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## AUTHORITY

## USNWC LTR, 24 MAR 1972

# COMPARISON OF WAVEGUIDE AND WAVE HOP TECHNIQUES FOR VLF PROPAGATION MODELING 




ABSTRACT. Several mathematical models for deecribing VLF radio wave propagation in the earth-Lonosphere waveguide have been presented in the literature. The Wave Hop model and the Waveguide mode model are investigated.

The computerised versions of these propagation models are examined by comparing the computed electric field atrengthe obteined for exch model when loing the same input parameters. It is found that the two models as they now exist do not produce exactly the same computational results, and that the degree of difference between the two computations is dependent upon propagation frequiency and the electrar-density profile used for the ionosphere.


# NAVAL $\dot{W}$ EAPONS CENTER AN AOTIVITY OF THE NAVAL MATERIAL COMMAND 

M. N. Fitheridge, CAFF. Used
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M. a. Willeon


## FOREWORD

er The pook dacertbod in this report was conducted an part of a conMminna/aVMHulheg questigation to evaluate VLI propagation modeling techmiquea* In. anam. The emank wee performad in the Space Geophyoice Diviolon of the Research Department, under the ponsorship of the Defenee Atomic Suppqrt Agqncy (DASA) (Code RMAD). Fundine was provided by MIPR 543-70.

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## ERRATA

Naval Weapoas Center. COMPARISON OF WAVEGUIDE AND WAVE HOP TECRNIQUES FOR VLF PROPAGATION MODELING, by D, G. Morfitt and R. F. Halley. Chin Lake, Calif., NWC, 4 Arugut 1970. 58 pp . (NWC TP 4952.)

Make the following pen and ink changes:

1. Page 36, Figure 13. In legend change frequency 28.125 kHz to 15.567 kHz .
2. Page 46. Equation (3) in the summation $\sum_{j=0}^{N} E_{j}$, charge $j=0$ to $j=1$.

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## INTRODUCTION

The Naval Weapons Center Corona Annex (NWCCA) has been tasked by the Defense Atomic Support Agency (DASA) to investigate, with the aid of the oblique-incidence VLF lonospheric sounder developed at the Corona Annex (aee Ref. 1), the accuracy with which VLF radio-wave propagation car be predicted. This is to be accomplished by correlating the sounder signals experimentally recorded at opecific locations along a propagation path with the values predicted by various existing theoretical VLF propagation models.

Several models Epplicable to VLF radio-wave propagation have been described in the literature. Some of these have been made available in the form of digital computer programs which provide a convenient means for obtaining full-wave calculation of VLF field atrengthe. An excellent summary of these existing VLF computer codes is presented in Ref. 2.

The objective of the VLF program at Corona is to determine which propagation environirients can be adequately represented or modeled by the existing computational techniques. Since certain environmental conditions cannot be handled with the computational techniques presently in use, an attempt is being made to determine the need for, or importance of, firther refinements in the propagation modele.

The VLF propagation model initially incorporated into the propa gation studies at the Corona Annex was the Wave Hop theory developed by L. A. Berry of the Envirommental Science Services Administration (ESSA) (Ref. 3). This model was chosen initially because it was readily available in the form of a FORTRAN language digital computer program referred to as WAVEHOP (Ref. 4). Results obtained at the Corona Annex for the VLF propagation environment using this model have been documented in Ref. $5,6,7$, and 8 .

Under certain propagation conditions major inaccuracies exist in the generally avallable WAVEHOP computer program (see Ref. 4). The WAVEHOP program presently being used at Corona has been modified
to correct for these deficiencies. A discussion to demonstrate the importance of these corrections is presented in the Appendix.

One of the inputs to the propagation model for predicting the VLF signal strength as a function of propagation range is an electron-density profile of the ionosphere. The "best-fit" profiles obtained in matching experimental sounder data for a variety of environmental conditions for daytime and nighttime propagation are discussed in the previously referenced Corona Annex publications.

It is of interest to compare the field-atrength levels computed by WAVEHOP to those predicted by VLF propagation models developed at other organizations. A second computerized propagation model (see Ref. 9), based on waveguide mode theory and developed at the Naval Electronics Laboratory Center (NELC), San Diege, has been adapted to the Corona Annex computer, an IBM 360/50, to make theoretical calculations of VLF fields.

This report presents a comparison of the electric field atrengths predicted by the ESSA WAVEHOP and the NELC WAVEGUIDE computer programs for the same set of environmental input conditions.

## THE PROPAGATION MODELS

## THE WAVE HOP MODEL

Of the several VLF radio propagation models described in the literature, most are based on waveguide mode theory. The Wave-Hop theory (see Ref. 3) provides full-wave solutions for the propagation of VLF radio waves between a homogeneous, spherical earth and an arbitrary, stratified ionosphere. This model is analytically and numer ically equivalent to VLF mode theory, but the concepts are asymptotically related to HF ray-hop theory or geometric optics. This model decomposes the mode solutions into more geometrically meaningful components referred to as wave hops. The field at some distant point is considered to be the sum of the individual rays or hops, such as the direct or ground wave, plus the ray that has been reflected once from the ionosphere, plus the ray that has been reflected twice from the ionosphere and once from the ground, and so on.

Since the different rays will arrive at the receiver at different times because of the different lengthe of the paths they hive traveled, they are sometimes called time-modes to indicate this separation in time.

It is important to point out that the formulas used to compute the wave hops can be derived rigorously from the same complex propagation integral as found in the usual waveguide napde theories. The individual wave hops are computed taking into account the effects of diffraction and surface wave propagation.

The actual computer program for computing the radial electric fields using the Wave-Hop model consiste of two parts, TUIK and WAVEHOP (see Ref. 4). The first part, TUIK, calculates the ionospheric reflection coefricients of an arbitrary stratified ionosphere as a function of the angle of the wave hop on the ionosphere, the propagation frequency, the magnitude and dip angle of the earth's magnetic field, the magnetic direction of propagation, a collision frequency profile, and an electron-density profile. The WAVEHOP program computes the vertical electric field at the receiver as the vector sum of individual hops and the ground wave. The inputs to WAVEHOP are the ionospheric reflection coefficients from TUIK, the propagation frequency, ground conductivity, relative dielectric constant of the earth, the propagation range, and the effective ionos pheric reflection height.

## THE WAVEGUIDE MODE MODEL

The waveguide moile theory as developed at NELC (see Ref. 10, 11, and 12) obtains the full-wave modal solution for a waveguide whose upper boundary has an arbitrary electron density distribution with height and whote lower boundary is a smooth homogeneous earth. In this theory, the electromagnetic waves are considered to propagate between the earth and the ionosphere as normal modes, analogous to microwave propagation in a lossy waveguide.

The modal equation for propagation within the earth ionouphere waveguide is solved for as many modes as desired. The eigenvalues (or eigenargles) so obtained are then used in a modal summation to compute the total field at some distant point from the transmitter. The effect of earth curvature are included in the calculation. The eigenangles are the angles of incidence at the height where the modified index of refraction becomes unity. The radial electric field is computed
as a function of the earth's magnetic field parameters, a collision frequency profile, an electron denstity profile, propagation frequency, and the relative dielectric constant and conductivity at the earth over the propagation path.

The computer program developed around this mode theory is documented in Ref. 9. The computer program will be referred to as WAVEGUIDE.

