Review

Comparison of Mechanical Impedance Methods for Vibration Simulation

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The work presented here explored the detrimental consequences that resulted when mechanical impedance effects were not considered in relating vibration test requirements with field measurements. The ways in which these effects can be considered were evaluated, and comparison of three impedance methods was accomplished based on a cumulative damage criterion. A test structure was used to simulate an equipment and support foundation system. Detailed finite element analysis was performed to aid in computation of cumulative damage totals. The results indicate that mechanical impedance methods can be effectively used to reproduce the field vibration environment in a laboratory test. The establishment of validated computer models, coupled with laboratory impedance measurements, can eliminate the overtesting problems inherent with constant motion, infinite impedance testing strategies.

INTRODUCTION

Realistic laboratory simulation of a structure's field vibration has been of major concern to design and test engineers for many years. The usual practice of basing vibration design and test specifications on an envelope of the equipment base acceleration levels experienced in the field environment has often resulted in excessive levels of overtesting. This results from the large differences between the mechanical impedance of the structure in the field configuration and that of a fully equalized vibration shaker. Mechanical impedance effects occur naturally in a field environment. Including such effects in a laboratory vibration test achieves more realistic conditions of similitude. A possible solution to the problem is the generation of design and test specifications that are based on the knowledge of both the acceleration and the forces transmitted to the equipment in the field environment.

BACKGROUND

Mechanical impedance applications for vibration testing had its inception in the 1960s (Morrow, 1960), predominantly driven by the requirements of the manned space program and for the desire to more accurately simulate field vibration in the laboratory test (Otts, 1965, 1970). The conceptual foundations of mechanical impedance were laid down and applied during this period of time. Many articles were generated detailing the benefits of mechanical impedance testing and the basic incorrectness of a constant motion, infinite impedance simulation (Plunkett, 1958; Ratz, 1966; Murfin, 1968). However, it was concluded that real-time control using impedance test methods was not easily achievable with the available test equipment of the time (Morrow, 1960).

The last decade has seen a revitalized interest in using mechanical impedance vibration testing. Much of this new found interest originates partly

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out of necessity for more accurate simulations and partly because of much improved force measuring instruments and test equipment. Advent of the high speed microprocessor and advancements in control technology have now rendered mechanical impedance testing fully achievable.

Overtest Problem

It is common practice to present field vibration data as some form of plot of motion amplitude (typically acceleration) versus frequency. When such data is obtained on airborne structures, it is almost universal practice to record the motion of the points of attachment of equipment to the supporting structure. These plots usually show one or more characteristic peaks along with characteristic valleys. After several such plots that are considered to be pertinent to a given equipment item have been gathered together, the data is usually synthesized into a single simpler spectrum for design and test requirements. Because a margin of safety is desirable, the synthesized spectrum is usually a smooth simple curve whose level is determined principally by the peaks of a spectrum that is a composite of the original field spectra.

It seems reasonable to require an equipment item to withstand the maximum intensities of vibration that are observed to occur during flight. However, this seemingly straightforward procedure must be viewed incredulously because it requires the neglect of the influence of the reactions of a mounted equipment item upon its supporting structure. Vibration specifications derived in the manner described require that a vibration shaker mounting platform deliver a prescribed motion regardless of the reaction of the unit under test. Input acceleration to the test specimen is maintained at the prescribed level regardless of the force magnitude required to sustain this acceleration. This amounts to testing with an infinite impedance vibration source and thus implies that the actual equipment support structure must have an infinite effective mass at all frequencies. This test method is referred to as infinite impedance vibration testing (Plunkett, 1958).

The support structure does not possess infinite effective mass at any frequency and the vibratory motion of the support can be significantly affected by the interface reactions of the coupled equipment item. This alteration of motion is known as dynamic loading. At a certain frequency an equipment item may exert an unusually large reaction, or load, working against support excitation. This opposing reaction force may be of sufficient magnitude to reduce force components existing in the excitation to relatively small values. If the support excitation is a random vibration, then the frequency response function (FRF) of the support will exhibit a notch at this frequency. The frequency at which an equipment exerts maximum reaction against an excitation by its support is called an antiresonance frequency. The equipment, from the viewpoint of its support, has a maximum value of mechanical impedance at this frequency.

