Comparison of σ° Obtained from the Conventional Definition with σ° Appearing in the Radar Equation for Randomly Rough Surfaces

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Abstract—A comparison is made of the radar cross section of rough surfaces calculated in one case from the conventional definition and obtained in the second case directly from the radar equation. The objective of the analysis is to determine how well the conventional definition represents the cross section appearing in the radar equation. The analysis is executed in the special case of perfectly conducting, randomly corrugated surfaces in the physical optics limit. The radar equation is obtained by solving for the radiation scattered from an arbitrary source back to a colocated antenna. The signal out of the receiving antenna is computed from this solution and the result put into a form recognizeable as the radar equation. The conventional definition is obtained by solving a similar problem but for backscatter from an incident plane wave. It is shown that these two forms for σ° are the same if the observer is far enough from the surface; However, the usual far-field criteria are not sufficient. For the two cross sections to be the same, the observer must be far from the surface compared to the radii of curvature of the surface at the reflection (specular) points. Numerical comparison of the two cross sections has been made for normally distributed surfaces and the difference can be significant.

INTRODUCTION

IN THE CONVENTIONAL definition of radar cross section (see [2], [11], [12]) a limit is taken as the observer recedes to infinity

$$\sigma(\theta) = \lim_{R \to \infty} 4\pi R^2 \left| \frac{\bar{e}_s \cdot \bar{e}_s^*}{\bar{e}_i \cdot \bar{e}_i^*} \right| \tag{1}$$

where $\bar{e}_s$ is the scattered field and $\bar{e}_i$ is the incident electric field. In this limit, all radiation has approximately plane phase structure. Of course, in an actual radar measurement the distance between the scattering element and the observer/transformer is finite and the incident radiation does not have a truly plane phase structure. The measurement is governed not by (1) but by the radar equation, and in the radar equation the scatterer/observer geometry and the phase structure of the incident radiation must be taken into account. The validity of the conventional definition for radar cross section and any simplifying assumptions such as the use of incident plane waves to calculate the scattered fields lies in their ability to yield the same cross section as appears in the radar equation.

The objective of this paper is to seek insight into conditions under which the conventional definition yields the same cross section as appears in the radar equation. This will be done by
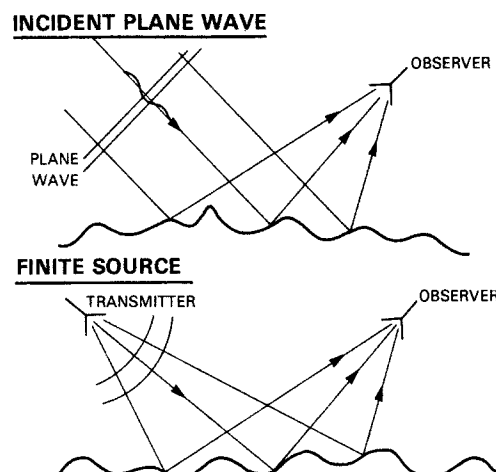
examining a special case in which both the conventional definition and the radar equation can be obtained analytically under an identical set of assumptions. The special case to be treated is that of a randomly rough conducting surface in the case of two dimensions (line sources and corrugated surfaces). This is an idealized model relevant in a first order to scattering from long crested ocean waves and perhaps to other rough surfaces such as plowed fields. A solution will be obtained in the physical optics limit, adopting a Kirchoff (i.e., tangent plane) approximation to obtain the fields on the surface and then evaluating the Helmholtz integral (Green's theorem) asymptotically in the high-frequency limit. This procedure has proven to be reasonable for microwave scattering from ocean surfaces [5], [6]. Two problems are to be solved using this approach (see Fig. 1). In the first case, a plane wave will be assumed to be incident on the surface and the fields scattered to an arbitrarily located observer will be computed. This result will then be used in the conventional definition to determine the radar cross section for the case of backscatter (monostatic cross section). In the second problem, the source of the radiation will be an antenna at an arbitrary position above the surface. The fields scattered from the surface back to a co-located receiving antenna will be obtained and will be used to compute the available power. It will be shown that this expression for power has the form of the radar equation and the term corresponding to radar cross section will be identified. It will be shown that the two forms for radar cross section are identical if the observer is far from the surface compared to the radii of

INCIDENT PLANE WAVE

FINITE SOURCE

Fig. 1. Geometry for the calculation of scattered fields.

curvature of the surface at the points of reflection (stationary points). This restriction is not a far field (e.g., Fraunhoffer) requirement. Rather it is a consequence of focusing of the scattered rays (caustics) which can occur in the physical optics approximation [14]. This focusing does not appear in the conventional definition because the observer is at infinity.

