Comparison of Robustness of PID Control and Sliding Mode Control of Robotic Manipulator

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ABSTRACT
High accuracy trajectory tracking is challenging topic in robotic manipulator control. This is due to nonlinearities and input coupling present in robotic arm. This paper is concerned with the problem of modelling and control of two degree of freedom robotic manipulator. PID controller and sliding mode controller is derived so that actual trajectory tracks desired trajectory as close as possible despite of highly nonlinear and coupled dynamics. The goal is to determine which control strategy exhibit more robustness. Simulation study has been done in Matlab/Simulink environment shows that both the controllers are capable to control robot manipulator successfully. The result shows that Sliding Mode Control (SMC) produce better response compared to PID Control strategy when payload is changed.

Keywords
Robotic manipulator, PID controller, Manipulator control, Sliding mode control

1. INTRODUCTION
The dynamic of robots is described by coupled second nonlinear differential equations and inertial parameter depends on the payload which is often unknown and changes during the task. Usually in a classical control we must have an accurate model, classical control cant compensate accurate model and robust model such as sliding mode control. So effort has been made for comparison of classical control such as PID control and sliding mode control in sense of robustness.

The theory of Variable structure control has been developed firstly in Soviet Union by Emelyanov[11], introduced after by Utkin[10] and more recently studied by several authors. The robust nature of VSS is proved by the sliding mode. When the sliding mode occurs, the system will be forced to slide along or near the vicinity of the switching surface. The system became then robust and insensitive to the interaction, disturbances and variations. In addition, this does not require an accurate model of the robot.

2. ROBOT MANIPULATOR
The dynamics of robot manipulator describes how the robot moves in response to these actuator forces which apply torques at the joint of robot. For simplicity, we will assume that the actuators do not have dynamics of their own and arbitrary torques can be commanded at the joint of the robot[4].

Fig.1 shows a two link planner robot arm manipulator. This arm simple enough to simulate, yet has all the nonlinear effects common to general robot manipulators.

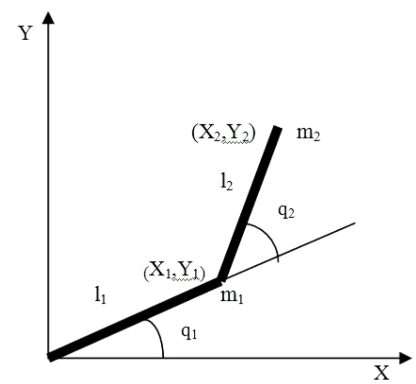

Fig.1 Two Link Planner Robotic Arm

To determine the arm dynamics, we assume that the link masses m_1 and m_2 concentrated at the ends of links of lengths l_1 and l_2, respectively. We define the angle of first link q_1 with respect to the inertial frame as depicted in fig.1. The angle of second link q_2 is defined with respect to the orientation of the first link. Torques τ_1 and τ_2 are applied by the actuators to control the angles q_1 and q_2, respectively.

The complete dynamics of two links arm[1,2] described as,

$$M(q)\ddot{q} + N(q,\dot{q}) = \tau \qquad (2.1)$$

Where the symmetric inertia matrix,

$$M(q) = \begin{pmatrix} \alpha + \beta + 2\eta\cos q_2 & \beta + \eta\cos q_2 \\ \beta + \eta\cos q_2 & \beta \end{pmatrix} \qquad (2.2)$$

and nonlinear terms,

$$N(q,\dot{q}) = V(q,\dot{q}) + G(q) \qquad (2.3)$$

Where,

$$V(q,\dot{q}) = \begin{pmatrix} -\eta(2\dot{q}_1\dot{q}_2 + \dot{q}_2^2)\sin q \\ \eta\dot{q}_1^2\sin q_2 \end{pmatrix}$$

$$G(q) = \begin{pmatrix} \alpha e_1\cos q_1 + \eta e_1\cos(q_1+q_2) \\ \eta e_1\cos(q_1+q_2) \end{pmatrix}$$

$$\left.\begin{aligned} \alpha &= (m1+m2)l_1^2 \\ \beta &= m_2 l_2^2 \\ \eta &= m_2 l_1 l_2 \\ e_1 &= g/l_1 \end{aligned}\right\} \quad (2.4)$$

This is a special form of state model called "Brunovsky canonical form". Many systems, like the robot arm are naturally in Brunovsky form.

