is given in Tab.1.

Table.1. Mechanical properties of seat structural elements materials.

seat part	material	elastic properties
upper seat layer	foam T3037	nonlinear as given in Fig.3a
seat filling (seat a)	foam T3037	nonlinear as given in Fig.3a
bodno	foam T3546	nonlinear as given in Fig.3b
auxetic spring (seat b)	silicon 75 Shore'a	Mooney-Rivlin hyperelastic model $E=9.32$ [MPa], $C10=1.05$, $C01=0.3$, $K=7.77$ [MPa]
disc(seat b)	Shore rubber 95	$E=4000$ [MPa], $\nu=0.3$
aligning layer	felt	$E=2.58$ [kPa], $\nu=0.3$

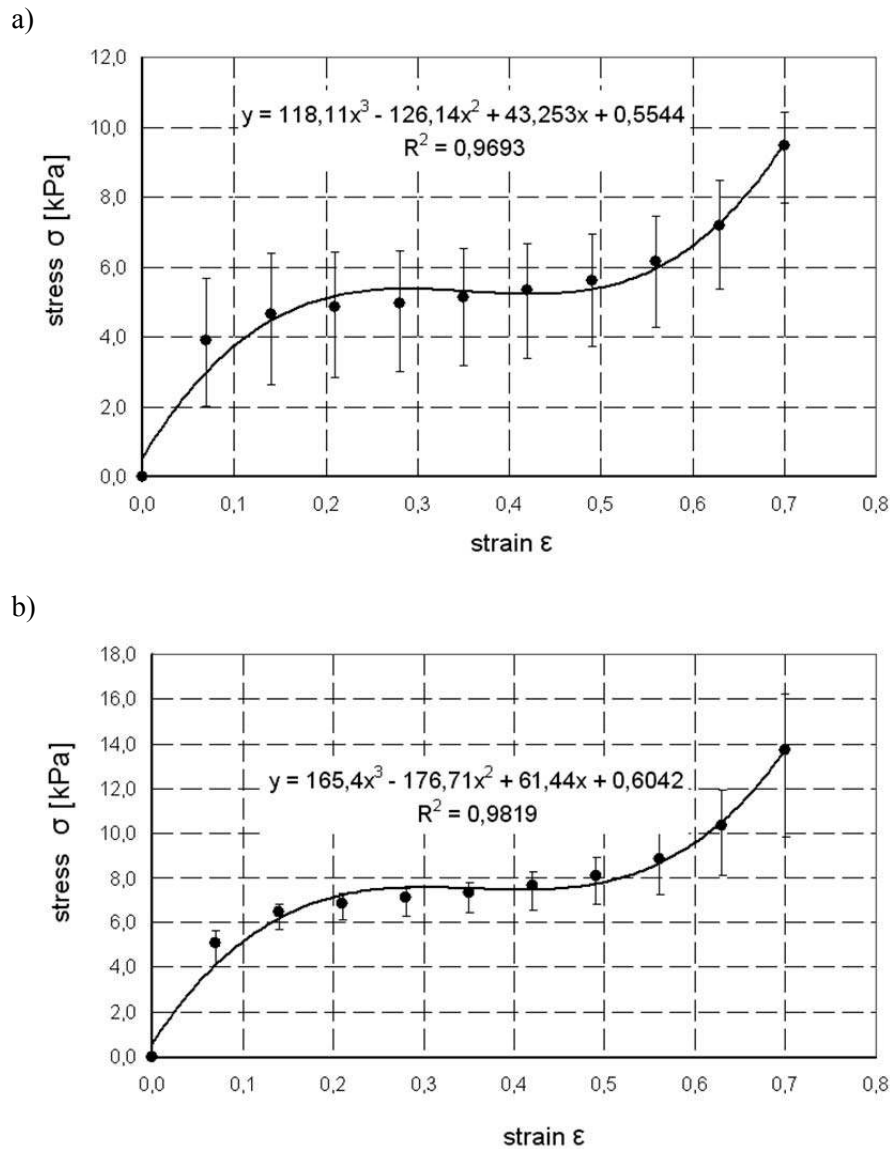


Fig.3. Nonlinear foam characteristics, a. foam T 3037, b. foam T 3546.

Seat dimensions: height 138mm, width 640mm, depth 580 mm.

6. Seat design requirements

The following two states are considered: ultimate limit stress state and serviceability limit state. The first state reads that the reduced von Mises stress at each point of the structure fulfills the condition: $\sigma^M \leq f_d$ where: f_d denotes limit stress value. The serviceability limit state concerns vertical deflection, which should not exceed 50% of seat height. Comfort condition means that contact stresses are limited by the value of 4.265 kPa - the limit value of pressure.

7. Numerical analysis - scope of the study

In this paper, we perform a numerical analysis to determine the nonlinear load displacement path for static loading (seat stiffness characteristics). We also present stress maps. The strength of given structures for static loading is checked by reduced Mises stress maps. Calculations are performed for utility load represented by an indenter of the shape of a human thigh and buttock.

7.1. Finite element mesh

a. Foam seat

56000 8-node linear brick elements, (reduced integration, enhanced hourglass control) modeling aligning foam and bodno foam,

2400 3-node triangular facet rigid elements for model of indenter's surface.

b. Auxetic spring seat

preliminary test on single spring -55600 4-node linear tetrahedron elements

Seat with auxetic skeleton – discretization of seat elements

spring skeleton – 94000 10-node quadratic tetrahedron elements

bodno and aligning foam- 82200 8-node linear brick elements

felt - 5100 4-node quadrilateral membrane elements

intender- 2400 3-node triangular facet rigid elements.