## PROGRAM COMPUTATIONS

Among the input parameters to these computerized propagation models is an electron density profile of the ionosphere. The profiles chosen for thes. comparisons are the exponential distributions described in Ref. 13. These profiles were chosen as they are the most commonly referenced in the literature for theoretical calculations of VLF propagation. The values of electron denuity $N(Z)$ as a function of hei,ght Z , in kilometers, are calculated froin the equation

$$
N(Z)=N_{0} \exp \left(\beta-0.15-\beta h^{\prime}+0.15(70)\right.
$$

where

$$
\begin{aligned}
\beta & =0.5 \mathrm{~km}^{-1} \\
\mathrm{~N}_{0} & =393 \text { electrons } / \mathrm{cm}^{3} \\
\mathrm{~h}^{\prime} & =70,75,80,85, \text { and } 90 \mathrm{~km}
\end{aligned}
$$

where $h^{\prime}=70 \mathrm{~km}$ corresponds to the ambient daytime profile and $h^{\prime}=90 \mathrm{~km}$ to the ambient nighttime profile. Figure 1 shows the height versus electron density relationship for the profiles $\beta=0.5 \mathrm{~km}^{-1}$, $h^{\prime}=70 \mathrm{~km}, \mathrm{~h}^{\prime}=75 \mathrm{~km}, \mathrm{~h}^{\prime}=80 \mathrm{~km}, \mathrm{~h}^{\prime}=85 \mathrm{~km}$, and $\mathrm{h}^{\prime}=90 \mathrm{~km}$.

The propagation path chosen for this compsrison was that used for experimental work at Corona Annex. A transmitter station is located on the island of Hawall ( $155.60^{\circ} \mathrm{W}, 19.642^{\circ} \mathrm{N}$ ) with a propagation path passing through receiver cuordinates in Southern California $\left(116.625^{\circ} \mathrm{W}, 34.533^{\circ} \mathrm{N}\right)$, with a path length of about 4.2 megameters. The path is considered to be entirely over sea water.


FIG. 1. Electron Density Profiles for a Conductivity Parameter of $\beta=0.5 \mathrm{~km}^{-1}$.

As described in Ref. 13 the collision frequency was chosen to be

$$
\nu=v_{0} \exp (-a Z)
$$

where

$$
\begin{aligned}
v_{0} & =1.82 \times 10^{11} \text { collisions } / \mathrm{sec} \\
a & =1.5 \times 10^{-4} \text { meters }-1 \\
\mathrm{z} & =\text { meters }
\end{aligned}
$$

The sea water propagation path parameters were permittivity and conductivity. The permittivity is

$$
\begin{array}{ll}
\epsilon & =7.172015 \times 10^{-10} \\
\frac{\text { farads }}{\text { metar }} \\
\epsilon_{0}=8.85434 \times 10^{-12} & \frac{\text { farads }}{\text { meter }}
\end{array}
$$

or

$$
\epsilon_{y}=\frac{c}{\epsilon_{0}}=81
$$

vihere

$$
\begin{aligned}
\epsilon & =\text { permittivity of the medium } \\
\epsilon_{0} & =\text { permittivity of free space } \\
\epsilon_{r} & =\text { relative permittivity }
\end{aligned}
$$

Conductivity is given as $\sigma=5.0$ mhos/meter. The propagation path used for these computations may be assumed to be a horizontally homogeneous waveguide, and thue the earth's magnetic-field parameter. along the propagation path may be assigned the values at midpath (see Ref. 14). The magnetir parameter at mid path were

$$
\begin{aligned}
\left|\mathrm{H}_{\mathrm{e}}\right| & =4.25 \times 10^{-5} \text { webers } / \text { meter }^{2} \\
\text { Dip } & =50 \mathrm{deg} \text { (WAVEHOP) } \\
\text { CO-Dip } & =40 \mathrm{deg} \text { (WAVEGUIDE) }
\end{aligned}
$$

$$
\text { Magnetic azimuth }=50.633 \text { deg }
$$

Propagation frequenciea were 15.567 kHz and 28.125 kHz .
The angles of incidence used in TUIK consister of a range from 41 deg to 82 deg. The increment between thise angles was chosen so that the phases of the ionospheric reflection coefficients obtained would be close enough to insure a smooth curve (no discontinuities because of $\pm 180$ deg changes). Interpolation is used to determine the reflection coefficient phases at the particular geometrical insidence angles of each hop on the propagation path; therefore, a continuous phafe is mandatory.

The determination of the effective ionos pheric reflection height used in WAVEHOP is based on the assumption that the upgoing field will be maximum when it enters the ionosphere. The field should then decrease with height due to reflection and absorption, until a value approximately 10 dB below the original field level remains. The height
so obtained is taken as the effective reflection height. This height also correnpoads to a level about one-skin depth above the height where the rate of reflection of the incident wave maximizes. The most important reflection of the wave occurs within an altitude range of several kilometers centered at or near the height of maximum reflection, as discussed by Field and Engle (see Ref. 15), and the actual choice of the reflection height to be an input to WAVEHOP may vary over a few kilometers without ignificantly affecting the output field atrength. For long VLF propagation pathe, the angle of incidence is about 82 deg for the mont important hops, and the reflection height which was determined for thim ang's was used in the computations. The WAVEHOP program was implemented uning the values as shown in Tables 1 and 2.

TABLE 1. Frequency and Reflection Height Parameters Used in WAVEHOP Computations.

| Profile | Frequency, <br> kHz | Reflection <br> height, <br> km |
| :---: | :---: | :---: |
| $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=70 \mathrm{~km}$ | 15.567 | 66.6 |
| $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=75 \mathrm{~km}$ | 28.125 | 68.2 |
|  | 15.567 | 72.8 |
|  | 28.125 | 73.5 |
| $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=80 \mathrm{~km}$ | 15.567 | 79.4 |
|  | 28.125 | 80.6 |
| $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=85 \mathrm{~km}$ | 15.567 | 86.3 |
|  | 28.125 | 88.1 |
| $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=90 \mathrm{~km}$ | 15.567 | 94.0 |

The choice of reflection height values shown in Table 1 is not too critical in that the field atrength valuea computed uning reflection heights of $65 \mathrm{~km} \pm 5 \mathrm{~km}$ for the proftle $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=70 \mathrm{~km}$ were found to be nearly equal. Also, for the profile $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=90 \mathrm{~km}$ profile, the values of the computed fields were found to be nearly equal for a reflection height of $95 \mathrm{~km} \neq 5 \mathrm{~km}$. The WAVEGUDE program was implemented using the values shown in Table 3.

TABLE 2. Distance and Number of Hops Used in WAVEHOP Computations.

| Profile | $\begin{gathered} \text { Distance, } \\ \mathrm{km} \end{gathered}$ | No. of hop: |
| :---: | :---: | :---: |
| $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=70 \mathrm{~km}$ | 1000-2000 | 5 |
|  | 2000-3000 | 6 |
|  | 3000-8000 | 9 |
| $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=75 \mathrm{~km}$ | 1000-2000 | 5 |
|  | 2000-3000 | 6 |
|  | 3000-8000 | 9 |
| $\beta=0.5 \mathrm{~km}^{-1}{ }_{\mathrm{H}} \mathrm{h}^{\prime}=80 \mathrm{~km}$ | 1000-2000 | 5 |
|  | 2000-3000 | 6 |
|  | 3000-8000 | 9 |
| $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=85 \mathrm{~km}$ | 1000-2000 | 5 |
|  | 2000-3000 | 7 |
|  | 4000-8000 | 9 |
| $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=90 \mathrm{~km}$ | 1000-2000 | 5 |
|  | 2000-4000 | 7 |
|  | 4000-8000 | 10 |

TABLE 3. Height Parameters Ured in WAVEGUIDE Computations.

| Profile | $\mathrm{D}^{\mathrm{a}}, \mathrm{km}$ | $\mathrm{H}_{1}^{\mathrm{b}} \mathrm{km}$ | REFLHT ${ }^{\mathrm{C}} \mathrm{km}$ |
| :---: | :---: | :---: | :---: |
| $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=70 \mathrm{~km}$ | 30 | 70 | 70 |
| $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=75 \mathrm{~km}$ | 40 | 75 | 75 |
| $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=80 \mathrm{~km}$ | 50 | 80 | 80 |
| $\beta=0.5 \mathrm{kr.}^{-1}, \mathrm{~h}^{\prime}=85 \mathrm{~km}$ | 50 | 85 | 85 |
| $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=90 \mathrm{~km}$ | 50 | 90 | 90 |

a | $\mathrm{D}=$ | The heigint below which ionospheric effecto |
| ---: | :--- |
| can be considered regligible relative to |  |

earthecurvature effects.