3. DESIGN OF CONTROLLER

Defining the state vector as,

$$X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} x_{11} \\ x_{12} \\ x_{21} \\ x_{22} \end{pmatrix} = \begin{pmatrix} q \\ \dot{q} \end{pmatrix} = \begin{pmatrix} q_1 \\ q_2 \\ \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} \quad (3.1)$$

Now,

$X_1 = \dot{q} = X_2$

$$X_2 = \ddot{q} = -M^{-1}(q)[V(q,\dot{q}) + G(q)] + M^{-1}(q)\tau \quad (3.2)$$

$$= -M^{-1}(x_1)[V(x_1,x_2) + G(x_1)] + M^{-1}(x_1)\tau$$

$$= f(x) + g(x)\tau \quad (3.3)$$

Where,

$f(x) = -M^{-1}(x_1)[V(x_1,x_2) + G(x_1)]$

$g(x) = M^{-1}(x_1)$

The control law is given by,

$$\tau = g^{-1}(x)[-f(x)+u] \quad (3.4)$$

where $\dot{x}_2 = u$

Defining the tracking error as

$$e(t) = q_d(t) - q(t) \quad (3.5)$$

$\dot{e}(t) = \dot{q}_d(t) - \dot{q}(t)$

Defining $\tilde{x}_1 = e$ and $\tilde{x}_2 = \dot{e}$ (3.6)

From (2.2) and (2.4)

$\tilde{x}_2 = \ddot{q}_d - \ddot{q}$

$= \ddot{q}_d - f(x) - g(x)\tau$

The control law

$$\tau = g^{-1}(x)[-f(x) - u + \ddot{q}_d] \quad (3.7)$$

where $u = -K_1\tilde{x}_1 - K_2\tilde{x}_2$

3.1 PID controller

From (3.5)

$$\tau = g^{-1}(x)[-f(x) + (K_P + K_I)\tilde{x}_1 - K_D\tilde{x}_2 + \ddot{q}_d] \quad (3.8)$$

where K_P, K_I and K_D are the proportional, Integral and derivative gains which are to be chosen so as to minimize the tracking error[5].

3.2 Sliding mode controller (SMC)

The design of variable structure sliding mode controller consist of two phases[3,5-9]:

- Sliding (switching) surface design so as to achieve the desired system behaviour, when restricted to the surface.
- Selecting feedback gain of the controller, so that the closed loop system is stable to the sliding surface.

Here we use linear sliding surface defined by,

$$\sigma(\tilde{x}) = \lambda\tilde{x}_1 + I\tilde{x}_2 = 0 \quad (3.9)$$

Where,

$\sigma(\tilde{x}) = [\sigma_1(\tilde{x})\ \sigma_2(\tilde{x})]^T$

$$\lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

$\tilde{x}_1 = [\tilde{x}_{11}\ \ \tilde{x}_{12}]^T$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\tilde{x}_2 = [\tilde{x}_{21}\ \ \tilde{x}_{22}]^T$

Combining the (3.3), (3.5), (3.6) and (3.9) we get

$\tilde{x}_2 = -\lambda\tilde{x}_1$

so,

$$\tilde{x}_1 = \lambda\tilde{x}_1 \quad (3.10)$$

(3.10) describe the system dynamics in sliding mode (observe the order reduction of system dynamic in sliding mode). The response of the system in sliding mode is completely specified by an appropriate choice of parameter λ_1 and λ_2 of the switching surface. While in sliding mode the system is not affected by model uncertainty.

