# COMPARISON OF MAMDANI AND SUGENO FUZZY INFERENCE SYSTEM MODELS FOR RESONANT FREQUENCY CALCULATION OF RECTANGULAR MICROSTRIP ANTENNAS

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**Abstract**—Models based on fuzzy inference systems (FISs) for calculating the resonant frequency of rectangular microstrip antennas (MSAs) with thin and thick substrates are presented. Two types of FIS models, Mamdani FIS model and Sugeno FIS model, are used to compute the resonant frequency. The parameters of FIS models are determined by using various optimization algorithms. The resonant frequency results predicted by FIS models are in very good agreement with the experimental results available in the literature. When the performances of FIS models are compared with each other, the best result is obtained from the Sugeno FIS model trained by the least-squares algorithm.

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## 1. INTRODUCTION

MSAs have many desirable features such as low profile, light weight, conformal structure, low production cost, and ease of integration with microwave integrated circuit or monolithic microwave integrated circuit components [1–5]. MSAs are therefore used extensively in a broad range of commercial and military applications.

In MSA designs, it is important to determine the resonant frequencies of the antenna accurately because MSAs have narrow bandwidths and can only operate effectively in the vicinity of the resonant frequency. So, a technique to compute accurately the resonant frequency is helpful in antenna designs. Several techniques [1–39] are available in the literature to calculate the resonant frequency of rectangular MSA, as this is one of the most popular and convenient shapes. These techniques can be broadly classified into two categories: analytical and numerical techniques. The analytical techniques offer both simplicity and physical insight, but depend on several assumptions and approximations that are valid only for thin substrates. The numerical techniques provide accurate results but usually require considerable computational time and costs.

The neural models trained by various algorithms were used in calculating the resonant frequency of rectangular MSAs [26, 28, 29, 31]. A neuro-fuzzy network was presented in [30] to compute the resonant frequencies of MSAs. In [30], the number of rules and the premise parameters of Sugeno FIS were determined by the fuzzy subtractive clustering method and then the consequent parameters of each output rule were determined by using linear least squares estimation method. The training data sets were obtained by numerical simulations using a moment-method code based on electric field integral equation approach. To validate the performances of the neuro-fuzzy network, a set of further moment-method simulations was realized and presented to the neuro-fuzzy network.

The concurrent neuro-fuzzy system models were proposed in [34, 35] to calculate simultaneously the resonant frequencies of the rectangular, circular, and triangular MSAs. The concurrent neuro-fuzzy system comprises an artificial neural network and a fuzzy system. In a concurrent neuro-fuzzy system, neural network assists the fuzzy system continuously (or vice versa) to compute the resonant frequency.

In this paper, the models based on Mamdani FIS [40, 41] and Sugeno FIS [41, 42] are presented for computing the resonant frequency of rectangular MSAs with thin and thick substrates. The FIS is a popular computing framework based on the concepts of fuzzy set theory, fuzzy if-then rules, and fuzzy reasoning [40–43]. FISs are

nonlinear systems capable of inferring complex nonlinear relationships between input and output data. The high-speed real-time computation feature of the FIS recommends its use in antenna computer aided design (CAD) programs.

In previous works [27, 33–35, 44–50], we successfully used Sugeno FISs for computing accurately the various parameters of the rectangular, triangular, and circular MSAs. In [27, 33], the tabu search algorithms and the hybrid learning algorithm were used to train the Sugeno FISs. However, in this paper, six different optimization algorithms, least-squares (LSQ) algorithm [51–53], nelder-mead (NM) algorithm [54, 55], genetic algorithm (GA) [56, 57], differential evolution algorithm (DEA) [58–60], particle swarm optimization (PSO) [61, 62], and simulated annealing (SA) algorithm [63–65], are used to train the Sugeno FISs. Furthermore, in this paper, Mamdani FIS models trained by LSQ, NM, GA, DEA, PSO, and SA are proposed to compute the resonant frequency of rectangular MSAs.

## 2. FIS MODELS FOR RESONANT FREQUENCY COMPUTATION

Figure 1 shows a rectangular patch of width $W$ and length $L$ over a ground plane with a substrate of thickness $h$ and a relative dielectric constant $\varepsilon_r$. A survey of the literature [1–39] clearly shows that only four parameters, $W$, $L$, $h$, and $\varepsilon_r$, are needed to describe the resonant frequency. In this paper, the resonant frequency of the rectangular MSA is computed by using FIS models. The FIS is a very powerful approach for building complex and nonlinear relationship between a set of input and output data [40–43].
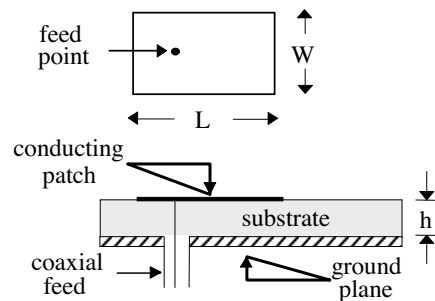


**Figure 1.** Geometry of rectangular MSA.

Three types of FISs, Mamdani FIS, Sugeno FIS, and Tsukamoto FIS, have been widely used in various applications [41]. The