7.2. Type of analysis

A geometric nonlinear analysis (large displacements and contact between seat elements) is performed. Material nonlinearity is also involved due to material nonlinearity of hyperelastic foams.

7.3. Boundary conditions and contact conditions

Half of the seat structure with load due to symmetry is analysed. Static loading is performed by kinematic exertion – indenter displacement. The indenter is modeled as a rigid body, contact with the seat is frictionless, bodno parts are stuck together, bodno and felt are also stuck together.

Seat a. frictionless contact between foam and bodno.

Seat b. additional conditions: spring and discs are stuck together, parts of bodno and felt are stuck together, friction coefficient disc-felt 0.1 or without friction, self contact of spring is frictionless.

7.4. Contact stress distribution and seat deformation

Foam seat

A nonlinear incremental analysis is performed for seat deflection equal to 50% of its height (with corresponding loading 300 N). Maps of contact stress distribution are given in Fig.4. For increasing loading the maximum contact stress also increases. The maximum stress value is higher than 4.265 kPa - the limit value of comfort condition. Figure 5 illustrates deformation in subsequent loading stages.

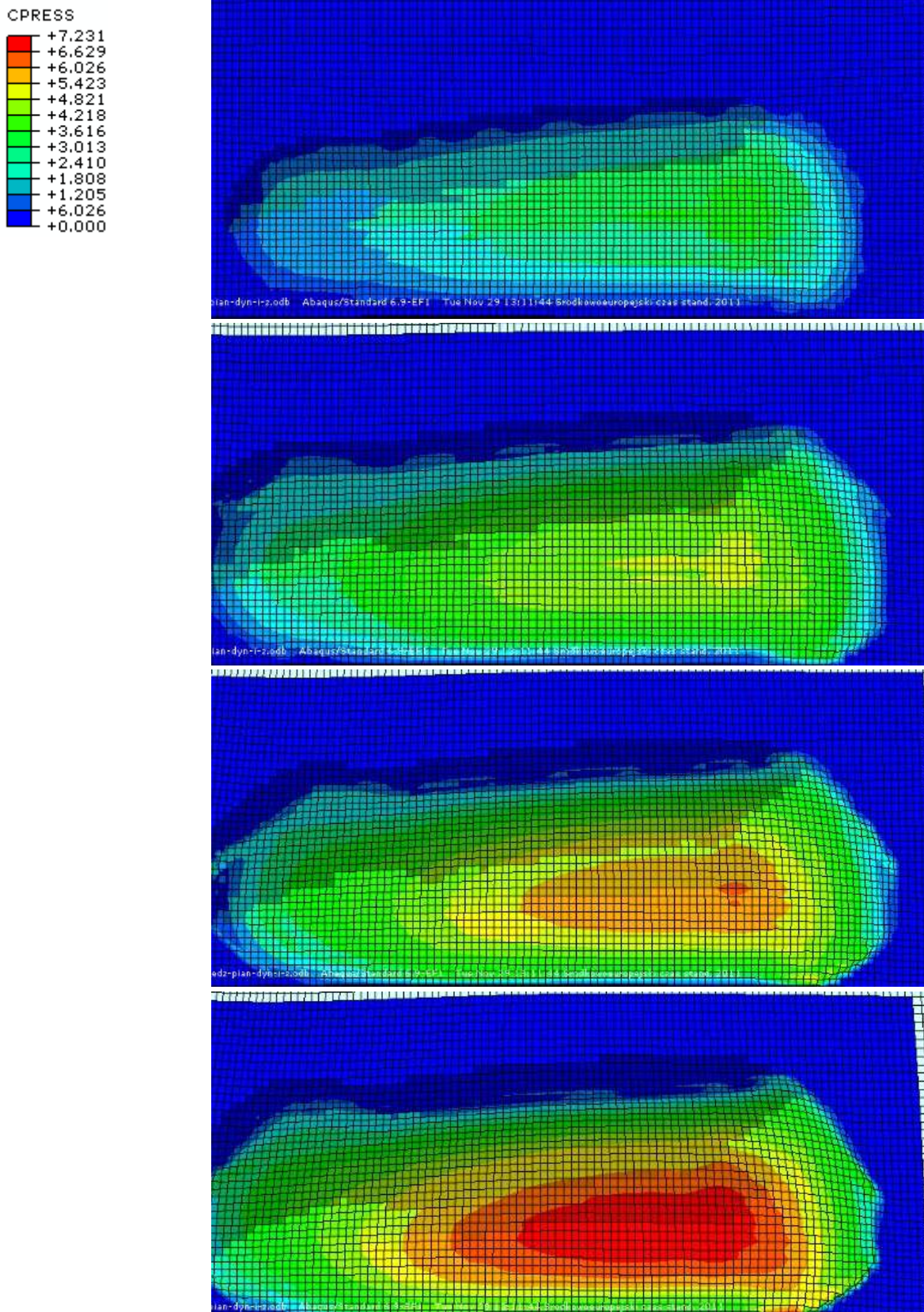


Fig.4. Contact stress distributions [kPa] in subsequent equal load increments to maximal loading (300N).

