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COMPARISON OF THE SOLAR SAIL WITH ELECTRIC PROPULSION SYSTEMS

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ABSTRACT

The propulsive efficiencies of the solar sail and of electric propulsion systems are compared on the basis of specific impulse. It is shown that the solar sail is more efficient at one a.u. for mission durations greater than about two months, that the advantage is increased for missions toward the inner planets, and that the same conclusions are reached when the comparison is based on maximizing the momentum transferred to the payload. Other factors that will influence the choice of a propulsion system for a specific mission are mentioned.

LIST OF SYMBOLS

A	surface area
C	specific power (P/m_p) at one a.u.
F_G	gravitational force of the sun on the sail at one a.u.
g	acceleration of gravity
G_s	gravitational attraction of the sun at one a.u. in Earth g's
I_{sp}	specific impulse
$\dot{m}$	mass flow rate
m_ℓ	total launch mass
m_p	fixed mass of the power plant
P	power contained in the beam of expelled particles
p_o	solar radiation pressure at one a.u.
T	thrust
t	time
U	distance from the sun in astronomical units
V^*	characteristic velocity, see Eq. (14)
W_ℓ	total launch weight
W_{ps}	total weight of propulsion system
α	tankage weight parameter, see Eq. (9)
β	$m_p/\dot{m}t$
γ	$\dot{m}t/m_\ell$
θ	incidence angle of the sun's rays
λ_s	lightness number of the sail

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The idea of using solar radiation pressure for the propulsion of space vehicles was explored by several authors in the technical literature of the late 1950's (refs. 1, 2, 3). The concept was not immediately given serious consideration because of the extremely large surface areas that are required (5.3×10^6 sq. ft. per pound of thrust at the Earth's distance from the sun). A more recent investigation (refs. 4 and 5) has described a technically feasible scheme for deploying, rigidizing and controlling the large required areas and has therefore enhanced the credibility of the concept.

Performance analyses of the solar sail have, in the past, been concerned mainly with specific missions. It is difficult to compare competing propulsion systems for specific missions unless care is taken to equalize boundary conditions (such as the characteristics of the launch vehicle) and to optimize each competing system to the same degree. This has not frequently been done, primarily because it is not easy to do. There would appear to be a need, therefore, for a simpler measure of comparison between competing space propulsion systems that could be used for gross estimates.

A standard measure of the efficiency of a propulsive device is its specific impulse, i.e., the ratio of the time integral of its thrust to its weight. Different propulsion systems can be compared on the basis of their specific impulse, although it is not the only important factor to be considered, nor is it necessarily the best measure of efficiency for any particular application. Specific impulse is, perhaps, a direct measure of efficiency only in those cases where the thrust is used to counteract other forces, as in the case of station keeping. For cases in which the thrust is used to change the velocity of the vehicle, the ratio of the momentum added to the payload to the total mass is a better performance indicator. This ratio can, however, be directly related to specific impulse as will be shown later. Other performance indicators, such as comparisons of travel times for equal payloads, generally require detailed calculations.

The specific impulse of the solar sail is computed as follows. The thrust is directed normal to the surface of the sail and has the magnitude

$$T = p_o A \cos^2\theta \frac{1}{U^2} \qquad (1)$$

where

p_o = solar radiation pressure at the Earth's solar radius

θ = incidence angle of the sun's rays

U = distance from the sun in astronomical units

A = area of the surface

The product, p_oA, is related to the lightness number of the sail, λ_s, by

$$p_oA = \lambda_s F_G \tag{2}$$

where F_G is the gravitational attraction of the sun on the sail at one a.u. Lightness number is the primary performance parameter for solar sails. It is inversely proportional to the mass per unit area. When $\lambda_s = 1.0$ the gravitational attraction of the sun is exactly balanced by the radiation pressure. The specific impulse is

$$I_{sp} = \frac{\int_o^t T\,dt}{W_{ps}} \tag{3}$$

where W_{ps} is the weight of the sail and its supporting structure. Substituting from Eqs. (1) and (2) into Eq. (3)

$$I_{sp} = \lambda_s G_s t \left(\frac{\cos^2\theta}{U^2}\right)_{avg.} \tag{4}$$

where $G_s = \frac{F_G}{W_{ps}}$ is the gravitational attraction of the sun at one a.u. expressed in Earth g's and where the quantity $\left(\frac{\cos^2\theta}{U^2}\right)$ is averaged over the mission. The value of the dimensionless constant, G_s, is $.611 \times 10^{-3}$. Note that the specific impulse is linearly proportional to lightness number and also to time. Note also that specific impulse decreases with the square of the distance from the sun.