Although quite common, the use of an envelope of spectral peaks to determine vibration test levels for use in standard test procedures does not account for antiresonances. In other words, the dynamic loading of the equipment against its support is neglected. At test, the use of an envelope test spectrum thus will result in rather accurate vibration responses of the equipment item only close to the resonance frequencies of the combined system in the field. At all other frequencies, the response levels may be grossly in error particularly close to the fixed base natural frequencies of the equipment item (antiresonance frequencies of the combined system).

To quantify the magnitude of error that can occur, a 5-degree of freedom (5-DOF) system (Gatscher, 1994) was used (see Fig. 1). The support structure masses (m_1 and m_2 of Fig. 1) were used to input the vibration forcing function. A test specification, based on the results of the 5-DOF system analysis, was developed and applied to the equipment-only subsystem (m_4 and m_5 of Fig. 1). Figure 2 shows the test specification as an envelope of the m_3 base spectral peaks from the 5-DOF analysis results. This envelope spectrum was then applied to the equipment-only subsystem as input for a hypothetical infinite impedance

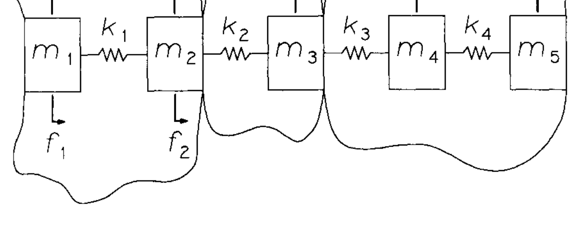

FIGURE 1 5-DOF, free–free, dynamic system, with 0.5% of critical damping implied.

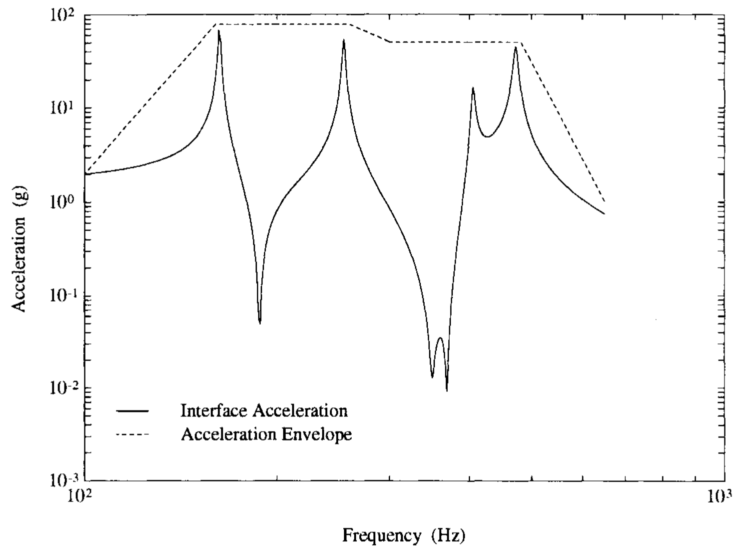

FIGURE 2 Acceleration test specification as an envelope of m_3 base field acceleration levels.

vibration test. Figure 3 displays the results of the equipment-only subsystem test superimposed against the analysis results from the original 5-DOF system. As can be observed, the amount of overtest is in error by a factor of 37 for m_4 response and by a factor of 52 for m_5 response. Obviously, an unacceptable amount of overtesting resulted for this lightly damped mechanical system. An increase of structural damping will attenuate the amount of overtest error.

MECHANICAL IMPEDANCE THEORY

The relation of driving force to the acceleration that a support structure imparts to a mounted equipment item is conveniently expressed through the use of mechanical impedance variables (Neubert, 1987). Impedance variables contain ratios of input force to resulting input motion. The manner in which impedance data is derived may be illustrated using a classical mass (m),

