Comparison of interval and Monte Carlo simulation for uncertainty propagation in atmospheric dispersion model

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Abstract— *In this paper, the problem of tackling uncertainty propagation in the estimation of the atmospheric dispersion of a toxic gas release is analyzed in order to assess the risk at the event of an accident. This estimation is based on an effect model associated with the studied dangerous phenomenon where some input variables and model parameters are known with imprecision. Two simulation approaches, Monte Carlo and interval analysis method, are applied and compared for estimating the confidence interval of risk intensity. Interval analysis method is superior in estimating all the possible values of intensity relative to the Monte Carlo simulation. A sensitivity analysis based on Sobol indices is applied in order to reduce the number of uncertain variables while conserving an acceptable precision of effect model. Furthermore, much less computational time is required for interval analysis method than for Monte Carlo simulation.*

Keywords: Risk assessment, sensitivity analysis, uncertainty propagation, interval analysis, Monte Carlo simulation.

1. Introduction

The risk assessment is a decision aid that aims to rank or quantify risks to human in order to prioritize management actions and the allocation of resources. Science-quality criteria require the assessment to be transparent, repeatable and systematic, and its estimations to be precise and accurate. Intensity estimations of accidental releases of hazardous gases have a significant impact on emergency planning around industrial plants and on the choice of risk prevention and mitigation barriers. This impact has a very high severity in urban areas and may be disastrous for the population [1]. Atmospheric dispersion simulations are dependent on a significant number of input variables (source term, weather conditions) as well as internal parameters of the dispersion model. This effect model includes parameters and variables which may be measured, estimated or deduced from a priori knowledge, but all of them are known with uncertainty [2], [3], [4], [5]. This leads to the inaccuracy in the results when computing the intensity of the dangerous phenomenon i.e the gas concentration. In order to perform this intensity, it is necessary to choose a suitable method able to express the uncertainty associated with parameters and variables of the dispersion model and after that, it is necessary to define a method for estimating the propagation of uncertainties in this model.

In the present study, two simulation approaches, Monte Carlo and interval analysis method are applied for estimating the confidence interval of intensity of the atmospheric dispersion. The obtained results by means of interval analysis method are compared here with Monte Carlo simulation results for uniform probability distributions in order to study the variability of uncertainty propagation in the two approaches and the computation time. A global sensitivity analysis based on Sobol indices is applied in order to determine how uncertainty in the model output can be apportioned to the different uncertain model inputs. This analysis allows reducing the number of uncertain model inputs while conserving an acceptable model precision.

The organization of this paper is as follows. In the next section the problem statement and the global sensitivity analysis are presented. In section 3, the Monte Carlo and interval analysis approaches for uncertainty propagation are explained. The application and the results obtained with the proposed approaches are reported in section 4. Finally, the conclusion is drawn in the section 5.

2. Sensitivity analysis

In this paper, the problem of tackling uncertainty in the estimation of the atmospheric dispersion of a toxic gas release is analyzed. For this reason, two simulation approaches, Monte Carlo and interval method are studied and compared in order to propagate uncertain inputs in a chosen analytical atmospheric dispersion model.

2.1 Uncertainty estimation

The concepts of risk and uncertainty are intimately linked. Risk occurs because the past, present and future are uncertain. A measurement is a process whereby the value of a quantity is estimated. When a measurement is made or when some quantity is calculated from the data, generally it is assumed that some exact or "true value" exists based on how is defined what is being measured (or calculated). A range of values, that should contain this "true value", is then usually specified. The most common way to define this values set is: Measured a calculated value = exact value $\pm$ uncertainty

2.2 Objective of sensitivity analysis

The sensitivity analysis is the study of how uncertainty on the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input [6]. This is useful as a guiding tool when the model is under development as well as to understand model behavior when it is used for estimation or for decision support. For mathematical models, sensitivity analysis is closely related to the study of error propagation, i.e. the influence that the lack of precision on model input will have on the output. Sensitivity studies can identify and prioritize the most influential inputs, decide which parameters need more investigations to be precisely determined and simplify the model by removing or making constant the less influential input factors. Two types of sensitivity analysis methods can be selected, local and global methods. The choice depends on the objective of the analysis, the number of uncertain input factors, the degree of regularity of the model, and the computing time for a single model simulation. In this study we are interested in global sensitivity analysis [7], specifically Sobol technique. Sobol method is a global sensitivity analysis (SA) technique which determines the contribution of each input (or group of inputs) to the variance of the output and they take into account the whole field of possible variation of the input variables. The usual Sobol sensitivity indices include the main and total effects for each input, but the method can also provide specific interaction terms [8], [9].