Frora the otandpoint of program running time, it is desirable to choose D as large as possible since this minimizes the time involved in integrating the modal equation through the ionosphere. This computation is the most time consuming part of the WAVEGUIDE program. Care must be exercised to insure that the value of $D$ be chosen low enough that the eigenvalue (eigenangle) solutions have stabilized (see Ref. 10).

The values of D, H, and REFLHT presented in Table 3 need not be precise, because different values may be used to obtain the same field strength values. For example, four sets of values for D, H, and REFLHT were nsed with the $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{1}=70 \mathrm{~km}$ proflle from which identical field strengths resulted. The four set of kilometer values for $\mathrm{D}, \mathrm{H}$, and REFLHT were, respectively, $20,50,70 ; 50,5 \mathrm{C}$, 70; 30, 50, 65; and 50, 50, 65.

A similar investigation was made for the $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=85 \mathrm{~km}$ profile. The values used for computing field strengthe were $3 \mathrm{C}, 85,85$
$\mathrm{km} ; 50,50,85 \mathrm{krn}$; and $50,50,90 \mathrm{~km}$, for $\mathrm{D}, \mathrm{H}$, and REFLHT, respectively. Again identical field strength results were obtained.

Summaries of the parameters obtained from the WAVEGUIDE computations are found in Tables 4 through 13 for the five electron-density profiles at the two frequencies shown. In these tables, the real and imaginary parts of the eigenangles of the various modee are listed under Theta. As shown in the tables, when the real part of Theta increases, the imaginary part generally increases in value. The ratio of phase velocity in the medium to that in free space is $V / C$. The ratio V/C is seen to decrease with increasing real eigenangle, indicating a decreasing phase velocity or increaning refractive index. The importance of a given mode is determined by the attenuation and excitation factors. The most dominant mode is described by a low attenuation value and a large ponitive excitation value. The tables illustrate that the attenuation ( $\mathrm{dB} /$ megameter)decreases with increasing real angle of incidence. The Polarization Magnitude term is a measure of the extent of polariation mixing (see Ref. 12), Values of this parameter which are much greater than unity indicate a nearly pure TM mode (vertical polarization) whilc values much less than unity indicate a pure TE mode (horizontal polarization), Values of the ratio close to unity indicate that a nearly equal mixture of TE and TM components comprise the mode. The tables indicate that for these frequencies, the modes which are quasi-TM are more important in the mode sum than those which are quasi-TE, in that the quasi-TM modes are more highly excited. A convenient rule of thumb in insuring that all modes are being used in the computation for the fields is that the polariation term should alternate between values greater than one and less than one. This relationship is illustrated in Tables 4 through 13. Table 9 does not follow the relationship at 82.661 deg and 82.890 deg. This would indicate that a mode was missed with a real eigenangle between these two values. It was not posaible, however, to find such an eigenvalue. The computations of Table 11 indicate a similar problem between 80.812 deg and 82.654 deg , while Table 13 shows that this occurs between 80.874 deg and 82.658 deg. This could be explained as follows: acme eigenangles, particularly at 89 deg, have small attenuation rates but also very small excitation factors and consequently have little effect on the total electric field strength value. These are called "earth-detached" modes, (see Ref. 16). The foregoing eigenangles, in particular 82.661 deg and 82.890 deg from Table 9, 82.654 deg from Table 11, and 82,658 deg from Table 13, may also be considered earth-detached and as such their behavior in the polarization magnitude scheme might be explained.
TABLE 4. Summary of WAVEGUIDE Parameters at 15.567 kHz for $\beta=0.5 \mathrm{~km}^{-1}$,
$h^{\prime}=70 \mathrm{~km}$.

| Theta, deg |  | V/C | Attenuation, <br> dB/mega <br> meter | Excitation <br> factor, <br> dB | Polarization |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Real | Imaginary |  | Magnitude | Angle, deg |  |  |
| 64.290 | -1.249 | 1.097415 | 27.099319 | -35.69444 | 0.187769 | 13.49584 |
| 67.608 | -1.015 | 1.069492 | 19.337585 | -0.28066 | 12.217490 | 52.68211 |
| 72.480 | -0.794 | 1.037018 | 11.953999 | -42.66788 | 0.115792 | 9.64833 |
| 75.773 | -0.572 | 1.020250 | 7.030396 | 0.76325 | 17.611496 | 52.02216 |
| 79.862 | -0.404 | 1.004669 | 3.556337 | -49.89520 | 0.062235 | 30.23956 |
| 81.724 | -0.172 | 0.999412 | 1.238154 | -1.63192 | 32.158966 | 88.96785 |
| 89.990 | 0.003 | 0.989008 | -0.000026 | -137.49515 | 0.005799 | 115.18188 |

TABL工 5. Summary of Waveguide Parameters at 28.125 kHz for $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\mathbf{\prime}}=70 \mathrm{~km}$.

| 'Thets, deg |  | V/C | Atteruation, dB/mege meter | Excitation factor, dB | Polarisation |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Real | Imaginary |  |  |  | Magnitude | Angle, deg |
| 67.999 | -0.863 | 1.066565 | 29.212234 | -0.46685 | 5.925715 | 69.64754 |
| 70.633 | -0.916 | 1.048195 | 27.447037 | -34.53508 | 0.133177 | 1.01856 |
| 72.526 | -0.671 | 1.036783 | 18.205078 | 0.12703 | 7.313920 | 69.01436 |
| 74.940 | -0.679 | 1.024112 | 15.940947 | -39.95148 | 0.094010 | 7.09345 |
| 76.786 | -0.476 | 1.015871 | 9.831292 | 0.97377 | 12.164526 | 75.22852 |
| 78.950 | -0.449 | 1.007658 | 7.775711 | -45.59265 | 0.062676 | 358.64966 |
| 80.447 | -0.258 | 1.002905 | 3.868734 | 2.24449 | 14.374891 | 90.04794 |
| 82.494 | -0.289 | 0.997543 | 3.411057 | -42.45749 | 0.122350 | 47.44543 |
| 82.778 | -0.155 | 0.996914 | 1.760603 | -7.72704 | 6.429727 | 169.88083 |
| 89.980 | 0.001 | 0.989008 | -0.000032 | -137.14377 | 0.0 | 89.99997 |

TABLE 6. Summary of Waveguide Parameters at $15.567 \mathrm{kHz} \mathrm{fve} \beta=0.5 \mathrm{~km}^{-1}$,
$\mathrm{h}^{\prime}=75 \mathrm{~km}$.