To obtain an indication of the significance of the difference between the two cross sections, numerical examples have been computed for a normally distributed random process. This is an example surface which has been used in studies of the scattering from fully developed ocean waves [3], [17]. The differences between the two cross sections can be significant, and values for representative situations are presented.

SCATTERED FIELDS

Solutions are desired for the electric field scattered from an irregularly corrugated, perfectly conducting surface in two cases: 1) the incident radiation is a plane wave, and 2) the incident radiation is produced by an antenna located at an arbitrary point above the surface. These two problems are illustrated in Fig. 1. The solutions will be specialized to the case in which there are no variations perpendicular to the plane of the figure (i.e., two dimensions) and results will be obtained explicitly for the case of backscatter.

The scattered electric field $\bar{e}_s(r, \nu)$ for both problems can be expressed in terms of the Helmholtz integral which in the case of perfectly conducting boundaries has the following form:

$$\bar{e}_s(\bar{r}, \nu) = -\iint_{\text{surface}} \frac{\partial \bar{e}}{\partial n} g(\bar{r}/\bar{r}') \, ds \tag{2}$$

where $g(\bar{r}/\bar{r}')$ is the two-dimensional Green's function $j/4 H_0^{(1)}$. $(k |\bar{r} - \bar{r}'|)$, and $\hat{n}$ is a unit vector normal to the surface $Z(y)$. The kernel $\partial \bar{e}/\partial n$ in this integral will be evaluated for both problems by making a Kirchoff approximation: That is, the fields at a particular point on the surface are assumed to be the same as would exist on an infinite plane tangent to the surface at that point. Assuming perpendicular polarization ($\bar{E}_0 = E_0 \hat{x}$), the Kirchoff approximation yields the following kernel for incident plane waves [14]:

$$\frac{\partial \bar{e}}{\partial n} = 2j (\bar{k} \cdot \hat{n}) E_0 \hat{x} e^{j\bar{k} \cdot \bar{r}} \tag{3a}$$

and in the case of radiation from the finite source the Kirchoff approximation yields the form [15]

$$\frac{\partial \bar{e}}{\partial n} = \frac{1}{2} j \sqrt{\epsilon/\mu} \, k^3 \cos(\phi - \alpha) \tilde{H}_0^{(1)}(kR) F(y, \nu) \hat{x}. \tag{3b}$$

In (3b), the function $F(y, \nu)$ is the Fourier transform of the source current density $J(z, y)$, evaluated at spatial frequencies $\nu_y = \nu/c \sin \phi(y)$ and $\nu_z = \nu/c \cos \phi(y)$. $R(y)$ is the distance from the source to the surface, $\alpha(y)$ is the slope of the surface, $\phi(y)$ is the angle $R(y)$ makes with the vertical (z-axis), and $k = 2\pi\nu/c$. The frequency of the incident radiation is ν and $\tilde{H}_0^{(1)}(kR) \cong \sqrt{2/\pi kR} \exp[j(kR - \pi/4)]$ is the asymptotic form for large kR of the Hankel function of first kind [1]. Equation (3b) was obtained by representing the source in terms of an equivalent current distribution, $\hat{x} J(z, y)$ (e.g., the

equivalent aperature illumination of the antenna). Then the Kirchoff approximation was used to find the fields on the surface taking into account radiation from the image of the source below the tangent plane. This results in an integral over the equivalent current distribution and its image which has been evaluated here using a Fraunhoffer approximation [15, appendix A and B]. This is an appropriate procedure when the antenna is small relative to the distance to the surface, a restriction which is to be distinguished from requiring that the antenna be in the far field of the scattering elements (surface). The latter restriction implies limitations on the relative size of the object while the former implies a restriction on the size of the antenna (for given distance and wavelength).

