Comparison of Logic-Based Switching Control Designs for a Nonlinear System

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***Abstract*— We consider an example of a second order nonlinear system with large parametric uncertainties. The two parameters of the system are assumed to belong to a finite set. The goal is to guarantee (practical) convergence of the system output to a given constant reference signal. Feedback linearization-based candidate controllers with pole placement are designed for each possible set of parameters. After that, we consider the design of a high-level logic-based supervisor to organize switching between these candidate controllers. Three different approaches are used and compared.**

I. PROBLEM FORMULATION

Consider the following second order system in strict-feedback form, investigated in [2, pp. 76–82] (see also [3]),

$$\dot{x}_1 = p_1^* x_1^3 + p_2^* x_2, \qquad \dot{x}_2 = u, \qquad y = x_1 - r, \qquad (1)$$

where x_1 and x_2 are the state variables, p_1^* and p_2^* are the unknown parameters, u is the control input, r is the constant reference, and y is the output error.

For all $p^* = (p_1^*, p_2^*) \in \mathcal{P} = \{-1, -0.9, \ldots, 0.9, 1\} \times \{-1, 1\} \subset \mathbf{R}^2$, a dynamic feedback control law needs to be designed to ensure that the solutions of the closed-loop system are bounded and for any given tolerance $\varepsilon_0 > 0$ and any given $r \in \mathbf{R} : \limsup_{t\to\infty} |y(t)| \le \varepsilon_0$.

In the next section, for each possible set of parameters we design a candidate controller to ensure acceptable performance of the closed-loop system, provided the real parameters are known. Then, we complete the control design by deriving a high-level system that is responsible for supervision of switching between the candidate controllers.

II. DESIGN OF CANDIDATE CONTROLLERS

When the parameters are known, the control design could be carried out using feedback linearization followed by pole placement. We transform the system into the normal form by employing the new state variables y and $v = p_1^* x_1^3 + p_2^* x_2$, so that $\dot{y} = v$ and $\dot{v} = 3p_1^*(y+r)^2 v + p_2^* u$.

The control law

$$u = -[\omega^2 y + 2\eta\omega v + 3p_1^*(y+r)^2 v]/p_2^*,$$

where $\omega > 0$ and $\eta > 0$ are chosen to ensure acceptable transient performance, leads to the closed-loop system

$$\dot{y} = v, \qquad \dot{v} = -\omega^2 y - 2\eta\omega v. \qquad (2)$$

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When p^* is unknown, we switch between the following 42 candidate controllers

$$u = u^{(j)} = -[\omega^2 y + 2\eta\omega v^{(j)} + 3p_1^{(j)}(y+r)^2 v^{(j)}]/p_2^{(j)}, \quad (3)$$

where $v^{(j)} = p_2^{(j)} x_2 + p_1^{(j)}(y+r)^3$, $j \in \mathcal{J} = \{1, \ldots, 42\}$, $p^{(j)} = (p_1^{(j)}, p_2^{(j)}) \in \mathcal{P}$, and $\mathcal{P} = \bigcup_{j\in\mathcal{J}} \{p^{(j)}\}$.

Next, we design a supervisor to decide when, and to which particular controller, to switch.

III. SWITCHING LOGIC DESIGNS

We define below three different ways to organize the switching. First, we follow the scale-independent hysteresis-based logic design [3] (extending S.A. Morse's supervisory control idea), then the Lyapunov function and HGO-based one [1], and, finally, combine these two procedures.

A. Scale-independent hysteresis switching

First, we design a multi-estimator [2]. The hardest part of this approach is to verify that the so-called "ij-injected system" is strongly detectable with a known gain function. Second, this gain function is used to define "performance indices" $\mu^{(i)}$, $i \in \mathcal{J}$.

Switching is organized as follows. We start with initial value $j = 1$ ($u = u^{(1)}$) at $t = 0$ and for $t > 0$ continuously check the inequality $\mu^{(j)}(t) \le (1+h)\min_{i\in\mathcal{J}}\{\mu^{(i)}(t)\}$, where $h > 0$ is a fixed hysteresis constant. As soon as the inequality fails, we redefine $j = \operatorname{argmin}_{i\in\mathcal{J}}\{\mu^{(i)}(t)\}$ and switch the candidate controller in the loop to $u = u^{(j)}$.

It was shown in [2] that the solutions of the closed-loop hybrid system are well-defined, switching has to stop in finite time (with some value $j = i_0 \in \mathcal{J}$ and it is possible but not necessary that $i_0 = i^*$), all signals are bounded, and $\lim_{t\to\infty} |y(t)| = 0$.

B. Lyapunov-based controller falsification

An alternative approach to switching was recently proposed by the authors [1] for a class of nonlinear systems that includes (1) as a special case. For this particular example there is no need to use continuous sliding mode control as in [1] and (3) can be used instead.

We need to define the Lyapunov function candidate for the "perfectly supervised" system. Assuming $\eta > 0.25$, let

$$V(y, v) = \omega(1+\eta)y^2 + yv + v^2/\omega,$$

so that along the trajectories of (2)

$$\dot{V} = -W = -\omega^2 y^2 - (4\eta - 1)v^2.$$

On the other hand, along the trajectories of (1), (3),

$$\dot{V} + W = \omega^2 y^2 + 4\eta v^2 + y\dot{v} + (2/\omega)v\dot{v} + 2\omega(1+\eta)vy,$$

and so the inequality $\dot{V}+W \leq 0$ must be satisfied, provided the right controller is in the loop (i.e. $j = i^*$). Furthermore, if it is satisfied with $u = u^{(j)}$ for some $j \neq i^*$, then the output vanishes at least as fast as when $u = u^{(i^*)}$.