**Table 1.** Measured resonant frequency results and dimensions for electrically thin and thick rectangular MSAs.

| Patch No | $W$ (cm) | $L$ (cm) | $h$ (cm) | $\varepsilon_r$ | $(h/\lambda_d) \times 100$ | Measured ($f_{me}$) (MHz) |
|---|---|---|---|---|---|---|
| 1 | 5.700 | 3.800 | 0.3175 | 2.33 | 3.7317 | 2310[+] |
| 2 | 4.550 | 3.050 | 0.3175 | 2.33 | 4.6687 | 2890[+] |
| 3 | 2.950 | 1.950 | 0.3175 | 2.33 | 6.8496 | 4240[+] |
| 4 | 1.950 | 1.300 | 0.3175 | 2.33 | 9.4344 | 5840[+] |
| 5* | 1.700 | 1.100 | 0.3175 | 2.33 | 10.9852 | 6800[+] |
| 6 | 1.400 | 0.900 | 0.3175 | 2.33 | 12.4392 | 7700[+] |
| 7 | 1.200 | 0.800 | 0.3175 | 2.33 | 13.3600 | 8270[+] |
| 8 | 1.050 | 0.700 | 0.3175 | 2.33 | 14.7655 | 9140[+] |
| 9 | 1.700 | 1.100 | 0.9525 | 2.33 | 22.9236 | 4730[+] |
| 10 | 1.700 | 1.100 | 0.1524 | 2.33 | 6.1026 | 7870[+] |
| 11 | 4.100 | 4.140 | 0.1524 | 2.50 | 1.7896 | 2228[Δ] |
| 12* | 6.858 | 4.140 | 0.1524 | 2.50 | 1.7671 | 2200[Δ] |
| 13 | 10.800 | 4.140 | 0.1524 | 2.50 | 1.7518 | 2181[Δ] |
| 14 | 0.850 | 1.290 | 0.0170 | 2.22 | 0.6535 | 7740 |
| 15* | 0.790 | 1.185 | 0.0170 | 2.22 | 0.7134 | 8450 |
| 16 | 2.000 | 2.500 | 0.0790 | 2.22 | 1.5577 | 3970 |
| 17 | 1.063 | 1.183 | 0.0790 | 2.55 | 3.2505 | 7730 |
| 18 | 0.910 | 1.000 | 0.1270 | 10.20 | 6.2193 | 4600 |
| 19 | 1.720 | 1.860 | 0.1570 | 2.33 | 4.0421 | 5060 |
| 20* | 1.810 | 1.960 | 0.1570 | 2.33 | 3.8384 | 4805 |
| 21 | 1.270 | 1.350 | 0.1630 | 2.55 | 5.6917 | 6560 |
| 22 | 1.500 | 1.621 | 0.1630 | 2.55 | 4.8587 | 5600 |
| 23* | 1.337 | 1.412 | 0.2000 | 2.55 | 6.6004 | 6200 |
| 24 | 1.120 | 1.200 | 0.2420 | 2.55 | 9.0814 | 7050 |
| 25 | 1.403 | 1.485 | 0.2520 | 2.55 | 7.7800 | 5800 |
| 26 | 1.530 | 1.630 | 0.3000 | 2.50 | 8.3326 | 5270 |
| 27 | 0.905 | 1.018 | 0.3000 | 2.50 | 12.6333 | 7990 |
| 28 | 1.170 | 1.280 | 0.3000 | 2.50 | 10.3881 | 6570 |
| 29* | 1.375 | 1.580 | 0.4760 | 2.55 | 12.9219 | 5100 |
| 30 | 0.776 | 1.080 | 0.3300 | 2.55 | 14.0525 | 8000 |
| 31 | 0.790 | 1.255 | 0.4000 | 2.55 | 15.1895 | 7134 |
| 32 | 0.987 | 1.450 | 0.4500 | 2.55 | 14.5395 | 6070 |
| 33* | 1.000 | 1.520 | 0.4760 | 2.55 | 14.7462 | 5820 |
| 34 | 0.814 | 1.440 | 0.4760 | 2.55 | 16.1650 | 6380 |
| 35 | 0.790 | 1.620 | 0.5500 | 2.55 | 17.5363 | 5990 |
| 36 | 1.200 | 1.970 | 0.6260 | 2.55 | 15.5278 | 4660 |
| 37 | 0.783 | 2.300 | 0.8540 | 2.55 | 20.9105 | 4600 |
| 38* | 1.256 | 2.756 | 0.9520 | 2.55 | 18.1413 | 3580 |
| 39 | 0.974 | 2.620 | 0.9520 | 2.55 | 20.1683 | 3980 |
| 40 | 1.020 | 2.640 | 0.9520 | 2.55 | 19.7629 | 3900 |
| 41 | 0.883 | 2.676 | 1.0000 | 2.55 | 21.1852 | 3980 |
| 42 | 0.777 | 2.835 | 1.1000 | 2.55 | 22.8353 | 3900 |
| 43 | 0.920 | 3.130 | 1.2000 | 2.55 | 22.1646 | 3470 |
| 44* | 1.030 | 3.380 | 1.2810 | 2.55 | 21.8197 | 3200 |
| 45 | 1.265 | 3.500 | 1.2810 | 2.55 | 20.3196 | 2980 |
| 46 | 1.080 | 3.400 | 1.2810 | 2.55 | 21.4787 | 3150 |

[+] These frequencies measured by Chang et al. [14]. [Δ] These frequencies measured by Carver [8]. The remainder measured by Kara [22, 23]. * Test data. $\lambda_d$ is the wavelength in the dielectric substrate.

differences between these three FISs lie in the consequents of their fuzzy rules, and thus their aggregation and defuzzification procedures differ accordingly. In this paper, Mamdani and Sugeno FIS models are used to calculate accurately the resonant frequency of rectangular MSAs. The data sets used in training and testing Mamdani and Sugeno FIS models have been obtained from the previous experimental works [8, 14, 22, 23], and are given in Table 1. The 37 data sets in Table 1 are used to train the FISs. The 9 data sets, marked with an asterisk in Table 1, are used for testing. For the FIS models, the inputs are $W$, $L$, $h$, and $\varepsilon_r$ and the output is the measured resonant frequency $f_{me}$.

In this paper, the grid partitioning method [41] is used for the fuzzy rule extraction. In the grid partitioning method, the domain of each antecedent variable is partitioned into equidistant and identically shaped membership functions (MFs). A major advantage of the grid partitioning method is that the fuzzy rules obtained from the fixed linguistic fuzzy grids are always linguistically interpretable. Using the available input-output data, the parameters of the MFs can be optimized.

In this paper, the number of MFs for the input variables $W$, $L$, $h$, and $\varepsilon_r$ are determined as 3, 3, 2, and 3, respectively. Each possible combination of inputs and their associated MFs is represented by a rule in the rule base of the Mamdani and Sugeno FIS models. So, the number of rules for FIS models is 54 ($3 \times 3 \times 2 \times 3 = 54$).

The application of the Mamdani and Sugeno FIS models to the resonant frequency calculation is given in the following sections.

## 2.1. Mamdani FIS Models for Resonant Frequency Computation

Mamdani FIS [40, 41] was proposed as the very first attempt to control a steam engine and boiler combination by synthesizing a set of linguistic control rules obtained from experienced human operators. The Mamdani FIS architecture used in this paper for the resonant frequency computation of rectangular MSAs is illustrated in Figure 2, in which a circle indicates a fixed node, whereas a rectangular indicates
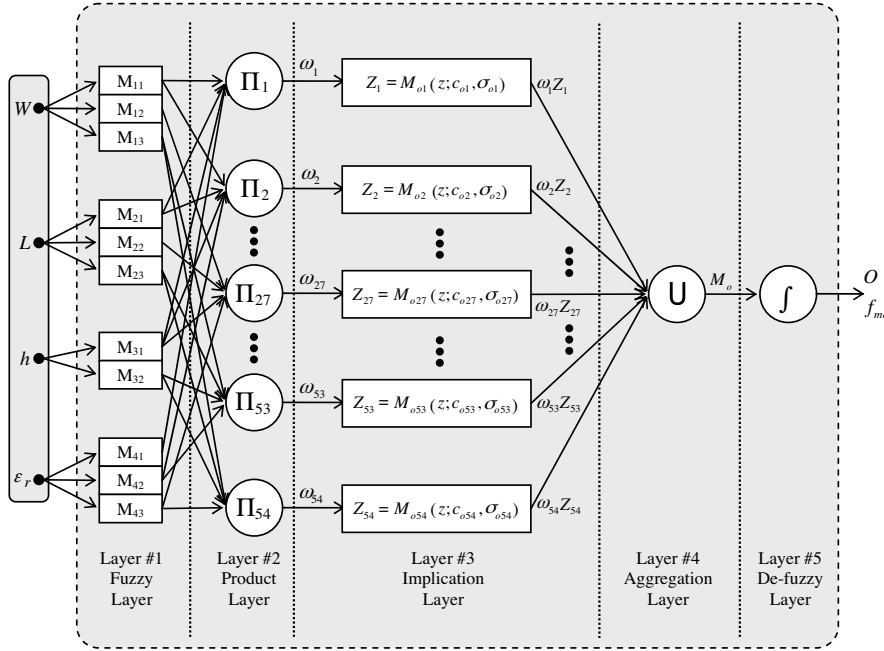
**Figure 2.** Architecture of Mamdani FIS for resonant frequency computation of rectangular MSA.

an adaptive node. The rule base for Mamdani FIS can be written as

1. if ($W$ is $M_{11}$) and ($L$ is $M_{21}$) and ($h$ is $M_{31}$) and ($\varepsilon_r$ is $M_{41}$)
   then $Z_1 = M_{o1}(z; c_{o1}, \sigma_{o1})$

2. if ($W$ is $M_{11}$) and ($L$ is $M_{21}$) and ($h$ is $M_{31}$) and ($\varepsilon_r$ is $M_{42}$)
   then $Z_2 = M_{o2}(z; c_{o2}, \sigma_{o2})$

3. if ($W$ is $M_{11}$) and ($L$ is $M_{21}$) and ($h$ is $M_{31}$) and ($\varepsilon_r$ is $M_{43}$)
   then $Z_3 = M_{o3}(z; c_{o3}, \sigma_{o3})$

4. if ($W$ is $M_{11}$) and ($L$ is $M_{21}$) and ($h$ is $M_{32}$) and ($\varepsilon_r$ is $M_{41}$) (1)
   then $Z_4 = M_{o4}(z; c_{o4}, \sigma_{o4})$

   $\vdots$                                    $\vdots$

53. if ($W$ is $M_{13}$) and ($L$ is $M_{23}$) and ($h$ is $M_{32}$) and ($\varepsilon_r$ is $M_{42}$)
    then $Z_{53} = M_{o53}(z; c_{o53}, \sigma_{o53})$

54. if ($W$ is $M_{13}$) and ($L$ is $M_{23}$) and ($h$ is $M_{32}$) and ($\varepsilon_r$ is $M_{43}$)
    then $Z_{54} = M_{o54}(z; c_{o54}, \sigma_{o54})$

with

$$Z_k = M_{ok}\left(z; c_{ok}, \sigma_{ok}\right) \qquad k = 1, \dots, 54 \qquad (2)$$

where $M_{ij}$, $Z_k$, and $M_{ok}$ represent the $j$th MF of the $i$th input, the output of the $k$th rule, and the $k$th output MF, respectively. In Eq. (1), $c_{ok}$ and $\sigma_{ok}$ are the consequent parameters that characterize the shapes of the output MFs.