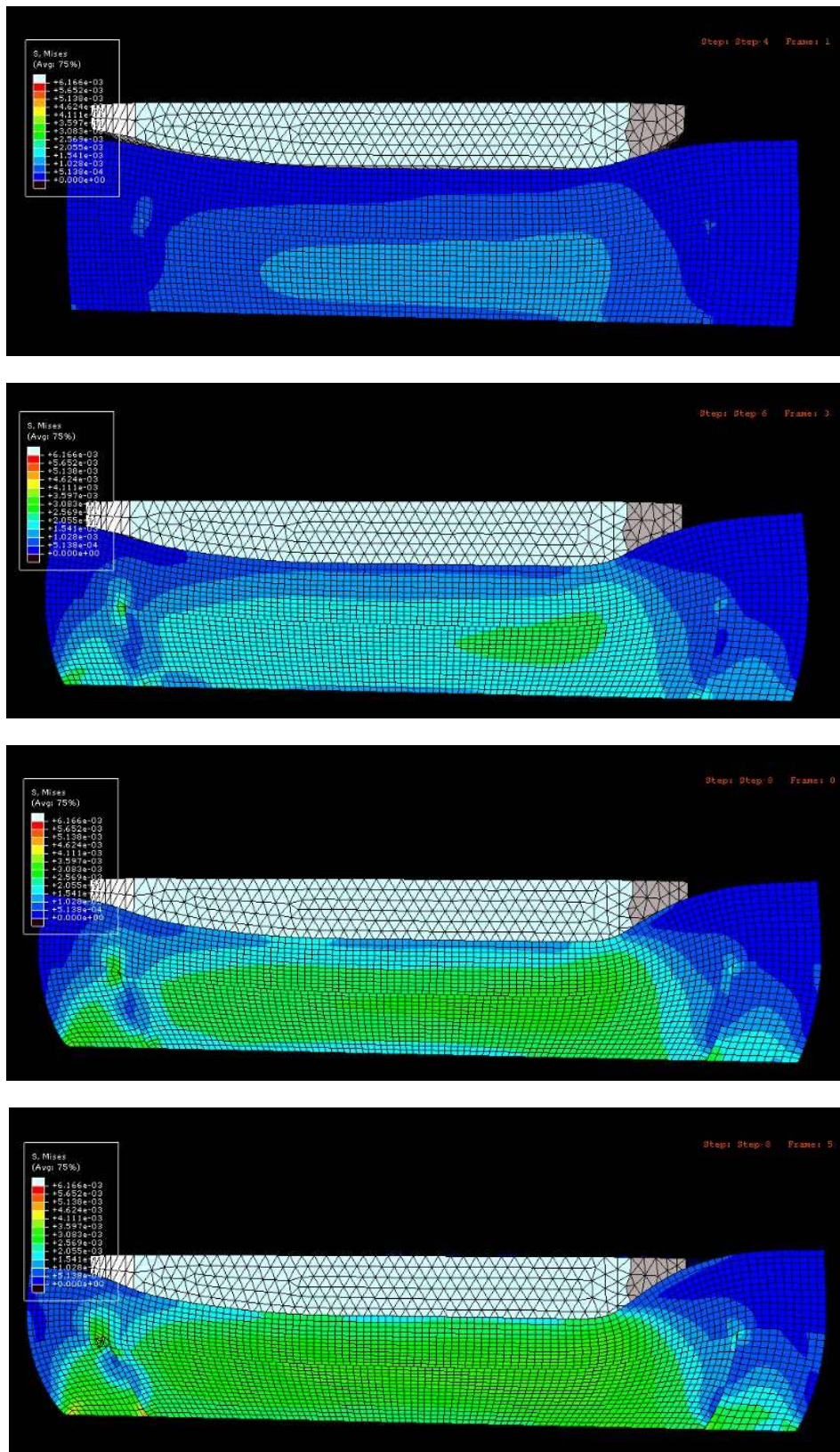


Fig.5. Deformation in subsequent equal load increments to maximal loading (300N).

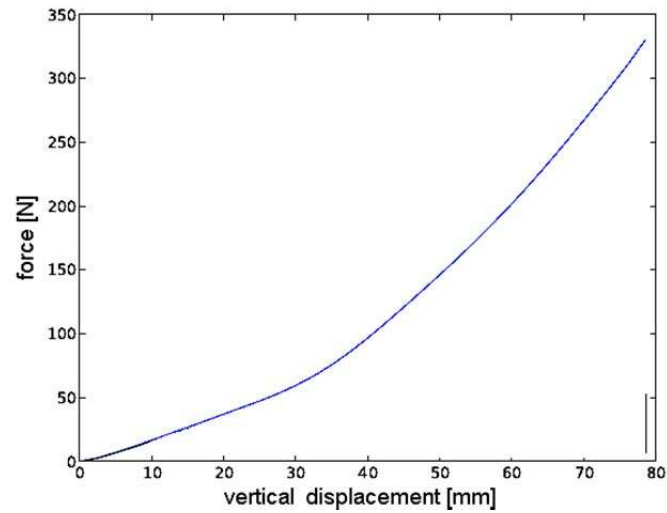


Fig.6. Load-displacement path.

Figure 6 shows the nonlinear load- displacement path.

