# Comparison of TRIAD+EKF and TRIAD+UKF Algorithms for Nanosatellite Attitude Estimation

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*Abstract:* - Two most commonly used sensors on nanosatellites are magnetometer and sun sensor. In this paper, magnetometer and sun sensor measurements are combined gyro measurements to produce enhanced attitude estimation. Tri-Axial Attitude Determination (TRIAD) algorithm is used with Kalman filter to form a complete attitude filter. Sun sensor and magnetometer measurements are selected as inputs to TRIAD algorithm and output is fed to Kalman filter as a measurement. Two different Kalman filters, extended and unscented, are used with TRIAD algorithm. A comparison is given between performances of both Kalman filter.

*Key-Words:* Nanosatellite, Magnetometer, Sun Sensor, Kalman Filter, Attitude Estimation.
Received: May 15, 2021. Revised: March 4, 2022. Accepted: April 8, 2022. Published: May 4, 2022.

## 1 Introduction

Increasing demand for the space operations, space industry turns its face to cost effective solutions. Small satellites, due to their size and cost, are receiving interest from many organizations. The amount of attitude determination and control equipment that can be placed in small satellites are considerably lower than the regular size satellite.

Kalman filter has been the backbone of the space industry since the publication of the Dr. Kalman's work [1]. Many different versions of the Kalman filter have been derived. Harold Black published an algorithm called Tri-Axial Attitude Determination (TRIAD) in 1964 [2]. It is the earliest algorithm that was published to find satellite attitude with two measurements. In 1965, Grace Wahba suggested her famous problem [3]. Solution methods such as q-method [4] and Quest algorithm have been widely used [5]. A computationally expensive method, SVD, is also published [6]. Many of the mentioned methods are coupled with Kalman filters for higher accuracy [7], [8]. In this work algebraic method is used with sun sensor and magnetometer measurements.

EKF is probably the most used version of the Kalman filter [9]. One of the earliest work of Kalman filter for attitude determination used Euler angle rotations [10]. It is known that all three-parameter representations of the special orthogonal group suffer from singularity and discontinuity problems. To overcome this challenge new representation methods have been studied [11]. Quaternions have become the most used form of the attitude representations. Euler angles, Rodrigues and modified Rodrigues parameters are avoided for most of the agile missions due to their singularity problems. In the last two decades new approaches have been suggested for replacing EKF. Unscented Kalman filter is one of them [12]. UKF uses the unscented transformation to achieve high-order approximations of the nonlinear functions in order to estimate mean and covariance of the state vector. Filter uses predefined number of sigma points to approximate Gaussian distribution. Each of the sigma points are propagated through the propagation functions [12, 13].

In this work, using common sensors, couple of filters are design to overcome to attitude determination problem. Two of most common sensors that are being used in nanosatellites are magnetometers and sun sensors. Magnetic dipole model is selected for magnetic field model. VSOP87 theory is used for sun direction vector [14]. Using these two models, sensor measurement models have been established. For attitude representation of the spacecraft Euler angles are selected. TRIAD algorithm is used for combining sun sensor and magnetometer measurements. Body angles that are produced by TRIAD are used as linear measurements to the Kalman filter. Algorithm is constructed for both Extended and Unscented Kalman filters for comparison.

# 2 Mathematical Model

Satellite kinematic, dynamic and sensor models are derived in this section.

## 2.1 Satellite Kinematic and Dynamic Model

Firstly, for kinematic model, rate equations are given below

$$\dot{\phi} = \omega_u \cos\psi - \omega_v \sin\psi \tag{1}$$

$$\dot{\theta} = (\omega_u \sin\psi + \omega_v \cos\psi)\sec\phi \tag{2}$$

$$\dot{\psi} = \tan\phi(\omega_u \sin\psi + \omega_v \cos\psi) + \omega_w \tag{3}$$

where $\phi$ is the roll angle about $x$ axis, $\theta$ is the pitch angle about $y$ axis, $\psi$ is the yaw angle about $z$ axis, $\omega_u$, $\omega_v$ and $\omega_w$ are the components of $\omega_{BR}$ vector in body frame with respect to the reference frame.

