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Comparison of Limit-State Seismic Earth Pressure Theories

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SYNOPIS: The paper considers how, for the limit analysis approach to the seismic design of earth retaining structures, both the magnitude of soil force and the center of pressure vary with the type of displacement of the structure in translation or rotation. Two approaches are used. The first assumes that for a rotating wall the apparent internal friction angle of the backfill will vary for purely geometrical reasons. The second considers the effect of the peaked form of the stress-strain curve for a dense cohensionless soil. Both approaches show that, compared with a wall rotating about its base, the center of pressure will rise for translational displacement and even more for rotation about the top. The paper gives figures showing the magnitudes of the shifts in center of pressure, and discusses the interrelationship between the two approaches.

INTRODUCTION

The classic upper-bound approach to limit analysis for seismic lateral pressures is a modification of Coulomb's fundamental static solution with its assumption of a straight slip surface mechanism, uniform soil properties and a sliding lateral boundary. Inertia forces are added to the static weight of a trial failure wedge, and the critical angle of the slip surface and corresponding lateral force are determined by a maximization or minimization of the active or passive thrust required to maintain equilibrium. The so-called Mononobe-Okabe (M-O) equations result (Mononobe 1929, Okabe 1926). This solution is now the basis for the seismic design of most retaining structures.

Many walls, however, fail with some rotational component of lateral movement. Such structures as cantilevered sheet piles and the sides of large drainage channels are constrained to rotate at their base (RB mode), while tied-back or braced walls often rotate about their top (RT mode).

A major problem in the seismic design of retaining walls of whatever type is the assessment of the point of action of the soil forces acting on the wall. In general terms it is well-understood that both the magnitude and distribution of the soil forces will depend on the nature and extent of wall movement. If there is sufficient movement to mobilize the backfill strength, the soil force can be calculated directly form the M-O solutions. However, for neither strength-governed nor displacement-controlled (Richards and Elms 1979) designs do they predict the point at which the resultant of the soil forces acts on the wall. An excellent summary of the situation is given by Whitman (1990).

It seems clear from the evidence of both static tests (Tschebotarioff, in Leonards 1962) and dynamic investigations (Nagel and Elms 1984; Richards and Elms 1987) that as it moves outwards a wall prefers, as it were, to remain upright. That is, compared with a sliding or translational motion, if the wall rotates outwards about the bottom, the center of pressure will become lower as the force distribution attempts to restore verticality, and if it rotates about the top, the center of pressure will become higher.

Two things are required: a theoretical explanation of the reasons for the changes in center of pressure, and a means of quantitatively predicting the magnitudes of the shifts. In what follows, the issues are addressed using two lines of argument. Firstly, an approach is used in which it is argued that the effective angle of friction (more presisely the orientation of the resultant force on tne slip surface) varies throughout the soil mass according to the geometry of the motion of the retaining wall, leading to pressure shifts. Secondly, it is shown that the effect of the peaked form of the stress-strain curve for a dense cohesionless material will also produce the observed effects. It is postualted that one or both effects occur in practice.

EFFECT OF GEOMETRICALLY-INDUCED VARIATIONS IN SOIL FRICTION ANGLE

Dubrova produced a theoretical explanation for the changes observed in the point of action of the soil forces acting on a static retaining wall (Dubrova 1963, Harr 1966). He argued that in the case of a retaining wall constrained to rotate about its center, the soil at the top would be in a passive state as the wall would be moving into the backfill, while at the bottom of the wall an active failure would take place. Thus at the top of the wall, the force resultants on quasi-rupture lines would be inclined towards the wall at an angle ψ equal to the soil friction angle ϕ while at the bottom they would be inclined at the same angle away from the wall. At the center, where no motion takes place, the force resultant would be normal to the quasi-rupture line, and $\psi = 0$. Assuming a linear variation in ψ between top and bottom of the wall, (that is, $\psi = 2\phi z/H - \phi$, where H is the height of the wall), Dubrova substituted ψ into the Coulomb solution for static force on a wall for different wall heights and differentiated to get a pressure distribution on the wall. This turned out to be roughly parabolic, with a center of pressure well above the one-third point to be expected for a linear (hydraulic) distribution. The same approach was applied to walls starting outwards at the top. In this case rotating the effective friction angle ψ was assumed to have a linear variation from zero at the top of the wall to ϕ at the bottom. Once again, the pressure distribution was roughly parabolic (Fig. 1). On the other hand, rotation about the bottom produced a linear distribution, again shown in Fig. 1, which gives the