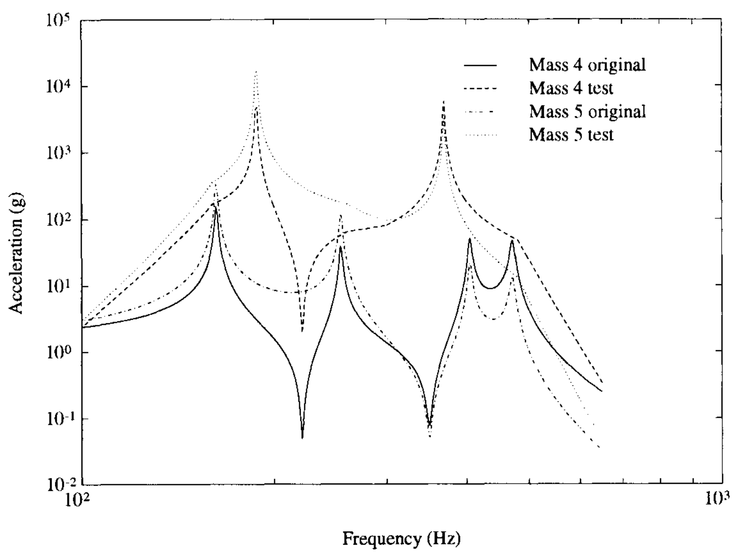

FIGURE 3 FRFs of m_4 and m_5 field results from original 5-DOF system versus hypothetical infinite impedance equipment test results.

spring (k), and damper (c) system. The general equation of motion for this system is

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t). \tag{1}$$

Now, suppose that the applied force is harmonic with a magnitude $|F|$ and a circular frequency ω. Then by using phasor notation both quantities can be described by a single parameter

$$f(t) = F(\omega)e^{i\omega t}. \tag{2}$$

Now, assume that the system responds with harmonic displacement at the same frequency ω but with a magnitude $|X|$. We may express this as

$$x(t) = X(\omega)e^{i\omega t}. \tag{3}$$

Substituting Eqs. (2) and (3) into Eq. (1), dividing through by $e^{i\omega t}$ and solving for the ratio $F/\ddot{X}$ yields

$$\frac{F(\omega)}{A(\omega)} = M(\omega) = \frac{\omega^2 m - i\omega c - k}{\omega_2}. \tag{4}$$

The complex quantity $M(\omega)$ fully defines the relationship between the harmonic force, $F(\omega)$, and the harmonic acceleration response, $A(\omega)$, in the frequency domain. The function $M(\omega)$ is commonly referred to as effective mass (or apparent weight).

Combined Structural System

Mechanical impedance concepts are also valid for complex multiple DOF systems. Consider a missile system consisting of two subsystems, joined at a common connection point, as shown in Fig. 4. It is assumed that *subsystem I* is an active vibration source, because input loading is applied to it, and *subsystem II* is a passive vibration load. Subscripts denote missile station location and superscripts denote subsystem designator. The matrix equation for the separate subsystems before coupling is (Gatscher, 1994)

$$\begin{Bmatrix} F_1^{\mathrm{I}} \\ F_2^{\mathrm{I}} \\ F_2^{\mathrm{II}} \\ F_3^{\mathrm{II}} \end{Bmatrix} = \begin{bmatrix} M_{11}^{\mathrm{I}} & M_{12}^{\mathrm{I}} & 0 & 0 \\ M_{21}^{\mathrm{I}} & M_{22}^{\mathrm{I}} & 0 & 0 \\ 0 & 0 & M_{22}^{\mathrm{II}} & M_{23}^{\mathrm{II}} \\ 0 & 0 & M_{32}^{\mathrm{II}} & M_{33}^{\mathrm{II}} \end{bmatrix} \begin{Bmatrix} A_1^{\mathrm{I}} \\ A_2^{\mathrm{I}} \\ A_2^{\mathrm{II}} \\ A_3^{\mathrm{II}} \end{Bmatrix}. \tag{5}$$

To connect the two subsystems, the boundary conditions are taken as

$$A_2^{\mathrm{I}} = A_2^{\mathrm{II}}, \quad F_2^{\mathrm{I}} = -F_2^{\mathrm{II}}. \tag{6}$$

That is, at the connection point the accelerations are equal and the sum of the internal reaction forces is equal to zero. If the effective masses of the subsystems are known, and the input acceleration is specified, then the interface acceleration at the connection point can be solved for in terms of effective mass and input acceleration. The interface acceleration at the connection point is

$$A_2^{\mathrm{I}} = A_2^{\mathrm{II}} = \frac{M_{21}^{\mathrm{I}}}{M_{22}^{\mathrm{I}} + M_{22}^{\mathrm{II}}}(A_1^{\mathrm{I}}). \tag{7}$$