Rotations need to be defined with respect to inertial frame.

$$\begin{bmatrix} \omega_u \\ \omega_v \\ \omega_w \end{bmatrix} = \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} - A \begin{bmatrix} 0 \\ -\omega_0 \\ 0 \end{bmatrix} \tag{4}$$

where $w_x, w_y$ and $w_z$ are the components of vector $w$ in body frame with respect to the inertial frame, A is the attitude transformation matrix composed of Euler angles, $\omega_0 = \sqrt{\frac{\mu}{r^3}}$ defines the orbital angular velocity with respect to the inertial frame, $\mu$ is the gravitational constant of the Earth and r is the distance between center of mass of the satellite and the Earth, $r = |\vec{r}|$.

Attitude dynamics is related with time derivative of the angular momentum vector. If the origin of the body frame is selected as the center of mass, then

$$h_c = J\vec{w} \tag{5}$$

where h is the angular momentum vector, J is the moment of inertia matrix.

Relation between time derivative of the angular momentum and the angular velocity is

$$\dot{\vec{h}}_c = \dot{\vec{h}}_b + \vec{w} \times (\vec{h}_c) \tag{6}$$

where $h_b$ is the time derivative of angular momentum in body-fixed frame.

Rewriting derivative equations for discrete time we have,

$$w_{x_{k+1}} = w_{x_k} + \frac{\Delta t}{J_x}\left[w_z w_y \left(J_y - J_z\right) + T_m\right] \tag{7}$$

$$w_{y_{k+1}} = w_{y_k} + \frac{\Delta t}{J_y}\left[w_x w_z \left(J_z - J_x\right) + T_m\right] \tag{8}$$

$$w_{z_{k+1}} = w_{z_k} + \frac{\Delta t}{J_z}\left[w_x w_y \left(J_x - J_y\right) + T_m\right] \tag{9}$$

where $T_m$ is the external torque and $\Delta t$ is the sampling time interval.

Equations above describe the satellite attitude motion.

## 2.2 Sensor Models

Magnetometer is the most commonly used sensor particularly in nanosatellite applications. For magnetic field vector, dipole model is used. Sensor model is given below,

$$\begin{bmatrix} B_x(\phi,\theta,\psi,t) \\ B_y(\phi,\theta,\psi,t) \\ B_z(\phi,\theta,\psi,t) \end{bmatrix} = A \begin{bmatrix} B_1(t) \\ B_2(t) \\ B_3(t) \end{bmatrix} + b_m + \eta_m \tag{10}$$

where $B_1(t)$, $B_2(t)$ and $B_3(t)$ indicates Earth magnetic field vector components in orbit frame, $B_x(\phi,\theta,\psi,t)$, $B_y(\phi,\theta,\psi,t)$ and $B_z(\phi,\theta,\psi,t)$ show the Earth magnetic field vector components in body frame as a function of body angles and time. The magnetometer bias vector is given as $b_m = \begin{bmatrix} b_x & b_y & b_z \end{bmatrix}^T$. Bias vector is modeled via the expression below:

$$\dot{b}_m = 0 \,. \tag{11}$$

The noise term, $\eta_m$, is added linearly to the model and modeled as zero mean Gaussian white noise with the characteristic of

$$E\left[\eta_{1k}\eta_{1j}^T\right] = I_{3x3}\sigma_m^2\delta_{kj} \tag{12}$$

where $I_{3x3}$ is the identity matrix, $\sigma_m$ is the standard deviation of magnetometer errors and $\delta_{kj}$ is the Kronecker symbol.