After designing the sliding surface we construct a feedback controller. The controller objective is to drive the plant state to the sliding surface, and maintain it on the surface for all subsequent time. We use a generalize lyapunov approach in constructing the controller. So that controller structure of the form

$$\tau = g^{-1}(x)\,[-f(x) + \dot{x}_{2d} + \lambda(\dot{x}_{2d} - \dot{x}_1) + K \quad (3.11)$$

Where $K=[k_1\,\mathrm{sgn}(\sigma_1)\ \ k_2\,\mathrm{sgn}(\sigma_2)]^T$

$k_1, k_2 > 0$ are the gains to be determined so that the condition $\sigma^T \dot{\sigma} < 0$ is satisfied.

4. RESULT AND ANALYSIS

In this section, the simulation results of the proposed controller, which is performed on the model of a two link robotic arm which is given in section 2 are presented. For angle q_1 and q_2 sine and cosine trajectories are chosen respectively. Comparative assessment of both control strategies to the system performance are also discussed in detail.

Table 1 Link Parameter

Link Parameter	Without Uncertainty	With Uncertainty
m_1	1 kg	1 kg
m_2	1 kg	3 kg
l_1	1 m	1 m
l_2	1 m	1 m
g	9.81	9.81

Using the values given in table.1 simulation is carried out for PID controller and SMC controller. Fig.2-5 shows the trajectory tracking when system is subjected to both the controller.

4.1 PID controller (without uncertainty)

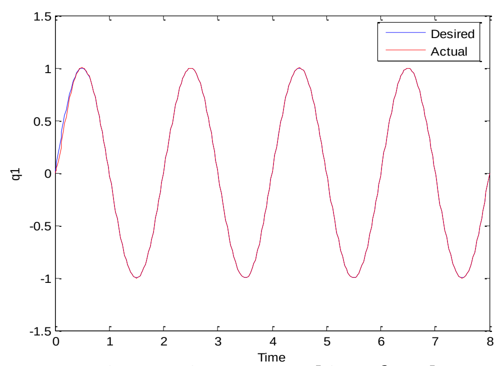

Fig.2 Trajectory Tracking of angle q_1

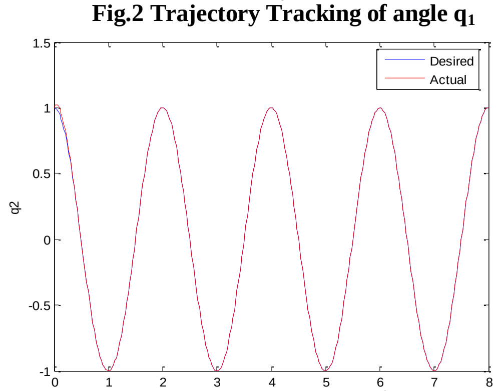

Fig.3 Trajectory Tracking of angle q_2

4.2 Sliding mode controller (without uncertainty)

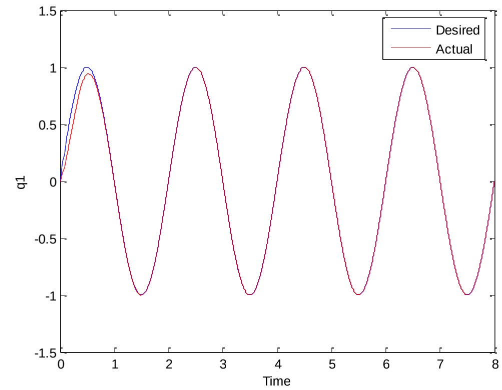

Fig.4 Trajectory Tracking of angle q_1

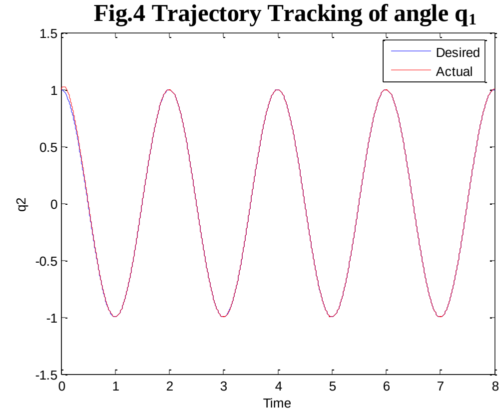

Fig.5 Trajectory Tracking of angle q_2

Now changing the values as given in table.1 the simulation result of PID controller and SMC controller are shown in fig.6, 7, 10, and 11.

