COMPARISON OF PARTICLE-FIELD INTERACTION THEORY WITH SOLAR PROTON DIFFUSION COEFFICIENTS

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The mean free path $\lambda_{||}$ of cosmic rays in interplanetary space is derived from particle-field interaction theory. For power law magnetic field spectra, $f(k) \sim k^q$, $\lambda_{||}$ is finite only for $q > -2$. A bounding factor at small wavenumbers yields finite $\lambda_{||}$ for all q. The asymptotic behaviour of $\lambda_{||}$ for small rigidities is logarithmic for $q = -2$ and constant for $q < -2$. The importance of the detailed structure of the magnetic field spectral tensor is stressed, particularly for steep spectra and at small wavenumbers. The agreement with the numerical values derived from solar proton propagation is good close to solar minimum. For higher solar activity the experimental diffusion coefficients are systematically larger than the theoretical values based on magnetic field power spectra.

Introduction

The wave-particle interaction formalism has been developed in the last years in numerous theoretical papers concerned with weak turbulence in a plasma. The special problem of diffusion of charged particles in interplanetary space through weak interactions with random electromagnetic fields has been considered by ROELOF [1], JOKIPII [2, 3], HASSELMANN and WIBBERENZ [4] (referred to in the following as I). The results of JOKIPII's computations have been applied to the solar modulation of galactic cosmic rays [5, 6, 7]. In I, the relativistic transport coefficients for pitch-angle and energy diffusion and parallel and perpendicular spatial diffusion were derived for an arbitrary electromagnetic spectral tensor; numerical examples were given for three specific field models. In the present paper we investigate in more detail the application of these results to the diffusive propagation observed for solar proton events.

The weak interaction theory is based on three conditions:

(i) The characteristic transfer time must be large compared with the gyration period. This is normally the case if the field fluctuations are small compared with the mean magnetic field.

(ii) The characteristic transfer time must be small compared with the sampling time (in the reference frame of the particle) needed to statistically define the field spectrum. This requires a resonably smooth spectrum.

(iii) Statistical moments higher than the second should not be "unduly large".

The first two conditions are the usual weak interaction conditions; they arise naturally in the two-timing treatment of interactions in random fields. If the conditions are satisfied, it is normally permissible to truncate the interaction analysis at

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the second moments, the contributions from higher moments then being small in proportion to a perturbation parameter characterising the weakness of the interactions. This is the case if the normalised higher order statistical moments are of the same order as the second moments, condition (iii). For most statistical fields this is a weak condition. However, the condition can become violated if the field is highly intermittent, for example, in the form of widely separated patches of turbulence, or isolated discontinuities.

For applications in interplanetary space, all three conditions can become critical. If conditions (i) and (ii) are not satisfied, it is no longer possible to describe particle diffusion by a Fokker—Planck equation. If (iii) is violated, the Fokker—Planck formalism can still be applicable, but the transport coefficients can no longer be determined by the field spectra alone. From our present knowledge of the statistical structure of interplanetary magnetic fields the limitations of the theory cannot yet be determined. Part of the discrepancy between experimental and theoretical diffusion coefficients at high levels of solar activity (see last section) may be due to violation of one or more of the conditions (i) . . . (iii).

The representation of magnetic field power spectra

We shall consider magnetic field data for 1964 and 1965. Spectral densities are presented by SISCOE et al. [8] for the Mariner 4 mission, by SARI and NESS [9] for the Pioneer 6 mission. The spectral density of a given magnetic field component is found to have approximately the same shape over periods of months, but the spectral power fluctuates up to an order of magnitude between extremly quiet and disturbed days. Over longer time intervals, the shape of the function is also found to vary. At the end of 1964 the spectra can be approximated by a $f^{-1.5}$ power law between $3 \cdot 10^{-4}$ and 0.5 Hz [8], at the end of 1965 by a $f^{-2.0}$ power law between $3 \cdot 10^{-4}$ and 0.02 Hz [9].

Experimental data on power spectra $P_\theta(k)$ for the field component perpendicular to the ecliptic plane are summarized in Fig. 1. The measured frequency spectra have been converted to wave number spectra by assuming a constant solar wind velocity $u_p = 400$ km/sec.