2.3 Estimated Sobol indices by Monte Carlo

In a general manner, the analytical model of atmospheric dispersion can be written in the form of a mathematical relation 1 describing the dangerous phenomenon at a given instant:

$$y = f(x_1, ..., x_p) \tag{1}$$

We assume in this study that the input variables $(x_1, ..., x_p)$ of the model are independent. To appreciate the importance of an input variable x_i on the variance of the output y , we study how the variance of y decreases if the variable x_i is fixed to a value w_i : $V(y|x_i = w_i)$ .The problem with this indicator is the choice of the w_i value of x_i which is solved by considering the expectancy of this quantity for all possible values of w_i : $E[V(y|x_i)]$. Thus, the variable x_i is more influent on the variance of y, when this amount is small. The formula of the total variance
$V(y) = V(E[y|x_i]) + E[V(y|x_i)]$,
leads to use in an equivalent manner the amount $V(E[y|x_i])$, which becomes larger when the variable x_i has a more important contribution to the variance of y. In order to use a standardized indicator, we define the sensitivity indices of y to x_i as [8]:

$$S_i = \frac{V(E[y|x_i])}{V(y)}.$$

Consider an N-sample $X_{(N)} = (x_{k1}, ..., x_{kp})_{k=1,...N}$ of realizations of the input variables $(x_1, ..., x_p)$, the index k denotes the k^{th} sample. The expectation of y, $E[y] = f_0$ and the variance $V(y) = V$ are classically estimated by:

$$f_0 = \frac{1}{N}\sum_{k=1}^{N} f(x_{k1}, ..., x_{kp}) \ , \ V = \frac{1}{N}\sum_{k=1}^{N} f^2(x_{k1}, ..., x_{kp}) - f_0^2 \tag{2}$$

The estimation of sensitivity indices requires a variance estimation of a conditional expectation. We remind a technique to estimate $V(E[y|x_i])$ due to Sobol [9].
Let us note : $V_i = V(E[y|x_i]) = E[E[y|x_i]^2] - E[E[y|x_i]]^2 = U_i - E[y]^2$ with $U_i = E[E[y|x_i]^2]$.
The variance of y being conventionally estimated by 2, Sobol proposes to estimate the quantity U_i, in other words the expectation of the square of the expectation of y conditional on x_i , as a conventional expectation where, all input variables can vary, except the variable x_i which is fixed. This requires two N samples of input variables, denoted $X^1_{(N)}$ and $X^2_{(N)}$:

$$U_i = \frac{1}{N}\sum_{k=1}^{N} f(x^{(1)}_{k1}, ..., x^{(1)}_{k(i-1)}, x^{(1)}_{ki}, x^{(1)}_{k(i+1)} x^{(1)}_{kp}) \times f(x^{(2)}_{k1}, ..., x^{(2)}_{k(i-1)}, x^{(1)}_{ki}, x^{(2)}_{k(i+1)} x^{(2)}_{kp}),$$

when the indexes (1) and (2) denote the associated N sample. The sensitivity indices of the first order of the x_i input are then estimated by:

$$S_i = \frac{V_i}{V} = \frac{U_i - f_0^2}{V}$$

3. General approach on uncertainty propagation

The aim of this approach is to make uncertainty evaluation internationally comparable. This methodology is also proposed by the new draft of the Guide to the expression Uncertainty in Measurement (GUM [10]). The methodology presented can be summarized in the following main steps:

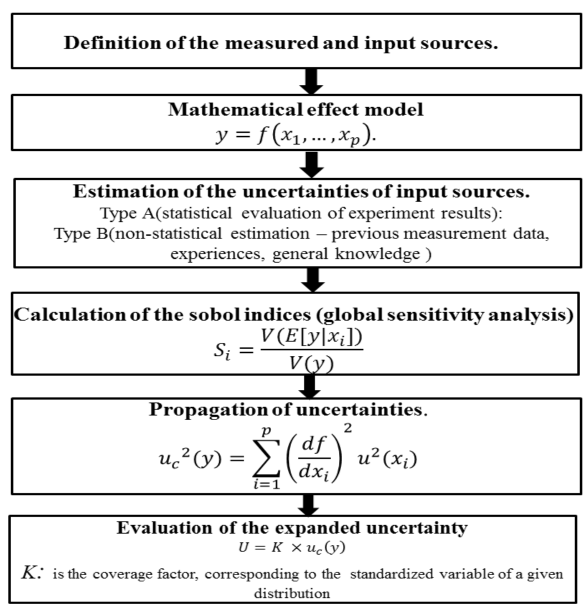

Fig. 1: Methodology for evaluating model uncertainty

3.1 Monte Carlo simulation for uncertainty propagation

Monte Carlo simulation [11], [12] is a computational mathematical technique, which performs model simulation by calculating the model output by substituting each uncertain model input by a particular feasible value. It then calculates outputs over and over, each time using a different set of random input values from the probability functions. Depending upon the number of uncertainties and the ranges specified for them, a Monte Carlo simulation could involve thousands or tens of thousands of recalculations before it is complete.

3.1.1 Monte Carlo simulation process

The Monte Carlo simulation process consists in two steps:

- Generation of a N sample $X_{(N)}$ of size p with uniformly distributed random values, where N is the number of simulations and p is the number of parameters. For each independent sample of size p the resulting model value of y is calculated.
- These N values of y are used to perform the propagation of uncertainties in the model output.

3.1.2 Implementation of the Monte Carlo simulation

Figure 2 present the calculation phase of uncertainty evaluation using Monte Carlo simulation to implement the uncertainty propagation.

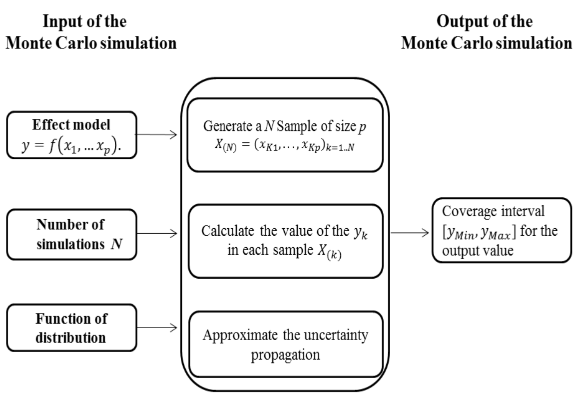

Fig. 2: The calculation phase of uncertainty propagation.

The inputs of the calculation phase of uncertainty propagation are firstly the mathematical effect model, secondly the number N of samples and thirdly the uniform distribution functions of the uncertain model inputs. Three main steps are executed during the implementation of the Monte Carlo simulation : generation of N samples of p input variables, evaluation of the model output for each sample and finally estimation of the model output and the associated uncertainty from the distribution function. The final result of uncertainty propagation is the coverage interval for the model output. Based from the N values $y_1, y_2, ..., y_N$, the uncertainty u is defined as:

$$u = \frac{y_{Max} - y_{Min}}{2} * \frac{100}{y_{NominalValue}}. \qquad (3)$$

With $y_{NominalValue}$ is the output value of the model without uncertainty on the model inputs. The y_{Min} and y_{Max} define respectively the minimal and maximal values of $y_{i,i=1,...,N}$.

3.2 Interval analysis approach in uncertainty calculation

The sources of uncertainty are multiple, i.e. mathematical models with uncertain parameters, representation of real numbers on digital computers with finite precision, uncertain initial data. In some applications, it is necessary to know the influence of these uncertainties on the computed solution. To solve such problems, techniques based on interval analysis have been developed in particular by Moore [13], [14]. This tool allows calculating an overestimated interval containing with guarantee the feasible values of the model output. Interval modeling consists in describing a model uncertainty by an unknown bounded variable, whose known support defines its feasible value set. The interval containing a real uncertain variable x, whose value is comprised between a lower bound x^- and an upper bound x^+, is written:
$[x] = [x^-, x^+] = \{x \in \mathbb{R} | x^- \leq x \leq x^+\}$.
Note that no distribution function is required.