Auxetic seat

As a preliminary task we analyze the deformation and load bearing capacity of a single spring compressed by a stiff plane surface. The friction coefficient between discs and the surface μ equals 0.2 .

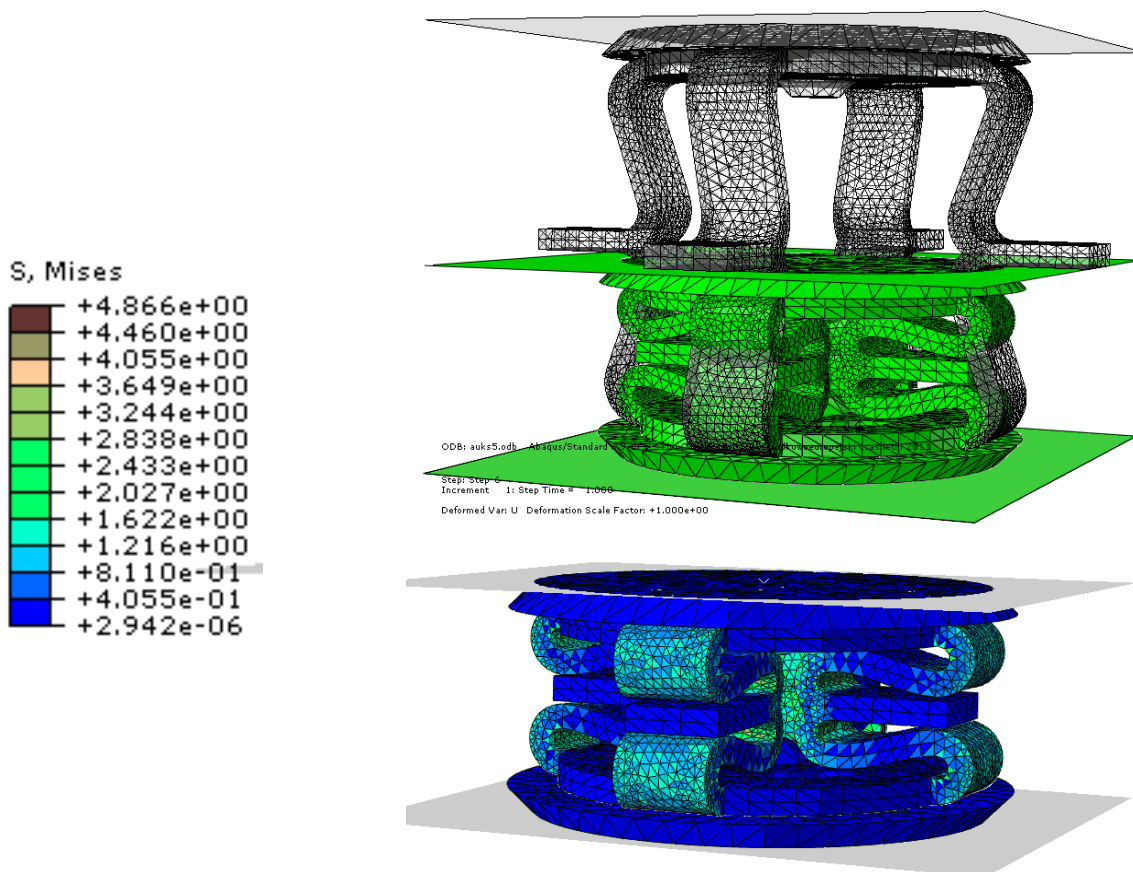


Fig.7. Deformation and stress distribution [MPa].

The average pressure simulated by the stiff surface loaded by a typical weight of a human body is $0.02 \cdot 10^6 \text{ N/m}^2$ (20 kPa) and maximum pressure is $0.072 \cdot 10^6 \text{ N/m}^2$ (72 kPa). It is recommended for the average pressure to deliver a vertical displacement of 20%-30% of spring height, and for the maximum pressure the vertical displacement should not exceed 40% of spring height.