The data are presented in smoothed form as a sequence of points. The Mariner 4 data from COLEMAN et al. [10] correspond to the period 29 Nov.—30 Dec., 1964; they are roughly the same as intermediate values from an earlier paper by SISCOE et al. [8]. The Pioneer 6 data from SARI and NESS [9] above $5 \cdot 10^{-11}$ cm^{-1} correspond to a period of medium activity ($\bar{K}_p = 1.5$, hours 0—12 on 21 Dec., 1965) and to a period of low activity ($\bar{K}_p = 0.83$, hours 0—12 on 23 Dec., 1965). The low frequency points are from the period 30 Dec. 1965—14 Jan. 1966. We have also included the Mariner 2 data for the end of 1962 [11]. For the calculations it will be represented by a power law $\sim k^{-1}$.

Also shown in Fig. 1 are analytical expressions corresponding to a scalar spectrum

$$f(k) = C \cdot k^{-q} \exp\left\{-\left(\frac{k_0}{k}\right)^A\right\} \tag{1}$$

The exponential factor produces a flattening of the power law for small wave-numbers, beginning at $k \sim k_0$. A controls the width of the flattening range. We have taken $A = 0.6$ in both cases.

The spectrum $f(k)$ determines the full magnetic field spectral tensor for the 3 field models (a), (b), and (c) considered in I. The relation between $f(k)$ and the spectral power $P_\theta(k)$ depends on the model. The curves in Fig. 1 correspond to an *isotropic* field model (c). For calculations of the particle transport coefficients we

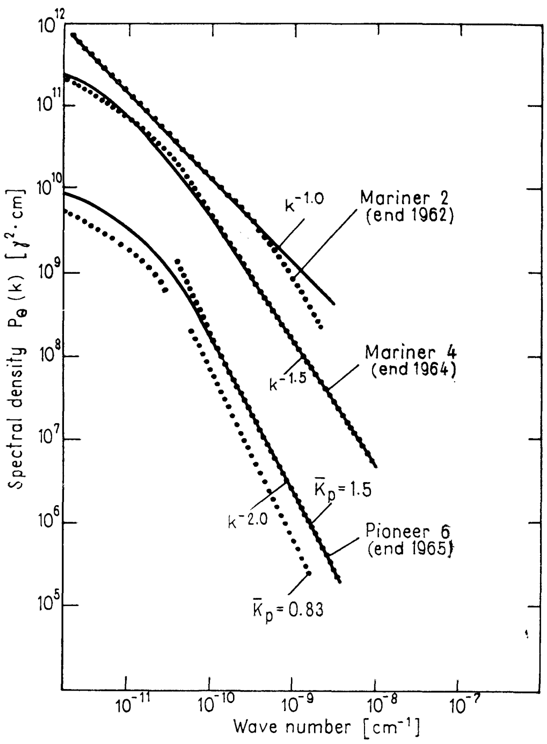

Fig. 1. Average magnetic field power spectra for different periods of time; experimental values are represented in smoothed form as a sequence of points. The full lines are analytical approximations (Eq. (1))

shall also consider model (a), an *axialsymmetric* field containing only wave numbers parallel to the mean field, with field fluctuations perpendicular to the mean field (e.g. Alfvén waves propagating in the direction of the mean field).

The rigidity dependence of the mean free path

The pitch angle diffusion coefficient D^{33} for an arbitrary spectral tensor is given in I, together with numerical values for the longitudinal spatial diffusion coefficient $K_{||}$ for different magnetic field models and pure power law spectra $f(k) \sim k^q$. It can be shown quite generally that the mean free path $\lambda_{||} \equiv \frac{3K_{||}}{v}$ is only a function of magnetic rigidity if the particle scattering occurs in an electromagnetic field with vanishing electric field components. For a pure power law with $q > -2$ the mean free path can be expressed as $\lambda_{||}(P) \sim P^{2+q}$. This simple dependence is an important consequence of the theory, as has been pointed out by several authors [1, 6].