4.3 PID controller (with uncertainty)

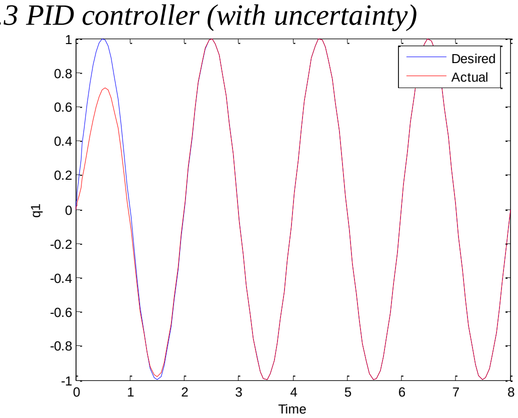

Fig.6 Trajectory Tracking of angle q_1

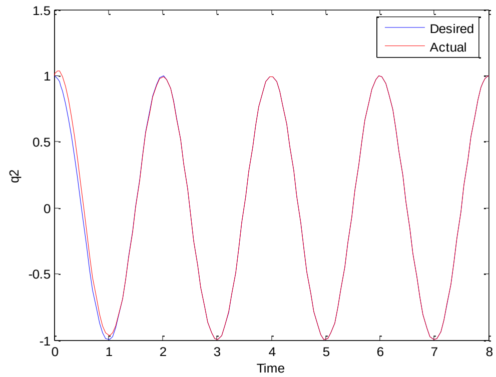

Fig.7 Trajectory Tracking of angle q_2

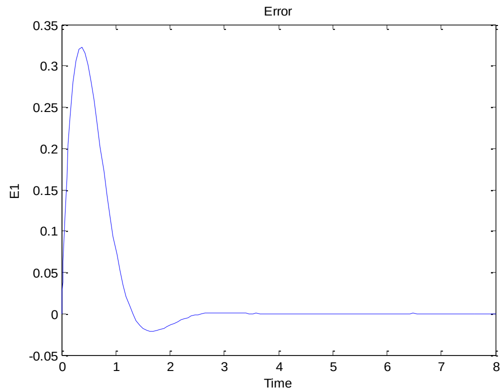

Fig.8 Tracking Error of q_1

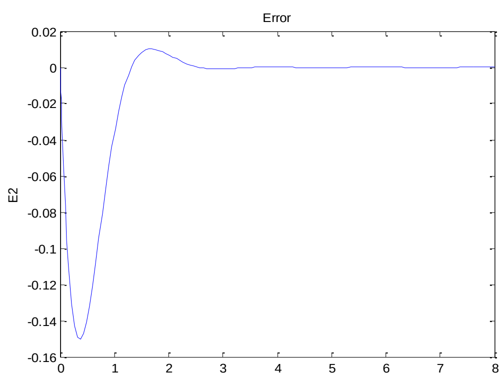

Fig.9 Tracking Error of q_2

4.4 Sliding mode controller (with uncertainty)

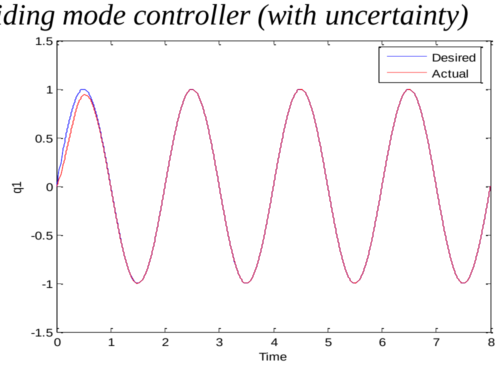

Fig.10 Trajectory Tracking of angle q_1

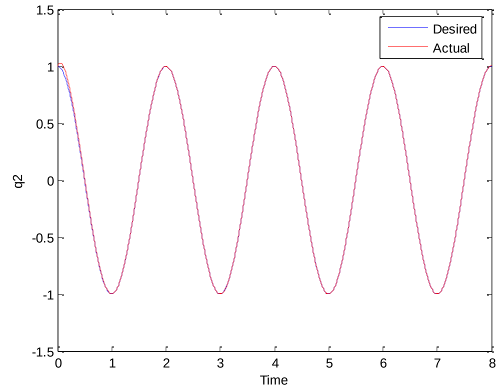

Fig.11 Trajectory Tracking of angle q_2

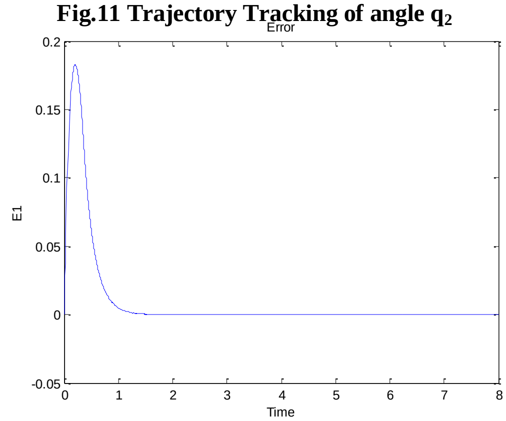

Fig.12 Tracking Error of q_1

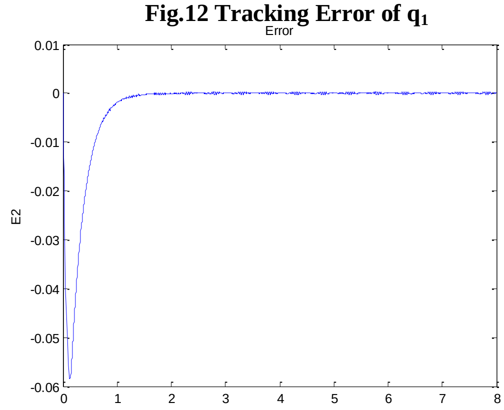

Fig.13 Tracking Error of q_2

Fig.8, 9, 12, 13 shows the tracking error. It is clear that the tracking error is increased when uncertainty is introduced. Tracking error are summarised in table.2

Without uncertainty the PID controller gives better performance than the Sliding Mode controller.

As payload is change or during uncertainty the tracking error in PID controller is increased and response is not desired at all, but in sliding mode controller we get almost response as previous. From this result it is clear that the sliding mode control is more robust then PID controller.

Fig.8, 9, 12, 13 shows the tracking error. It is clear that the tracking error is increased when uncertainty is introduced. Tracking error are summarised in table.2

Table.2 Tracking Error (with Uncertainty)

Error	PID Controller	SMC Controller
Maximum of E_1	0.3221	0.1825
Minimum of E_1	-0.0214	-2.6370e-004
Maximum of E_2	0.0101	2.6385e-004
Minimum of E_2	-0.1504	-0.0584

5. CHATTERING ELIMINATION

In order to eliminate the control input chattering problem, the boundary layer is used. The sgn(σ) in (3.11) is replaced by the sat(σ/Δ) function where Δ is boundary layer,

$$\mathrm{sat}(\xi) = \begin{cases} 1, & \xi \geq 1 \\ \xi, & -1 \leq \xi \leq 1 \\ -1, & \xi \leq -1 \end{cases}$$

Where $\xi = \sigma/\Delta$
Fig.14-17 shows the control torque input to the arm.