In order to construct a sun sensor model, sun direction vector is used. Using VSOP87 theory, a direction cosine matrix is calculated which shows the sun's position relative to Earth in ECI frame [14].

$$\begin{bmatrix} s_{B_x} \\ s_{B_y} \\ s_{B_z} \end{bmatrix} = A \begin{bmatrix} s_{E_1} \\ s_{E_2} \\ s_{E_3} \end{bmatrix} + \eta_s \tag{13}$$

Construction of sun direction vector, $s_E$, requires two assumptions. Comparing the distance between Sun-Earth, 1 AU, and Earth-satellite, satellite altitude is negligible. Therefore, satellite's sun direction vector is always parallel to Earth's sun direction vector. The other assumption is taking the right ascension node of Sun's orbit as zero. The sun direction vector,

$$s_E = \begin{bmatrix} \cos\lambda_{ecliptic} \\ \sin\lambda_{ecliptic}\cos\varepsilon \\ \sin\lambda_{ecliptic}\sin\varepsilon \end{bmatrix} \tag{14}$$

The noise term, $\eta_s$, is added linearly to the model and modeled as zero mean Gaussian white noise with the characteristic of

$$E\left[\eta_{1k}\eta_{1j}^T\right] = I_{3x3}\sigma_s^2\delta_{kj} \tag{15}$$

where $I_{3x3}$ is the identity matrix, $\sigma_s$ is the standard deviation of sun sensor errors.

The gyro model is constructed with satellite dynamic equations. A commonly used model for gyro measurements given by

$$\hat{\omega}_{BI} = \omega_{BI} + b_g + \eta_g \tag{16}$$

where $b_g$ is the gyro bias vector and the $\eta_g$ modeled as zero mean Gaussian white noise with the characteristic of

$$E\left[\eta_{1k}\eta_{1j}^T\right] = I_{3x3}\sigma_g^2\delta_{kj} \tag{17}$$

where $I_{3x3}$ is the identity matrix, $\sigma_g$ is the standard deviation of gyro errors. Gyro bias vector is modeled as,

$$\dot{b}_g = 0\,. \tag{18}$$

# 3 Filter Design

## 3.1 TRIAD

A rotation matrix describes the attitude of a spacecraft with respect to a known reference frame. It takes at least two measured vectors to determine the orientation of the vehicle. The algebraic method constructs two triads of orthonormal vectors, two triads are expressed by sun sensor and magnetometer unit vectors in body and reference frames. Let magnetometer measurement vector is denoted by B and sun sensor vector is denoted by S. For initial base vector, sun sensor is selected.

$$\begin{aligned} u &= S \\ v &= S\times B / \left|S\times B\right| \\ r &= u\times v \end{aligned} \tag{19}$$

It is important to note that two vectors, S and B cannot be parallel, $\left|\vec{S}.\vec{B}\right| < 1$. Constructing the direction cosine matrix,

$$A_{rb} = \left[\vec{u}_r \quad \vec{v}_r \quad \vec{w}_r\right]\left[\vec{u}_b \quad \vec{v}_b \quad \vec{w}_b\right]^T \tag{20}$$

Final step of the TRIAD is to find error covariance of the algorithm

$$P = \frac{\sigma_m^2\vec{S}_b\vec{S}_b^T + \sigma_s^2\vec{B}_b\vec{B}_b^T}{\left\|\vec{S}\times\vec{B}\right\|} + \sigma_s^2\vec{v}_b\vec{v}_b^T \tag{21}$$

Body angles and error covariances that are obtained from TRIAD algorithm are used as measurement inputs to the Kalman filter. These results are combined with gyro measurements in order to achieve better accuracy.

## 3.2 Extended Kalman Filter

In 1960, Dr. Kalman published his famous paper "A new Approach to Linear Filtering and Prediction". It represented a sequential solution to the time-varying filtering problem. The filter is particularly good for dynamical systems and also removed the non-dynamical requirements of Weiner filter [1]. The extended Kalman filter uses the first order Taylor series linearization for non-linear systems. This process requires two assumptions. Propagation and measurement functions must be differentiable. Filter algorithm is given below. Propagation of state vector is conducted with equation,

$$\hat{x}_k^- = f\left(\hat{x}_{k-1}^+\right) \tag{22}$$

Equation of state estimation,

$$\hat{x}_k^+ = \hat{x}_k^- + K\left(z_k - h\left(x_k^-\right)\right) \tag{23}$$

Here, h is the measurement function and K is the Kalman gain,

$$K = P_k^- H^T \left[H P_k^- H^T + R\right]^{-1} \tag{24}$$

where R is the covariance matrix of measurement error, H is the measurement matrix of system, $P_k^-$ is the covariance matrix of extrapolation error.