As shown in Figure 2, the Mamdani FIS architecture consists of five layers: fuzzy layer, product layer, implication layer, aggregation layer, and de-fuzzy layer. Layered operating mechanism of the Mamdani FIS can be described as follows:

**Layer 1:** In this layer, the crisp input values are converted to the fuzzy values by the input MFs. In this paper, the following generalized bell, trapezoidal, and gaussian MFs for the inputs are used:

i) Generalized bell MFs for $(i = 1$ or $i = 4)$, $(j = 1, 2, 3)$, $(x = W$ or $x = \varepsilon_r)$:

$$M_{ij}\left(x\right) = Gbell\left(x; a_{ij}, b_{ij}, c_{ij}\right) = \frac{1}{1 + \left|\frac{x - c_{ij}}{a_{ij}}\right|^{2b_{ij}}} \qquad (3a)$$

ii) Trapezoidal MFs for $i = (2)$, $j = (1, 2, 3)$, $(x = L)$:

$$M_{ij}\left(x\right) = Trap\left(x; a_{ij}, b_{ij}, c_{ij}, d_{ij}\right) = \begin{cases} 0, & x \leq a_{ij} \\ \dfrac{x - a_{ij}}{b_{ij} - a_{ij}}, & a_{ij} \leq x \leq b_{ij} \\ 1, & b_{ij} \leq x \leq c_{ij} \\ \dfrac{d_{ij} - x}{d_{ij} - c_{ij}}, & c_{ij} \leq x \leq d_{ij} \\ 0, & d_{ij} \leq x \end{cases} \qquad (3b)$$

iii) Gaussian MFs for $(i = 3)$, $(j = 1, 2)$, $(x = h)$:

$$M_{ij}\left(x\right) = Gauss\left(x; c_{ij}, \sigma_{ij}\right) = e^{-\frac{1}{2}\left(\frac{x - c_{ij}}{\sigma_{ij}}\right)^2} \qquad (3c)$$

where $a_{ij}$, $b_{ij}$, $c_{ij}$, $d_{ij}$, and $\sigma_{ij}$ are the premise parameters that characterize the shapes of the input MFs.

**Layer 2:** In this layer, the weighting factor (firing strength) of each rule is computed. The weighting factor of each rule, which is expressed as $\omega_k$, is determined by evaluating the membership expressions in the antecedent of the rule. This is accomplished by first converting the input values to fuzzy membership values by using

the input MFs in the layer 1 and then applying the "*and*" operator to these membership values. The "*and*" operator corresponds to the multiplication of input membership values. Hence, the weighting factors of the rules are computed as follows:

$$\omega_1 = M_{11}(W) M_{21}(L) M_{31}(h) M_{41}(\varepsilon_r)$$
$$\omega_2 = M_{11}(W) M_{21}(L) M_{31}(h) M_{42}(\varepsilon_r)$$
$$\omega_3 = M_{11}(W) M_{21}(L) M_{31}(h) M_{43}(\varepsilon_r)$$
$$\omega_4 = M_{11}(W) M_{21}(L) M_{32}(h) M_{41}(\varepsilon_r)$$

$$\vdots \qquad\qquad \vdots$$

$$\omega_{53} = M_{13}(W) M_{23}(L) M_{32}(h) M_{42}(\varepsilon_r)$$
$$\omega_{54} = M_{13}(W) M_{23}(L) M_{32}(h) M_{43}(\varepsilon_r)$$

(4)

**Layer 3:** In this layer, the implication of each output MF is computed by

$$M_{imp,k} = \omega_k Z_k \qquad k = 1, \ldots, 54 \tag{5}$$

where $M_{imp,k}$ represents the implicated output MFs.

**Layer 4:** In this layer, the aggregation is performed to produce an overall output MF, $M_o(z)$, by using the union operator:

$$M_o(z) = \bigcup_{k=1}^{54} M_{imp,k} = \bigcup_{k=1}^{54} \omega_k Z_k$$
$$= \bigcup_{k=1}^{54} \omega_k M_{ok}(z; c_{ok}, \sigma_{ok}) = \bigcup_{k=1}^{54} \omega_k e^{-\frac{1}{2}\left(\frac{z-c_{ok}}{\sigma_{ok}}\right)^2} \tag{6}$$

The types of the output MFs ($M_{ok}$) are Gaussian. Here, the "*union*" operation is performed using the maximum operator.

**Layer 5:** In this layer, the defuzzification is performed by using the centroid of area method:

$$O = \frac{\int_Z M_o(z) \, z \, dz}{\int_Z M_o(z) \, dz} \tag{7}$$

It is clear from Eq. (3) that the generalized bell, trapezoidal, and gaussian MFs are specified by three, four, and two parameters, respectively. Therefore, the Mamdani FIS used in this paper contains a total of 142 fitting parameters, of which 34 ($3\times3+3\times4+2\times2+3\times3 = 34$) are the premise parameters and 108 ($2 \times 54 = 108$) are the consequent parameters.
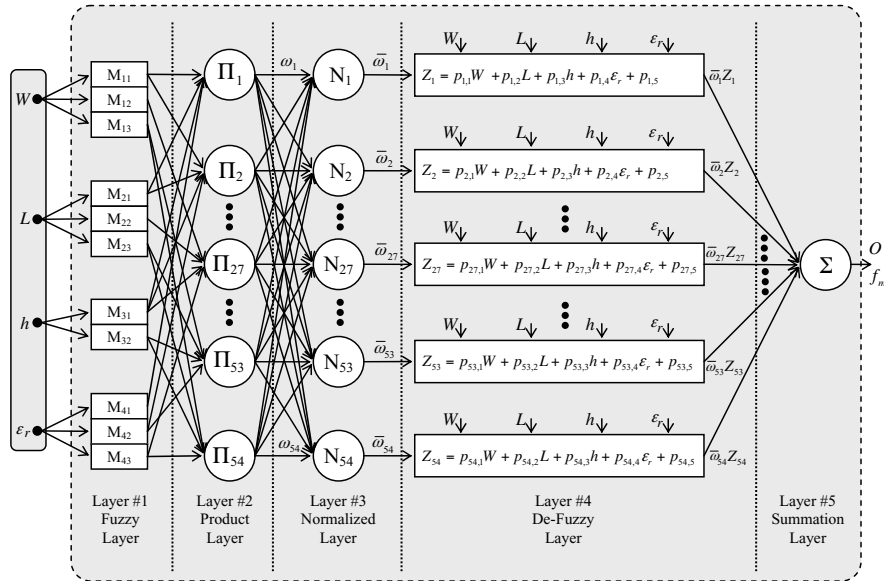
**Figure 3.** Architecture of Sugeno FIS for resonant frequency computation of rectangular MSA.