3.2.1 Interval Arithmetic

Interval arithmetic in its modern form was introduced by Moore [13], [14] and is based on arithmetic conducted on closed sets of real numbers. Mathematics elementary operations are extended to intervals. The operation result between two intervals is an interval that contains all the results of this operation between the different values contained in these intervals. The operation result on finite intervals is defined by two bounds which are obtained by working only on their bounds. In this way, interval arithmetic is an extension of real arithmetic. For a real arithmetic operation $\circ \in \{+, -, *, /\}$, the corresponding interval operation on intervals $[x]$ and $[y]$ is defined by:

$$[x] \circ [y] = \{x \circ y | x \in [x], y \in [y]\}. \qquad (4)$$

Interval arithmetic considers the whole range of possible instances represented by an interval model. In the classic set-theory interval analysis, given a $\mathbb{R}^p$ to $\mathbb{R}$ continuous function $y = f(x_1, ...x_p)$, the interval united extension $[f]$ of f corresponds to the range of f-values on its interval argument $([x_1], ...[x_p])$ in $I(\mathbb{R}^p)$:
$[f]([x_1], ...[x_p]) = \{f(x_1, ...x_p) | x_1 \in [x_1], ...x_p \in [x_p]\} = [min\{f(x_1, ...x_p) | x_i \in [x_i]\}, max\{f(x_1, ...x_p) | x_i \in [x_i]\}] i = 1, ...p.$

3.2.2 Pessimism

Generally, the result of a series of operations between two or more intervals is not minimal; the obtained interval is pessimistic. This problem is mainly due to the dependence problem [15]. Considered a no degenerate interval $[x] = [x^-, x^+]$ and an arithmetic operation $\circ \in \{+, -, *, /\}$, then using the definition 4, we obtain:

$$[x] \circ [x] = \{x \circ y | x \in [x], y \in [x]\}. \tag{5}$$

According to 5, we see that bounded variables x and y are considered different despite the fact that we manipulate the same interval. So, dependency between bounded variables cannot always be taken into account when their interval supports are manipulated and this problem is called dependency phenomenon. For example, let $[x] = [-1, 1]$, then $[x] - [x] = [-1, 1] - [-1, 1] = [-2, 2] \neq \{0\}$, the interval operation overestimates the exact domain $\{0\}$. In a general manner, pessimism depends on the occurrence of interval variables in the expression of $[f]$. It leads to the very interesting guarantee property (reliable computing) of interval tool, but the overestimation may be important if unsuited interval extensions are manipulated. The interval computation can be considered as a semantic extension of f, since it admits the logical interpretation:
$(\forall x_1 \in [x_1])...(\forall x_p \in [x_p])(\exists y \in [f]([x_1], ..., [x_p]))y = f(x_1, ..., x_p)$.
This logical interpretation contains the set of all trajectories that verify the model equation.

3.2.3 The implementation of the interval analysis

Figure 3 presents the calculation phase of uncertainty evaluation using interval analysis method to implement the uncertainty propagation

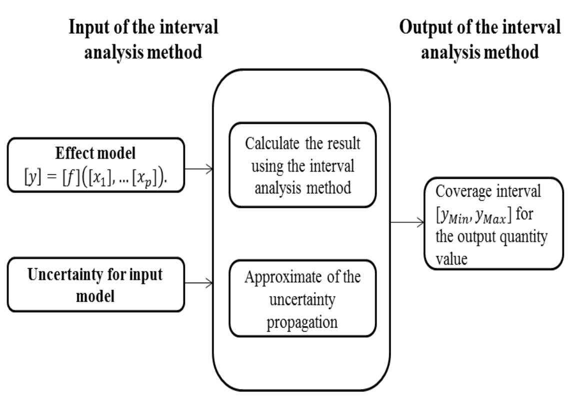

Fig. 3: The calculation phase of uncertainty evaluation

From the result value calculates by the interval analysis method, the propagation uncertainty u is defined as 3. The y_{Min} and y_{Max} are the lower and upper bounds of the caculated of $[y]$.