In many cases $\lambda_{||}$ is observed to be independent of rigidity over a fairly large interval. This can be "explained" by extending the above law formally to $q = -2$, [12, 13]. It is important to note, however, that the P^{q+2}-law breaks down for $q = -2$, at least for the three general models (including isotropy) considered in (I). The reason is different for models (a) and models (b), (c). In the axialsymmetric model (a), the pitch-angle diffusion coefficient D^{33} tends very rapidly towards zero for pitch-angles near 90° as $q \to -2$ (I, Fig. 2). As a result, particles with pitch-angles close to 90° are weakly scattered, and the mean free path tends to infinity. This is caused by the behaviour of the spectrum at high wave-numbers; the divergence is not removed by limiting the spectrum at small wave-numbers. In the case of model (b) (not considered here) and the isotropic model (c) the pitch-angle scattering for pitch angles close to 90° becomes so effective as $q \to -2$ that the pitch-angle diffusion coefficient becomes infinite, and the mean free path zero. Here the divergence is caused by the steepness of the power law spectrum for small wave numbers and can be removed by including a flattening factor for low wave numbers (Eq. (1)).

The calculations in paper I for pure power law spectra have been extended to exponentially bounded, isotropic spectra of the type (1). We define $\lambda_{||}(P) = G(P/P_0) \cdot P^{q+2}$, so that the factor G describes the effect of the deviation from a power law. $P_0 = B/k_0$ (B = mean magnetic field) is a characteristic rigidity corresponding to the k_0-value in (1). The influence of the flattening factor is basically different for $q > -2$, $q = -2$ and $q < -2$.

$q > -2$

$G(P/P_0)$ approaches a constant for $P \ll P_0$; i.e., the spectral flattening at low wave numbers does not influence the scattering at high wave numbers (small rigidities).

The complete curve derived from the average Mariner 4 spectrum in Fig. 1 is shown as "(c), end 1964" in Fig. 2. The power law yields the dotted line "$q = -1.5$". The difference between the two curves is almost exactly a factor $e = 2.72$ at $P = P_0 = B/k_0$.

Jokpii and Coleman [7] have presented results for the same Mariner 4 spectrum based on the calculations of Jokpii [3, 6, 14]. For high rigidities the results are almost the same; for small rigidities, the authors anticipate a constant mean free path equal to the correlation length of the fluctuating field, which they estimate as $2 \cdot 10^{11}$ cm. However, in our view the correlation length is not a limitation of the scattering formalism (see the introductory discussions).

The results derived for the Mariner 2 data in 1962 [11] are plotted in Fig. 2 as a straight line, $\lambda_{||} \sim P$, labelled "(c), 1962".

$q = -2$

For a pure power law k^{-2} the parallel diffusion coefficient becomes zero for isotropic fluctuations. For the flattened spectrum (1), the factor G tends towards const/log (P_0/P) for $P \ll P_0$. This is indicated schematically in Fig. 2 by the dashed portion of the curve "(c), end 1965" below 100 MV. Above 100 MV, the curve has been calculated numerically from the average Pioneer 6 spectrum (see Fig. 1).

For rigidities below P_0 the dependence of $\lambda_{||}$ on P is less marked than in the case $q = -1.5$, but the result $\lambda_{||}$ = const anticipated by extrapolation of the law for $q > -2$ is not confirmed. In practice, however, it may sometimes be difficult to distinguish a constant value from a logarithmic dependence.

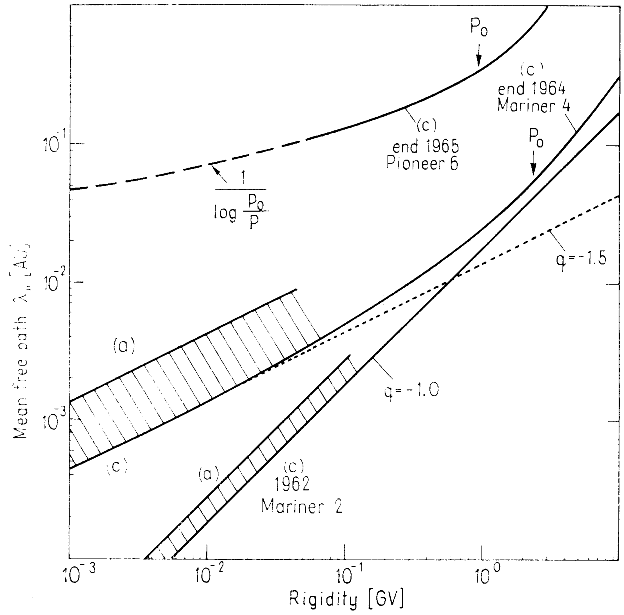

Fig. 2. Computation of the parallel mean free path based on particle-field interaction mode for the spectra shown in Fig. 1