## 2.2. Sugeno FIS Models for Resonant Frequency Computation

Sugeno FIS [41, 42] was proposed to develop a systematic approach to generate fuzzy rules from a given input-output data. The Sugeno FIS architecture used in this paper for the resonant frequency calculation is shown in Figure 3. The rule base for Sugeno FIS is given by

1.  if ($W$ is $M_{11}$) and ($L$ is $M_{21}$) and ($h$ is $M_{31}$) and ($\varepsilon_r$ is $M_{41}$) then $Z_1 = r_1\,(W, L, h, \varepsilon_r)$

2.  if ($W$ is $M_{11}$) and ($L$ is $M_{21}$) and ($h$ is $M_{31}$) and ($\varepsilon_r$ is $M_{42}$) then $Z_2 = r_2\,(W, L, h, \varepsilon_r)$

3.  if ($W$ is $M_{11}$) and ($L$ is $M_{21}$) and ($h$ is $M_{31}$) and ($\varepsilon_r$ is $M_{43}$) then $Z_3 = r_3\,(W, L, h, \varepsilon_r)$

4.  if ($W$ is $M_{11}$) and ($L$ is $M_{21}$) and ($h$ is $M_{32}$) and ($\varepsilon_r$ is $M_{41}$)  (8) then $Z_4 = r_4\,(W, L, h, \varepsilon_r)$

    $\vdots$                              $\vdots$

53.    if ($W$ is $M_{13}$) and ($L$ is $M_{23}$) and ($h$ is $M_{32}$) and ($\varepsilon_r$ is $M_{42}$)
        then $Z_{53} = r_{53}(W, L, h, \varepsilon_r)$

54.    if ($W$ is $M_{13}$) and ($L$ is $M_{23}$) and ($h$ is $M_{32}$) and ($\varepsilon_r$ is $M_{43}$)
        then $Z_{54} = r_{54}(W, L, h, \varepsilon_r)$

with

$$Z_k = r_k(W, L, h, \varepsilon_r) \qquad k = 1, \ldots, 54 \tag{9}$$

where $M_{ij}$, $Z_k$, and $r_k$ represent the $j$th MF of the $i$th input, the output of the $k$th rule, and the $k$th output MF, respectively.

As shown in Figure 3, the Sugeno FIS structure consists of five layers: fuzzy layer, product layer, normalized layer, de-fuzzy layer, and summation layer. It is clear that **Layer 1** and **Layer 2** of Sugeno FIS are the same as those of Mamdani FIS. The operating mechanism of the other layers for Sugeno FIS can be described as follows:

**Layer 3:** The normalized weighting factor of each rule, $\bar{\omega}_k$, is computed by using

$$\bar{\omega}_k = \frac{\omega_k}{\sum\limits_{i=1}^{54} \omega_i} \qquad k = 1, \ldots, 54 \tag{10}$$

**Layer 4:** In this layer, the output rules can be written as:

$$\begin{aligned}
\bar{\omega}_k Z_k &= \bar{\omega}_k r_k(W, L, h, \varepsilon_r) \\
&= \bar{\omega}_k (p_{k1}W + p_{k2}L + p_{k3}h + p_{k4}\varepsilon_r + p_{k5}) \quad k = 1, \ldots, 54
\end{aligned} \tag{11}$$

where $p_k$ are the consequent parameters that characterize the shapes of the output MFs. Here, the types of the output MFs ($r_k$) are linear.

**Layer 5:** Each rule is weighted by own normalized weighting factor and the output of the FIS is calculated by summing of all rule outputs:

$$O = \sum_{k=1}^{54} \bar{\omega}_k Z_k = \frac{\sum\limits_{k=1}^{54} \omega_k Z_k}{\sum\limits_{k=1}^{54} \omega_k} \tag{12}$$

The Sugeno FIS used in this paper contains a total of 304 fitting parameters, of which 34 ($3 \times 3 + 3 \times 4 + 2 \times 2 + 3 \times 3 = 34$) are the premise parameters and 270 ($5 \times 54 = 270$) are the consequent parameters.

**Table 2.** Comparison of measured and calculated resonant frequencies obtained by using Mamdani FIS models presented in this paper for electrically thin and thick rectangular MSAs.

| Patch No | Measured $(f_{me})$ (MHz) | Present Mamdani FIS Models | | | | | |
|---|---|---|---|---|---|---|---|
| | | LSQ | NM | GA | DEA | PSO | SA |
| 1 | 2310 | 2309.9 | 2309.9 | 2309.8 | 2310.0 | 2309.8 | 2309.5 |
| 2 | 2890 | 2889.8 | 2889.8 | 2889.7 | 2889.7 | 2889.5 | 2889.1 |
| 3 | 4240 | 4239.7 | 4239.6 | 4239.5 | 4239.4 | 4239.2 | 4238.4 |
| 4 | 5840 | 5839.5 | 5839.4 | 5839.3 | 5839.1 | 5838.9 | 5837.8 |
| 5 | 6800 | 6799.4 | 6799.3 | 6799.2 | 6798.9 | 6798.7 | 6797.4 |
| 6 | 7700 | 7699.4 | 7699.2 | 7699.1 | 7698.8 | 7698.5 | 7697.1 |
| 7 | 8270 | 8269.3 | 8269.2 | 8269.0 | 8268.7 | 8268.4 | 8266.9 |
| 8 | 9140 | 9139.2 | 9139.1 | 9138.9 | 9138.5 | 9138.3 | 9136.6 |
| 9 | 4730 | 4729.6 | 4729.5 | 4729.4 | 4729.2 | 4729.1 | 4728.2 |
| 10 | 7870 | 7869.3 | 7869.2 | 7869.0 | 7868.7 | 7868.5 | 7867.0 |
| 11 | 2228 | 2227.9 | 2227.8 | 2227.8 | 2227.9 | 2227.7 | 2227.4 |
| 12 | 2200 | 2200.0 | 2199.9 | 2199.9 | 2200.2 | 2199.9 | 2199.8 |
| 13 | 2181 | 2181.2 | 2181.2 | 2181.3 | 2182.1 | 2181.4 | 2181.8 |
| 14 | 7740 | 7739.4 | 7739.2 | 7739.1 | 7738.7 | 7738.5 | 7737.1 |
| 15 | 8450 | 8449.3 | 8449.1 | 8449.0 | 8448.6 | 8448.4 | 8446.8 |
| 16 | 3970 | 3969.7 | 3969.6 | 3969.5 | 3969.4 | 3969.3 | 3968.6 |
| 17 | 7730 | 7729.4 | 7729.2 | 7729.1 | 7728.8 | 7728.5 | 7727.1 |
| 18 | 4600 | 4599.8 | 4599.8 | 4599.7 | 4600.0 | 4599.5 | 4599.1 |
| 19 | 5060 | 5059.6 | 5059.5 | 5059.4 | 5059.2 | 5059.0 | 5058.1 |
| 20 | 4805 | 4804.6 | 4804.5 | 4804.4 | 4804.2 | 4804.1 | 4803.2 |
| 21 | 6560 | 6559.5 | 6559.3 | 6559.2 | 6558.9 | 6558.8 | 6557.5 |
| 22 | 5600 | 5599.5 | 5599.4 | 5599.3 | 5599.1 | 5598.9 | 5597.9 |
| 23 | 6200 | 6199.5 | 6199.4 | 6199.3 | 6199.0 | 6198.8 | 6197.7 |
| 24 | 7050 | 7049.4 | 7049.3 | 7049.1 | 7048.9 | 7048.7 | 7047.4 |
| 25 | 5800 | 5799.5 | 5799.4 | 5799.3 | 5799.1 | 5798.9 | 5797.8 |
| 26 | 5270 | 5269.6 | 5269.5 | 5269.4 | 5269.2 | 5269.0 | 5268.0 |
| 27 | 7990 | 7989.3 | 7989.2 | 7989.0 | 7988.7 | 7988.5 | 7987.0 |
| 28 | 6570 | 6569.5 | 6569.3 | 6569.2 | 6568.9 | 6568.8 | 6567.5 |
| 29 | 5100 | 5099.6 | 5099.5 | 5099.4 | 5099.2 | 5099.0 | 5098.1 |
| 30 | 8000 | 7999.3 | 7999.2 | 7999.0 | 7998.7 | 7998.5 | 7997.0 |
| 31 | 7134 | 7133.4 | 7133.3 | 7133.1 | 7132.9 | 7132.6 | 7131.3 |
| 32 | 6070 | 6069.5 | 6069.4 | 6069.3 | 6069.0 | 6068.9 | 6067.7 |
| 33 | 5820 | 5819.5 | 5819.4 | 5819.3 | 5819.1 | 5818.9 | 5817.8 |
| 34 | 6380 | 6379.5 | 6379.4 | 6379.2 | 6379.0 | 6378.8 | 6377.6 |
| 35 | 5990 | 5989.5 | 5989.4 | 5989.3 | 5989.0 | 5988.9 | 5987.8 |
| 36 | 4660 | 4659.6 | 4659.5 | 4659.4 | 4659.3 | 4659.1 | 4658.3 |
| 37 | 4600 | 4599.6 | 4599.5 | 4599.5 | 4599.3 | 4599.1 | 4598.3 |
| 38 | 3580 | 3579.7 | 3579.6 | 3579.6 | 3579.5 | 3579.3 | 3578.7 |
| 39 | 3980 | 3979.7 | 3979.6 | 3979.5 | 3979.4 | 3979.3 | 3978.5 |
| 40 | 3900 | 3899.7 | 3899.6 | 3899.5 | 3899.4 | 3899.3 | 3898.6 |
| 41 | 3980 | 3979.7 | 3979.6 | 3979.5 | 3979.4 | 3979.3 | 3978.5 |
| 42 | 3900 | 3899.7 | 3899.6 | 3899.5 | 3899.4 | 3899.3 | 3898.6 |
| 43 | 3470 | 3469.7 | 3469.7 | 3469.6 | 3469.5 | 3469.4 | 3468.7 |
| 44 | 3200 | 3199.7 | 3199.7 | 3199.6 | 3199.5 | 3199.4 | 3198.9 |
| 45 | 2980 | 2979.8 | 2979.7 | 2979.7 | 2979.6 | 2979.5 | 2978.9 |
| 46 | 3150 | 3149.7 | 3149.7 | 3149.6 | 3149.5 | 3149.4 | 3148.9 |

