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COMPARISON OF FRESNEL ZONE PLATES AND UNIFORMLY REDUNDANT ARRAYS

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Abstract

Several imaging systems in laser fusion, e-beam fusion, and astronomy employ a Fresnel zone plate (FZP) as a coded aperture. The recent development of uniformly redundant arrays (URAs) promises several improvements in these systems. The first advantage of the URA is the fact that its modulation transfer function (MTF) is the same as the MTF of a single pinhole, whereas the MTF of an FZP is an erratic function including some small values. This means that if inverse filtering is used, the URA will be less susceptible to noise. If a correlation analysis is used, the FZP will produce artifacts whereas the URA has no artifacts (assuming planar sources). Both the FZP and URA originated from functions which had flat MTFs. However, practical considerations in the implementation of the FZP detracted from its good characteristics whereas the URA was only mildly affected. The second advantage of the URA is that it better utilizes the available detector area. With the FZP, the aperture should be smaller than the detector in order to maintain the full angular resolution corresponding to the thinnest zone. The cyclic nature of the URA allows one to mosaic it in such a way that the entire detector area collects photons from all of the sources within the field of view while maintaining the full angular resolution. If the FZP is as large (or larger) than the detector, all parts of the source will not be resolved with the same resolution. The FZP does have some advantages, in particular its radial symmetry eases the alignment problem; it has a convenient optical decoding method; and higher diffraction order reconstruction might provide better spatial resolution.

Introduction

For many situations in which an x-ray image is sought, one is faced with the problem that the x-rays neither refract nor reflect. Thus, normal optics can not be used. Two systems which can be used are the single pinhole camera and the rastering collimator. Both of these systems usually require very long exposure times due to the inherently weak nature of many x-ray sources. Given the same resources of time and available detector area, both the pinhole and the rastering collimators produce images with approximately the same quality, that is, the same signal to noise ratio (SNR). Coded aperture imaging[1] is a technique which seeks to overcome the normally poor SNR in x-ray imaging.

In coded aperture imaging, the pinhole of the simple pinhole camera is replaced by many pinholes arranged in some pattern. The recorded image consists of many overlapping images of the x-ray source, one image from each pinhole. The overlapping is so severe that the recorded picture usually bears no resemblance to the x-ray object. This necessitates some form of processing of the picture in order to reconstruct the x-ray object.

A main goal of coded aperture imaging is to improve the image by increasing the collecting area with the use of many pinholes, but maintain the same angular resolution as a single pinhole. In order to accomplish this goal, a suitable choice for the pinhole pattern and decoding method must be made.

If S(x,y) is the x-ray source and A(x,y) is the aperture, then[1]

$$P(x,y) = S(x,y) * A(x,y) \qquad (1)$$

is the recorded picture where * is the correlation operator. In order to obtain the reconstructed object, R(x,y), one "decodes" by correlating with the decoding function, G, that is,

$$R(x,y) = P(x,y) * G(x,y) = S(x,y) * (A(x,y) * G(x,y)) \qquad (2)$$

In order for R(x,y) to be the original source, A*G should be a delta function.

Several pairs of As and Gs have been suggested. One of the earliest suggestions was based on the Fresnel sine function.[1] The Fresnel sine function (FF) is defined as

$$FF = \sin[2 (x^2 + y^2)/R_1^2] \qquad (3)$$

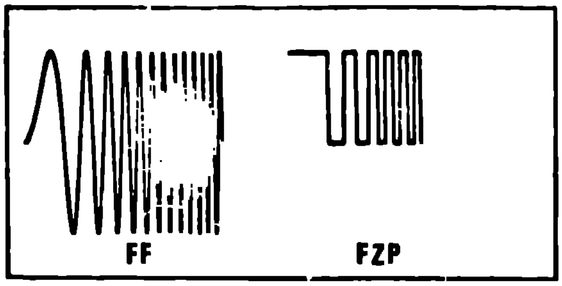

Fig. 1. This shows how the Fresnel function was modified by practical considerations to form the Fresnel zone plate aperture. These modifications adversely affected the MTF.