3.3 The interest of the sensitivity analysis

In one hand, identifying the most influential inputs from the sensitivity analysis aids to decide which uncertain inputs need more investigations in order to reduce their interval supports for the IAM and so to improve model accuracy. On the other hand to reduce the number of uncertain parameters by imposing less influential model inputs to their nominal values leads to reduce the occurrence of some interval variables, and thus the pessimistic in the result value.

4. Results and application

4.1 Mathematical effect model

In order to assess the severity of the risk when an undesirable and unexpected event occurs, a mathematical model can be used to compute physical effects coming from the considered event. In this study, an effect model is used to estimate or to predict the downwind gas concentration emitted from sources such as industrial plants, vehicular traffic or accidental chemical releases. This model represents the relationships between the inputs of the atmospheric dispersion model (wind speed, conditions emission point, release flow ....) and the gas concentration in the air at a specific point [16], [17], [18]. The concentration c_k of the released gas at a position x_k, y_k, z_k from a continuous source is given by the following Gaussian plume model:

$$c_k = f(x_k, y_k, z_k, u_{ref}, z_{ref}, h, q, a_y, a_z, b_y, b_z, c_y, c_z)$$
$$= \frac{q z_{ref}^{0.33}}{2\pi u_{ref} h^{0.33}(a_y x_k^{by} + c_y)(a_z x_k^{bz} + c_z)} * exp\left[-\frac{1}{2}\left(\frac{y_k}{a_y x_k^{by} + c_y}\right)^2\right] *$$
$$\left\{exp\left(-\frac{1}{2}\left(\frac{z_k - h}{a_z x_k^{bz} + c_z}\right)^2\right) + exp\left(-\frac{1}{2}\left(\frac{z_k + h}{a_z x_k^{bz} + c_z}\right)^2\right)\right\} \tag{6}$$

Where:
c_k is the concentration of the emission (in micrograms per cubic meter) at any point x_k meters downwind of the source, y_k meters laterally from the centerline of the plume, and z_k meters above ground level. The index k denotes different evaluations of the model output. q is the quantity or mass of the emission (in grams) per unit of time (seconds). u_{ref} is the wind speed (in meters per second) measured at a given altitude z_{ref}. h is the height of the source above ground level (in meters). The terms $a_y x_k^{by} + c_y$ and $a_z x_k^{bz} + c_z$ represent the dispersion parameters and depends on the distance x_k. They represent the standard deviations of a statistically normal plume in the lateral and vertical dimensions, respectively. The values of a_y, a_z, b_y, b_z, c_y and c_z , may be determined for each atmospheric stability class defined by Pasquill, by using the table given in [17].

4.2 Modelling of uncertain model inputs

Instead of representing an uncertain parameter or variable by a constant nominal value, this one can be defined as a bounded variable. In other words, its real value is unknown, but it belongs to a set of feasible values defined as an interval whose bounds are known. In the following, an imprecision ρ_v means that an uncertain positive variable v is represented by the interval value set

$$[v(1-\rho_v), v(1+\rho_v)]. \tag{7}$$

This study has been applied to an example of accident involving nitric oxide gas. This gas is toxic and has a density relative to air of 1.04, so a Gaussian model is well suited to model dispersion of such a gas.

For a chosen stability class of C, the nominal values of dispersion parameters are: $a_y = 0.105, a_z = 0.066, b_y = b_z = 0.915$, and $c_y = c_z = 0$ [17]. We assume that the height of leakage point is $h = 2m$. The measured wind speed is $u_{ref} = 4.58m/s$, at a height of $z_{ref} = 40m$. The theoretical nominal value of the release flow is $q = 2216g/s$. In the following, the inaccuracy on some parameters and variables are considered: the release flow q with an uncertainty of $\rho_v = 5\%$, the wind speed u_{ref} with $\rho_v = 2.5\%$, the dispersion parameters a_y, a_z with $\rho_v = 2.5\%$ and b_y, b_z with $\rho_v = 1\%$, then for each parameter is defined an interval support of feasible values. These intervals are directly used to calculate the result of the interval analysis method. To compare this result with the Monte Carlo simulation result, we need on the one hand to generate random values for each model input contained in the same bounded support according to a uniform distribution function. On the other hand, we need the same indicator to express the propagation of uncertainties; for this reason, the used indicator for the both approaches is presented in the next section.