**Table 3.** Comparison of measured and calculated resonant frequencies obtained by using Sugeno FIS models presented in this paper for electrically thin and thick rectangular MSAs.

| Patch No | Measured $(f_{me})$ (MHz) | Present Sugeno FIS Models | | | | | |
|---|---|---|---|---|---|---|---|
| | | LSQ | NM | GA | DEA | PSO | SA |
| 1 | 2310 | 2310.0 | 2310.2 | 2310.1 | 2310.1 | 2310.1 | 2309.5 |
| 2 | 2890 | 2890.0 | 2890.3 | 2890.1 | 2890.1 | 2890.0 | 2890.2 |
| 3 | 4240 | 4240.0 | 4240.0 | 4240.1 | 4240.1 | 4240.1 | 4239.4 |
| 4 | 5840 | 5840.0 | 5839.8 | 5839.8 | 5839.4 | 5839.7 | 5839.4 |
| 5 | 6800 | 6800.0 | 6800.1 | 6800.1 | 6799.9 | 6800.3 | 6800.9 |
| 6 | 7700 | 7700.0 | 7700.0 | 7699.7 | 7700.1 | 7699.6 | 7701.4 |
| 7 | 8270 | 8270.0 | 8270.0 | 8269.7 | 8270.1 | 8269.7 | 8270.5 |
| 8 | 9140 | 9140.0 | 9140.0 | 9139.7 | 9140.1 | 9139.8 | 9140.9 |
| 9 | 4730 | 4730.0 | 4730.0 | 4730.0 | 4730.0 | 4729.9 | 4728.6 |
| 10 | 7870 | 7870.0 | 7869.8 | 7869.8 | 7869.6 | 7869.9 | 7869.1 |
| 11 | 2228 | 2228.0 | 2228.0 | 2228.1 | 2228.0 | 2228.0 | 2228.1 |
| 12 | 2200 | 2200.0 | 2200.0 | 2200.1 | 2200.0 | 2200.0 | 2200.0 |
| 13 | 2181 | 2181.0 | 2181.0 | 2181.1 | 2181.0 | 2181.0 | 2181.0 |
| 14 | 7740 | 7740.0 | 7739.7 | 7739.8 | 7739.4 | 7739.7 | 7738.5 |
| 15 | 8450 | 8450.0 | 8449.9 | 8450.0 | 8449.8 | 8450.5 | 8449.3 |
| 16 | 3970 | 3970.0 | 3970.1 | 3970.0 | 3970.1 | 3970.0 | 3970.1 |
| 17 | 7730 | 7730.0 | 7729.3 | 7729.2 | 7728.6 | 7728.3 | 7725.4 |
| 18 | 4600 | 4600.0 | 4600.0 | 4600.0 | 4600.0 | 4600.0 | 4600.5 |
| 19 | 5060 | 5060.0 | 5060.1 | 5060.3 | 5059.9 | 5061.3 | 5061.3 |
| 20 | 4805 | 4805.0 | 4805.5 | 4805.2 | 4804.9 | 4804.4 | 4803.9 |
| 21 | 6560 | 6560.0 | 6559.0 | 6559.5 | 6558.5 | 6560.5 | 6561.7 |
| 22 | 5600 | 5600.0 | 5599.3 | 5598.9 | 5598.1 | 5597.5 | 5596.3 |
| 23 | 6200 | 6200.0 | 6203.0 | 6203.5 | 6202.6 | 6204.7 | 6203.6 |
| 24 | 7050 | 7050.0 | 7050.1 | 7050.7 | 7050.1 | 7052.9 | 7054.6 |
| 25 | 5800 | 5800.0 | 5798.6 | 5798.8 | 5797.9 | 5799.7 | 5799.9 |
| 26 | 5270 | 5270.0 | 5270.5 | 5270.3 | 5269.7 | 5270.2 | 5270.6 |
| 27 | 7990 | 7990.0 | 7990.3 | 7991.0 | 7991.3 | 7992.1 | 7989.9 |
| 28 | 6570 | 6570.0 | 6568.8 | 6568.1 | 6567.4 | 6564.6 | 6560.1 |
| 29 | 5100 | 5100.0 | 5100.2 | 5100.1 | 5099.8 | 5100.1 | 5099.7 |
| 30 | 8000 | 8000.0 | 7999.5 | 7999.2 | 7999.3 | 7998.5 | 7998.6 |
| 31 | 7134 | 7134.0 | 7134.5 | 7134.7 | 7134.6 | 7135.6 | 7138.1 |
| 32 | 6070 | 6070.0 | 6069.6 | 6069.6 | 6069.4 | 6069.1 | 6065.0 |
| 33 | 5820 | 5820.0 | 5820.5 | 5820.4 | 5820.4 | 5821.2 | 5828.1 |
| 34 | 6380 | 6380.0 | 6379.5 | 6379.0 | 6379.1 | 6377.7 | 6379.1 |
| 35 | 5990 | 5990.0 | 5989.9 | 5989.8 | 5990.0 | 5989.0 | 5988.3 |
| 36 | 4660 | 4660.0 | 4660.0 | 4659.9 | 4660.1 | 4659.6 | 4658.7 |
| 37 | 4600 | 4600.0 | 4600.1 | 4600.2 | 4600.2 | 4600.1 | 4599.6 |
| 38 | 3580 | 3580.0 | 3580.0 | 3580.0 | 3580.0 | 3579.9 | 3579.3 |
| 39 | 3980 | 3980.0 | 3979.7 | 3979.9 | 3979.9 | 3979.4 | 3978.5 |
| 40 | 3900 | 3900.0 | 3900.2 | 3900.3 | 3900.4 | 3900.8 | 3899.5 |
| 41 | 3980 | 3980.0 | 3979.9 | 3979.7 | 3979.7 | 3978.8 | 3977.1 |
| 42 | 3900 | 3900.0 | 3900.0 | 3900.3 | 3900.3 | 3900.1 | 3899.5 |
| 43 | 3470 | 3470.0 | 3470.0 | 3470.2 | 3470.2 | 3470.2 | 3469.4 |
| 44 | 3200 | 3200.0 | 3200.1 | 3199.8 | 3199.8 | 3198.9 | 3199.4 |
| 45 | 2980 | 2980.0 | 2980.0 | 2980.1 | 2979.9 | 2979.9 | 2980.0 |
| 46 | 3150 | 3150.0 | 3149.9 | 3150.5 | 3150.5 | 3151.0 | 3150.3 |