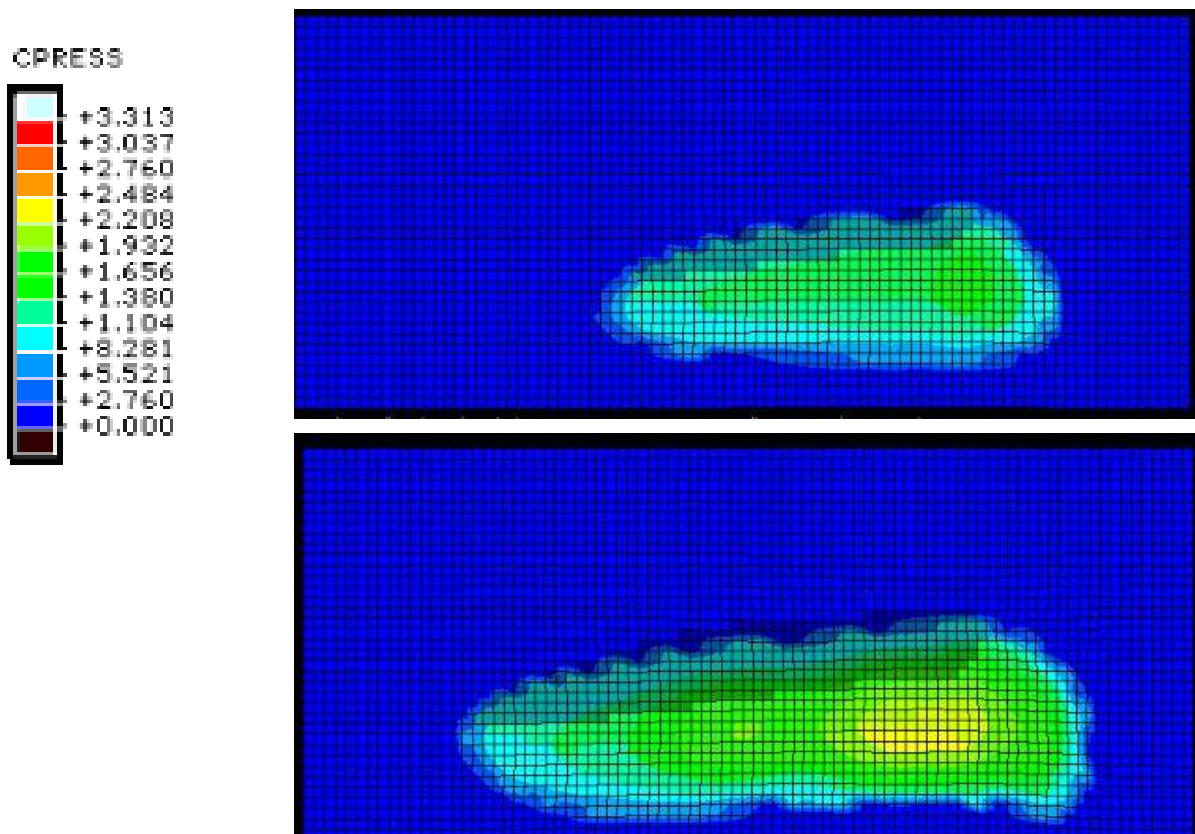
The maximum deformation and corresponding stress distribution are given in Fig.7 to check if a material fulfills the first design condition ($f_d = 20 \text{ MPa}$).

Seat with auxetic spring skeleton

We consider two cases with respect to friction coefficient.

Case 1. Friction coefficient between the rubber discs and felt $\mu = 0.1$

Distributions of contact pressure stress [kPa] and deformation in subsequent equal load increments (for maximal seat deflection equal to 50% of its height) are illustrated in Fig.8. It is clearly visible that redistribution of contact pressure (after fourth increment) gives lower pressure maximum.