Applying Norton's theorem (Neubert, 1987), the input acceleration can be related to the free accel-

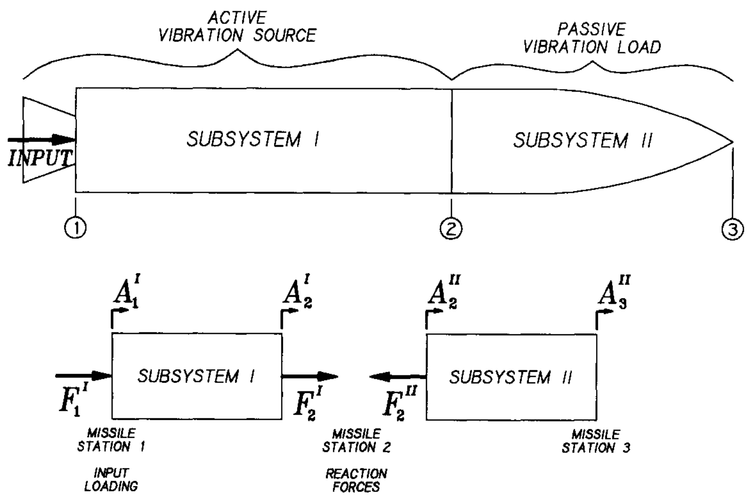

FIGURE 4 Combined structural system with an active vibration source and passive vibration load.

eration at the connection point as

$$(A_2^{\mathrm{I}})_{fr} = -\frac{M_{21}^{\mathrm{I}}}{M_{22}^{\mathrm{I}}}(A_1^{\mathrm{I}}). \tag{8}$$

This represents the free acceleration of subsystem I when subsystem II is removed completely. Then by substituting Eq. (8) into Eq. (7), the interface acceleration of the combined system, in terms of effective mass and free acceleration, is

$$A_2^{\mathrm{I}} = A_2^{\mathrm{II}} = \frac{(A_2^{\mathrm{I}})_{fr}}{\dfrac{M_{22}^{\mathrm{II}}}{M_{22}^{\mathrm{I}}} + 1}. \tag{9}$$

Examination of Eq. (9) yields considerable insight into the inherent problems associated with infinite impedance testing. When subsystem I is a vibration shaker ready to perform an infinite impedance vibration test, the effective mass of the structural support (shaker) approaches infinity,

$$|M_{22}^{\mathrm{I}}| \to \infty. \tag{10}$$

This implies from Eq. (9) that the interface acceleration of the combined system approaches the value of the free acceleration of subsystem I when subsystem II is removed. This is the root source of the overtest error; in other words, the subsystem test is carried out under the inaccurate assumption that

$$A_2^{\mathrm{I}} = (A_2^{\mathrm{I}})_{fr} \tag{11}$$

is always true. In theory, Eq. (11) is never true and can be significantly unequal at the fixed base natural frequencies of the subsystem (antiresonances of the combined system).

Undertesting is also a possibility. For most structures, the plot of driving point effective mass versus frequency may be divided into two general regions. At high frequencies, the structure is predominantly springlike, and the effective mass declines with increasing frequency; at low frequencies, the structure is masslike, and the effective mass increases with frequency. Suppose that over a given frequency range the vibration source effective mass M_{22}^{I} is compliant and the vibration load effective mass M_{22}^{I} is inertial in nature, and that there is some frequency, f_c, where these two quantities resonate,

$$\frac{M_{22}^{\mathrm{II}}(f_c)}{M_{22}^{\mathrm{I}}(f_c)} = -1. \tag{12}$$

The result in the field is a resonant peak of f_c with the field amplitude increased many times over that was implied in the Eq. (9) test condition (Ratz, 1966). Infinite impedance testing in this case leads to serious undertesting.

In the combined mechanical system, there is an interplay of forces and accelerations due to the mechanical impedance of test object and support structure. This interplay renders infinite impedance testing completely unrealistic as a means of simulating the field vibration environment. A vibration simulation test for a subsystem mounted to a vibration shaker, must therefore account for both interface force and acceleration.

MECHANICAL IMPEDANCE METHODS

Several techniques have been suggested, taking into account the dynamic interaction between the test item and its supporting structure. For example, many proposals have been made in the past to introduce force control (Witte, 1970; Witte and Rodeman, 1970) or combinations of force and acceleration control (Scharton, 1991a,b; Smallwood, 1989) to reduce the gross errors that occur when using only infinite impedance testing. The most promising of the investigated methods are force-acceleration product method, dual-extremal control method, and transmissibility correction method.