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where R_1, is some conveniently chosen constant. The Fresnel sine function was chosen because, if both A and G are Fresnel functions, A*G is the sought after delta function.[1] However, there are two features of the Fresnel function which limit one's capability to implement it in a practical coded aperture system: it has infinite extent and it requires aperture transmission values which vary continuously between -1.0 and 1.0. Figure 1 demonstrates how the Fresnel function (on left) is modified during implementation. The Fresnel zone plate (FZP) aperture is a binary function based on the Fresnel function according to the relationship

$$A(x,y) = \begin{cases} 1 & \text{if } \sin[2\pi(x^2 + y^2)/R_1^2] > 0 \\ 0 & \text{if } \sin[2\pi(x^2 + y^2)/R_1^2] < 0 \\ & \text{or } x^2 + y^2 > R_N^2 \end{cases} \qquad (4)$$

where the FZP has been restricted to the finite distance R_N. The properties of FZPs have been reviewed by Barrett and Horrigan.[1]

The same motivation involved with the FZP prompted the suggestion of using uniformly redundant arrays (URAs)[2]. Namely, the URA is a pattern whose autocorrelation is like a delta function. As with the FZP, there are practical details of implementation which will affect how well the URA can produce a delta function response and therefore how well the reconstructed object (R) resembles the original object (S). The purpose of this paper is to compare the FZP with the URA in order to evaluate which provides the better image.

It is very natural to compare a FZP with a URA because both would be likely candidates for many x-ray imaging situations. Both have transmissions of ~1/2, that is, 1/2 of the available area is open. Coded apertures with transmissions of 1/2 are likely candidates in such fields as x-ray astronomy or the imaging of laser fusion targets. Indeed, FZPs have been used for laser fusion,[3] e-beam fusion and were originally suggested for x-ray astronomy.[5] URAs were only recently suggested and implementation in laser fusion imaging is currently underway.

The conditions for this comparison will be similar to those found in x-ray astronomy and laser fusion. The sources are assumed to be far enough away that they are effectively planar sources. Thus, the tomographical capabilities will not be considered although these are investigated separately.[6,7] This comparison will concentrate only on the first diffraction order of the FZP because the higher orders contain severe artifacts which makes their usefulness questionable. In addition, geometric optics will be assumed, that is, no diffraction. If diffraction is allowed, the FZP would act as a lens for the x-rays and not as a coded aperture.

MTF of the URA

The modulation transfer function (MTF) of a system characterizes how well objects will be imaged. The sought after delta function response implies a perfectly flat MTF which means that all frequencies are passed equally. However, the MTF will not be flat out to infinite frequencies in a practical aperture. Figure 2 shows a one-dimensional slice through a practical URA aperture. It can be considered to have been formed from

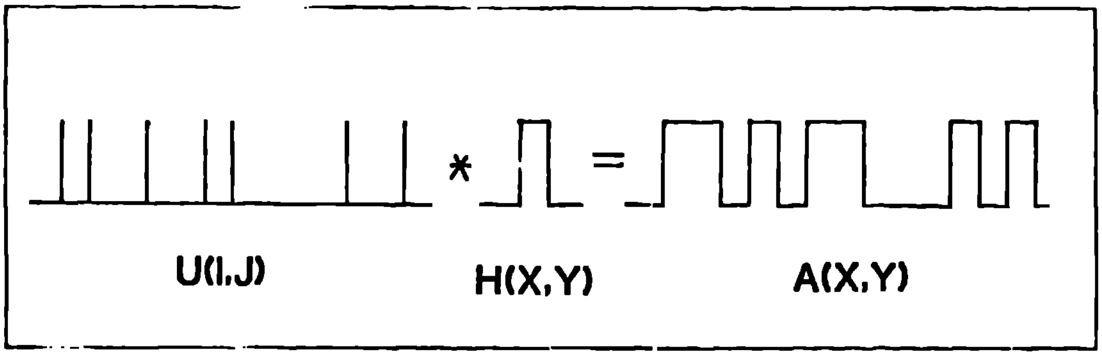

Fig. 2. This shows how a function with the separations of the uniformly redundant array is modified to form a practical aperture. This has little effect on the MTF.

the convolution of a square pinhole with a function [U(i,j)] which is a set of delta functions whose separations follow the URA pattern. Although the Fourier transform of U(i,j) can be shown to be flat, the practical aperture will not have a flat Fourier transform.

A 41 by 43 URA aperture was sampled at 512 by 512 locations and then Fourier transformed. Figure 3 shows a one-dimensional slice through the resulting MTF. Except for the DC term, the curve has the same expected (sin x)/x shape as for an individual pinhole of the same size used in the aperture. This leads to the important conclusion that the URA aperture has the same spatial frequency response as a single pinhole. Although the spatial resolution is the same, the SNR can be much better due to the larger collecting area.