The specific impulse of an electric propulsion device including, for example, a solar power supply and an ion engine, is computed as follows. The thrust is related to the power expended and the mass flow rate by

$$T = (2\,P\dot{m})^{\frac{1}{2}} \tag{5}$$

where $\dot{m}$ is the mass flow rate and P is the useful power that goes into the kinetic energy of the expelled particles. The useful power is related to the mass of the power plant by

$$P = Cm_p \tag{6}$$

in the case of a nuclear power plant or by

$$P = Cm_p \frac{1}{U^2} \tag{7}$$

in the case of a solar electric power plant. m_p is the fixed mass of the powerplant, U is the distance from the sun in astronomical units and C is the specific power. It includes the conversion efficiency of the power plant as a factor. Substituting Eq. (7) into Eq. (5)

$$T = \frac{(2Cm_p\dot{m})^{\frac{1}{2}}}{U} \tag{8}$$

The total launch weight of the propulsive system is

$$W_{ps} = g(m_p + (1+\alpha)\dot{m}t) \tag{9}$$

where g is the acceleration of gravity, and α is a parameter that accounts for the weight of the tankage, assumed to be proportional to the weight of expellant.

The specific impulse of the propulsive system is

$$I_{sp} = \frac{\int_o^t T\,dt}{W_{ps}} = \frac{(2Cm_p\dot{m})^{\frac{1}{2}}}{(m_p + (1+\alpha)\dot{m}t)g} \int_o^t \frac{dt}{U} \tag{10}$$

where, in the second form, a constant mass flow rate is assumed. Rearrange Eq. (10) slightly to obtain

$$I_{sp} = \frac{(2C)^{\frac{1}{2}}}{\left[\left(\frac{m_p}{\dot{m}}\right)^{\frac{1}{2}} + (1+\alpha)\left(\frac{\dot{m}}{m_p}\right)^{\frac{1}{2}} t\right]g} \int_o^t \frac{dt}{U} \tag{11}$$

The quantity, $\frac{m_p}{\dot{m}}$, is a design parameter that can be varied by changing the voltage in the ion engine. The maximum value of I_{sp} is obtained when

$$m_p = mt(1 + \alpha) \tag{12}$$

i.e., when the fixed power plant weight is equal to the fuel weight. Thus the maximum value of I_s is

$$I_{sp} = \frac{1}{\sqrt{2(1+\alpha)}\ g} \left(\frac{C}{t}\right)^{\frac{1}{2}} \int_0^t \frac{dt}{U} = \frac{(Ct)^{\frac{1}{2}}}{\sqrt{2(1+\alpha)}\ g} \left(\frac{1}{U}\right)_{av.} \tag{13}$$

Note that the specific impulse is proportional to the <u>square root</u> of the product of the specific power of the power plant and the mission time.

Comparing Eqs. (4) and (13), it is evident that the solar sail has the advantage over electric propulsion for very long missions, $t \longrightarrow \infty$.

Equations (4) and (13) are plotted in Fig. 1 for the conditions: $\theta = 30°$ in Eq. (4); $\alpha = 0$ in Eq. (13); and $U = 1.0$ in Eqs. (4) and (13). The lower limit of the range of sail lightness number, $\lambda_s = 0.50$, represents currently available 0.08 mil polycarbonate film with 1500 A° of aluminum coating, Ref. 6. $\lambda_s = 1.0$ represents a modest increase in performance over current state-of-the-art, and $\lambda_s = 2.0$ represents a substantial increase. The lower limit of the specific power of solar electric propulsion (C = 25 watts/Kg) represents the current state-of-the-art of solar electric power plants (Refs. 7, 8). The upper limit (200 watts/Kg) represents projected capabilities for nuclear-electric and other advanced systems (Refs. 9, 10). The specific impulse ranges for chemical and non-electric nuclear propulsion systems (Ref. 11) are also indicated in Fig. 1.

If the present state-of-the-art is assumed ($\lambda_s = 0.50$, C = 25 watts/Kg), the solar sail is more efficient than solar electric propulsion for mission durations greater than about 30 days. For a mission duration of 1000 days the solar sail is about 6 times as efficient as solar electric propulsion.