$q < -2$

For power spectra steeper than k^{-2} the factor

$$G \to \text{const} \left(\frac{P_0}{P}\right)^{q+2} \Big/ \Gamma\left(-\frac{q+2}{A}\right) \quad \text{for } P/P_0 \to 0.$$

This yields $\lambda_{||} = GP^{q+2}$ = const. Thus a constant $\lambda_{||}$ is not quite attained for $q = -2$, but for all values $q < -2$. The constant depends on the shape of the spectrum at small wave numbers, even for small P/P_0 (large k/k_0). $\lambda_{||}$ increases with A. For $A \to 0$ (pure power law), $\lambda_{||} \to 0$, as expected.

The above discussion is based on numerical calculations for an isotropic spectrum, model (c). If the axialsymmetric model (a) is used instead, the $\lambda_{||}$-values increase by a factor 1.5 for $q = -1$, and 3.0 for $q = -1.5$. This is indicated in Fig. 2 by the hatched areas. The differences between the two models become more pronounced with increasing $|q|$. (We had already mentioned that for a pure power law the limiting case $q = -2$ yields $\lambda_{||} = 0$ for model (c) and $\lambda_{||} = \infty$ for model (a)).

The main difference between the models lies in the available resonances. In (a), only one resonance is possible ($p = -1$ in the notation of paper I, the cyclotron resonance between the motion of the particle parallel to the mean field and waves parallel to the mean field). For isotropic fluctuations the scattering at small wave numbers is governed by the $p = 0$ resonance, (Fig. 4, I) which becomes very effective if the power spectrum increases indefinitely for decreasing wave numbers. Thus the results for the particle transport coefficients depend rather critically on the spectral shape at small wave numbers. The numerical results given in Fig. 2 should therefore be regarded as illustrative. Clearly, more precise information about the structure of the magnetic fluctuations at small wave numbers is needed.

In summary, we find:

1. The numerical results depend strongly on the type of magnetic field fluctuations.

2. Exponentially bounded power law spectra give finite results for the mean free path also for large negative values of q.

3. The asymptotic behaviour of $\lambda_{||}$ for sufficiently small P is logarithmic for $q = -2$ and constant for $q < -2$.

Selection of experimental data

Time histories of solar events have been successfully described by various diffusion models [15–20]. It is still an open question, however, to which extent the models uniquely determine the value of the parallel diffusion coefficient $K_{||}(r_E)$ near the orbit of the earth.

The strong dependence of solar event intensities and time profiles on the solar longitude indicates that the diffusion must be anisotropic somewhere between the sun and the earth. The various models differ mainly in the assumed radial distribution of anisotropy.

Let us define as "solar magnetic longitude" $\theta_0 = |\theta_F - \theta_R|$ the angular distance between the location θ_F of the flare and the root θ_R of the spiral field line connecting the solar surface with the observer in space. For simplicity, we shall assume a constant value $\theta_R = 60°$. For large values of θ_0 the time profile of an event is essentially determined by the *azimuthal* diffusion. The local value of $K_{||}(r_E)$ therefore has little immediate relation to the observed intensity-time-profile. Consequently, it is not possible to determine $K_{||}(r_E)$ reliably from the diffusion models for $\theta_0 \gtrsim 50°$.

For small values of θ_0, on the other hand, the time profile up to and shortly after the maximum is determined mainly by the radial dependence of the parallel component of the diffusion tensor, $K_{||}(r)$, between sun and earth; the magnetic field is essentially radially directed, and the diffusion across the field lines has only a weak influence. This can be seen, for example, from the t_m-values obtained from the models of AXFORD [19], an extension of REID's [21] assumption, and BURLAGA [20], which can be regarded as limiting cases. For small θ_0 we have in both cases $t_m \sim r_E^2/K_{||}$ for radially directed field lines, the proportionality constant differing only by a factor 1.67.