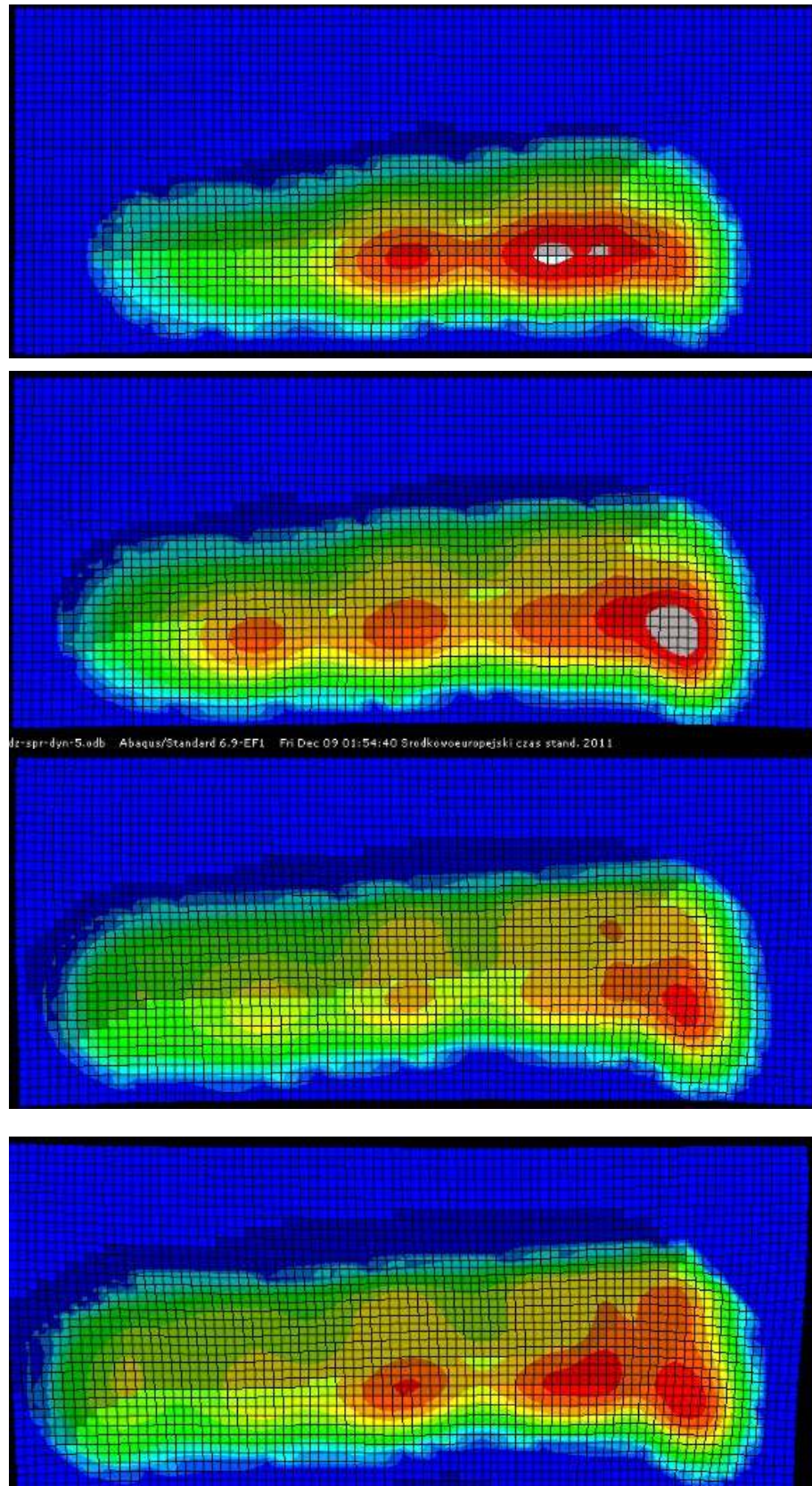
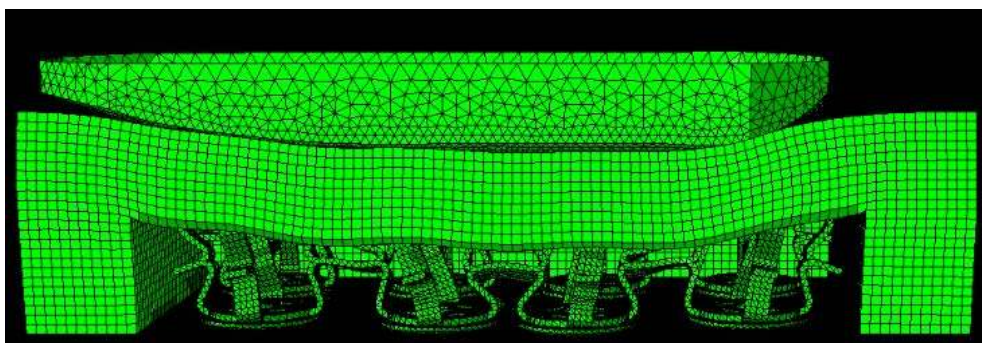
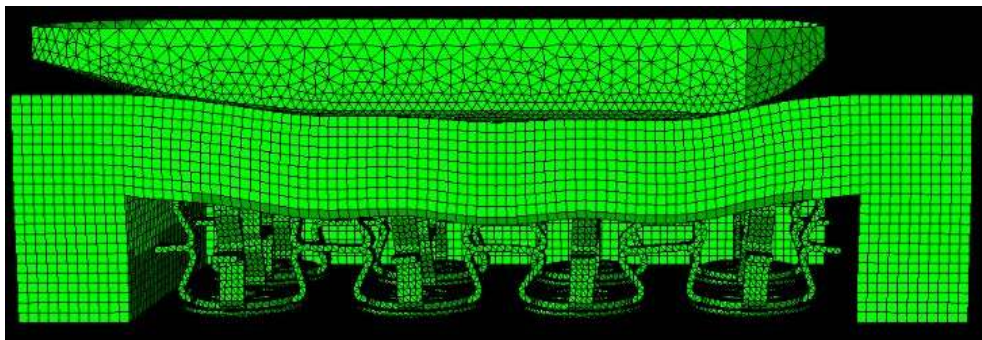
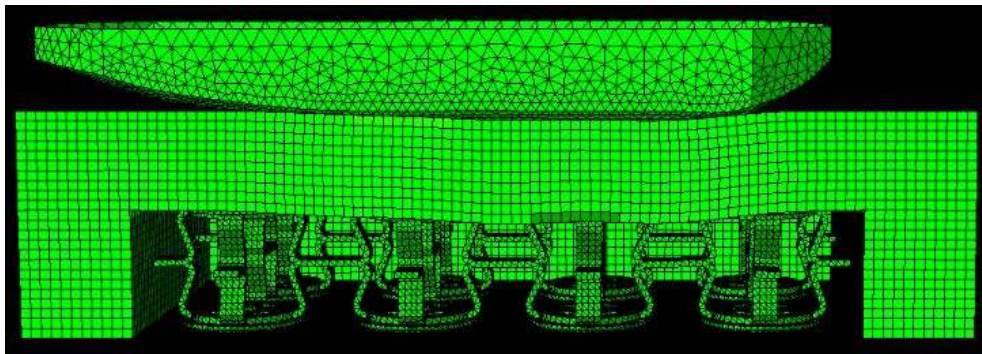
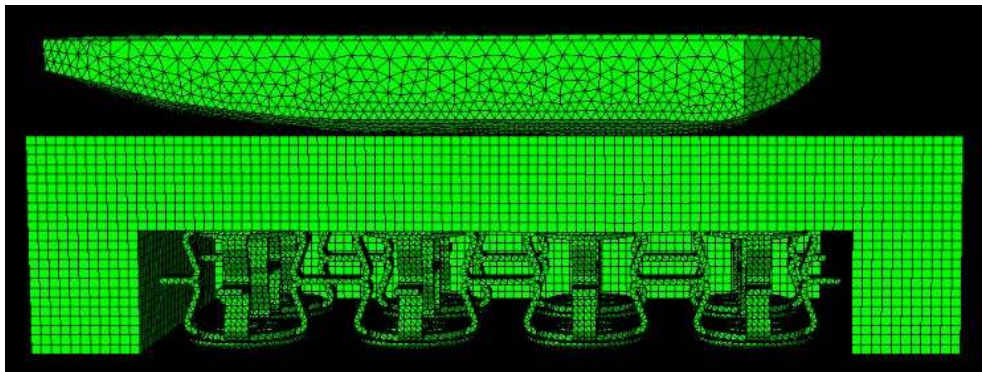


Fig.8. Distribution of contact pressure stress [kPa] in subsequent equal load increments.