It is seen by comparing Eqs. (4) and (13) that the advantage of the solar sail is increased for missions toward the inner planets, $U < 1$, and is decreased for outward bound missions, $U > 1$. For outward bound missions an indirect trajectory with an initial inward loop (similar to those described in Ref. 12 for solar electric systems) may improve the relative efficiency of the solar sail.

For all applications except orientation control and station keeping, the thrust of the propulsion system acts to change the translational velocity of the vehicle. In these applications a more direct measure of efficiency than specific impulse is the ratio of the momentum added to the payload divided by the total launched mass of the vehicle. Thus we define the "characteristic velocity" of the propulsion system to be

$$V^* = \frac{(W_\ell - W_{ps})}{W_\ell} \Delta V = \frac{W_\ell - W_{ps}}{W_\ell} \int_0^t \frac{T}{m} dt \qquad (14)$$

where W_ℓ is the launch weight and W_{ps} is the weight of the propulsion system.

The characteristic velocity can be directly related to the specific impulse. In the case of the solar sail, the mass is constant with time so that

$$\Delta V = \frac{1}{m} \int_0^t T\, dt = \frac{g}{W_\ell} W_{ps} I_{sp} \qquad (15)$$

and

$$V^* = g \frac{(W_\ell - W_{ps})W_{ps}}{(W_\ell)^2} I_{sp} \qquad (16)$$

The maximum value of V^*, obtained when $W_{ps} = 1/2\, W_\ell$, is:

$$V^*_{max} = .25\, g\, I_{sp} \qquad (17)$$

In the case of electric propulsion, the mass of the system changes with time. Assume that the mass flow rate and the thrust are constant with time so that

$$V^* = \frac{W_\ell - W_{ps}}{W_\ell} \int_0^t \frac{T}{m} dt$$

$$= \left(1 - \frac{W_{ps}}{W_\ell}\right) \frac{T}{\dot{m}} \log \left| \frac{m_\ell}{m_\ell - \dot{m}t} \right|$$

$$= \left(1 - \frac{m_p + (1+\alpha)\dot{m}t}{m_\ell}\right) \frac{(2Cm_p\dot{m})^{\frac{1}{2}}}{\dot{m}} \log \left| \frac{m_\ell}{m_\ell - \dot{m}t} \right| \qquad (18)$$

The assumption that the thrust is constant with time particularizes Eq. (18)

to a nuclear power plant or to a solar power plant in Earth orbit. Equation (18) can be simplified by means of the parameters

$$\beta = \frac{m_p}{\dot{m}t} \qquad \text{and} \qquad \gamma = \frac{\dot{m}t}{m_\ell}$$

which can be selected by the designer of the spacecraft.

In terms of these parameters

$$V^* = (2Ct)^{\frac{1}{2}} (1 - \gamma(1 + \alpha + \beta))(\beta)^{\frac{1}{2}} \log \left| \frac{1}{1 - \gamma} \right| \qquad (19)$$

For the special case of $\alpha = 0$, the optimum values of the design parameters are $\gamma = 1/3$ and $\beta = 2/3$. The corresponding maximum value of the characteristic velocity is

$$V^*_{max} = .2944 \left(\frac{Ct}{2}\right)^{\frac{1}{2}} = .2944\, g(I_{sp})_{max} \qquad (20)$$

where Eq. (13) has been used in obtaining the second form.

Comparing Eq. (20) and (17), it is seen that the ratio of characteristic velocities for the optimized solar sail and the optimized electric propulsion system is approximately equal to the ratio of the optimized specific impulses, with a small advantage (18%) accorded to the electric propulsion system.

Efficiency is, of course, not the only factor to be considered. Factors that favor electric propulsion over the solar sail are:

1. Ability to vary the direction of the thrust without changing its magnitude, and vice-versa.

2. More compact configuration, and perhaps, therefore, greater reliability of deployment.

3. Angular momentum of the vehicle may be small, or zero.

Factors that favor the solar sail over electric propulsion are:

1. Less complicated hardware, and, therefore, greater reliability during operation.

2. Additional uses of the large surface area (micrometeroid flux measurement, communications, etc.).

3. Ability to exploit unexpected increases in mission duration, i.e., to continue to produce thrust for an indefinite time.

4. Lower cost. *In this connection it is worth noting that* .08 mil polycarbonate film can be obtained commercially for a price of 2 cents per sq. ft.