According to REID and AXFORD, azimuthal diffusion is concentrated in the surface layers of the sun. In interplanetary field, the diffusion is entirely parallel to the field. This yields a diffusion equation (formally identical with radial diffusion in an isotropic medium)

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left\{ r^2 K_r(r) \frac{\partial \rho}{\partial r} \right\} = 0$$

where $K_r = K_{\parallel} \cos^2 \phi$ and ϕ = streaming angle between the field and the radial direction.

We shall base our estimates of diffusion coefficients on the simplest possible model $K_r(r) = \text{const} = K_0$. Inclusion of azimuthal diffusion yields somewhat higher K_r-values. For $\Theta_0 \lesssim 50°$ the difference should be within a factor of 2 (see [20] for the θ_0-dependence of t_m).

Results from solar events where the fit has been made according to other model assumptions are converted to the above simple case. Where the fit has been made according to KRIMIGIS' [18] model, $K_r(r) = K_E(r/r_E)^s$, we have set $K_0 = \frac{2-s}{2} K_E$. The relation is found to give the same t_m-values for events analyzed according to both models.

We estimate that the choice of model yields for small θ_0 uncertainties in $K_r(r_E)$ of the order of a factor 2 in either direction. The K_r-values obtained for events with $\theta_0 \gtrsim 50°$ should be considered with some caution.

Comparison with theory

We have multiplied the $K_{\parallel} = \frac{1}{3} v \lambda_{\parallel}$ values derived from particle-field interaction theory by $\cos^2 \phi = 0.5$ to obtain an equivalent theoretical K_r. This relation is in accordance with the REID/AXFORD model of negligible diffusion across the field lines and $u_p = 430$ km sec.

The results are grouped into two time intervals. Data points from 1963–1967 are plotted in Fig. 3 together with the theoretical curves from Fig. 2 for conditions close to solar minimum (1964 and 1965). The experimental results are rather well confined between the two theoretical curves. The general trend of the experimental points as a function of energy seems to be equally well described by either theoretical curve, $K_r \sim \frac{\beta}{\log \frac{P_0}{P}}$ or $K_r \sim \beta \cdot P^{1/2}$.

A one-to-one correspondence between experimental and theoretical values cannot be expected. The magnetic spectral density relevant for a given event fluctuates by at least a factor of 3 in both directions from the average values within a given period. There are further uncertainties due to the structure of the magnetic field tensor, as already discussed.

The interpretation of the March 24, 1966 event is especially interesting. The diffusion coefficient derived from the measured time of maximum intensity is so large that there are only about two mean free paths between sun and earth. (At this point, of course, the diffusion picture breaks down.) The magnetic spectral density corresponding to this mean free path in the scattering formalism is just that measured by SARI and NESS [9] during very quiet times (see the lowest curve in Fig. 1).

For periods of higher solar activity (Fig. 4), the general trend of the experimental points is not very different from Fig. 3. Some bias may be introduced by the small amount of GeV data at solar minimum and the relatively few results around 10 MeV close to solar maximum.

The values derived from the 1962 Mariner 2 data lie systematically below the experimental points. (MeV data should perhaps be omitted from the discussion. In this range, convection and adiabatic deceleration can no longer be neglected.)