| Theta, deg |  | V/C | Attenuation, dB/mega meter | Excitation factor, dB | Polarization |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Real | Imaginary |  |  |  | Magnitude | Angle, deg |
| . 66.215 | -1.042 | 1.079768 | 21.034241 | -28.53926 | 0.239821 | 2.63871 |
| 69.196 | . 0.674 | 1.057074 | 11.981479 | -0.16209 | 6.773071 | 68.17528 |
| 73.624 | -0.665 | 1.029938 | 9.384047 | -35.26738 | 0.160650 | 3.26129 |
| 76.546 | -0.392 | 1.016083 | 4.564807 | 0.88612 | 11.006417 | 64.01772 |
| 80.268 | -0.350 | 1.002632 | 2.961166 | -40.94304 | 0.087120 | 22.19275 |
| 81.706 | -0.125 | 0.998666 | 0.962479 | -2.42902 | 15.877316 | 128.70358 |
| 89.988 | 0.002 | 0.988222 | -0.000021 | -141.37369 | 0.008398 | 115.13184 |

TABLE 7. Summary of Waveguide Parameters at 28.125 kHz for $\beta=0.5 \mathrm{~km}^{-1}$, $h^{\prime}=75 \mathrm{~km}$.

| Theta, deg |  | V/C | Attenuation, dB/megameter | Excitation factor, dB | Polarisation |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Real | Imaginary |  |  |  | Magnitude | Angle, deg |
| 67.989 | -0.906 | 1.065782 | 3G.705536 | -21.08469 | 0.280973 | 336.80908 |
| 69.526 | -0.616 | 1.054795 | 19.483932 | -0.45940 | 3.009709 | 69.35957 |
| 72.042 | -0.731 | 1.038746 | 20.380737 | -25.64145 | 0.205898 | 338.16431 |
| 73.623 | -0.494 | 1.029974 | 12.595168 | 0.21603 | 4.116412 | 70.10898 |
| 75.904 | -0.555 | 1.018853 | 12.222857 | -30.65016 | 0.129614 | 338.09302 |
| 77.452 | -0.359 | 1.012383 | 7.052810 | 1.19597 | 5.608160 | 78.65514 |
| 79.460 | -0.377 | 1.005159 | 6.235909 | -34.58418 | 0.104922 | 338.43628 |
| 80.623 | -0.188 | 1.001600 | 2.769808 | 2.21072 | 7.220531 | 84.08716 |
| 82.620 | -0.279 | 0.996465 | 3.240633 | -29.94334 | 0.306434 | 8.13619 |
| 82.764 | -0.140 | 0.996152 | 1.594564 | -12.54633 | 1.853441 | 206.61401 |

TABLE 8. Summary of Waveguide Parameters at 15.567 kHz for $\beta=0.5 \mathrm{~km}^{-1}$, $h^{\prime}=80 \mathrm{~km}$.

| Thets, deg |  | V/C | Attenuation, dB/mega meter | Excitation factor, dB | Polarization |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Real | Imaginary |  |  |  | Magnitude | Angle, deg |
| 63.293 | -0.577 | 1.105305 | 12.989402 | -1.36378 | 3.129542 | 80.29333 |
| 67.865 | -0.821 | 1.065894 | 15.495461 | -23.01541 | 0.310205 | 344.79517 |
| 70.615 | -0.442 | 1.046748 | 7.348512 | -0.24807 | 4.407783 | 76.61931 |
| 74.609 | -0.516 | 1.024125 | 6.859793 | -29.19182 | 0.205012 | 348.31396 |
| 77.214 | -0.266 | 1.012533 | 2.948713 | 0.98845 | 7.614028 | . 76.52620 |
| 80.611 | -0.285 | 1.000833 | 2.328875 | -33.13902 | 0.128496 | 8.11715 |
| 81.686 | -0.087 | 0.997924 | 0.630129 | -3.4868: | 6.653981 | 141.95859 |
| 89.991 | -0.003 | 0.987438 | 0.000024 | -153.57190 | 0.017737 | 344.88843 |

TABLE 9. Summary of Waveguide Parameters at 28.125 kHz for $\beta=0.5 \mathrm{~km}^{-1}$.
$h^{\prime}=80 \mathrm{~km}$.

| Theta, deg |  | V/C | Attenuaiion, <br> dB/mega meter | Excitation factor. dB | Polerication |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Real | Imaginary |  |  |  | Magnitude | Angle, deg |
| 67.036 | -0.167 | 1.072392 | 16.488831 | -2.15791 | 1.708636 | 65.26321 |
| 69.525 | -0.607 | 1.053967 | 19.215393 | -13.62779 | 0.513407 | 316.42432 |
| 70.898 | -0.418 | 1.C44949 | 12.379257 | -1.10132 | 2.108592 | 64.07761 |
| 73.229 | -0.482 | 1.031266 | $12.586 \$ 76$ | -17.26471 | 0.359327 | 315.85596 |
| 74.598 | -0.348 | 1.024201 | 8.364258 | -0.06498 | 2.722632 | 70.19466 |
| 76.723 | -0.368 | 1.014533 | 7.648388 | -21.23329 | 0.259620 | 310.16772 |
| 78.026 | -0.257 | 1.009390 | 4.825250 | 1.27671 | 3.200137 | 73.46654 |
| 79.888 | -0.259 | 1.003008 | 4.115238 | -23.65295 | 0.219967 | 320.79077 |
| 80.740 | -0.127 | 1.000473 | 1.849419 | 1.65285 | 4.279541 | 91.19252 |
| 82.661 | -0.227 | 0.995587 | 2.624161 | -22.67204 | 0.905500 | 346.35791 |
| 82.890 | -0.122 | 0.995087 | 1.366551 | -24.83273 | 0.727605 | 204.11745 |

TABLE 10. Summary of Waveguide Parameters at 15.567 kHz for $\beta=0.5 \mathrm{~km}^{-1}$,
$\mathrm{h}^{\prime}=85 \mathrm{~km}$.

| Theta, deg |  | V/C | Attenuation, dB/mega meter | Excitation factor. dB | Polarization |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Real | Imaginary |  |  |  | Magnitude | Angle, deg |
| 62.509 | -0.960 | 1.112085 | 22.215591 | -14.60308 | 0.577910 | 329.73120 |
| 65.242 | -0.403 | 1.086492 | 8.460461 | -2.01493 | 2.605300 | 84.24753 |
| 69.238 | -0.628 | 1.055109 | 11.159905 | -18.85110 | 0.403090 | 326.81079 |
| 71.845 | -0.316 | 1.038326 | 4.935863 | -0.52766 | 3.408094 | 84.65630 |
| 75.420 | -0.384 | 1.019457 | 4.845797 | -24.24103 | 0.289466 | 329.55957 |
| 77.767 | -0.191 | 1.009568 | 2.028769 | 0.96218 | 5.466550 | 80.95462 |
| 80.869 | -0.230 | 0.999307 | 1.829692 | -25.84442 | 0.221352 | 2.02758 |
| 81.678 | -0.058 | 0.997151 | 0.420824 | -5.09335 | 4.040819 | 155.89731 |

TABLE il. Summary of Waveguide Parameters at 28.125 kHz for $\beta=0.5 \mathrm{~km}^{-1}$,
$h^{\prime}=85 \mathrm{~km}$.