It has been shown that, other factors being equal, the solar sail is superior to electric propulsion for mission durations greater than about two months. In spite of its evident advantages very little effort has been expended to date on the development of the solar sail concept into a practical propulsion system.

The MacNeal-Schwendler Corporation
Los Angeles, California, May 1, 1970

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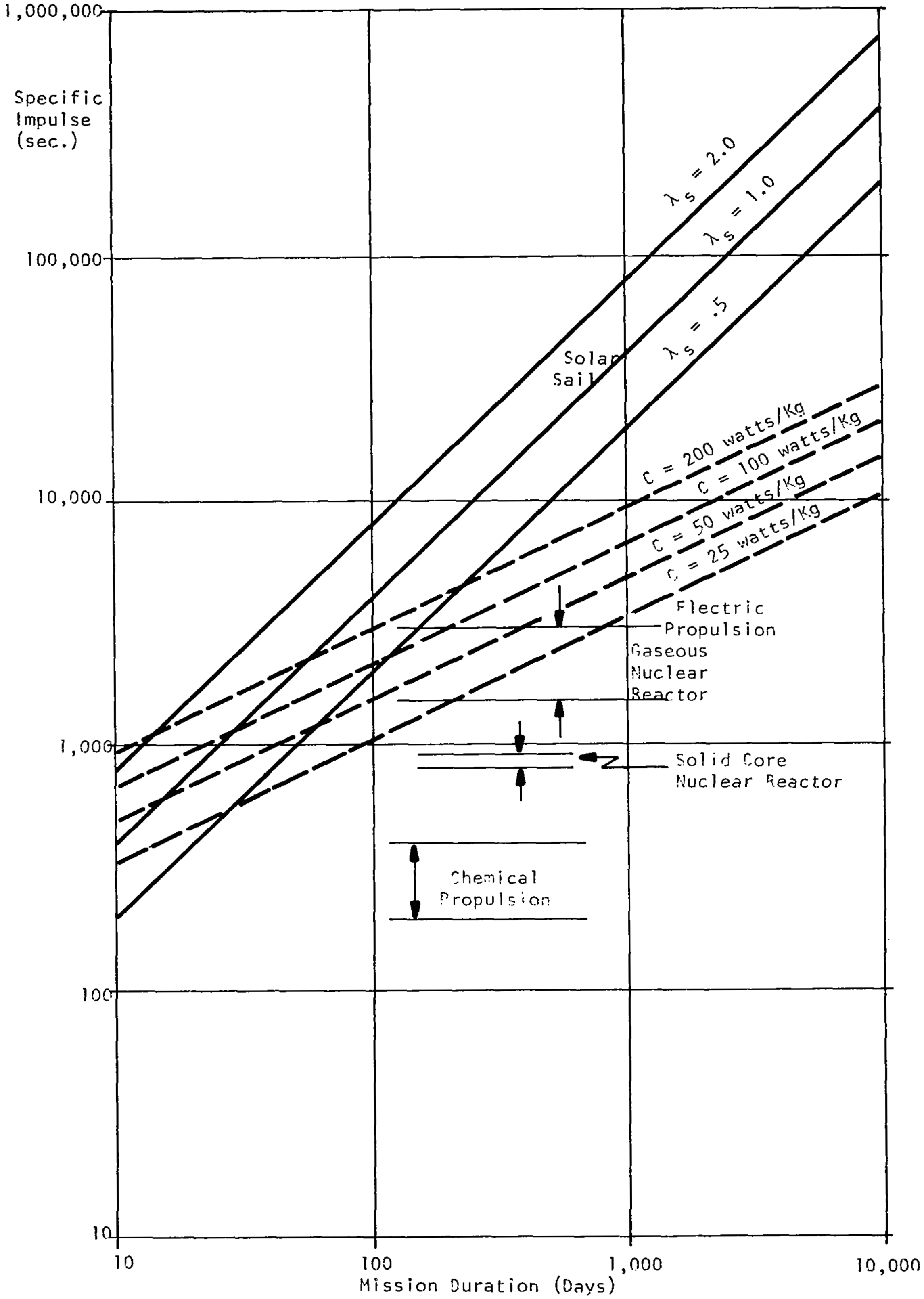

FIG. 1. SPECIFIC IMPULSE VS MISSION DURATION FOR VARIOUS PROPULSION SYSTEMS OPERATING IN THE VICINITY OF THE EARTH'S ORBIT