**Table 4.** Resonant frequencies obtained from the Sugeno FIS models available in the literature for electrically thin and thick rectangular MSAs.

| Patch No | Measured ($f_{me}$) (MHz) | Sugeno FIS Models in the Literature [27] | | |
|---|---|---|---|---|
| | | ITSA | MTSA | CTSA |
| 1 | 2310 | 2310.0 | 2310.0 | 2310.0 |
| 2 | 2890 | 2890.0 | 2890.0 | 2890.0 |
| 3 | 4240 | 4240.1 | 4240.1 | 4240.2 |
| 4 | 5840 | 5839.5 | 5839.7 | 5840.0 |
| 5 | 6800 | 6800.7 | 6800.7 | 6799.8 |
| 6 | 7700 | 7698.7 | 7698.4 | 7699.1 |
| 7 | 8270 | 8271.7 | 8272.1 | 8271.3 |
| 8 | 9140 | 9139.2 | 9139.1 | 9139.3 |
| 9 | 4730 | 4730.0 | 4730.0 | 4730.0 |
| 10 | 7870 | 7870.0 | 7869.9 | 7869.7 |
| 11 | 2228 | 2228.0 | 2228.0 | 2228.0 |
| 12 | 2200 | 2200.0 | 2200.0 | 2200.0 |
| 13 | 2181 | 2181.0 | 2181.0 | 2181.0 |
| 14 | 7740 | 7740.1 | 7740.1 | 7740.0 |
| 15 | 8450 | 8449.7 | 8449.6 | 8449.6 |
| 16 | 3970 | 3970.1 | 3970.1 | 3970.2 |
| 17 | 7730 | 7730.3 | 7730.4 | 7730.7 |
| 18 | 4600 | 4600.0 | 4600.0 | 4600.0 |
| 19 | 5060 | 5060.0 | 5060.4 | 5061.9 |
| 20 | 4805 | 4804.9 | 4804.4 | 4802.9 |
| 21 | 6560 | 6559.8 | 6560.0 | 6560.5 |
| 22 | 5600 | 5600.1 | 5600.2 | 5600.3 |
| 23 | 6200 | 6200.0 | 6199.2 | 6197.0 |
| 24 | 7050 | 7050.1 | 7050.6 | 7050.5 |
| 25 | 5800 | 5800.5 | 5801.1 | 5801.9 |
| 26 | 5270 | 5269.6 | 5269.0 | 5268.5 |
| 27 | 7990 | 7990.2 | 7990.6 | 7991.8 |
| 28 | 6570 | 6569.6 | 6569.0 | 6570.0 |
| 29 | 5100 | 5100.0 | 5098.3 | 5095.1 |
| 30 | 8000 | 7999.5 | 7998.7 | 7997.0 |
| 31 | 7134 | 7134.9 | 7136.1 | 7137.9 |
| 32 | 6070 | 6069.5 | 6066.6 | 6068.8 |
| 33 | 5820 | 5821.6 | 5829.2 | 5838.2 |
| 34 | 6380 | 6378.9 | 6377.0 | 6369.3 |
| 35 | 5990 | 5989.7 | 5987.9 | 5986.5 |
| 36 | 4660 | 4660.1 | 4660.4 | 4660.8 |
| 37 | 4600 | 4599.9 | 4600.0 | 4600.2 |
| 38 | 3580 | 3579.5 | 3579.4 | 3579.5 |
| 39 | 3980 | 3978.0 | 3975.5 | 3974.5 |
| 40 | 3900 | 3901.7 | 3903.8 | 3904.7 |
| 41 | 3980 | 3981.2 | 3981.7 | 3982.0 |
| 42 | 3900 | 3899.0 | 3899.0 | 3898.6 |
| 43 | 3470 | 3471.2 | 3470.8 | 3471.0 |
| 44 | 3200 | 3200.7 | 3200.3 | 3200.3 |
| 45 | 2980 | 2980.6 | 2980.4 | 2980.2 |
| 46 | 3150 | 3148.1 | 3149.0 | 3149.0 |

**Table 5.** Resonant frequencies obtained from the conventional methods available in the literature for electrically thin and thick rectangular MSAs.