| Theta, deg |  | V/C | Attenuation, dB/mega meter | Excitation factor. dB | Polarisation |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Real | Imaginary |  |  |  | Magritude | Anglc, deg |
| 67.211 | -0.520 | 1.070148 | 18.242294 | -6.99214 | 0.992478 | 306.76660 |
| 68.576 | -0.344 | 1.059868 | 11.380239 | -4.00963 | 1.382699 | 63.15662 |
| 70.785 | -0.405 | 1.044834 | 12.072172 | -9.13138 | 0.770695 | 302.56445 |
| 72.077 | -0.306 | 1.036959 | 8.528738 | -2.30005 | 1.570908 | 63.54758 |
| 74.184 | -0.308 | 1.025458 | 7.602866 | -11.95107 | 0.584410 | 305.66626 |
| 75.425 | -0.256 | 1.019449 | 5.834623 | -0.71824 | 2.159298 | 66.22102 |
| 77.361 | -0.227 | 1.011146 | 4.498512 | -14.86057 | 0.388796 | 300.64893 |
| 78.492 | -0.184 | 1.006887 | 3.324704 | 1.20421 | 2.423215 | 79.39552 |
| 80.186 | -0.169 | 1.001299 | 2.608955 | -15.02133 | 0.411545 | 309.40625 |
| 80.813 | -0.077 | 0.999474 | 1.113542 | 0.01832 | 2.167155 | 101.62277 |
| 82.654 | -0.204 | 0.994811 | 2.362394 | -25.89392 | 1.344164 | 353.85425 |
| 83.067 | -0.065 | 0.993918 | 0.710613 | -37.15074 | 0.658765 | 190.38757 |

TABLE 12. Summary of Waveguide Parameters at 15.567 kHz for $\beta=0.5 \mathrm{~km}^{-1}$, $h^{\prime}=90 \mathrm{~km}$.

| Theta, deg |  | v/c | Attenuation, dB/mega meter | Excitation factor, dB | Polarization |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Real | Imaginary |  |  |  | Magnitude | Angle, deg |
| 60.472 | -0.376 | 1.133006 | 9.296992 | -5.06141 | 1.702432 | 87.11026 |
| 64.212 | -0.709 | 1.094825 | 15.474374 | -12.09672 | 0.753823 | 314.26318 |
| 66.896 | -0.322 | 1.071816 | 6.339072 | -2.81639 | 2.323418 | 88.29460 |
| 70.423 | -0.463 | 1.046321 | 7.783310 | -15.83295 | 0.502564 | 311.98071 |
| 72.876 | -0.251 | 1.031588 | 3.707755 | -0.91193 | 2.964911 | 89.46432 |
| 76.102 | -0.283 | 1.015586 | 3.410278 | -20.57822 | 0,385714 | 312.08350 |
| 78.205 | -0.147 | 1.007128 | 1.507507 | 0.90061 | 4.239605 | 84.27917 |
| 81.053 | -0.191 | 0.998004 | 1.490262 | -20.28026 | 0.394766 | 357.58643 |
| 81.695 | -0.038 | 0.996315 | 0.275371 | -7.80828 | 2.620982 | 166.78784 |
| 89.989 | 0.003 | 0.985867 | -0.000029 | -153.39684 | 0.031168 | 3.41930 |

TABLE 13. Summary of Waveguide Parameters at 28.125 kHz for $\beta=0.5 \mathrm{~km}^{-1}$,
$h^{\prime}=90 \mathrm{~km}$.

| Theta, deg |  | V/C | Attenuation, <br> dB/mega, <br> meter | Excitation <br> factor, <br> dB | Polarization |  |
| :--- | :---: | :---: | :---: | ---: | ---: | ---: |
| Real | Imaginary |  | Magnitude | Angle, deg |  |  |
| 61.560 | -0.509 | 1.121130 | 21.972153 | -2.85852 | 2.444879 | 303.43652 |
| 63.142 | -0.341 | 1.105053 | 13.964008 | -11.82327 | 0.813592 | 47.13593 |
| 65.147 | -0.433 | 1.086455 | 16.495590 | -3.51609 | 1.778311 | 298.00513 |
| 66.554 | -0.299 | 1.074574 | 10.783415 | -8.96648 | 0.866296 | 54.23936 |
| 68.586 | -0.356 | 1.058949 | 11.781331 | -4.74999 | 1.371325 | 298.36230 |
| 69.862 | -0.268 | 1.050049 | 8.363271 | -6.33167 | 1.076632 | 58.66864 |
| 71.876 | -0.276 | 1.037321 | 7.782177 | -6.46030 | 0.962279 | 294.50171 |
| 73.060 | -0.237 | 1.030573 | 6.259215 | -3.90550 | 1.225620 | 66.90196 |
| 74.994 | -0.206 | 1.020666 | 4.834593 | -8.70418 | 0.809712 | 294.94971 |
| 76.106 | -0.198 | 1.015574 | 4.309546 | -1.69150 | 1.634636 | 63.95689 |
| 77.880 | -0.148 | 1.008339 | 2.816595 | -10.86768 | 0.507472 | 300.27026 |
| 78.853 | -0.137 | 1.004819 | 2.400707 | 0.78766 | 2.146343 | 85.72569 |
| 80.382 | -0.127 | 0.999920 | 1.923321 | -9.27415 | 0.578400 | 308.43750 |
| 80.874 | -0.039 | 0.998506 | 0.560677 | -3.90118 | 1.166057 | 126.94872 |
| 82.658 | -0.186 | 0.994012 | 2.154495 | -31.03848 | 1.505057 | 358.31055 |
| 83.237 | -0.028 | 0.992775 | 0.298878 | -47.32495 | 0.648848 | 184.53 .349 |
| 89.987 | 0.004 | 0.985866 | -0.000083 | -176.49695 | 0.007158 | 16.00693 |

Examples illustrating the relative field -atrength magnitudes for the individual waveguide mode components and the wave-hop componente of the total field are presented in Fig. 2 and 3, respectively. These resulte are determined for the daytime ionospheric profile $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=70 \mathrm{~km}$, at 28.125 kHz . It is thown in Fig. 2 that on'y three modes, all of which are of the quasi-TM type, are necessary to completely describe the propagated field for this case; and at ranges beyond about 4 megameters two modes are sufficient. Table 5 illustrates that many modes can be computed for this profile and propagation frequency, but only those modes with low attenuation rates and high excitation factors are important in comprising the total mode sum of the fields.

A similar breakdown of the wave -hop computation zs shown in Fig. 3. For the range 1.0 to 8.0 megameters, it is observed that energy traveling by the way of the ground wave, and the one-hop path, two-hop path, and so on up to a seven-hop path, can be important in the total propagated field. The actual aignificance of any hop path depends on the particular range of interest. At 1 megameter only the groundwave, one-., twon, and three-hop paths are important. At a distance of 4 megameter: the ground wave is completely negligible, but the four and five hop pathe have becorne aignificant. At 6 megameters the onehop path is no longer important, but the six-hop path must be tacluded in the total field-atrength computation. Beyond 8 megameters the twohop path is no longer of value, but the seven-hop path must be inciuded to obtain the total field.

In general, for the propagation ranges considered here, it may be inferred that for the waveguide model at least two but no more than three modes must be included in the computation for the total field atrength levels for the daytime ionosphere. On the other hand, as many as five wave-hop components are needed to describe the same fields by the wavehop theory.

The waveguide mode and wave hop comparison for nighttime signal levels is illustrated in Fig. 4 and 5, respectively, for the profile $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=85 \mathrm{~km}$ at 28.125 kHz , It is observed for the waveguide model in Fig. 4 that as many as eight modes are importunt at 1.0 megameter for the inghttime condition, whereas for the dinytime situa tion only three modes were significant. It is apparent, however, that most of these modes attenuate rapidly with distance so that beyond 5 megumeters only four modes are really significunt to the makeup of the total field level. An examination of Table 11 demonetrates that the



FIG. 4. Wavaguide Mode Field for Nighttime Profile $\left(\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=85 \mathrm{~km}\right)$ at 28.125 kHz .


FIG. 5. Wavehop Fielde for Nighttime Profile ( $\beta=0.5 \mathrm{~km}^{-1}$, $h^{\prime}=85 \mathrm{~km}$ ) at 28.125 kHz .
two most dominant modes (that is, No. 1 and 3 of Fig, 4) are not 2 s purely quasi-TM as was found for the daytime condition of Table 5.