To compare these three impedance methods it was necessary to calculate the cumulative damage each method imparts to a test specimen. To accomplish this task a simple structural system was used to perform vibration testing and to calculate the resulting cumulative damage for each test method. A steel beam/block structure was fabricated to serve as the structural test system (see Fig. 5). The beam structure was designed such that the bottom base plate bolts to a shaker mounting platform and the top beam/block assembly can be detached from the combined structure and mounted directly to the shaker platform [Fig. 5(b)]. In this regard, the top beam/block assembly can be considered as an equipment item for which a vibration test is to be developed. The detached top beam/block assembly will be referred to herein as the subsystem test structure.

The combined test structure was instrumented using six accelerometers and a constant ampli-

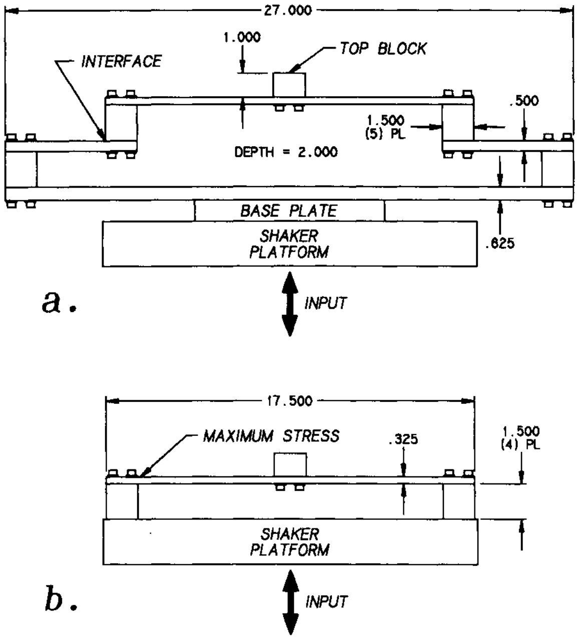

FIGURE 5 Beam/block structure assemblies for vibration testing: (a) combined system and (b) subsystem.

tude (3 g input) logarithmic sine sweep vibration test was performed from 20 to 2,000 Hz. The sine sweep test was run for a total of 1,500 s (to establish a damage baseline) and represents the field vibration environment for the combined structural system.

A detailed computer finite element model was generated of the combined structural system. A harmonic analysis was performed on the computer model to duplicate the sine sweep experimental testing. The computer model was optimized to closely match the experimental results by modifying alpha and beta damping constants (Rayleigh proportional damping) while minimizing the difference between experimental and calculated transmissibilities. The agreement was very good (see Table 1). The subsystem was removed from the support structure and directly attached to the shaker platform. A preliminary low level sine sweep subsystem test (1 g input from 20 to 2,000 Hz) was performed to avoid overstressing the subsystem's steel beam. Again, finite element analysis was performed for the subsystem, duplicating the vibration testing. A comparison of experimental and calculated results for the subsystem structure is summarized in Table 2.

The finite element solutions, for both combined and subsystem structures, compare quite favorably to the experimental data. The benefit of these numerical calculations can now be realized. Implementation of the force-acceleration product method requires subsystem accelerance measurements (reciprocal of effective mass) and implementation of the dual-extremal control method requires interface force measurements. Cumulative damage calculations require determination of beam bending stresses for all three methods.

Table 1. Experimental Versus Calculated Results for Combined Test Structure

	Experimental		Calculated	
Mode No.	Frequency (Hz)	Top Block Q	Frequency (Hz)	Top Block Q
1	85	75.5	84	76.5
2	200	94.7	198	78.3
3	518	11.5	522	11.1
4	673	33.1	678	28.7
5	1203	38.2	1166	37.2
6	1441	15.9	1474	20.9

Table 2. Experimental Versus Calculated Results for Subsystem Test Structure

	Experimental		Calculated	
Mode No.	Frequency (Hz)	Top Block Q	Frequency (Hz)	Top Block Q
1	169	118.2	169	118.3
2	1120	55.8	1120	56.1