Substituting (3) into (2) and performing the integration in the limit $kR \to \infty$ by means of a saddle point approximation [8] yields the physical optics solution for the scattered fields. Assuming in the plane-wave case that the wave is incident at angle θ and that scattering is to an observer at ($y = 0, z = H$), one obtains

$$\bar{e}_s(\bar{r}, \nu) = -E_0 \hat{x} \sum_{\text{all } y_n} \frac{\cos(\theta - \alpha)}{\cos(\beta - \alpha)} e^{jk\Phi(y_n)}$$
$$\cdot \left[1 - \frac{\cos(\theta) + \cos(\beta)}{\cos \alpha \cos^2(\beta - \alpha)} \cdot \frac{R(y_n)}{R_c(y_n)} \right]^{-1/2} \tag{4a}$$

where

$$\Phi(y_n) = R(y_n) + y_n \sin \theta - Z(y_n) \cos \theta \tag{4b}$$

$$R(y) = \sqrt{[H - Z(y)]^2 + y^2} \tag{4c}$$

$$\beta = \tan^{-1} [y/(H - Z(y))] \tag{4d}$$

and $R_c(y)$ is the radius of curvature of the surface at y. The y_n are the stationary points defined by $\partial \Phi/\partial y = 0$. To obtain the radar cross section for backscatter, (4a) will be evaluated in the limit that the observer recedes to infinity ($R(y) \to \infty$) along a ray parallel to the direction of incidence θ of the plane wave. In this limit, scattering occurs at points where $\alpha = \beta = \theta$ and the result for very large $R(y)$ is

$$\bar{e}_s(\bar{r}, \nu) \cong -jE_0 \hat{x} \sum_{\text{all } y_n} \sqrt{\frac{R_c(y_n)}{2R(y_n)}} \, e^{jk\Phi(y_n)}. \tag{5}$$

These are cylindrical waves propagating from the stationary points back in the direction of the incident plane wave.

The equivalent result in the case of radiation from the antenna is obtained by substituting (3b) into (2). Assuming a source and observer co-located at ($y = 0, z = H$) one obtains the following result for backscatter:

$$\bar{e}_s(\bar{r}, \nu) = -\tilde{E}_0 \hat{x} \sum_{\text{all } y_n} \frac{\tilde{F}(y_n, \nu)}{\sqrt{2R(y_n)}} e^{j2kR(y_n)}$$
$$\cdot \left[1 - \frac{R(y_n)}{R_c(y_n)} \right]^{-1/2} \tag{6a}$$

where

$$\tilde{E}_0 = -\sqrt{\mu/\epsilon} \, e^{-j\pi/4} \sqrt{k/8\pi} \, F(\theta, \nu) \tag{6b}$$

$$\tilde{F}(y_n, \nu) = F(y_n, \nu)/F(\theta, \nu). \tag{6c}$$

The amplitude in (6a) has been normalized so that the electric field radiated by the antenna in the direction θ, the angle of incidence of the plane wave, is equal to the amplitude of the plane wave. In this normalization $F(y_n, \nu)$ is the far-field radiation pattern of the antenna. $F(\theta, \nu)$ is a constant; and if $F(\theta, \nu)$ is the maximum value of $F(y_n, \nu)$, then $\tilde{F}(y_n, \nu)$ is the relative field pattern of the source [7].