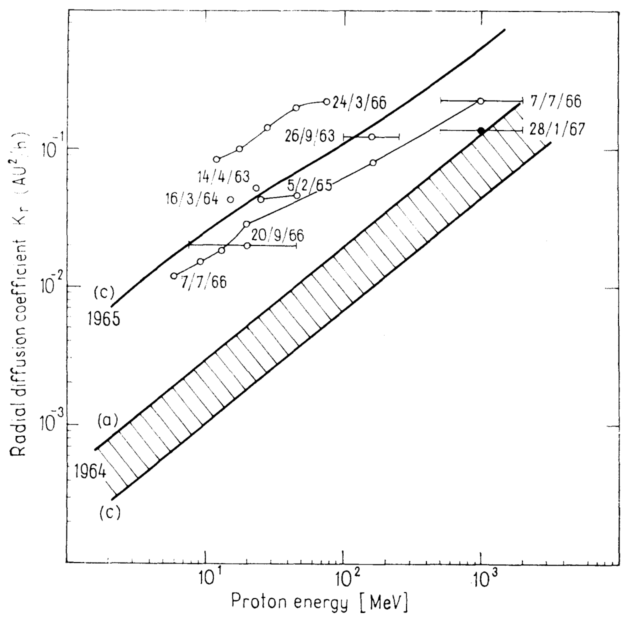

Fig. 3. Selection of experimental radial diffusion coefficients derived from solar particle propagation for the years 1963—1967 (date of event attached to the points). Results obtained during the same event are connected by thin lines. Data in Figs. 3 and 4 are taken from HOFMANN and WINCKLER [23], BRYANT et al. [24], KRIMIGIS [18], CHARACKCHYAN et al. [25], HERISTCHI et al. [26], BURLAGA [20], FICHTEL and MCDONALD [27], BOSTROM et al. [28], KRIMIGIS and VAN ALLEN [29], MCCRACKEN et al. [30], KAHLER et al. [31], LIN et al. [32], LOCKWOOD [33]

We suggest two possible explanations for the discrepancy:

a) The diffusion models used so far assume scattering throughout the path between sun and earth. If the particles can move freely along the field lines between the sun and a critical distance r_k, one can fit the observed time profile with a smaller value of the local diffusion coefficient $K_r(r_E)$. Model calculations show that the experimental values can be reduced up to a factor of 4 without extreme modification of the diffusion model.

Very low scattering in interplanetary space close to the sun would be extremely interesting with respect to the origin of magnetic instabilities in the solar wind.

b) The weak interaction theory may no longer be applicable. For the Mariner 2 spectrum and possibly also for the low frequency end of the Mariner 4 spectrum the wave-particle interaction can no longer be regarded as weak, since the characteristic transfer times are only slightly larger than the Larmor periods. Another restriction may be the patchiness of the field (higher moments non negligible). For a strongly intermittent field, the particle mean free path will be determined by the scale between patches [1] or by the mean distance between large discontinuities [22, 9] rather than the relaxation time of pitch angle scattering. (It should be noted that such an "intermittence length" is not identical with the correlation length of the field autocorrelation.) This would imply a return to the classical picture of "scattering centers".

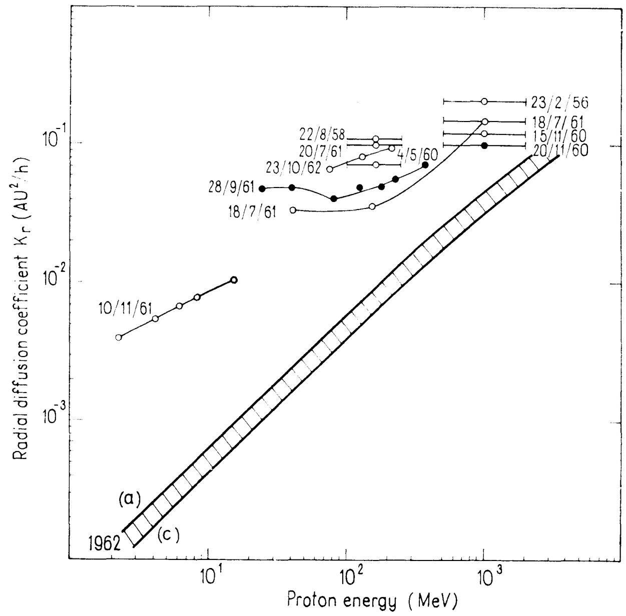

Fig. 4. As Fig. 3 for the years before 1962. Only a few events with $\theta_0 \gtrsim 50°$ have been included and are indicated as full points

In conclusion it appears that the good agreement between experimental and theoretical diffusion coefficients for periods close to minimum solar activity is rather encouraging for the weak interaction theory, but that the theory probably attains its limits for periods of high solar activity.

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