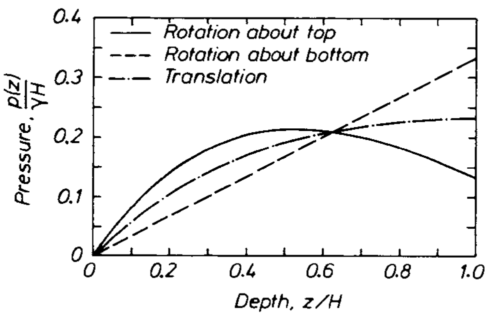


Fig. 1: Static pressure distribution after Harr, 1966 : $\phi=30°$, $\delta=0$.

center of pressure observed in static tests of walls and bulkheads (Leonards, 1962) at one third the height of the wall. Dubrova assumed that the pressure distribution for a wall sliding outwards with no rotation would be the average of those for the cases of rotation at the top and bottom, arguing that sliding was a combination of the two rotational motions.

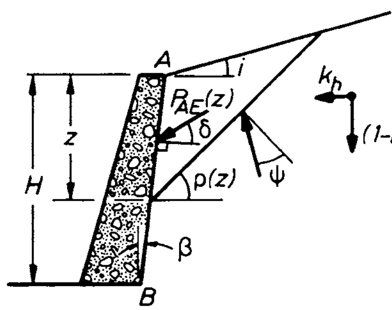


Fig. 2: Wall with intermediate quasi-rupture surface

We now follow Dubrova's basic approach, and apply it to the seismic case. For the top part z of a retaining wall subjected to vertical and horizontal accelerations $k_v g$ and $k_h g$, the M-O expression for the force exerted on the wall by the soil is

$$P_{AE}(z) = \frac{\gamma(1-k_v)}{2} \frac{z^2 \cos^2(\psi-\theta-\beta)}{\cos\theta \cos^2\beta \cos(\delta+\beta+\theta)(1+m)^2} \tag{1}$$

where γ is the unit weight of the soil,

$$m = \sqrt{\frac{\sin(\psi+\delta)\sin(\psi-\theta-i)}{\cos(\delta+\beta+\theta)\cos(i-\beta)}} \tag{2}$$

$$\theta = \tan^{-1}\left(\frac{k_h}{1-k_v}\right) \tag{3}$$

and β, i and δ are as defined in Fig. 2. The definition of ψ depends on the assumed motion of the retaining wall. This is of no concern for a wall tilting outwards and rotating about point B in Figure 2, as we can follow Dubrova and take the pressure distribution to be linear. However, for a wall rotating about the top (point A), we cannot use Dubrova's relation

$$\psi = \frac{\phi z}{H} \tag{4}$$

as ψ must be equal to or greater than $(\theta+i)$ otherwise the contents of the square root in Eq. (2) will become negative. We therefore assume a linear variation between top and bottom of the wall of

$$\psi = (\theta+i) + (\phi-\theta-i)\, z/H \tag{5}$$

It is important to understand the implications of Eq. 5. The inclination $\rho(z)$ (Fig. 2) of the local quasi rupture surface corresponding to Eq. 1 is

$$\rho(z) = (\psi-\theta) + \tan^{-1}\left\{\frac{[a(a+b)(1+bc)]^{1/2} - a}{1+c(a+b)}\right\} \tag{6}$$

where

$a = \tan(\psi - \theta - i)$
$b = \cos(\psi - \theta - \beta)$
$c = \tan(\delta + \beta + \theta)$

The inclination becomes zero when $\psi = \theta + i$ at the top of the wall, and the quasi rupture surface becomes horizontal. The implication is that at the top of the wall, the soil is in a "fluidized" state (Richards et al 1990), and can carry no horizontal shear. This is

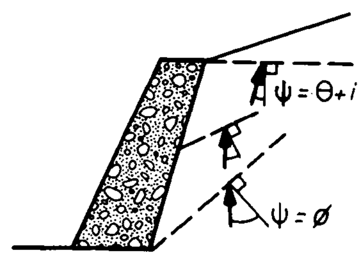


Fig.3: Quasi rupture surfaces

reasonable as no movement has been allowed at the top of the wall to allow the soil strength to develop. For the special case of horizontal backfill (i = 0) and zero acceleration (θ = 0) the situation reverts to Dubrova's static case (Harr 1966) where at the top of the wall, ψ = 0.