| Patch No | Measured ($f_{me}$) (MHz) | Conventional Methods in the Literature | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | [6] | [7] | [8] | [1] | [2] | [11] | [15] | [16] | [19] | [23] | [23] | [32] |
| 1 | 2310 | 2586 | 2381 | 2373 | 2452 | 2296 | 2458 | 2389 | 2377 | 2323 | 628 | 786 | 2450 |
| 2 | 2890 | 3222 | 2911 | 2893 | 3013 | 2795 | 3042 | 2915 | 2908 | 2831 | 963 | 1219 | 3016 |
| 3 | 4240 | 5039 | 4327 | 4239 | 4529 | 4108 | 4681 | 4296 | 4331 | 4191 | 2294 | 2983 | 4539 |
| 4 | 5840 | 7559 | 6085 | 5928 | 6448 | 5700 | 6918 | 5965 | 6089 | 5919 | 5032 | 6712 | 6449 |
| 5 | 6800 | 8933 | 6958 | 6806 | 7405 | 6467 | 8109 | 6761 | 6909 | 6798 | 6955 | 9375 | 7398 |
| 6 | 7700 | 10919 | 8137 | 7956 | 8711 | 7482 | 9836 | 7820 | 7889 | 7528 | 10261 | 14005 | 8701 |
| 7 | 8270 | 12284 | 8905 | 8986 | 9579 | 8132 | 11054 | 8508 | 8402 | 8254 | 12894 | 17725 | 9605 |
| 8 | 9140 | 14038 | 9831 | 11330 | 10621 | 8894 | 12588 | 9306 | 8709 | 9141 | 16706 | 23151 | 10753 |
| 9 | 4730 | 8933 | 5101 | 1094597 | 5614 | 4320 | 7958 | 4525 | 5109 | 4922 | 19581 | 28126 | 7391 |
| 10 | 7870 | 8933 | 7829 | 7694 | 8154 | 7463 | 8341 | 7795 | 7825 | 7585 | 3494 | 4500 | 8171 |
| 11 | 2228 | 2292 | 2209 | 2232 | 2259 | 2175 | 2248 | 2245 | 2222 | 2197 | 281 | 266 | 2252 |
| 12 | 2200 | 2292 | 2208 | 2204 | 2241 | 2158 | 2228 | 2221 | 2206 | 2180 | 281 | 266 | 2221 |
| 13 | 2181 | 2292 | 2208 | 2184 | 2230 | 2148 | 2216 | 2204 | 2210 | 2168 | 281 | 266 | 2192 |
| 14 | 7740 | 7804 | 7697 | 7750 | 7791 | 7635 | 7737 | 7763 | 7720 | 7717 | 295 | 412 | 7765 |
| 15 | 8450 | 8496 | 8369 | 8431 | 8478 | 8298 | 8417 | 8446 | 8396 | 8389 | 349 | 488 | 8451 |
| 16 | 3970 | 4027 | 3898 | 3949 | 3983 | 3838 | 3951 | 3950 | 3917 | 3887 | 358 | 510 | 3977 |
| 17 | 7730 | 7940 | 7442 | 7605 | 7733 | 7322 | 7763 | 7639 | 7551 | 7376 | 1775 | 1610 | 7730 |
| 18 | 4600 | 4697 | 4254 | 4407 | 4641 | 4455 | 4979 | 4729 | 4614 | 4430 | 14548 | 113 | 4618 |
| 19 | 5060 | 5283 | 4865 | 4989 | 5070 | 4741 | 5101 | 4958 | 4924 | 4797 | 1294 | 1621 | 5077 |
| 20 | 4805 | 5014 | 4635 | 4749 | 4824 | 4520 | 4846 | 4724 | 4688 | 4573 | 1169 | 1460 | 4830 |
| 21 | 6560 | 6958 | 6220 | 6421 | 6566 | 6067 | 6729 | 6382 | 6357 | 6114 | 2719 | 2550 | 6563 |
| 22 | 5600 | 5795 | 5270 | 5424 | 5535 | 5158 | 5625 | 5414 | 5374 | 5194 | 1907 | 1769 | 5535 |
| 23 | 6200 | 6653 | 5845 | 6053 | 6201 | 5682 | 6413 | 5987 | 5988 | 5735 | 3019 | 2860 | 6193 |
| 24 | 7050 | 7828 | 6566 | 6867 | 7052 | 6320 | 7504 | 6682 | 6769 | 6433 | 4942 | 4792 | 7030 |
| 25 | 5800 | 6325 | 5435 | 5653 | 5801 | 5259 | 6078 | 5552 | 5586 | 5326 | 3399 | 3259 | 5787 |
| 26 | 5270 | 5820 | 4943 | 5155 | 5287 | 4762 | 5572 | 5030 | 5081 | 4842 | 3281 | 3383 | 5273 |
| 27 | 7990 | 9319 | 7334 | 7813 | 7981 | 6917 | 8885 | 7339 | 7570 | 6822 | 8153 | 8674 | 8101 |
| 28 | 6570 | 7412 | 6070 | 6390 | 6550 | 5794 | 7076 | 6135 | 6264 | 5951 | 5236 | 5486 | 6543 |
| 29 | 5100 | 5945 | 4667 | 4993 | 5092 | 4407 | 5693 | 4678 | 4830 | 4338 | 5457 | 5437 | 5193 |
| 30 | 8000 | 8698 | 6845 | 7546 | 7519 | 6464 | 8447 | 6889 | 7160 | 6367 | 8089 | 8067 | 7948 |
| 31 | 7134 | 7485 | 5870 | 6601 | 6484 | 5525 | 7342 | 5904 | 6179 | 5452 | 7241 | 7242 | 7169 |
| 32 | 6070 | 6478 | 5092 | 5660 | 5606 | 4803 | 6317 | 5125 | 5341 | 4735 | 6113 | 6103 | 6026 |
| 33 | 5820 | 6180 | 4855 | 5423 | 5352 | 4576 | 6042 | 4886 | 5100 | 4513 | 5881 | 5875 | 5817 |
| 34 | 6380 | 6523 | 5101 | 5823 | 5660 | 4784 | 6453 | 5122 | 5396 | 4729 | 6529 | 6546 | 6515 |
| 35 | 5990 | 5798 | 4539 | 5264 | 5063 | 4239 | 5804 | 4550 | 4830 | 4196 | 5950 | 5976 | 6064 |
| 36 | 4660 | 4768 | 3746 | 4227 | 4141 | 3526 | 4689 | 3770 | 3949 | 3479 | 4600 | 4600 | 4613 |
| 37 | 4600 | 4084 | 3201 | 3824 | 3615 | 2938 | 4209 | 3168 | 3446 | 2921 | 4556 | 4603 | 4550 |
| 38 | 3580 | 3408 | 2668 | 3115 | 2983 | 2485 | 3430 | 2670 | 2845 | 2461 | 3554 | 3574 | 3628 |
| 39 | 3980 | 3585 | 2808 | 3335 | 3162 | 2590 | 3668 | 2790 | 3015 | 2572 | 3920 | 3955 | 3956 |
| 40 | 3900 | 3558 | 2785 | 3299 | 3133 | 2573 | 3629 | 2771 | 2987 | 2555 | 3863 | 3895 | 3907 |
| 41 | 3980 | 3510 | 2753 | 3294 | 3112 | 2522 | 3626 | 2721 | 2966 | 2509 | 3940 | 3982 | 3922 |
| 42 | 3900 | 3313 | 2608 | 3147 | 2964 | 2364 | 3473 | 2554 | 2823 | 2356 | 3852 | 3903 | 3747 |
| 43 | 3470 | 3001 | 2358 | 2838 | 2675 | 2146 | 3129 | 2317 | 2549 | 2137 | 3450 | 3493 | 3381 |
| 44 | 3200 | 2779 | 2183 | 2623 | 2474 | 1992 | 2889 | 2151 | 2357 | 1983 | 3160 | 3197 | 3123 |
| 45 | 2980 | 2684 | 2102 | 2502 | 2370 | 1936 | 2752 | 2086 | 2259 | 1924 | 2954 | 2982 | 2972 |
| 46 | 3150 | 2763 | 2168 | 2600 | 2453 | 1982 | 2863 | 2139 | 2338 | 1972 | 3125 | 3160 | 3096 |

**Table 6.** Train, test, and total absolute errors between the measured and calculated resonant frequencies for FIS models.

| Models | Algorithms | Train Absolute Errors (MHz) | Test Absolute Errors (MHz) | Total Absolute Errors (MHz) |
|---|---|---|---|---|
| Present Mamdani FIS Models | LSQ | 16 | 4 | 20 |
| | NM | 20 | 5 | 25 |
| | GA | 24 | 5 | 29 |
| | DEA | 31 | 7 | 38 |
| | PSO | 37 | 9 | 46 |
| | SA | 73 | 17 | 90 |
| Present Sugeno FIS Models | **LSQ** | **0** | **0** | **0** |
| | NM | 10 | 5 | 15 |
| | GA | 15 | 5 | 20 |
| | DEA | 19 | 4 | 23 |
| | PSO | 31 | 9 | 40 |
| | SA | 57 | 16 | 73 |
| Sugeno FIS Models in the Literature [27] | ITSA | 20 | 4 | 24 |
| | MTSA | 36 | 14 | 50 |
| | CTSA | 52 | 30 | 82 |

**Table 7.** Total absolute errors between the measured and calculated resonant frequencies for the conventional methods.

| Conventional Methods in the Literature | [6] | [7] | [8] | [1] | [2] | [11] |
|---|---|---|---|---|---|---|
| Errors (MHz) | 36059 | 26908 | 1104916 | 19179 | 32930 | 23746 |
| Conventional Methods in the Literature | [15] | [16] | [19] | [23] | [23] | [32] |
| Errors (MHz) | 23761 | 19899 | 31436 | 108707 | 126945 | 10132 |

## 2.3. Determination of Design Parameters of Mamdani and Sugeno FIS Models

The determination of the design parameters of the FIS is of vital importance. An optimization algorithm is used to determine these design parameters. The parameter optimization is done in a way such that the error measure between the target and the actual output is minimized. During the learning process of the FIS, the premise parameters and the consequent parameters are tuned until the desired response of the FIS is achieved. In this paper, six different optimization

algorithms, LSQ, NM, GA, DEA, PSO, and SA, are used to determine the optimum values of the design parameters and adapt the FISs. Basic optimization framework steps of these algorithms for the calculation of resonant frequency by using Mamdani and Sugeno FIS models are summarized below:

**Step 1:** Choose the initial FIS structure. In this step, the number and the types of MFs for the input variables $W$, $L$, $h$, and $\varepsilon_r$ are found. Thus, the numbers of the premise parameters $(a_{ij}, b_{ij}, c_{ij}, d_{ij}, \text{ and } \sigma_{ij})$ and the consequent parameters $(c_{ok} \text{ and } \sigma_{ok} \text{ or } p_k)$ are determined.
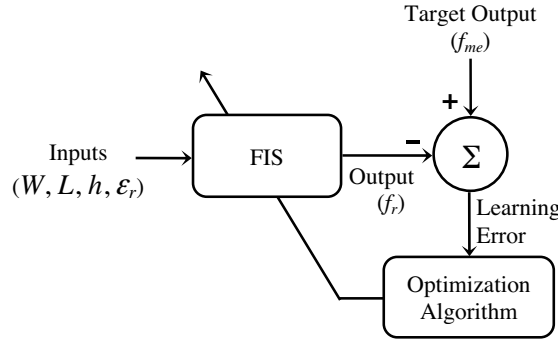


**Figure 4.** FIS model for calculating the resonant frequency.

**Step 2:** Initialize the values of the premise and consequent parameters. Initial values of the premise and consequent parameters are produced. Hence, the input MFs are equally spaced and made to cover the universe of discourse.

**Step 3:** Compute the output (resonant frequency $f_r$). FIS produces the output $(f_r)$ for a given input data $(W, L, h, \text{ and } \varepsilon_r)$ using the premise and the consequent parameters in the inference mechanism.

**Step 4:** Evaluate the performance of FIS. The learning error between the target (measured resonant frequency $f_{me}$) and the actual output is computed. In this paper, mean square error (MSE) criterion is used for this purpose.

**Step 5:** Continue or terminate the process. In this step, termination criterion of the optimization algorithm is checked. If the learning error is equal to/small from the predetermined error value or iterations reach to a final value, the process is stopped. Otherwise, the process continues to Step 6.

**Step 6:** Update the parameter values using the optimization algorithm. In this step, an optimization algorithm (LSQ, NM, GA,
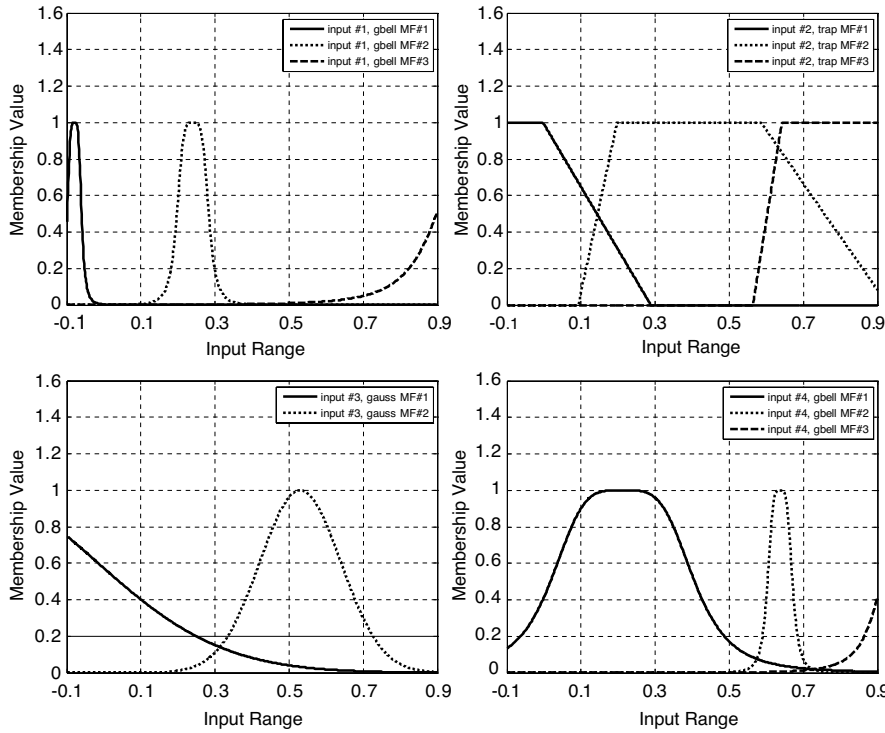
**Figure 5.** Shapes of the MFs of input variables (Input $\#1 = W$, Input $\#2 = L$, Input $\#3 = h$, and Input $\#4 = \varepsilon_r$) for the Sugeno FIS model trained by the LSQ.

DEA, PSO, and SA) is used for updating the values of the premise and the consequent parameters so as to decrease the learning error. Then, the process continues from Step 3.

The process explained above is simply given in Figure 4.

## 3. RESULTS AND CONCLUSIONS

The resonant frequencies calculated by using Mamdani and Sugeno FIS models presented in this paper for electrically thin and thick rectangular MSAs are given in Tables 2 and 3, respectively. LSQ, NM, GA, DEA, PSO, and SA in Tables 2 and 3 represent, respectively, the resonant frequencies calculated by the FIS models trained by LSQ, NM, GA, DEA, PSO, and SA. For comparison, the resonant frequency results obtained by using the Sugeno FIS models [27] and the conventional methods [1, 2, 6–8, 11, 15, 16, 19, 23, 32] are listed in

Tables 4 and 5, respectively. ITSA, MTSA, and CTSA in Table 4 represent, respectively, the resonant frequencies calculated by the Sugeno FIS models trained by the improved tabu search algorithm (ITSA), the modified tabu search algorithm (MTSA), and the classical tabu search algorithm (CTSA). The resonant frequency results in twelfth and thirteenth columns of Table 5 are obtained by using the curve-fitting formula [23] and by using the modified cavity model [23], respectively. The sum of the absolute errors between the theoretical and experimental results for FIS models and conventional methods is listed in Tables 6 and 7.

When the performances of Mamdani and Sugeno FIS models are compared with each other, the best result is obtained from the Sugeno FIS model trained by the LSQ algorithm, as shown in Tables 2, 3, 4 and 6. The final shapes of the input MFs are illustrated in Figure 5 for the Sugeno FIS model trained by the LSQ algorithm. For brevity, the final shapes of the MFs of other FIS models are not given.

It is clear from Tables 5 and 7 that the conventional methods [1, 2, 6–8, 11, 15, 16, 19, 23, 32] give comparable results. Some cases are in very good agreement with measurements, and others are far off. When the results of FIS models are compared with the results of the conventional methods, the results of all FIS models are better than those predicted by the conventional methods. The very good agreement between the measured resonant frequency values and the computed resonant frequency values of FIS models supports the validity of the FIS models and also illustrates the superiority of FIS models over the conventional methods.

In this paper, the FIS models are trained and tested with the experimental data taken from the previous experimental works [8, 14, 22, 23]. It is clear from Tables 5 and 7 that the theoretical resonant frequency results of the conventional methods are not in very good agreement with the experimental results. For this reason, the theoretical data sets obtained from the conventional methods are not used in this work. Only the measured data set is used for training and testing the FIS models.

As a consequence, Mamdani and Sugeno FIS models are used to accurately calculate the resonant frequency of electrically thin and thick rectangular MSAs. Six optimization algorithms, LSQ, NM, GA, DEA, PSO, and SA, are used to determine the design parameters of the FIS models. In order to validate the performances of the FIS models, comprehensive comparisons have been made. The results of FIS models are in very good agreement with the measurements. The best result is obtained from the Sugeno FIS trained by LSQ algorithm.

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