Figure 5 illustrates the wave-hop field components for the night... time case. It is observed here, as was demonstrated for the daytime situation, that several wave-hop pathe must be considered when deter mining the total signal strength available at any given range. Comparison with the daytime results shows that, in general, a larger number of wave hops are important for nighttime propagation, where as many as seven wave -hop componente are aignificant at 5 megameters. Also, where only four or less modes would be considered significant beyond 5 megameters, in the waveguide model, it is apparent from Fig. 5 that at least seven wave-hop components would need to be included in determining the total field for these ranges.

## DISCUSSION OF RESULTS

The field-atrengta valuen computed from the WAVEHOP and the WAVEGUIDE compurer programs, using the input parameters as described previously, are illustrated in Fig. 6 througl. 19 as a function of propagation range. The field-strength values were computed at 25 kkm intervals. The frequencies of propagation are 15.567 kHz and 28.125 kHz . The electron-denaity profiles are identified as $\beta=0.5 \mathrm{~km}^{-1}$, $\mathrm{h}^{\prime}=70,75,80,85$, and 90 km . The propagation path is entirely over sea water. The computed fields are normalized to 1 kW of radiated power at each frequency.

Figure 6 shows the comparison between the field strength values computed using WAVEHOP and WAVEGUIDE for the $\beta=0.5 \mathrm{~km}^{-1}$, $\mathrm{h}^{\prime}=70 \mathrm{~km}$, (daytime) profile at 15.567 kHz . In general, the agreement between the two calculations appears to be good. The average attenua tion rates of both signals are almost identical, and the amount of modal interference atructure predicted is essentially the same for the two propagation models. This structure disappars, however, beyond 6 megameters, where the two computations differ by a maximum of 2 dB . At the 4.2 megameter distance, which is the length of the path used by the Corona Annex in obtaining VLF memsurements, the value of the field strength computed using WAVEHOP is seen to be 1.5 dB higher than that obtained from WAVEGUIDE. The most noticeable discrepancy between the two curves is that the signal levels computed by WAVEHOP tend to occur at distances of about 150 to 200 km greater than where the same values of aignal levels are computed by WAVEGUIDE. The actual value

of this difference interval varies with the propagation range, with the smallest discrepancy occurring closest to the transmitter.

Figure 7 illustrates the comparison between field atrength levels for the profile $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=70 \mathrm{~km}$, for a frequency of 28.125 kHz . It is of interest that the modal interference computed for this frequency is much greater than that found for 15.567 kHz . The agreement between the field atrength obtained by the two computational methods is fairly good, except in the vicinity of the signal nulls. These nulls are the result of destructive interference in the vector summation of the field hops (or modes). The nulls computed by WAVEHOP occur at a slightly greater range than the correaponding nulls computed by WAVEGUIDE. As for the $15.567-\mathrm{kHz}$ fields, the values computed by WAVEHOP occur at approximately 150 -to 200 -km greater distances than do similar values computed by WAVEGUIDE. Alsc, the location of the null containing the deepest signal fade does not coincide for the two models. The deepest null found from the WAVEGUIDE computation is located at 4.50 megameters with a shallower null located at 6.25 megameters. The WAVEHOP computation, however, shows the shallower null at 4.75 megameters and the deeper null positioned at 6.50 megameters. At the 4.2 megameter distance the WAVEHOP signal level is computed to be 4.0 dB higher than that computed using WAVEGLIDE. For a receiver loceted at the $4.5-\mathrm{meg}$ ameter distance, the two modela yield signal levels which differ by as much as 20 dB .

In a first attempt to determine the reason for the differencea in the field levels obtained for the two models, the ionosphere electron density profile used in the WAVEHOP program was lowered 1 km in height. This lowered profile was then input to the WAVEHOP program with the result as shown in Fig. 8. The signal-level nulls from this calculation are seen to line up much better (in range) with those deter mined from WAVEGUIDE. The relative depth of the aignal nulls, however, continues to differ considerably. The observation that the WAVEGUIDE computation gives a field-atrength level 15 dB below that of WAVEHOP for the aignal null at 4.5 megameters and 15 dB above that of WAVEHOP for the null at 6.25 megameters indicates thnt fundamental differences do exist between the two propagation theories.

The comparisons of the field atrength values computed using the profile $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=75 \mathrm{~km}$ for 15.567 kHz and 28.125 kHz are diaplayed in Fig. 9 and 10, respectively. The aignal levels computed at 15.567 kHz by the two models are quite similar as a function of propagation distance. The average attenumion rates of the two field


curves appear to be ensentially equal. Both calculations show deeper mode structures which increase to greater ranges than those found for the $h^{\prime}=70 \mathrm{~km}$ profile at 15.567 kkiz . A distance interval of 150 to 200 km exists between equal datin values computed by the two methods for this profile as was found for the previous profile. At the 4.2 -megameter distance the fields computed using the two models differ by 2 dB . There is also a difference of approximately 5 dB in the depth of the null computed by the two methods in the vicinity of 1.5 megameters. For the $28.125-\mathrm{kHz}$ sigmal, the $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{1}=75 \mathrm{~km}$ proflle does not show as good an agreement between the signal levels computed by the two models as was determined for the cases so far considered. In general, WAVEHOP is seen to predtet alightly higher aignal levele beyoad 4.0 megameters than WAVECUIDE. The largest doviation between the two computations is 6 dB at 6.75 megameters, while at 4.2 megameters the difference is 2 dB . Again, the values computed by WAVEHOP tend to lead those of WAVEGUIDE by diatance intervals ranging from 150 km at 1.5 megameters io 250 km at 6 megameters. A lowrering of the electron-density profile as input to the WAVEHOP computation would tend to improve the comparison between the two progrems, but the relative depth of the interference nulls which are characteriotic of either model will remain unequal. This result is an indication of the presence of some dissimilarity in theory between the two propagation models.

The comparisome batween the computed signal levels obtained for the proflle $\beta=0.5 \mathrm{~km}^{-1}, h^{\prime}=80 \mathrm{~km}$ are shown in Fig. 11 and 12 . The petterns of field strength as a function of distance for the two frequenctes are similar for the WAVEHOP and WAVEGUIDE methods. In fact, a shift inward of the WAVEHOP values would agree very well with the WAVEGUIDE results. The fielde as now cmputed, however, can give large deviations between the two propagation modele at a givan distance. In particuler, the discrepancy for 15.567 kHz (see Fig. 11) is 3.0 dB at $4.50,5.0,5.75$, and 6.25 megameters ; 2 dB at 4.2 megameters, and as much as 18 dB at 1.9 megameters. For 28.125 kHz (see Fig. 12) the discrepancy between the two models reachen 2 dB at soveral ranges. It is of interest that model interference atructure is atill prevalent as far out as 8.0 megameters in the fielde computed for 15.567 kHz , whereas the existence of interference structure hel essentially dieappeared beyond 7 megameters for $28,125 \mathrm{kkls}$. This reault may be contrasted to the daytime proflle, $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=70 \mathrm{~km}$, where considerable interference structure was found throughout the 8.0 megameter range at 28.125 kHz and vary little interference atructure was present beyond 5.5 megameters for 15.567 kHz .

FIG. 11. Field Strength Vereue Distance for WAVEHOP and WAVEGUIDE Comparison at $15.567 \mathrm{kHz}\left(\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=80 \mathrm{~km}\right)$.