A variation on this approach might be to consider the possibility that although the wall is assumed physically to rotate about the top, the effective point of rotation might be lower for the reason that, in the seismic situation, the horizontal acceleration drives the backfill into the wall such that in the top layer of soil, passive failure takes place locally, even though the wall itself is constrained from moving at the top. Localized passive failure of this nature has been observed during tests of a top-rotating wall (Neelakantan, G. et al 1990). Thus the assumption that $\psi = \theta + i$ at the top of the wall can be looked on as a limiting case.

Substituting (5) into (1) and differentiating with regard to z gives the pressure $p_T(z)$ as a function of z, thus:

$$p_T(z) = \frac{K\cos(\psi-\theta-\beta)}{(1+m)^2}\left(\frac{z}{H}\right)\left\{\cos(\psi-\theta-\beta) - (\phi-\theta-i)\left(\frac{z}{H}\right)\sin(\psi-\theta-\beta) - \frac{(\phi-\theta-i)}{2}\left(\frac{z}{H}\right)\frac{m\cos(\psi-\theta-\beta)}{(1+m)}\left[\frac{1}{\tan(\psi+\delta)} + \frac{1}{\tan(\psi-\theta-i)}\right]\right\} \quad (7)$$

where

$$K = \frac{\gamma(1-k_v)\,H}{\cos\theta\,\cos^2\beta\cos(\delta+\beta+\theta)} \quad (8)$$

For rotation about the bottom, assuming a triangular distribution, the pressure is

$$p_B(z) = K\left(\frac{z}{H}\right)\;\frac{\cos^2(\phi-\theta-\beta)}{(1+m)^2} \quad (9)$$

For translating (sliding) walls, we can follow Dubrova and take the pressure as the average between those for top and bottom rotation; that is,

$$p_s(z) = 1/2(p_T(z) + p_B(z)) \quad (10)$$

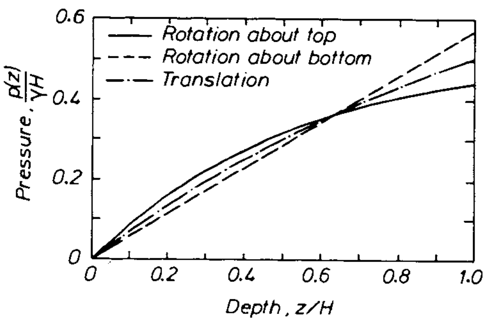


Fig. 4: Seismic Pressure Distribution: $\phi=30°$, $\delta=0$, $k_h=0.3$

Figure 4 shows the pressure distributions for the translational and the two rotational cases for ϕ = 30° and k_h = 0.3. The results have been made non-dimensional by dividing by γH. It can be seen that the distributions more nearly approximate in shape to the hydrostatic distribution than the equivalent static case shown in Fig. 1. This is to be expected, as in the limit when the acceleration reaches a level such that $\phi - \theta - i = 0$, (k_h = 0.577 in this case) the backfill is completely fluidized and can carry no horizontal shear. Thus, no matter what the movement of the wall, the pressure distributions must be hydrostatic.

Fig. 5 shows the variation in the increase in the height of the center of pressure on the wall above the hydrostatic center of pressure at one third of the height of the wall, for both the rotation-about-the-top and the translation cases. As stated previously, it is assumed that the center of pressure for rotation about the bottom occurs at the third-point of the wall. The graphs in Fig. 5 are plotted for zero wall friction (δ = 0). The effect of δ on the height of the center of pressure is negligible: for δ = 30° the resultant force is about 3% higher than for δ = 0, if k_h = 0.3. The results show the expected trends and exhibit the appropriate limiting behavior, as, for instance, when $\phi = \theta$. However, the Dubrova analysis is based on assumptions which are, while intuitively