Prior to using the scattered fields given by (4) and (6) to obtain expressions for radar cross section, some comments are in order. First, the solutions for $\bar{e}_s(\bar{r}, \nu)$ are applicable to surfaces which are large compared with the distance $R(y)$, to the observer. In particular, although the assumption $kR \gg 1$ was used to justify an asymptotic expansion of the Helmholtz integral, it was not necessary to make a sagital approximation for $R(y)$ in the evaluation of (2). That is, it has not been necessary to expand $R(y)$ in a power series in y and $Z(y)$ keeping only terms up to first order in y and $Z(y)$. As a result, (4) and (6) are applicable as long as the observation point is many wavelengths above the surface (kR large) regardless of the size of the surface. This is in contrast to making a Fraunhoffer approximation in which it is necessary to impose restrictions on the size of the surface relative to the distance to the observer. (Typically $L/R < 1$ and $kL^2/R < 2\pi$ where L is a dimension characteristic of the surface.) Secondly, in the limit $R(y_n) \to \infty$ backscatter from the plane wave and finite source ((5) and (6), respectively) become very similar. If the size of the surface is restricted so that one can also do a binomial expansion of $R(y)$ about R_0, the distance from the observer to the center of the illuminated surface, then one can show that

$$\phi(y) = 2R(y) - R_0$$

and in this case, the phase factors in (5) and (6) are equal to within a constant (which is arbitrary for the plane wave). Thus, as $R(y_n) \to \infty$, one obtains

$$\bar{e}_s(\bar{r}, \nu)_{\text{Plane Wave}}$$
$$\cong -jE_0\hat{x} \sum_{\text{all } y_n} \sqrt{\frac{R_c(y_n)}{2R(y_n)}} e^{j[2kR(y_n) - R_0]} \qquad (7a)$$

$$\bar{e}_s(\bar{r}, \nu)_{\text{Source}}$$
$$\cong -j\tilde{E}_0\hat{x} \sum_{\text{all } y_n} \frac{\tilde{F}(y_n, \nu)}{\sqrt{R(y_n)}} \sqrt{\frac{R_c(y_n)}{2R(y_n)}} e^{j2kR(y_n)}. \qquad (7b)$$

Hence, in this far-field (Fraunhoffer) approximation, except for the arbitrary phase, the two solutions differ only by the factors $\tilde{F}(y_n, \nu)/\sqrt{R(y_n)}$ which are the antenna pattern ($\tilde{F}$) and the cylindrical spreading ($1/\sqrt{R(y_n)}$) present in the case of radiation from a finite source. If this ratio is kept constant as $R \to \infty$, the two solutions are directly proportional.

As a final comment consider the singular case of a flat surface, $Z(y) = 0$. In this case there is only one stationary point in both (4) and (6) and this corresponds to the ray normally incident on the surface. Of course $R_c(y_n) \to \infty$ in this case. Setting $\alpha = 0$ in (4) and letting $R_c \to \infty$ yields a reflected plane wave $\bar{e}_s = -E_0\hat{x} \exp(jkR(0))$ as expected. In (6) set-

ting $R_c \to \infty$ also yields one solution, which in this case is just radiation from the image of the antenna below the surface.

RADAR CROSS SECTION

With minor modifications to specialize the results to two dimensions, the conventional formula for radar cross section may be written [2], [10], [11]

$$\sigma(\theta) = \lim_{R \to \infty} 2\pi R \left| \frac{\bar{e}_s(\bar{r}, \nu) \cdot \bar{e}_s^*(\bar{r}, \nu)}{\bar{e}_i(\bar{r}, \nu) \cdot \bar{e}_i^*(\bar{r}, \nu)} \right| \qquad (8)$$

where $\bar{e}_s(\bar{r}, \nu)$ is the scattered field and $\bar{e}_i(\bar{r}, \nu)$ is the incident electric field. For distributed targets such as land and ocean surfaces in which there are many randomly located scattering elements per radar footprint, it is convenient to define a normalized cross section $\langle \sigma^\circ(\theta) \rangle$ [16], [18] as follows:

$$\langle \sigma^\circ(\theta) \rangle \triangleq \langle \sigma(\theta) \rangle / \text{Length}$$
$$= \frac{1}{L} \lim_{R \to \infty} 2\pi R \left\langle \left| \frac{\bar{e}_s(\bar{r}, \nu) \cdot \bar{e}_s^*(\bar{r}, \nu)}{\bar{e}_i(\bar{r}, \nu) \cdot \bar{e}_i^*(\bar{r}, \nu)} \right| \right\rangle \qquad (9)$$

where the brackets $\langle \; \rangle$ denote an ensemble (statistical) average.