Figure 13 illustrate the fields computed for the profile $\beta=0.5 \mathrm{~km}^{-1}$, $\mathrm{h}^{\prime}=85 \mathrm{~km}$ at 15.567 kHz . Both propagation models are seen to produce considerable interference structure throughout the range shown. The depth of the null which occurs close to 4 megameters is seen to be as great as 20 dB for both models. The differences in the signal level, between the two computations, is 12 dB at 4.2 megameters, and as much as 15 dB at 5.7 megameters. The curves are generally similar in appearance, and if the WAVEHOP fields were to be shifted inward by approximately 200 km , the agreament between the two computations would be much improved; however, some disaimilarities would remain. In particular the ahapes of the nulls obtained from the two programe at approximately 2.5 and 4.0 megameters would not produce exact duplication.

The field strength comparison for the $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=85 \mathrm{~km}$ at 28.125 kHz is represented in Fig. 14. The signal levels of the two models are very similar throughout the propagation range. The deviation at 4.2 km is found to be 2 dB . In contrast to the daytime profile $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=70 \mathrm{~km}$, where a much deeper mode structure was found for the $28.125-\mathrm{kHz}$ aignal than for the $15.567-\mathrm{kHz}$ aignal, this profile, which is a fair approximation of the electron densities expected at night, shows the greatest mode atructure at the lower of the two frequencies. In fact, the interference structure at 8.0 megameter: for 15.567 kHz is very significant, whereas for 28.125 kHz at 8.0 mega metera mode interference appears to be quickly appromehing a range where its effect will be negligible to the total field.

Figure: 15 and 16 portray the fielde computed by the WAVEHOP and WAVEGUIDE propagation models for the $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=9: \mathrm{km}$ profile. In Fig. 15 the modal interference structure is shown to be very atrong throughout the range for 15.567 kHz . A close observation of the $\mathrm{h}^{\prime}=90-\mathrm{km}$ curves verifies that the two computations yield similar signal-level characteristic! throughout the range. In particular, both modele yield deop, narrow nulls at approximately 1.25 megameters, narrow relative maximums at about 1.75 megameters, and somewhat aimilar nulls at 2.75-3.0, 4.50-4.75, and 6.25-6.50 megameters. Also, the relative miximums have the same general shape at all ranges. The discrepancies tretween ficld-atrength values obtained from the two computation methods for a given range can very great. The differences are found to be 6 dB at 4.2 megameters, and as great as 18 dB at 6.25 megameters. Again, if the WAVEHOP fielo levels could be shifted inward in range by an interval of 200 to 250 cm , the comparison of the


FIG. 15. Field Strength Versus Distance for WAVEHOP and WAVEGUIDE Comparison at $15.567 \mathrm{kHz}\left(\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=90 \mathrm{~km}\right)$.

fields computed by WAVEHOP and WAVEGUDE would be much improved, although the exact shape and depth of the nulls would still not be identical.

Figure 16 shows the field computations at 28.125 kHz for the profile $\beta=0.5 \mathrm{~km}^{-1}, h^{\prime}=90 \mathrm{~km}$. The two computation models produce a similar field atrength pattern with range, but the actual field strength levels at a given range may be drastically different because of the - apparent shift with distance between the two computations. In particular, the difference in signal level determined by the two methods is as much as 19 dB at 4.2 megameters. It is appareat from Fig. 16 that if the field values computed with WAVEHOP could be transformed inward approximmtely $200-250 \mathrm{~km}$ then the field levels computed as a function of propagation range would line up almost exactly with those computed by WAVEGUIDE.

In order to determine why the differences in the field patterns exiat, the electron density in the WAVEHOP model was first lowered by 1 km , as was doae for the $\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=70 \mathrm{~km}$ proftle. The result was that the fields obtained for the $h^{\prime}=90-\mathrm{km}$ profile from the two models had a greater aimilarity, but still did not coincide. A further lowering of the profile used in WAVEHOP of 2 km yielded the field-streagth resulte as shown in Fig. 17 where the agreement between the field-atrength levels computed using the two computer programs is much improved. An adjustment of -1.5 km would probably give the optimum comparison between the two computations. Appareatly, if the proflles as used in the WAVEHOP model are adjusted downward in height by some optimum amount, the two propagation models can be made to provide resultant fields with much more similarity. However, as the models presentl'/ exist, the resulting field strength levels obtained as a function of propagation distance can give drasticelly different rasults for a partitular receiver range. The major ramification of this fact, in relation to the ionumphereic studies being conducted at the Corona Annex, is that a different ionospheric electron-density proflle would be needed to fit experimastally recorded data, depending on whether WAVEHOP 0: WAVECUIDE were being used in the investigation.

The reason for the diycrepancies between the field strengths calculated by the two computer programs is not fully understood at this time. The results of Fig. 8 and 17 show that much improvement in the agreement between the two computations is achieved by lowering the ionospheric electron-density profile used as an input to the WAVEHOP program. An explanation as to the reason that this procedure gives field-

strength values which compare favorably with thooe obtained friom WAVEGUIDe is possibly related to the method used within atch of the computer programa to determine loac spheric reflection cosfficiente.

In the computer program TUIK, which is used to compute reflection coefficients for the WAVEHOP propagation model, the continuous lonosphere is approximated by one that is stratified, where the input electron-dc-aity profile is divided up into a series of equally apaced layers, ef.ch of which is considered to be homogeneoun throughout. The electron denaity of any layer is then assumed to be that of the midpoint. The value of collisi afrequency corresponding to a particular leyer is also assigned the value at the midpoint. These midpoint values of electron density asd collision irequency are then used in a procedure which matches the upgoing and downgoing wave iield at each layer boundary. The method used computes the fields at the highest layer and then couples the results into each successively lower layer until the bottom of the profile is reached. At this height the reflection coefficipnts characteristic of the particular electron-density protile and propagation Irequency are determined, Layer widtha of lens than 0.5 krr for daytime and 0.75 km for nightime are unually sufficient to satiafy the required convergence critaria (soe Ref, 4).

In the WAVEGUIDE programa, the modal differential equation is integrated by a Runge-Kutta technique starting at sorne height above which negligible retlection ia assumed to take place. The integration is carried thr ough the ionosphere using a variable step size dependent un how rapidl:, the reflect!on coefficients at each step change with height. The size of these steps reaches values of 0.01 km at the point of reflec. tion, and in so doing, the denoity profile more closely approximates a continuously varying ionosphere than is found by using the TUIK method.

Because af the existing differences in layer (or slab) thicioness used in the two models, further computations using TUIK at smaller layer incremeats were attempted. Slas widthe of 0.1 km , which are the -mallent posible with the Corona Annex version of the computer program, did not renult in any inprovement in agreement between the fields obtained from WAVEHOP and WAVEGUIDE. Further study will be required if the discrepancies in signal level computed by the two propagation models are to be explained and crmected.

## CONCLUSIONS

In genaral, it is apparent that the ESSA WAVEHOP and the NELC WAVEGUIDE computerised propagation models, as they now exist, do not produce oxactly the same computational result when using the same input parameters. The degree of difference in the computed fieldatrongth levels in found to be depencient upon the propagation frequency and the input olectron-demaity profile. The field-atrangth levels computed by the two programa have baen shown in some instances to differ considerably when thay are compared at a particular propagation distance. The significance of the diocropancy between the two computational mothods depends on how the ignal-level result are to be used, In come VLI communications applications whore the field etreagth versus distance relationehip is not needed in detail, either program may be used with an oqual degree of confidence. However, for atudies pertaining to the dotormination of characteriatics of the propagation medin, a precise knowledge of the ralationehip between field-ntreagth level and propegation distance is required.