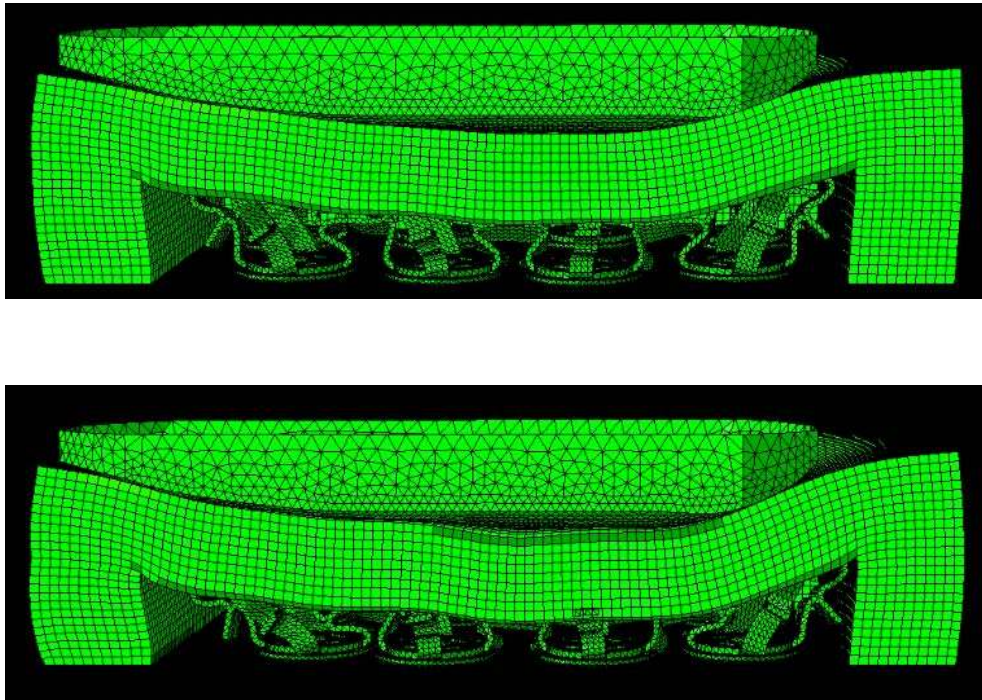


Fig.9. Seat deformation in subsequent equal load increments.

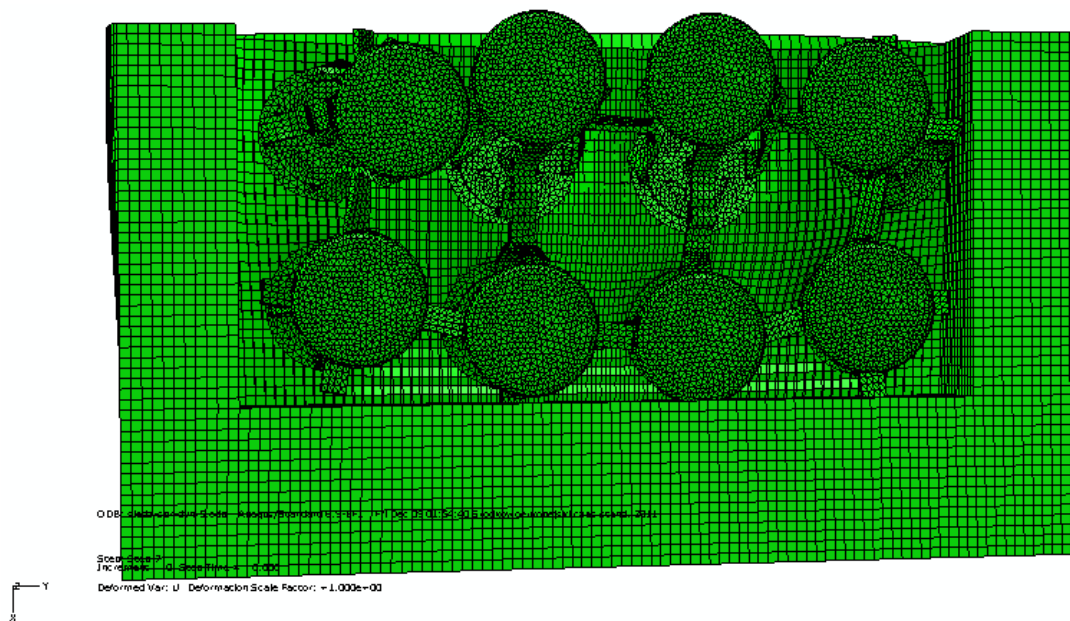


Fig.10. Horizontal displacements of springs (for maximal loading).

Seat characteristics is illustrated below (in Fig.11).