Alternatively, the radar equation itself may also be regarded as the definition of radar cross section. In the case of distributed targets, one may write the radar equation for the received power, $P_r(\nu)$, in the following form in two dimensions [18]:

$$\langle P_r(\nu) \rangle = \int_{\text{surface}} \frac{4}{k} \frac{P_t(s) G_t(s) G_r(s)}{(2\pi R)^2} \langle \sigma^\circ(\theta) \rangle \, ds \qquad (10)$$

where P_t is the transmitted power and G_t and G_r are the gains of the transmitting and receiving antennas, respectively.

The objective of this paper is to compare the conventional definition for $\langle \sigma^\circ \rangle$ (see (9)) with the normalized cross section appearing in the radar equation (equation (10)). This will be done in the special case of rough, corrugated conducting surfaces using the solutions for the scattered fields obtained in the previous section. To obtain the cross section from the definition, denoted here by $\langle \sigma^\circ(\theta) \rangle_{PW}$, (4a) will be substituted into (9). To obtain the form for the normalized cross section as it appears in the radar equation, to be denoted here by $\langle \sigma^\circ(\theta) \rangle_{RE}$, first, the expression for the scattered fields (see (6)) will be used to determine the power available from the receiving antenna and then this solution will be cast into the form of a radar equation and the terms corresponding to $\langle \sigma^\circ \rangle$ identified. Finally, to make the required averages tractable, it will be assumed that the fields scatter incoherently and that the stationary points (specular points) are homogeneously distributed over the surface (i.e., spatially stationary). These latter assumptions have been used in the past to analyze scattering from rough surfaces [3], [13]. Ideally, $\langle \sigma^\circ(\theta) \rangle_{PW}$ will be identical to $\langle \sigma^\circ(\theta) \rangle_{RE}$. However, as will be shown, this is only true if the receiving antenna is far from the illuminated surface compared to the radii of curvature of the surface at the stationary points: $R(y_n) \gg R_c(y_n)$.

To begin, consider the radar cross section obtained from the definition (see (9)). Substituting (4a) into (9) and assuming that the scattered fields $\bar{e}_s(\bar{r}, \nu)$ are incoherent, one obtains

the following:

$$\langle \sigma^\circ(\theta)\rangle_{PW} = \frac{2\pi}{L} \sum_{\text{all } y_n} \left\langle \frac{\cos^2(\theta - \alpha)\cos\alpha}{\cos(\theta) + \cos(\beta)} \left| R_c(y_n)\right| \right\rangle. \tag{11}$$

In the case of backscatter, the incident and reflected rays are locally normally incident on the surface and so $\alpha = \beta = \theta$. In this case, (11) simplifies to

$$\langle \sigma^\circ(\theta)\rangle_{PW} = \frac{\pi}{L} \sum_{\text{all } y_n} \langle |R_c(y_n)|\rangle \tag{12}$$

and when the surface is homogeneous, this result may be written

$$\langle \sigma^\circ(\theta)\rangle = \pi n \langle |R_c(y_n)|\rangle \tag{13}$$

where n is the number of stationary points per unit length. This is the two-dimensional equivalent of results which have appeared in the literature on scattering from rough surfaces [3], [13]. It is a well-known result which in three dimensions has shown agreement with data on ocean surfaces. The dependence of this cross section on the radii of curvature of the surface is a characteristic of the physical optics solution [2].

To obtain an expression for the radar cross section from the radar equation, it is necessary to first compute the power available from the receiving antenna (using (6) to express the fields incident on the antenna) and then to put the result in the form of (10). To do this note that the time average power, $P_r(\nu)$ available from the receiving antenna can be written in terms of the scattered fields, $\bar{e}_s(\bar{r}, \nu)$, incident on the antenna in the form

$$P_r(\nu) = \sqrt{\epsilon/\mu}\,[\bar{e}_s(\bar{r}, \nu) \cdot \bar{e}_s^*(\bar{r}, \nu)]\,\frac{4G_r(\bar{r}, \nu)}{k} \tag{14}$$