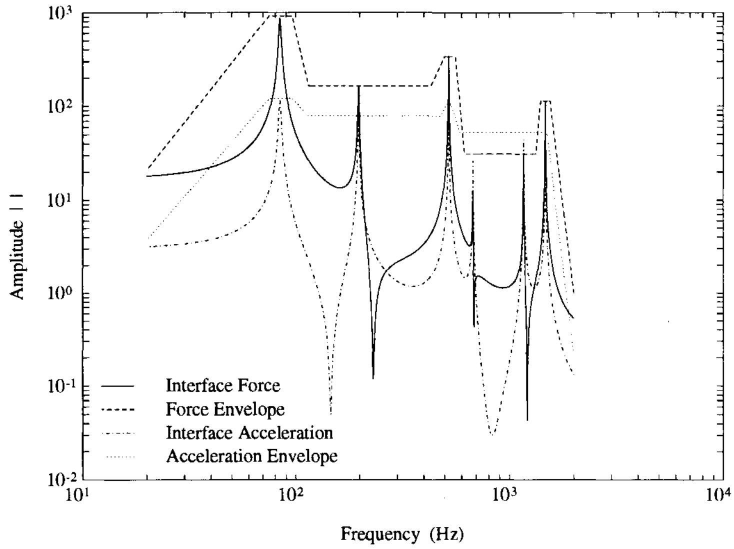

FIGURE 6 Acceleration and force FRFs from interface of combined system with envelopes of peak amplitudes.

These requirements were easily obtained from the finite element analysis results. Because the computer models were correlated to the experimental results, a high level of accuracy was assured in using the numerical results to compare the three impedance test methods.

RESULTS

To implement the force-acceleration product method (Witte, 1970; Witte and Rodeman, 1970), two pieces of information are required: maximum expected interface acceleration levels in the field environment and driving point accelerance function for the subsystem structure. The field acceleration levels were those obtained during the combined structure analysis. The peak amplitudes of the interface acceleration were enveloped and are displayed in Fig. 6. Driving point accelerance for the subsystem structure was obtained from the finite element results and the peak amplitudes were enveloped and plotted in Fig. 7. The square root of the ratio of driving point

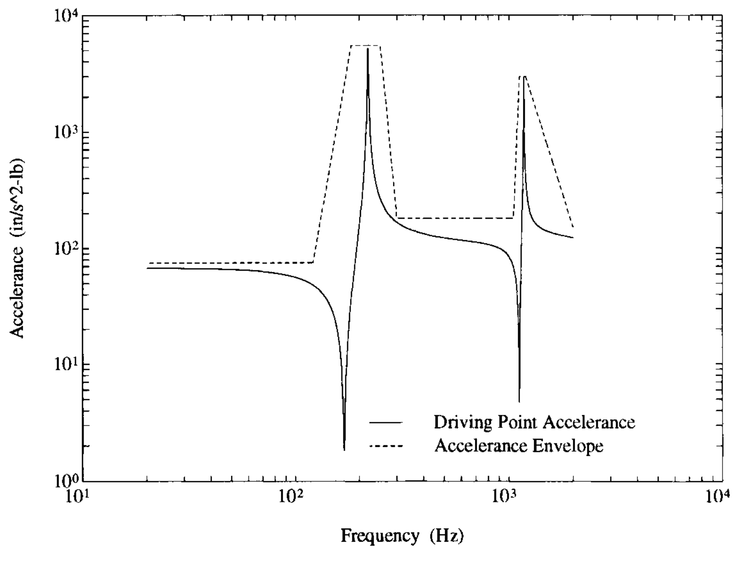

FIGURE 7 Driving point accelerance FRF of subsystem structure with envelope of peak amplitudes.