5.1 With SIGNUM function

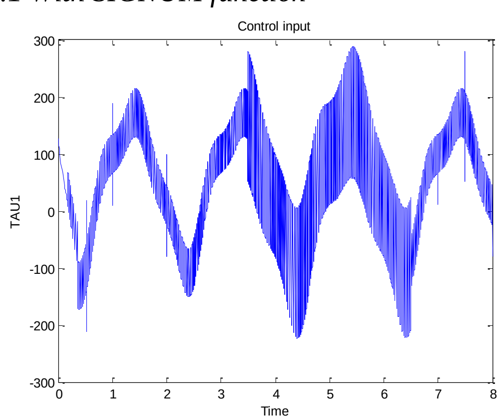

Fig.14 Control Input for Angle q_1

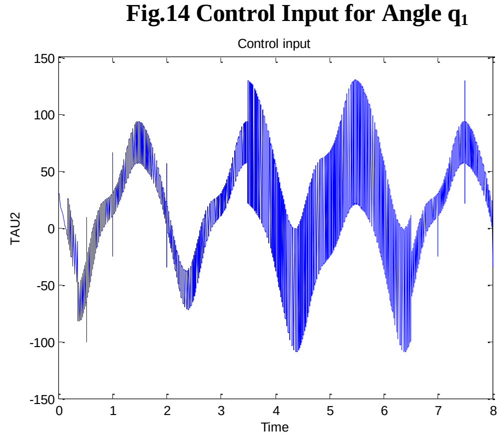

Fig.15 Control Input for Angle q_2

5.2 With SATURATION Function

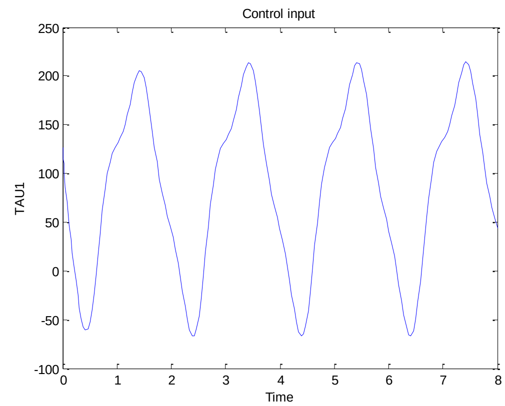

Fig.16 Control Input for Angle q_1

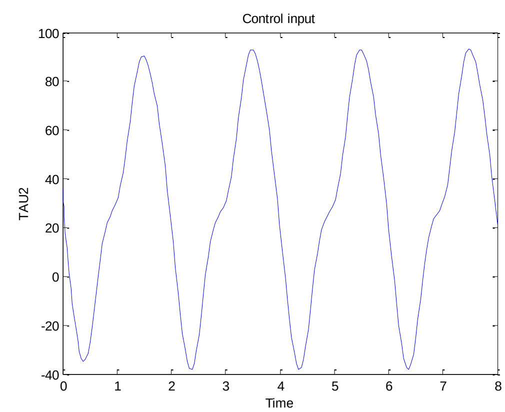

Fig. 17 Control Input for Angle q_2

6. CONCLUSION

In this paper two controller such as SMC and PID are designed successfully. Based on result and analysis conclusion has been made that both of the control method modern controller (SMC) and conventional controller (PID) are capable of controlling the robotic manipulator. Table 2 shows the value of tracking error it is clear from numerical values that SMC gives better performance than PID. Also simulation result shows that SMC controller has better performance compared to PID controller. In case of uncertainty when PID controller is used tracking error is increased but in case of SMC performance remains same so SMC controller is more robust than PID controller. The chattering phenomenon is overcome by the use of a saturation function in place of a pure signum function in the control input.

7. ACKNOWLEDGMENTS

We are very much thankful to Mr. P. K. Trivedi, Phd Scholar, IIT, Bombay and Mr.Ankit Patel, MTech, VJTI, Bombay for giving us valuable guidance in this paper.

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