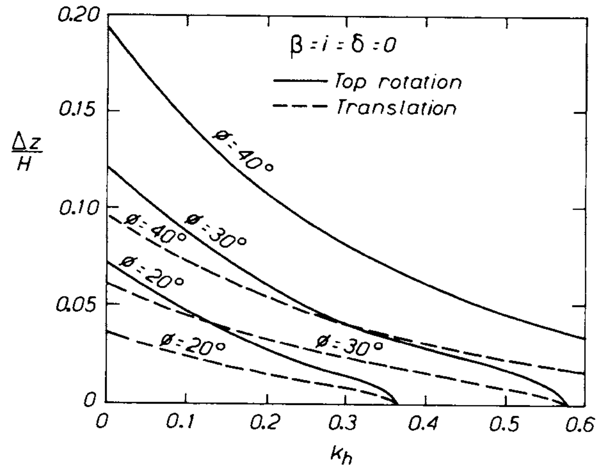


Fig. 5: Increase in height of center of pressure above hydrostatic third-point: Dubrova effect.

appealing and based on rational argument, nevertheless rather sweeping in nature. It cannot therefore be expected to give a complete description of the seismic pressure distributions on a retaining wall as the wall moves.

EFFECT OF THE STRESS-STRAIN CURVE SHAPE

Figure 6 shows a typical shear stress-strain curve for a dense cohesionless soil. As strain is increased, shear stress increases up to point A, where the effective friction angle reaches a peak value, ϕ_P. With further strain, the stress drops and ϕ reaches a residual value ϕ_R at B. The result is that as stresses build up gradually in the material, at first, while on segment OA in Fig. 6, the strains will be distributed broadly throughout the material. When however the falling or "softening" portion AB of the curve is reached, a local increase of strain leads to a reduction of stress. Local instability occurs and instead of distortion being distributed throughout a region it becomes concentrated on a failure surface on which the strain becomes infinite.

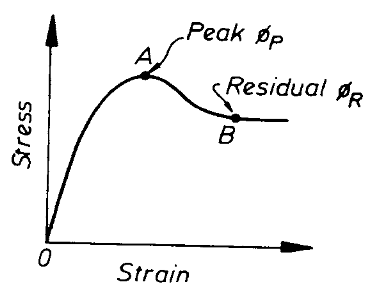
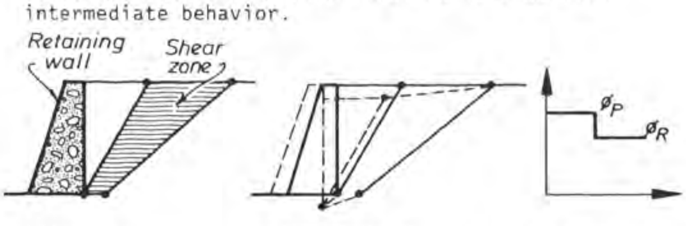


Fig. 6: Typical stress-strain curve for dense cohesionless soil

During seismic translational displacement of a retaining wall, the strain pattern in the backfill exhibits both stable and unstable modes of behavior. Figure 7 is a double-exposure photograph of the initial stages of outward wall movement in a small-scale test (Aitken et al 1982, Elms and Richards 1990). The figure shows the positions of vertical lines of white sand before and after displacement due to a single acceleration pulse. Where the two parallel lines occur, the backfill has moved out as an undistorted block. Where the pairs of lines are at an

Fig. 7: Initial development of shear zone (Aiken et al 1982)

angle to one another, distributed shear strain has taken place, while behind the heel of the wall, kinks in the lines indicate instability and the formulation of a local slip surface. When further acceleration pulses were applied to the wall, the shear strain region narrowed and the slip surface propagated upwards until, when it reached the surface, the sliding block behavior assumed by displacement-controlled design approaches (Richards and Elms 1979, Elms and Richards 1979, Whitman 1990) took place, with displacements predictable using the residual soil friction angle ϕ_R. However, in this case we are concerned not so much with final failure as with intermediate behavior.