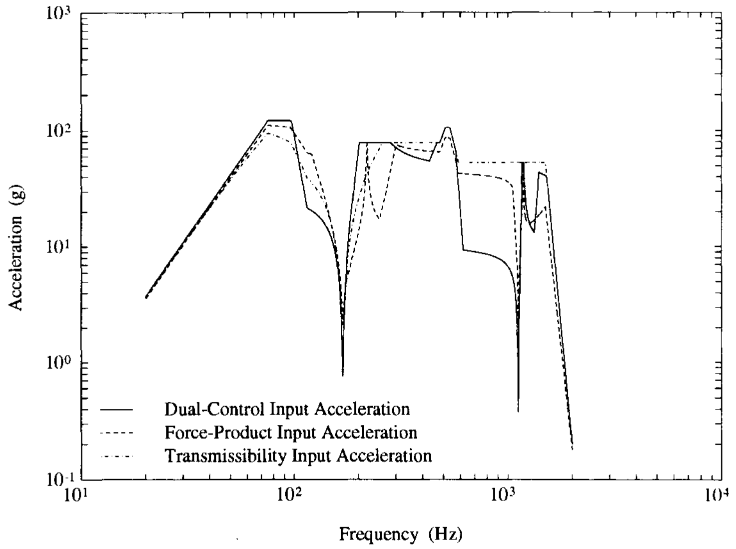

FIGURE 8 Final input acceleration FRFs for subsystem test using the three mechanical impedance correction methods.

accelerance over enveloped accelerance was calculated and this ratio was multiplied by the field acceleration envelope to arrive at the final subsystem test input acceleration (see Fig. 8).

The field acceleration envelope is effectively reduced or notched based on the dynamic characteristics of the subsystem structure (reductions occur at 169 and 1120 Hz). The corrected input acceleration can be applied using a standard constant motion test. Maximum subsystem structure bending stress occurs at the beam ends [see Fig. 5(b)]. The subsystem stress FRF using the force-acceleration product technique is plotted in Fig. 9.

Implementation of the dual-extremal control technique (Scharton, 1991a,b; Smallwood, 1989) also requires two pieces of information: maximum expected interface acceleration levels in the field environment and maximum expected interface force levels in the field environment. The acceleration control spectrum is the same as that used in the force-acceleration product method. The maximum expected interface force levels were taken from the finite element results for the

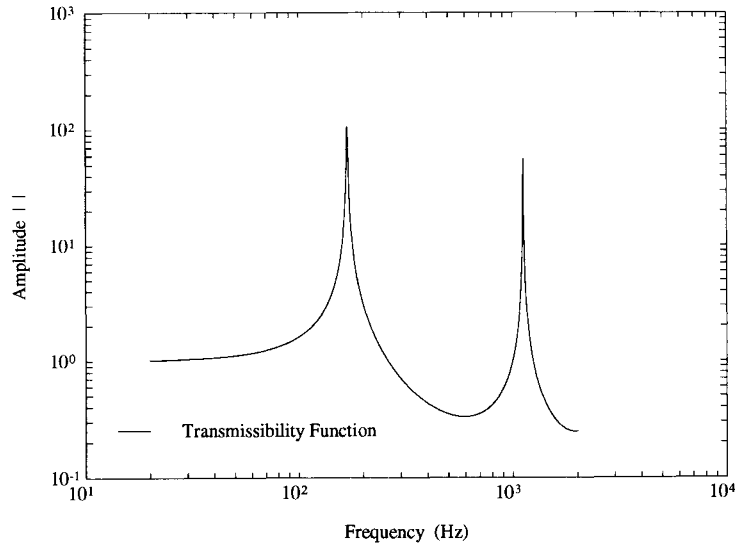

FIGURE 9 Subsystem structure bending stress FRFs comparing the three impedance methods versus field levels and an infinite impedance test.

combined system. Figure 6 contains a plot of the force control spectrum shown as an envelope of maximum interface force levels from the combined system results.

In the dual control scheme, input control to the test item is performed such that neither input force nor acceleration exceeds its control spectrum. If at a certain frequency the input force exceeds its maximum level, then the shaker controller reduces the input acceleration until the input force magnitude is less than or equal to the force control spectrum. This allows the test item to affect its own input vibration levels during the test operation. The resulting final input acceleration into the subsystem structure is shown in Fig. 8. The subsystem bending stress FRF using the dual control technique is displayed in Fig. 9.

The transmissibility correction method (Sweitzer, 1987) requires no force or accelerance measurements. All that is required is a transmissibility function for a damage sensitive item within the subsystem. The transmissibility of the top block was used for this purpose and is displayed in Fig. 10. Mechanical impedance correction is defined as the nominal interface acceleration envelope divided by the square root of the transmissibility function over a frequency correction range. The frequency correction range starts at the lowest test frequency and continues to the upper correction frequency where the transmissibility curve becomes less than one or to the frequency equal to the $\sqrt{2}$ times the fundamental frequency, whichever comes first. The resulting corrected input acceleration is displayed in Fig. 8. The subsystem bending stress FRF using the transmissibility correction method is contained in Fig. 9.