where $4G_r(\bar{r}, \nu)/k$ is the equivalent area of the receiving antenna in two dimensions (a line source) and $G_r(\bar{r}, \nu)$ is its gain [10]. Now substituting (6a) for $\bar{e}_s(\bar{r}, \nu)$ and assuming that the scattered fields are incoherent, one obtains

$$\langle P_r(\nu)\rangle = \sum_{\text{all } y_n} \left\langle \frac{\tilde{E}_0^2 FF^*}{2R(y_n)} \left|1 - \frac{R(y_n)}{R_c(y_n)}\right|^{-1} \frac{4G_r}{k}\right\rangle. \tag{15}$$

Examination of the definition of $\tilde{E}_0$ and F (see (6b), (6c)) and comparison with the form taken by radiation from a two-dimensional source in the far field yields [14]

$$|\tilde{E}_0 F|^2 = \sqrt{\mu/\epsilon}\,\frac{1}{2\pi}\,P_t G_t \tag{16}$$

where P_t is the transmitted power. Now using (16) and assuming that the stationary points are homogeneous with density per unit length, n, and replacing the summation in (15) formally by an integral, one obtains

$$\langle P_r(\nu)\rangle = \int_{\text{surface}} \left\langle \frac{4}{k}\,\frac{P_t G_r G_t}{(2\pi R)^2}\,\left\{\pi n\left|1 - \frac{R(s)}{R_c(s)}\right|^{-1} R(s)\right\}\,ds\right\rangle. \tag{17}$$

Equation (17) is the desired form for the scattered power seen by the observer in the case of backscatter. It is to be compared with the radar equation (equation (10)) to obtain an expression for the normalized radar cross section. Comparing (17) and (10) one obtains

$$\langle \sigma^\circ(\theta)\rangle_{RE} = \pi n \left\langle R(\theta)\left|1 - \frac{R(\theta)}{R_c(\theta)}\right|^{-1}\right\rangle. \tag{18}$$

INTERPRETATION

The solutions obtained in the previous section for the fields scattered from irregular, conducting, corrugated surfaces have been used to compute the radar cross section of the surface. This was done using the conventional definition of cross section in one case and by deriving a radar equation in the other. The results in the case of incoherent scattering and homogeneous surface statistics are as follows.

Definition:

$$\langle \sigma^\circ(\theta)\rangle_{PW} = \pi n \langle |R_c(\theta)|\rangle. \tag{19}$$

Radar Equation:

$$\langle \sigma^\circ(\theta)\rangle_{RE} = \pi n \left\langle R(\theta)\left|1 - \frac{R(\theta)}{R_c(\theta)}\right|^{-1}\right\rangle. \tag{20}$$

Clearly the two forms of the radar cross section are not the same. However, when $R \gg R_c$, the denominator in (20) approaches unity and the two results agree. That is, the cross sections are equivalent if the observer is far from the surface compared to the radii of curvature of the surface at the specular points.

Notice the singularity appearing in (20) (and also in (4a) and (6a)). This singularity is a manifestation of focusing of the scattered rays which can occur in the physical optics solution (e.g., [14]). In the physical optics solution the reflected rays emerge from the surface as if reflected from a small mirror with focal length proportional to the radius of curvature of the surface at that point. If the surface is concave in the direction of the observer, the rays will converge at a point above the surface. When the normalized cross section is computed from the conventional definition, the limit is taken as $R \to \infty$. Consequently, the observer is removed beyond any potential caustic (focusing) and as a result there is no singularity appearing in the solution (see (19)). However, when the radar equation is used to determine the cross section, the observer is maintained at a finite distance from the surface, and in this case the influence of focused rays appears explicitly in the solution (see (20)). The magnitude and phase of the reflected rays depend on the focal length of the equivalent mirror and on the phase of the incident wave which illuminates the mirror. Thus, for a given mirror, incident plane waves and cylindrical waves are reflected differently. This accounts for differences in the radical in (4) and (6). Also, the location of the stationary points and therefore the radii of curvature are different in (4) and (6), and as a result the averages in (19) and (20) are not over the same points. This is so because the stationary points occur where the local angle of incidence equals the angle of reflection, and the incidence angles are different in

the two cases. When $R \gg R_c$ the observer is removed beyond any possible singularity and in this case the incident radiation is nearly a plane wave for surfaces of finite extent in both cases and the two solutions for the radar cross section are identical.