(a) Shear zone (b) Mechanism (c) Simplified stress-strain curve

Fig. 8: Translational failure shear-zone mechanism

Figure 8 outlines an idealized failure mechanism for translational wall movement which would roughly duplicate the behavior observed in Fig. 7. Instead of the more usual assumption of a single slip surface, a tapered shear zone is proposed, narrow at the base and wider at the top (Fig. 8a). A constant-volume deformation of the shear zone (Fig 8b) would mean a very much greater shear strain at the lower end of the shear zone than at the upper. If the simplified stress-strain curve of Fig. 8c is adopted, then the residual shear strength ϕ_R would apply, initially, in the lower part of the shear zone so causing a failure surface to form there. As deformation progressed, the failure surface would move upwards, with the shear zone decreasing in width, until it reached the top of the backfill.

Thus at the beginning of movement the force exerted on the wall would be given by Eq. 1 using ϕ_P and with $z = H$. The force would then increase until, when the failure plane had fully developed, the force corresponding to the same equations but using ϕ_R would be achieved.

Consider now the case of a wall constrained to fail by rotation about the bottom. Its failure mechanism is shown in Fig. 9. (Nadim and Whitman 1984). A wedge-shaped shear zone OAB will form, and at constant volume, AA' will be parallel to OB. OB is not a failure surface, but merely the boundary of the shear zone. Shear strain will be uniformly distributed throughout the wedge. It will not be concentrated, and for this reason, the shear

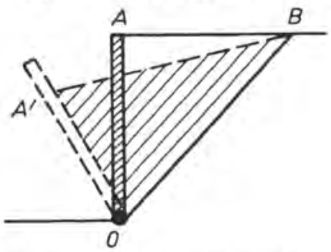
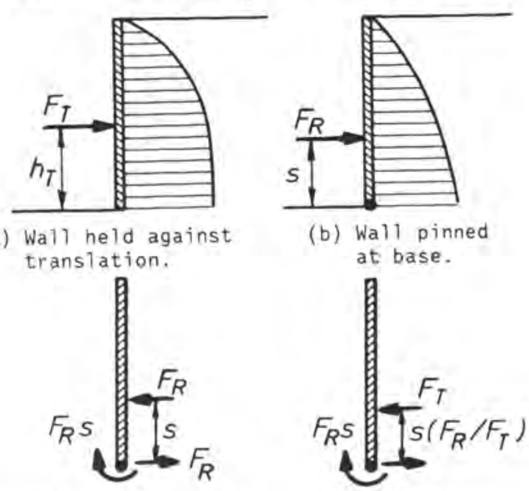


Fig. 9: Rotational failure mechanism

strength will be represented by ϕ_P throughout, and not ϕ_R. The force exerted by the soil will therefore be lower than that for translation with a fully-developed failure surface. Let us call the horizontal resultant force F_R.

Next, imagine that the wall is idealized to a narrow rigid wall held in place against translational movement by a horizontal force F_T as shown in Fig. 10a, applied at height h_T above the base, at the



(c) Free body diagram of (b)

(d) Translational restraining force must move down for rotation.

Fig. 10: Rigid wall, free to translate and rotate.

center of pressure. The wall may be supposed to be free to translate and rotate, so that F_T is applied at the correct point to allow the wall to translate with no rotation. Imagine, now, that the point of application of the restraining force is moved downwards. A certain amount of outwards rotation would begin to take place in addition to translation, until with the force at some height s above the base, the point of rotation of the wall would be at its base. The situation would be the same as if the wall were pinned at its base with a moment restraint $F_R s$, as shown in the free-body diagram of Fig. 10c. But if sufficient translational movement has been allowed to enable a slip surface to develop, the resultant horizontal soil force will have a magnitude of F_T. Assuming the moment restraint is still $F_R s$, which it must be to allow the wall to rotate, the moment arm must reduce to $s(F_R/F_T)$. (Fig. 10d). The center of pressure for rotational movement must therefore be lower than that for translation; or else (to relate to the previous section) the center of pressure for translation must be above that for rotation about the bottom by a factor of F_T/F_R.

It must be emphasized that this is a different effect from that explored in the previous section, even though the general tendency of the center of pressure to move to counteract rotation is the same. And whereas the Dubrova effect diminishes for high values of horizontal acceleration, the present effect -- let us call it the "peak strength" effect -- does not.