Covariances of both extrapolation and filtering errors are given below. For error equations, Jacobian matrix F is needed. Jacobian matrix can be constructed by taking partial derivatives of propagation function with respect to state variables

$$P_k^- = F P_{k-1}^+ F^T + Q \tag{25}$$

$$P_k^+ = \left(I - KH\right) P_k^- \tag{26}$$

where F is the Jacobian matrix and Q is the covariance matrix of process noise.

### 3.3 Unscented Kalman Filter

EKF, due to nature of the first-degree linearization, is not as accurate for highly linear systems. Jacobians are hard to derive and the linearization needs very short time intervals otherwise filter becomes unstable. But this comes with the computational power gets higher. The main idea behind the UKF is distributions are easier to approximate from nonlinear functions [13]. Therefore, it introduces “sigma points”. With these points, filter removes need for derivation of Jacobian matrix and it is more robust to the initial estimation errors [15]. Sigma point can be obtained by [16]

$$\chi_0 = x \tag{27}$$

$$\chi_i = x + \left(\sqrt{\left(L+\lambda\right) P_k}\right)_i \quad i = 1,....L \tag{28}$$

$$\chi_i = x + \left(\sqrt{\left(L+\lambda\right) P_k}\right)_{i-L} \quad i = L+1,....2L \tag{29}$$

$\chi$ is denotation for the sigma vector. In order to capture the true nature of the reflection, sigma points are weighted

$$W_0^{(m)} = \frac{\lambda}{\left(L+\lambda\right)} \tag{30}$$

$$W_0^{(c)} = \frac{\lambda}{\left(L+\lambda\right)} + \left(1 - \alpha^2 + \beta\right) \tag{31}$$

$$W_i^{(m)} = W_i^{(c)} \tag{32}$$

where $\lambda = \alpha^2\left(L+\kappa\right) - L$ is the scaling parameter, $\kappa$ is the secondary scaling parameter, $\alpha$ determines how spread the sigma points are and is used to incorporate prior knowledge for x. Using the weights, propagated state matrix can be determined by,

$$\hat{x}_k^- = \sum_{i=0}^{2L} W_i^{(m)} Y_i \tag{33}$$

And propagated covariance

$$P_k^- = \sum_{i=0}^{2L} W_i^{(c)} \left[Y_i - \hat{x}_k^-\right]\left[Y_i - \hat{x}_k^-\right]^T + Q \tag{34}$$

where Q is the process noise. Obtaining the Kalman gain is similar to EKF. Hence,

$$K = P_{xy} P_{yy}^{-1} \tag{35}$$

where $P_{xy}$ and $P_{yy}$ are,

$$P_{yy} = \sum_{i=0}^{2L} W_i^{(c)} \left[Z_i - \hat{z}_k^-\right]\left[Z_i - \hat{z}_k^-\right]^T + R \tag{36}$$

$$P_{xy} = \sum_{i=0}^{2L} W_i^{(c)} \left[Y_i - \hat{z}_k^-\right]\left[Z_i - \hat{z}_k^-\right]^T \tag{37}$$

The last part is to update both state and covariance matrix with measurements

$$\hat{x}_k = \hat{x}_k^- + K\left(z_k - \hat{z}_k^-\right) \tag{38}$$

$$P_k = P_k^- - K P_{yy} K^T \tag{39}$$

where $z_k$ is the measurement vector.