4.3 Uncertainty propagation before sensitivity analysis

Let note:

C_{Nom} : Concentration in studied point with the nominal value of the model inputs i.e. without uncertainty on these inputs.

$[MinC, MaxC]_{C-MC}$: Confidence interval of concentration in the studied point with the Monte Carlo simulation (MCS). The bounds $MinC$ and $MaxC$ define respectively the minimal and maximal values of the concentration c_k computed for $N = 100,000$ samples.

$[C_a, C_b]_{C-IA}$: Interval support of concentrations in the studied point computed with the interval analysis method(IAM).

$U-MC$: This indicator defines the uncertainty on the concentration in studied point with the Monte Carlo simulation, $U-MC = \dfrac{MaxC - MinC}{2} * \dfrac{100}{C_{Nom}}$.

This indicator expresses the margin value (width) relative to the nominal gas concentration according to uncertainty on the model parameters. More precisely it represents the same quantity in percent than the imprecision ρ_v defined in 7 for uncertain model inputs.

$U-IA$: This indicator defines the uncertainty on the concentration in studied point computed with the modal interval analysis. It is defined in the same way by the following relation:

$U-IA = \dfrac{C_b - C_a}{2} * \dfrac{100}{C_{Nom}}$.

Case study

The objective is to determine the confidence interval of gas concentration in order to determine if it is lower (safety zone) or bigger (danger zone) than a given regulatory threshold that leads to avoid for example lethal or health irreversible effects. In our study the gas concentration is estimated in 5 points placed in the downwind of a source emitting nitric oxide gas. Concentration estimations given by IAM in these 5 points of study are compared with the outcome of a Monte Carlo approach. The number of samples for the latter is increased until no significant changes in the upper and lower bounds are observed. This leads to $N = 100,000$ samples which is a reasonable and classical choice according to the number $p = 6$ of uncertain model inputs. Uncertainties on model inputs in the MCS are represented in terms of uniform probability distributions for comparison with IAM. Multiplicative congruential random generation is used to return successive pseudo-random numbers. A looping program is implemented in java using the class random (). Table 1 illustrates the values of the studied model inputs with the added uncertainties,

Table 1: Model inputs

$q \pm 5\%$	$a_y \pm 2.5\%$	$a_z \pm 2.5\%$	$b_y \pm 1\%$	$b_z \pm 1\%$	$u_{ref} \pm 2.5\%$	x	y	z	z_{ref}	h
2216	0.105	0.066	0.915	0.915	4.58	350	8	2	40	2
						200	10			
						150	5			
						50	5			
						40	5			

Table 2 illustrates the concentration C_{Nom} with the nominal values of the inputs studied, ranges of concentrations with Monte Carlo approaches $[MinC, MaxC]_{C-MC}$ and modal interval analysis $[C_a, C_b]_{C-IA}$, finally the computed indicators $(U-MC, U-IA)$ of the gas concentration on the 5 points studied.

Table 2: Computed confidence intervals and indicators

$Point(x_k, y_k, z_k)$	C_{Nom}	$[MinC, MaxC]_{C-MC}$	$[C_a, C_b]_{C-IA}$	$U-MC$	$U-IA$
(350, 8, 2)	1.22	[1.09, 1.37]	[1.06, 1.41]	11,47%	14.34%
(200, 10, 2)	2.65	[2.39, 2.96]	[2.27, 3.09]	10,75%	15,47%
(150, 5, 2)	5.10	[4.56, 5.65]	[4.40, 5.89]	10,68%	14,63%
(50, 5, 2)	12.04	[10.91, 13.29]	[9.84, 14.64]	9,88%	19,93%
(40, 5, 2)	10.46	[9.25, 11.76]	[8.36, 12.98]	11,99%	22.10%

Figure 4 represents the propagation of uncertainties with MCS and IAM methods for the 5 points of coordinates (x_k, y_k, z_k).