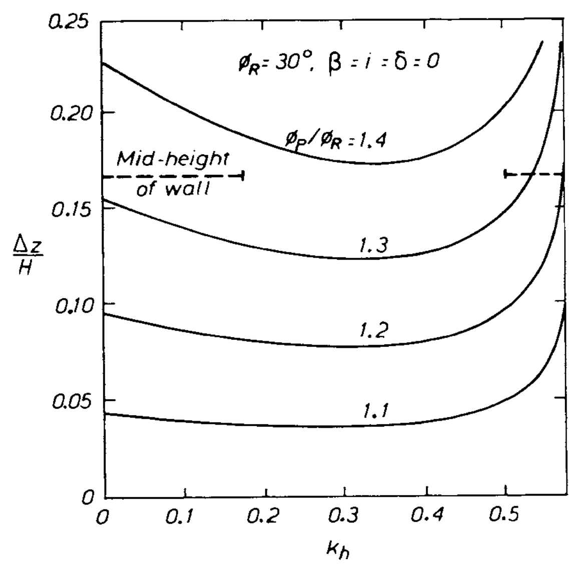


Fig. 11: Increase in height of center of pressure above hydrostatic third point for a sliding wall peak strength effect

Figure 11 shows the increase in height of the center of pressure for a translating wall relative to that for a wall rotating about the base, where the center of pressure is assumed to be at the third point of the wall as in the previous section. The figure is complementary to Fig. 5, except that instead of varying ϕ, the curves are plotted for different values of the peak strength ratio ϕ_p/ϕ_R. The effect of wall friction is negligible (of the order of 1%), so a smooth wall has been assumed. Except for high values of k_h (and the limiting, general fluidization value of k_h for ϕ = 30 is 0.577), the curves are flat and the variation with regard to k_h is small. Roughly speaking, an increase of 0.1 in peak strength ratio gives an increase in the height of the center of pressure of about 0.05 H, with a little more if ϕ_R = 35° and a little less where ϕ_R = 25°.

It is important to realize that the results of Fig. 11 are limiting results based on an assumption that the wall has translated sufficiently for a complete slip surface to have formed. At intermediate states described by the shear zone model of Fig. 8 the increase in center of pressure height will be less; and in any case, if no translation at all takes place, the M-O assumptions do not apply (Whitman 1990).

DISCUSSION AND CONCLUSION

So far, we have discussed the Dubvrova and peak strength effects separately. For a real situation they must be combined and we must consider the nature of their interrelationship.

What seems to happen is a combination of the two dependant on the wall movement. If a retaining wall is subjected to seismic shaking, then if it is rigidly fixed and does not move, Wood's elastic solutions give a reasonable estimate of the pressure distributions (Wood 1975). If the wall moves outwards, soil strength is mobilized and the backfill pressure reduces. However, the distribution of pressure as well as its magnitude depends very much on both the extent of the movement and also its nature; that is, on the combination of translation and rotation that characterize its displacement.

For relatively small movements the Dubrova effect gives an indication of the shifts in center of pressure to be expected and Fig. 5 applies. The overall magnitude of force is, however, not affected by the nature of the displacement.

If translational motion is increased, then the shear zone mechanism of Fig. 8 occurs which introduces the peak strength effect for dense backfills. (Unpublished tests at the University of Canterbury have indicated that loosely-placed backfills will densify and slump during initial shaking, which precedes and also enables formation of a shear zone mechanism in later excitation). As displacement increases a failure surface grows and the peak strength effect begins to lead to the shifts in center of pressure shown in Fig.11. At the same time the soil force on the wall increases, reaching the maximum given by the M-O equations with $\phi=\phi_R$ and full development of the failure surface.

The Dubrova and peak strength effects developed in this paper can therefore explain to a large extent the observed shifts in center of pressure during retaining wall movement, both static and seismic. They also give numerical values. At this stage however the results should be used with care both because several of the assumptions used are somewhat imprecise and general and also because there are still uncertainties in the overall puzzle.

Some of the more obvious questions remaining are these. First, there should be a more thorough exploration of the shear zone mechanism, to see experimentally how it develops and changes with time. Secondly, it is not at all clear at what strain levels transition between the different modes of behavior takes place, or what is meant, for instance, by a "sufficiently large movement". Finally, passive behavior, essential for understanding, say, tied-back walls,has not yet been considered.

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