## 4 Simulation

Simulations are performed with a hypothetical nanosatellite with principal moment of inertia J = diag(2.1x10e-3 2x10e-3 1.9x10e-3).

Simulations are conducted with a sampling time of 1 second. Orbit of the satellite is a circular orbit with an altitude of 400 km. Argument of perigee of the orbit is 207.4 degrees and inclination of the orbit is 97.65 degrees.

Figures below give attitude estimations in terms of Euler angles. First figure shows estimation with only TRIAD algorithm. Figure 1 and 2 show attitude estimations combining TRIAD and two different Kalman filters

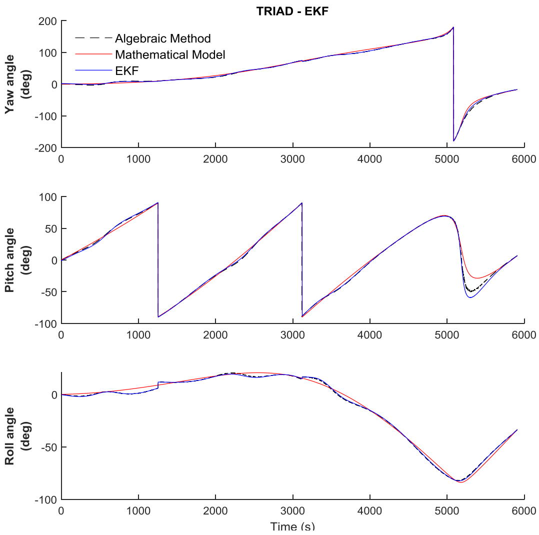


Fig. 1: TRIAD-EKF attitude estimation

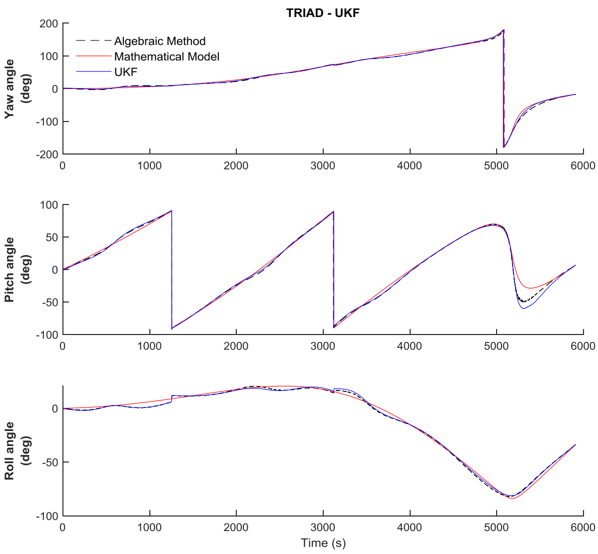


Fig. 2: TRIAD - UKF attitude estimation

Mean square errors of each method are given in Table 1.

Table 1: RMSE values

| | Roll (deg) | Pitch (deg) | Yaw (deg) |
|---|---|---|---|
| **TRIAD** | 6.2395 | 7.2175 | 11.4611 |
| **EKF** | 4.5339 | 5.4990 | 10.5363 |
| **UKF** | 3.4718 | 6.1306 | 10.7370 |

RMSE results clearly show that Kalman filters increase quality of estimation compared to TRIAD-only algorithm. Both extended and unscented Kalman filters estimated similarly. The only significant difference in RMSE values is at roll estimation.

## 5 Conclusion

Algebraic method, even though it is an aging algorithm, can estimate satellite attitude well. The sun sensor and magnetometer are selected for inputs to algebraic method because of their wide usage in space industry. Many different filtering algorithms are presented to this day but proven algorithms are still getting attention from engineers. Extended Kalman filter proves itself on many missions. Therefore, selecting the EKF for this work was a must. As expected, it performs really well on simulations especially in low initial angular velocity cases. UKF is known for its superiority to EKF when dealing with nonlinear functions. In many simulations attempts, UKF performed relatively same as EKF.

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