The formulas for radar cross section (see (19) and (20)) have been derived for one special example—corrugated, statistically homogeneous, conducting surfaces. However, the differences which appear in the cross sections are the result of features inherent in the method of analysis (the physical optics approximation) and not to the specific scattering object (surface) chosen. It would seem reasonable, therefore, to expect such differences in real three-dimensional configurations when the scattering is predominately specular (situations amenable to analysis by physical optics). An example is scattering at angles near nadir from the ocean surface or from rough soil [5].

Granted differences between the two cross sections, the question remains as to the significance of these differences. It is possible that $R \gg R_c$ is typical of measurements of real surfaces, in which case the differences would be of no practical consequence. To obtain an estimate of the possible order of magnitude of the difference between $\langle \sigma^\circ \rangle_{PW}$ and $\langle \sigma^\circ \rangle_{RE}$ the averages in (19) and (20) have been carried out in the case of a surface $Z(y)$ which is a Gaussian random process with a Gaussian correlation function. This is a model which has been used for the analysis of scattering from ocean waves [3], [17] and might apply to plowed fields. Since $R_c = [1 + (dZ/dy)^2]^{3/2}/[d^2Z/dy^2]$ in two dimensions, the joint densities for Z, dZ/dy and d^2Z/dy^2 are required to evaluate the averages. The calculations of these densities is straightforward for this surface [17] and one can show that given

$$f(Z) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[-\frac{Z^2}{2\sigma^2}\right] \qquad (21a)$$

$$\langle Z(y_1)Z(y_2)\rangle = \sigma^2 \exp\left[-\frac{(y_1 - y_2)^2}{l^2}\right] \qquad (21b)$$

then the joint density function for Z, dZ/dy, and d^2Z/dy^2 is

$$f(Z, Z', Z'') = f(Z')f(Z, Z'')$$

$$= \frac{l^3}{\sqrt{2}\,(4\pi)^{3/2}\sigma^2} \exp\left[-\frac{(Z')^2 l^2}{4\sigma^2}\right]$$

$$\exp\left[-\frac{12Z^2 + 4l^2 ZZ'' + l^4(Z'')^2}{16\sigma^2}\right] \qquad (22)$$

where l is the correlation length of the surface and σ is its standard deviation. The averages required to evaluate (19) and (20) are still difficult because of the singularities at $Z'' = 0$ and in (20) when $R_c = R$. These problems are avoided here by making the assumption that $\langle 1/A \rangle = 1/\langle A \rangle$. This is an ad hoc assumption made to simplify the mathematics which at least is consistent with the approximations made in arriving at (20) [3]. In particular, the asymptotic evaluation of the Helmholtz integral (equation (2)) employed here does not apply at the singular points $R = R_c$. At stationary points for which $R = R_c$ it is necessary to modify this asymptotic form by taking higher order terms [8], [9]. Doing so, one obtains finite fields at the singular points but the focused character
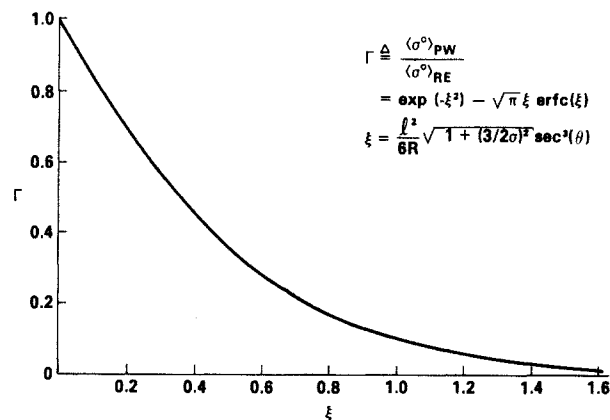

Fig. 2. The ratio Γ of the radar cross sections computed from the conventional definition and the radar cross section obtained from the radar equation in the case of a normally distributed surface with Gaussian correlation function.