The $\mathcal{F}(A)$ can be used to determine the system MTF including the effects of the decoder, G. In coded aperture imaging there are two general classes of G, those for correlation and those for inverse filtering. However, in the case of the URA, the correlation type decoder is identical to the inverse filter.[2] The original basis for the statement is the fact that balance correlation[2] with the URA (i.e. G(x,y) = 2 A(x,y) - 1) gives

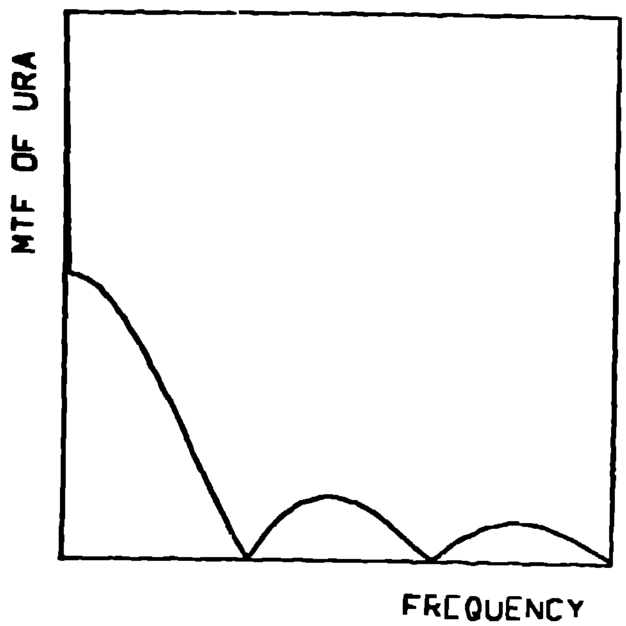

Fig. 3. A one-dimensional slice through the MTF for a 41 by 43 URA. It has the same shape as the MTF for a single pinhole.

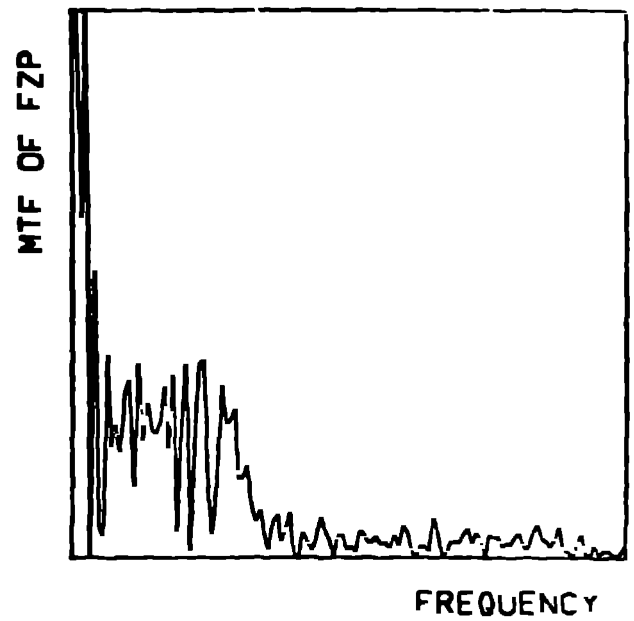

Fig. 4. A one-dimensional slice through the MTF for a ten ring FZP. The erratic structure prevents inverse filtering and tends to produce artifacts in the reconstructed image.

A*G equal to a delta function. Thus G is also the inverse filter of A. This can also be shown directly by evaluating $\mathcal{F}^{-1}(1/\mathcal{F}(A))$. Since G is just a scaled version of A, $\mathcal{F}(G)$ will be similar to $\mathcal{F}(A)$ [illegible] for a different DC term. The MTF for the URA is $\mathcal{F}|(A*G)|$ which is just the square of the curve in Fig. [illegible]. That the frequency response of the whole system is equivalent to producing a picture of the source with a single [illegible] pinhole and then viewing the picture through another similar pinhole. The first pinhole comes from the [illegible] cal pinholes in the aperture. The second factor pinhole comes from the decoder. Note that the [illegible] implementation of the URA causes the MTF to fall off at higher frequencies rather than being perfectly flat. The fall off is equivalent to the response of two convolved pinholes.

MTF for FZP

The Fourier transform of the FZP was found experimentally by sampling a 10 ring FZP at 512 by 512 locations and then Fourier transforming. Figure 4 presents a one-dimensional slice through $\mathcal{F}(A)$ for [illegible]. Basically, the procedure was the same as was used for the URA in Fig. 3. Ten rings were chosen to [illegible] the thinnest ring of a 10 ring FZP has approximately the same width as a single pinhole in a 41 by 43 [illegible]. A larger number of rings will tend to improve the MTF for the FZP, but it does not appear that it will ever be as good as the MTF for the URA.