Damage Assessment

Comparison of the three impedance methods along with the test results for an infinite impedance simulation, superimposed against the field measurements, is depicted in Fig. 9. Visual determination of the impedance method that most closely simulates the field environment is not discernable from these frequency response plots. However, by application of Miner's cumulative damage criterion (Miner, 1945), total damage resulting from each method was easily calculated. The subsystem structure was fabricated from ANSI 1018 carbon steel. The S–N curve for this material was used to calculate damage totals. The damage fraction at each frequency point was calculated and then summed across the entire frequency range.

$$D = \sum_{i=1}^{m} \frac{n_i}{N_i} = 1 \quad \text{(at failure)}, \tag{13}$$

where n_i = the number of cycles experienced by the specimen at load i, and N_i = the number of cycles to failure at load i obtained from material S–N curve. The cumulative damage totals plus the peak bending stress resulting from each method are listed in Table 3.

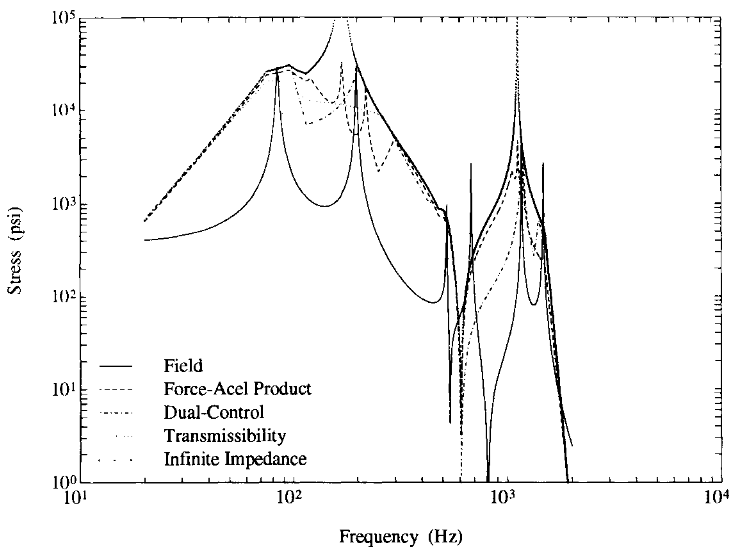

FIGURE 10 Transmissibility FRF of top block subsystem structure for use in transmissibility correction test method.

Table 3. Cumulative Damage Results and Maximum Bending Stress for Subsystem Testing

Vibration Type	Damage Total	Maximum Stress (psi)
Field environment	1.385×10^{-4}	29,601
Force-acceleration test	9.624×10^{-4}	33,796
Dual control test	2.198×10^{-3}	30,951
Transmissibility correction test	4.0318	76,301
Infinite impedance test	4.224×10^{13}	1.209×10^{6}

The force-acceleration product method most closely simulates the original field vibration environment. The dual-extremal control method also closely reproduces the original vibration damage, with a peak stress less than that resulting from the force-acceleration product method. The transmissibility correction method has a damage total greater than one. The majority of damage resulting from the transmissibility method occurs over the uncorrected portion of the input spectrum. Extending the frequency correction range would have resulted in an acceptable damage total. The infinite impedance test method would have obviously caused structural failure with a prodigious damage total.

CONCLUSIONS

Mechanical impedance methods can be effectively used to account for the dynamic loading interaction between equipment and support while performing an equipment-only vibration test. Realistic vibration tests are a function of the field vibration data available for reference, the techniques used in deriving the test specifications, and the test techniques used in the laboratory. Mechanical impedance methods offer a rational means of eliminating the costs and schedule delays associated with overdesign and overtesting to meet conventional infinite impedance vibration specifications.

Mechanical impedance methods have been sparingly utilized over the past 30 years. The equivocation associated with these methods lies in the tacit assumption that infinite impedance testing is a conservative, reasonable approach to laboratory vibration simulation. It is hoped that this argument will lose support and allow engineers to concentrate on producing equipment designs to withstand the field vibration environment, in lieu of overdesigning to survive an unrealistic test artifact.

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