TABLE I
ξ AS A FUNCTION OF l FOR $\sqrt{1 + (3/2\sigma)^2}\, s^3\, (\theta) = 1$

		l (meters)					
		500	100	50	10	5	1
R (kilometers)	0.01	4000	170	40	1.7	0.4	0.017
	0.1	400	17	4.0	0.17	0.04	0.0017
	1.0	40	1.7	0.4	0.017	0.004	0.00017
	10	4.0	0.17	0.04	0.0017	0.0004	—
	100	0.4	0.017	0.004	0.00017	—	—
	1000	0.04	0.0017	0.0004	—	—	—
	10,000	0.004	0.00017	—	—	—	—

of the radiation when R is not close to R_c remains as described above. The assumption $\langle 1/A \rangle = 1/\langle A \rangle$ avoids this subtelty and makes it possible to do the average analytically. It is imposed here in a purely ad hoc way without attempt to bound the error. Making this approximation one obtains in a straight forward manner

$$\Gamma \triangleq \frac{\langle \sigma^\circ(\theta)\rangle_{PW}}{\langle \sigma^\circ(\theta)\rangle_{RE}} \cong \frac{1/\langle |R/R_c| \rangle}{1/\langle |1 - R/R_c| \rangle}$$

$$= \exp(-\xi^2) - \sqrt{\pi}\,\xi\,\text{erfc}\,(\xi) \qquad (23)$$

where

$$\xi = \frac{l^2}{6R}\sqrt{1 + \left(\frac{3}{2\sigma}\right)^2}\, s^3\,(\theta). \qquad (24)$$

It is clear from (23) that the ratio of the two cross sections decreases from unity at $\xi = 0$ to zero at $\xi = \infty$. The behavior of $\Gamma(\xi)$ has been plotted in Fig. 2. In addition, values of ξ are listed in Table I for a range of the parameters l and R representative of the extremes one might encounter in observations of natural surfaces. Consider, as an example, a situation representative of observations from a low, earth orbiting satellite (e.g., $R = 1000$ km). In this case, it is clear from Table I that ξ is very small even for large values of the correla-

tion length l: for example, $\xi = 0.04$ when $l = 500$ m. Using $\xi = 0.04$ in Fig. 2, one finds $\Gamma \cong 1$. Consequently for observations from space it is unlikely that the differences between the cross section calculated from the conventional definition and that actually measured by a radar will be significant. In the case of observations from aircraft (e.g., 1 km $\leqslant R \leqslant 20$ km) the situation is less clear. In this case, one sees from Table I that ξ can be large for large values of the correlation length (e.g., $\xi = 1.7$ at $R = 1$ km and $l = 100$ m) in which case differences between $\langle\sigma^\circ\rangle_{PW}$ and $\langle\sigma^\circ\rangle_{RE}$ could be important, or ξ can be quite small (e.g., an observation at $R = 10$ km over plowed fields with $l = 1$ m) in which case $\Gamma \cong 1$. For tower-based operations, on the other hand, R is much smaller and differences between $\langle\sigma^\circ\rangle_{PW}$ and $\langle\sigma^\circ\rangle_{RE}$ are much more likely. For example, at $R = 10$ m and $l = 5$ m, one finds from Table I that, $\xi = 0.4$. In this case using Fig. 2 one finds $\Gamma = 0.45$ which is about 6 dB. Clearly, significant differences between the two cross sections are to be expected in such cases.

CONCLUSIONS

The results of the preceding section, and of this paper in general, are not reflections on measurement techniques. Rather, they are an indication to those concerned with the calculation of radar cross sections that the classical definition may not apply equally well in all cases. Specifically, in the case of remote sensing of rough surfaces where the scattering is predominately specular, it would appear that more specific attention needs to be given to the geometry under which an actual experiment might be performed before routinely using the classical definition to compute radar cross section.

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D. M. LeVine, photograph and biography not available at the time of publication.

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