Based on Fig. 4, one realizes that inverse filtering is not practical with the FZP. If one were to use inverse filtering, it would be necessary to divide the Fourier transform of the encoded picture by a [illegible] whose amplitude is like that in Fig. 4. This would give the correct reconstructed source if the [illegible] contained no noise. However, in actual situations with noise, the noise at those frequencies where the MTF is low would have a very strong contribution to the reconstruction because the power [illegible] is being multiplied by the reciprocal of a small number. In fact, such noise usually completely dominates the reconstructed image.

Rather than inverse filtering, the FZP encoded picture is usually reconstructed using a correlation analysis.[1] One of two types of decoding function is used. One type is to use a scaled version of the aperture, that is, $2A_{FZN} - 1$. However, then the system point-spread function (SPSF) is not a delta function and artifacts are produced.

Another type of decoder for the FZP is the Fresnel function (see Eq. 5). Use of the Fresnel function is a compromise between the inverse filter and the autocorrelation decoder. It has less artifacts than the autocorrelation analysis, but is more susceptible to noise.

No matter which decoder is chosen, the FZP will always have the basic problem that the Fourier transform of its aperture is not flat. This prevents inverse filtering whenever noise is present and forces the use of a decoder which introduces artifacts into the image. The URA function does have a flat Fourier transform which is not adversely affected by the details of implementation. Thus, inverse filtering (which coincidentally is also a correlation analysis) can be used without introducing any artifacts or excessive noise.

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Mosaicking

Figure 5 shows the usual way of implementing the FZP. The x-ray source cast a shadow of the FZP on the detector which (for the time being) we have assumed to be about the same size as the aperture. If the source happens to be on the system's optical axis, the entire FZP pattern is recorded and analysis will provide the resolution that the FZP is capable of. An off-axis source will produce a shadow which is not completely recorded. This has two effects, a smaller portion of the detector contributes to the signal and not all of the outermost rings are recorded. The first effect would tend to give a smaller SNR and the second effect produces poorer resolution.

A solution to the poor resolution would be to make the FZP smaller than the detector such that the outermost ring is always observed. However that would mean that the whole detector could never contribute to the signal. To improve the signal, a FZP larger than the detector could be used, but then the complete outermost ring would never be observed. The basic problem is that the complete FZP can not (for off axis sources) simultaneously fill the detector area and have the complete resolution typical of the outermost ring.

The URA is by nature a periodic function which can make better use of the available detector area. Figure 6 is analogous to Fig. 5 except a mosaicked (fictitious) URA pattern is used. The off-axis source casts a shadow which is a cyclic version of the basic URA pattern. (The basic URA pattern is one cycle and is what is observed for on-axis sources.) The cyclic version not only allows the entire detector to contribute, but the observed pattern always contains the same distribution of holes and thus has the same angular resolution regardless of the position of the source.

Although not normally done, the FZP could also be mosaicked to improve its performance. However there is a problem that the mosaicking would be done in a square lattice whereas the FZP is a round pattern. In order to fill the area the FZP would have to be completed into the square corners. In that case, the thinnest rings would be in the corners and only a fraction of them will be present in the pattern. Thus, the resolution of the thinnest ring present would not be achieved. The URA can set its smallest element (i.e., a pinhole) equal to the resoluton of the detector and have all of those (smallest) elements recorded for all off-axis sources.

Summary

The URA function has a Fourier transform which is flat therefore giving excellent frequency response. In practice, when it is implemented with finite size pinholes and decoded with a decoder representing finite size pinholes, the spatial frequency response is modified from being flat to being like that of two convolved pinholes. The Fresnel function also has a flat Fourier transform. However, when it is implemented the function must be truncated and forced to be binary (see Fig. 1). The result is that the MTF of the FZP is not flat and can contain small terms which preclude using inverse filtering (see Fig. 4). Without inverse filtering, the reconstructed object from a FZP will contain artifacts which are usually the dominant source of error. The URA removes the basic limitation involved with the FZP because the URA does not have artifacts. The new basic limitation depends on how well the URA passes the noise. An investigation shows that the URA has excellent noise handling characteristics, being equal to two convolved pinholes.

The fact that the URA is a periodic function leads naturally to implementing a mosaic of URAs. The mosaicked URA can better utilize the available detector area because the entire URA pattern is always recorded. In contrast, it is difficult to always record the complete outermost ring of a FZP from which its resolution is obtained.