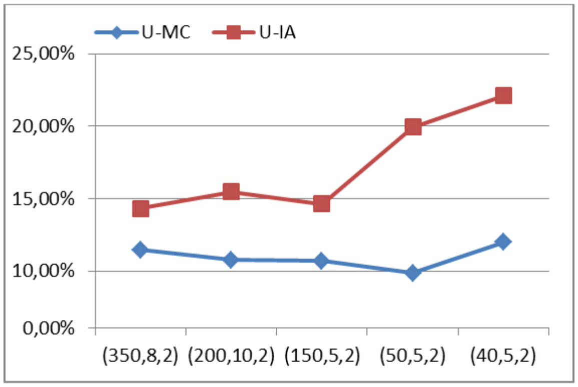

Fig. 4: Uncertainty propagation according to studied points

Interpretation of results

With the Monte Carlo simulation, the obtained results show that the uncertainty on model output varies between 9.88% and 11.99% in the different points studied. With the interval analysis method, the indicator varies between 14.34% and 22.10% and the uncertainty increases when the distance decreases between the source and the point studied. Concerning the execution time with the IAM, the calculation script needs 1.5 ms as execution time to obtain the concentration at a given point. With the MCS the execution time is 128 ms, so it can be deduced that the reduction time with the IAM is almost 98.8% compared to the MCS. These results show that the IAM provides larger confidence intervals relative to the MCS when some model inputs are uncertain. Several reasons explain the difference between the both approaches. The first one is due to the problem of pessimism of IAM because of multiple occurrences of some uncertain model inputs such as a_y, a_z, b_y, b_z. The second reason is that the Monte Carlo simulation needs to randomly generate each model input in its interval support. On one hand the MCS does not guarantee to take all the values in these bounded supports and on the other hand to take all the possible combinations of model input values. For comparison, the IAM takes into account all the feasible combinations which guarantees the results. In others word, IAM and MCS leads respectively to outer and inner approximations of the exact confidence interval on gas concentration. Concerning the execution time, the principal reason of the difference is the large number N of samples used by MCS.

4.4 Sensitivity analysis

Table 3 presents the results of the global sensitivity analysis for the studied uncertain model inputs. The first order indices are computed for 100 repetitions and $N = 100,000$ samples:
The result shows, that the less influential model inputs on the model output are a_z, b_y and b_z.

Table 3: Sensitivity analysis using Sobol indices

	Si :Sobol index of the first order	Confidence interval of Si
u_{ref}	0.14	[0.09, 0.23]
q	0.50	[0.44, 0.54]
a_y	0.33	[0.29, 0.41]
a_z	0.07	[-0.02, 0.12]
b_y	0.02	[-0.05, 0.10]
b_z	0.02	[-0.05, 0.10]

4.5 Uncertainty propagation after the sensitivity analysis

Based on the results of the sensitivity analysis, uncertainty on a_z, b_y and b_z has been removed, in other terms these model inputs are fixed on their nominal values. All the other model inputs q, a_y and u_{ref} are considered uncertain and can vary on their respective bounded supports.
Table 4 illustrates the values of the studied model inputs with the added uncertainties only on q, a_y and u_{ref}. Table 5 represents the obtained results for uncertainty propagation.

Table 4: Model inputs

$q \pm 5\%$	$a_y \pm 2.5\%$	a_z	b_y	b_z	$u_{ref} \pm 2.5\%$	x	y	z	z_{ref}	h
2216	0.105	0.066	0.915	0.915	4.58	350	8	2	40	2
						200	10			
						150	5			
						50	5			
						40	5			

Table 5: Computed confidence intervals and indicators

$Point(x, y, z)$	C_{Nom}	$[MinC, MaxC]_{C-MC}$	$[C_a, C_b]_{C-IA}$	$U - MC$	$U - IA$
(350, 8, 2)	1.22	[1.11, 1.34]	[1.10, 1.35]	9,42 %	10.24%
(200, 10, 2)	2.65	[2.44, 2.88]	[2.36, 2.97]	8,30 %	11,47%
(150, 5, 2)	5.10	[4.66, 5.58]	[4.58, 5.66]	9,01 %	10.60%
(50, 5, 2)	12.04	[10.96, 13.14]	[10.40, 13.87]	9,05 %	14,41%
(40, 5, 2)	10.46	[9.34, 11.63]	[8.82, 12.31]	10,94 %	16,69%

Figure 5 presents the comparison of the uncertainty propagation with MCS and IAM.