The inequality cannot be checked directly because neither $v = \dot{y}$ nor $\dot{v}$ is available to measure. So we estimate them using the third-order high-gain observer (HGO)

$$\dot{\hat{z}}_1 = \hat{z}_2 + \frac{3(y-\hat{z}_1)}{\varepsilon}, \quad \dot{\hat{z}}_2 = \hat{z}_3 + \frac{3(y-\hat{z}_1)}{\varepsilon^2}, \quad \dot{\hat{z}}_3 = \frac{y-\hat{z}_1}{\varepsilon^3}, \quad (4)$$

where $\varepsilon > 0$ is a sufficiently small tuning parameter. The estimates provided by this HGO are close to the derivatives of y as soon as peaking is over. Therefore, we start with $j = 1$ $(u = u^{(1)})$ at $t = 0$ and wait for a certain dwell-time $\tau > 0$ (another sufficiently small parameter to be tuned), that must be greater then the peaking time. After that, we continuously check the inequality

$$\omega^2 y^2 + 4\eta \hat{z}_2^2 + y\hat{z}_3 + (2/\omega)\hat{z}_2\hat{z}_3 + 2\omega(1+\eta)\hat{z}_2 y \leq a_0, \quad (5)$$

where $a_0 > 0$ is a small parameter aimed at dealing with possible non-vanishing small observation errors. As soon as the inequality fails, we increase the value of j by 1 and switch to the next candidate controller in the loop $u = u^{(j)}$. The procedure is repeated thereafter.

Following [1], it could be shown that there exists $\bar{\tau}$ such that for any $\tau \in (0, \bar{\tau})$ there exists $\bar{\varepsilon}$ such that for $\varepsilon \in (0, \bar{\varepsilon})$ there are no more then (i^*-1) switchings and all the trajectories of the closed-loop system are bounded, enter in finite time an invariant set where $|y(t)| \leq \varepsilon_0$ and stay thereafter.

C. *Combined approach*

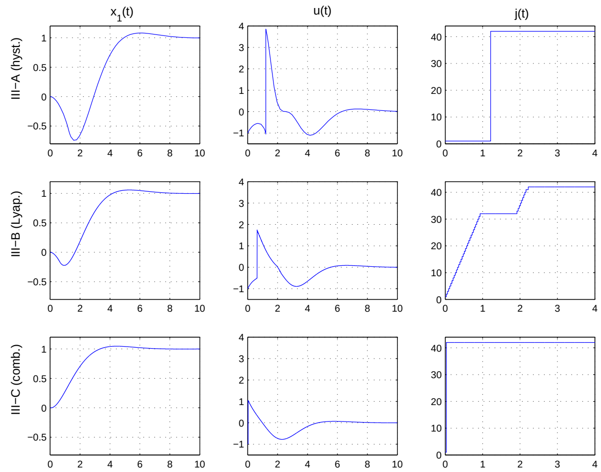
Clearly, it is not hard to invent a combination of the two logics described above. We will explain why this is a good idea and show that it does lead to a superior performance. However, we would like to remark that other combinations are possible as well.

We start with $j = 1$ as above and wait for the dwell-time $\tau > 0$. After that, we continuously check the inequality (5). Once it fails, we remove the index j from $\mathcal{J}$ and redefine $j = \operatorname{argmin}_{i \in \mathcal{J}}\{\mu^{(i)}(t)\}$ as in section III-A. New controller $u = u^{(j)}$ is put in the loop and used for the dwell-time period and for as long as the inequality is satisfied.

IV. SIMULATION RESULTS

The results for $r = 1.0$, $\omega = 1.0$ and $\eta = 0.7$ are shown in the figure. For each row, the switching logic described in section III-A ($h = 0.01$ and $\lambda = 0.5$), in section III-B ($\tau = 0.03$, $\varepsilon = 0.001$, and $a_0 = 0.01$), and in section III-C is used, correspondingly. We show the system's regulated state $x_1(t)$ (column 1), the generated control input $u(t)$ (column 2), and the index, $j(t)$, of the controller put in the loop (column 3).

The figure represents the worst possible case for the Lyapunov-based logic. Here, the correct controller is the last



one ($i^* = 42$) and this leads to 41 switchings. The scale-independent hysteresis switching logic, on the contrary, produces only one switching. Notice, however, that staying too long with $j = 1$ results in the worst transient performance and a control effort that is twice as large. The performance obtained combining the two approaches is remarkable in this case. There is only one switching as well and it is to the correct controller.

Other cases are not shown here due to space limitation. However, we would like to summarize what we have observed. The Lyapunov-based logic results in many switchings but produces better transient performance and smaller control effort than in the case of the one switching with the hysteresis-based logic. The best performance with just a few switchings is obtained by combining the two.

V. CONCLUSION

We have considered the design of an output feedback control law for a second order parametrically uncertain nonlinear system. It was possible to design a high-gain observer and Lyapunov-based switching regulator, following the idea recently proposed in [1], as well as a multi-estimator and scale independent hysteresis logic-based regulator [3]. The former has the following advantages: lower dynamic order; the ability to determine quickly whether the wrong regulator is currently in the loop so that switching is in order; and calculation of the gain function is not needed. The latter has the ability to determine which regulator is most likely the right one, independently of which one is currently in the loop. Therefore, a combination of these two approaches seems to hold a promise of superior performance. Simulation results confirm this intuitive idea and suggest the need to determine the intersection of the classes of nonlinear systems studied in [3] and [1].

REFERENCES

[1] L.B. Freidovich & H.K. Khalil, *Proc. of CDC*, 2003, pp. 1059–1064.
[2] J.P. Hespanha, *Ph.D. dissertation*, Yale Univ., 1998.
[3] J.P. Hespanha, D. Liberzon, & A.S. Morse, *Systems and Control Letters*, Vol. 49, pp. 49–65, 2003.