The FZP does have some advantages over the URA. The radial symmetry of the FZP eliminates an alignment problem. Also, the FZP has optical properties which can be used to obtain real time images. (It should also be possible to do spatial filtering with the URA although this has not yet been accomplished.) In addition, the FZP (when analyzed with coherent light) produces the reconstructed image in several diffraction orders. Each order is at a higher resolution although these orders apparently have low SNR and very severe artifacts.

Acknowledgment

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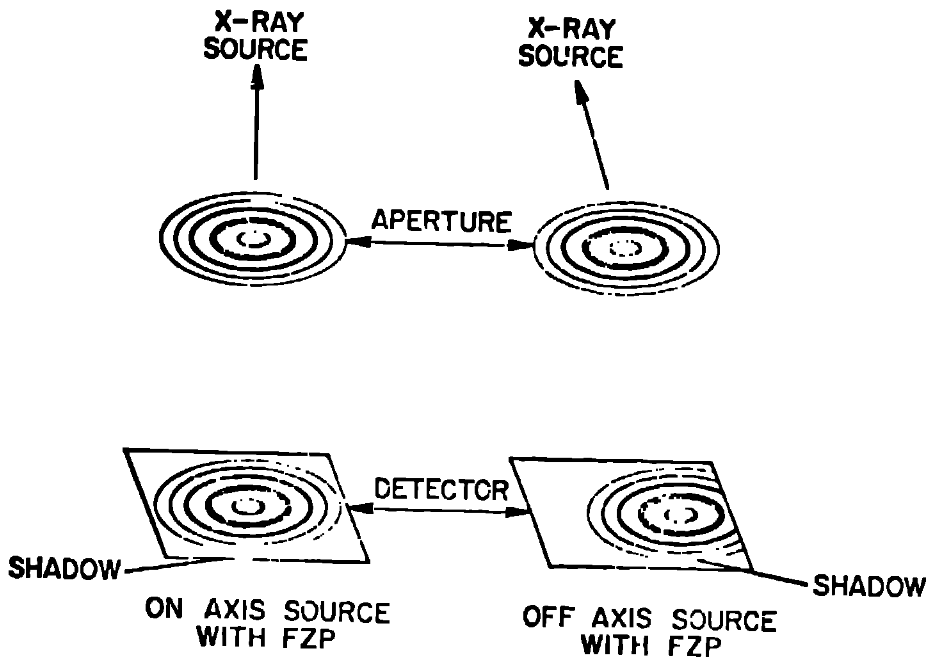

Fig. 5. On-axis and off-axis cases for imaging with the FZP. It is hard for the FZP to use all of the available area while recording the complete outermost ring, which is responsible for the resolution.

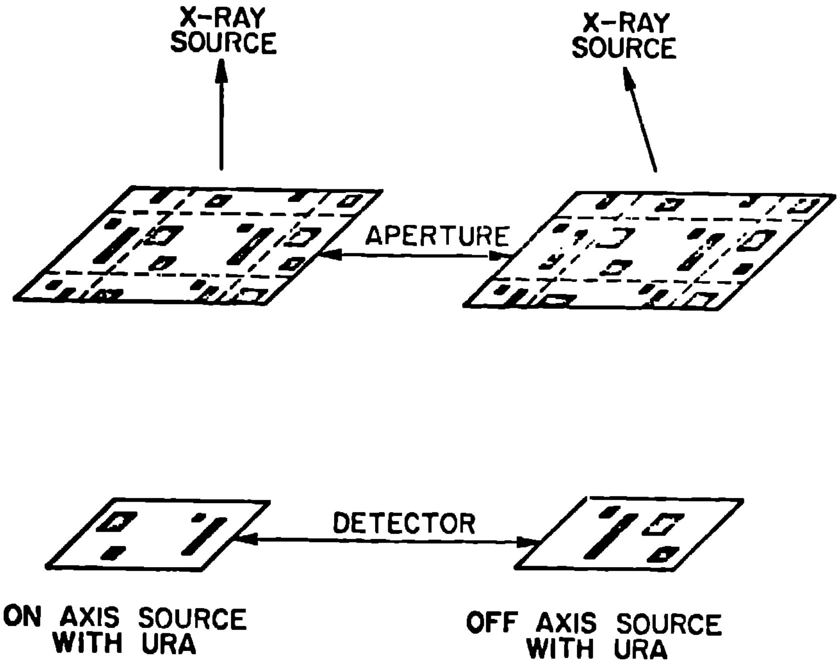

Fig. 6. On-axis and off-axis cases for imaging with the URA. Since the URA is a periodic function, it can be mosaicked such that all of the detector area is always used while recording a complete (although a cyclic version) URA pattern.