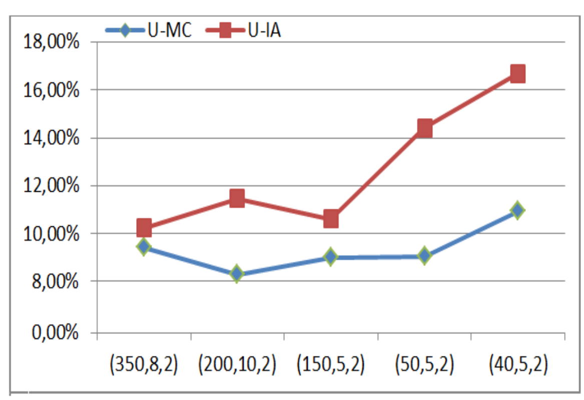

Fig. 5: Uncertainty propagation according to studied points

Interpretation of results

With the Monte Carlo simulation, the obtained result shows that the uncertainty on output model varies between 8.30% and 10.94% for the different points studied. With the interval analysis method, the indicator varies between 10.24% and 16.69% and the uncertainty increases when the distance decreases between the source and the point studied. With the IAM, the calculation script needs 0.7 ms as execution time to obtain the concentration of a given point. With the MCS the execution time is 105 ms, so the reduction time with the IAM is almost 99.3% compared to the MCS. The sensitivity analysis helps in fixing the less influential parameters as constant values. This in turn reduces the computation time which leads to a faster treatment. While some solutions may be loosed in the MCS method, the sensitivity analysis carried out is pertinent because this loss is reasonable. For the interval method, the number of lost solutions is greater than those in the MCS method, i.e. the reduction of the uncertainty on model output is more important. This is an expected result because some of the multi-occurrent variables (e.g. a_z, b_y, b_z) are fixed to constant and nominal values. Accordingly, this leads to a reduction in the dependence phenomenon between uncertain model inputs, so the reduction of the pessimism in the interval method and produces more accurate results. It is worth noting that the results obtained with IAM are almost equal to the confidence interval of MCS before carrying out the sensitivity analysis (see Table 2). Therefore, the sensitivity analysis may lead to an interesting simplification by improving the precision of the IAM model. In the context of risk assessment in the transport of hazardous materials, it is better to get all the possible estimations of gas concentration as with the method of analysis interval, instead of getting a part of the values as in the Monte Carlo simulation. An inner estimation of the interval confidence may lead to an inadequate and insufficient evacuation operation from the danger area, and then leads to serious injuries.

5. Conclusion

From this study we can conclude that the interval analysis method is a significant tool for estimating the propagation of uncertainties. In this study where several model inputs of the analytical model studied are uncertain , we find that the IAM provides larger confidence intervals relative to the MCS. Moreover the computation time is smaller with IAM than with the Monte Carlo simulation. The sensitivity analysis helps in fixing the less influential parameters as constant values. This in turn reduces the computation time which leads to a faster treatment and on the other hand leads to a reduction of the pessimism in the interval method and produces more accurate results. At last, the notion of reliable or guaranteed computation is crucial for risk assessment. The next objective is to extend the proposed approach which may be also used to determine all the geographical region in which gas concentration is less than a given regulatory threshold or used for other types of dangerous phenomenon like the explosion of dangerous goods.

Acknowledgment

This study is part of the Geofencing DG research project, and was initiated under the program Transport System of the French competitiveness cluster Lyon Urban Truck & Bus (LUTB). The authors would like to thank the Region Rhône-Alpes for funding this work in the Geofencing project.

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