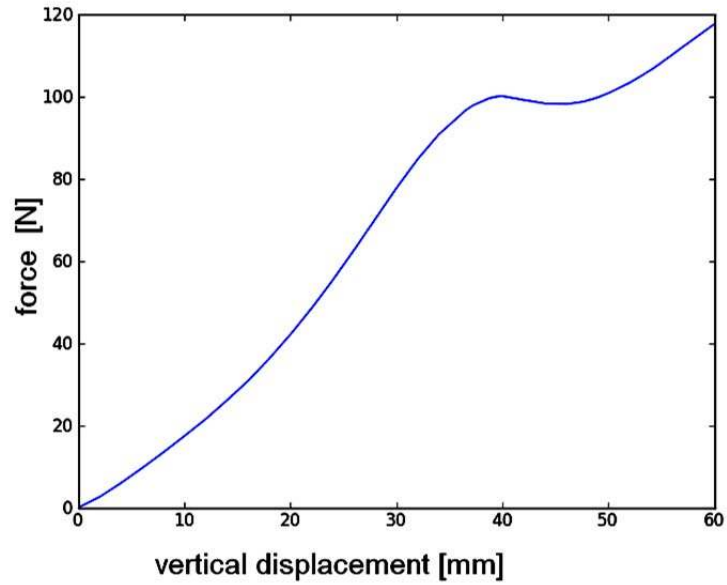


Fig.11. Load-vertical displacement path.

Case 2. Frictionless contact between the rubber disc and felt ($\mu = 0$).

For comparison with the previous example distributions of contact pressure stress [kPa] and deformation are illustrated in figure below. An advantageous redistribution of contact pressure is also visible.

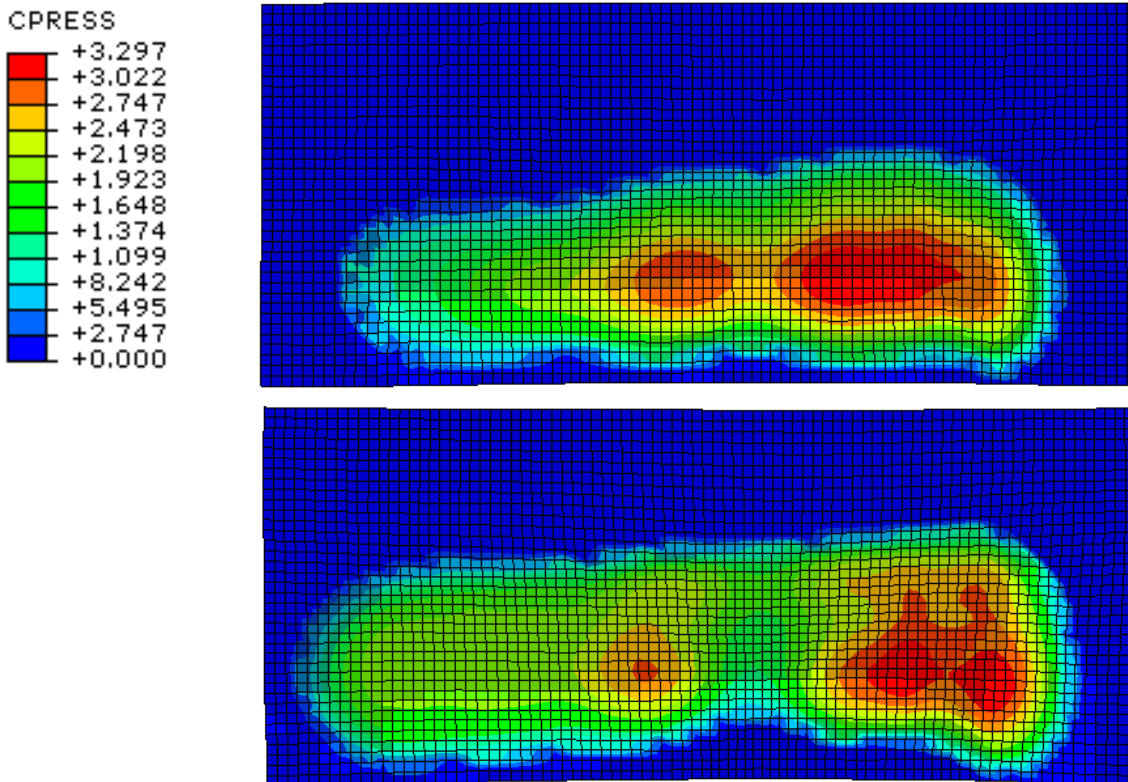


Fig.12. Distribution of contact pressure [kPa] for deflections of 30mm and 50mm.

nonlinearity of seat components. A short parameter study shows that seat pressure distributions are sensitive to variations in the seat properties. The structures are not adjusted to full bearing capacity. Changes of geometrical and material parameters can optimize the seat structure. The insight gained from the present analysis may be of use in designing new seats, and also provide guidelines for future analysis which can incorporate more complexities of the problem.

Nomenclature

- E_1, ν_1 – material Young's modulus and Poisson's ratio for the halfspace
 E_2, ν_2 – material Young's modulus and Poisson's ratio for the sphere
 F – load of intender
 f_d – limit stress value
 $p(x, y)$ – contact pressure distribution
 p_{\max} – maximal contact pressure
 μ – friction coefficient
 σ^M – Mises stress

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