Inventigationa prosently being made at the Corona Annex to deter mine the electron-deasity profile which exiets over a given propagation path for a givon time interval throughout the day and/or night are atrongly dependent upon the detail which exiets for signal levels as a function of propagation range. For the Corons studies, the fields computed by the propagation model are used in coajuaction with experimentally recorded VLF data by comparing the computed and measured signal levels as obtained at a particular diutance from the transmitter. Differences in the field strengthe as computed by the WAVEHOP and WAVEGUIDE computer programs of ? dB and greater at the 4.2 megameter range, as described in this report, result in the determination of two unequivalent electron-density proflles belag charecteriatic of the ionosphere during the period for which the experimental data was acquired.

The original objective of this investigation was to determine which of the two computational methode was the more suttable for une in the interpretation of the VLF multifrequency propagation data being acquired at the Coronil Annex in terms of computor efficiency, cont, and ease of operation. These origtmil factor: are now outweighed by the ovidont differences in computational resulte. Tomporarily ignoring this fector and addresaing the original objectives, it has been determined that use of the WAVEGUIDE method can be more economical for certain typee
of computations if the eigenanglen needed for the modal solution can be found without extensive searching. Finding solutions for eigenangles is a trisl and error procens which for the inexperienced user can be difficult and time-connuming. The renulting computer costs are directly dependent upon this procedure. Because of this factor, direct comparison of costs with the WAVEHOP calculations is very difficult and cannot be quantitatively expressed at this time, except to indicate that the WAVEGUIDE results could easily straddle the WAVEHOP costs, varying from aignificantly leas to significantly greater, depending upon the akill of the operator.

A note of caution is ccinstered in order regarding the use of elther of these programs by the "noz-expert" Inventigator. Indiscriminate use of either program by inexperienced personnel without clone communication with originators is not recommended. Erromeous resulte from improper operation of WAVEGUIDE and WAVEHOP programe could occur.

It has been the experience at the Corona Annex that the adaptation and une of WAVEHOP is more straightforward and that it requires less akill to use, and the operator is leas likely to mise an important signal component in the vector summation procer \%. In view of the differences in computational resulte, it is evident that a further inventigation is warranted in terms of general theory, approximations used in the thesry, and apecial computational technique: uned for computer adaptation. The inttial approach to this inventigation should be to make appropriate, comparative calcuiations which would clearly illustrate the source of the differences. If these differences cannot be easily explained on the basis of techniques, a propagation experiment could be deaigned which would make it possible to test the representativeness of either model. It is also important to carefully select the frequencios and distances that would produce the greateat discrimination between the two models.

## Appendix <br> CORRECTIONS TO THE WAVEHOP PROGRAM

In the oxiginal development of the wave-hop propagation model (Ref. 3 and 4) the jth wave-hop term was computed as

$$
\begin{equation*}
E_{j}=\gamma_{j} I_{j} \tag{1}
\end{equation*}
$$

where $y_{j}$ is the effective iomonpheric reflection coefficient for a plane, stratified, aniootropic tononphere, and $I_{j}$ to a path tategral which accounte for the offects of ground conductivity, reflection, height, earth curvature, and propagation range. In the following Eq, 2, Yj is computed from:

$$
\left(\begin{array}{ll}
y_{j} & a_{12}  \tag{2}\\
a_{21} & a_{22}
\end{array}\right)=\left(\begin{array}{ll}
T_{\text {ee }} & T_{e m} \\
-R_{m} T_{m e} & -R_{m 1} T_{m m}
\end{array}\right)
$$

where

$$
\mathbf{R}_{m}=\text { the Freanel ground reflection coefficient for horimontal }
$$

$T_{\text {ee }}=$ the ratio of the incident field in the plane of inciannce to the rellected field in the ame plane
$T_{\mathrm{mm}}$ = the ratio of the incidont field perpendicular to the plane of incidence to the roflected field perpendicular to the plane of incidence
$T_{\text {em }}=$ the ratio of the incident field in the plane of incidence to the reflected field perpeadicular to it
$\mathrm{T}_{\mathrm{me}}=$ the ratlo of the incident field perpondicular to the plane of incidence to the reflected field in the plane of incidence

It has recently been established, however, that the original version of the wave-hop theory contained several limitations and inaccuracies that were not previoully recognixed or fully apprectated. In particuler, the formulation of the offective ionompheric reflection coefficiont for the second and higher order hops was incorrect for an aninotroplc ionosphere. The corrected equation should read (see Rel. 17):

$$
\begin{equation*}
E_{x}=E_{0}+\sum_{j=0}^{N} E_{j} \tag{3}
\end{equation*}
$$

where

$$
\begin{aligned}
& \mathbf{E}_{0}=\text { the ground wave } \\
& \mathbf{E}_{\mathbf{j}}=\text { the field for the jth wavehop } \\
& \mathbf{N}=\text { the number of wavehops }
\end{aligned}
$$

For $\mathrm{j}=1$,

$$
\begin{equation*}
E_{1}=\int F_{j}(t) T e e^{d t} \tag{4}
\end{equation*}
$$

For $\mathrm{j}>1$.

$$
\begin{equation*}
E_{j}=\int F_{j}(t)\left(a_{j} T e e^{Y}+C_{j} T m^{d t}\right. \tag{5}
\end{equation*}
$$

where

$$
\begin{align*}
& a_{j}=R_{e}\left(T_{e e^{a_{j-1}}}+T_{e m} C_{j-1}\right)  \tag{6}\\
& C_{j}=R_{m}\left(T_{m e} a_{j-i}+T_{1 n m} C_{j-1}\right) \tag{7}
\end{align*}
$$

with $a_{1}=1, C_{1}=0$, also
$R_{e}=$ the Fresnel ground reflection coefficient for vertical polarization
$F_{j}=$ a new function of the propagation path

The deficiencies in the model have been corrected and the individull wavehops are now calculated with the proper formulation of the effective tonospheric reflection coefficient.

The exror wall not found to be significant for daytime propagation, at least at the ranges used by the Corom Annex for the Hawail to Southern California propagation path. This is not the case for aighttime propagation where the computations were found to be in consider able error. Comparisons of the olectric fields computed using the original version of WAVEHOP (see Ref, 4) and the corrected vercion are 111 ustrated in Fig. 18 and 19 for a propagation path over sea water. In these figures the computations obtained using the original WAVEHOP are plotted as WAVEHOP (original) while the corrected values are plotited a: WAVEHOP (corrected).

Jisure 18 showe the daytime comparison for the profile $\beta=0.5$ $\mathrm{km}^{-1}$. $\mathrm{h}^{\prime}=70 \mathrm{~km}$ for 28.125 kHz . It it observed that the values compare favorably out to about 4.5 magameters. At greater diatances the degree of comparison reduces quiclely, eapecimlly in the vicinity of the field nulls. It is important to notice that the two computational methode agree at the Hawail to Southern Californta propagation range of 4,200 km . Because of this, the results obtained in previou Corom Annex reporte for daytime propagation remain valid (see Ref, 5 and 6).

Figure 19 for 28.125 kHz illustrates the nighttime comparison for the proftle $\beta=6.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=90 \mathrm{~km}$. It is seen in this comparison that the resulte from the two computntional methode vary drastically. The publications (Ref. 7 and 8) for aighttime comparison were written using the corrected veraion of WAVEHOP, so the results were valid.

In the original veraion of the WAVEHOP program, the path integral $I_{j}$ of Eq. 1 was ovaluated by ote of three methods, depending on propa gation range relative to the poaition of the caustic. In general, the aaddle-point approximation was used to evaluate the intogral in the lit region of the propagation path except near the caustic; numerical integration was used in the vicinity of the casatic on the lit side, and a residue series was used in the shadow region.

In the corracted version of the WAVEHOP computer program, the above methods may again be used to evaluate the inteqrals in Eq. 4 and 5. In the shadow region, however, the fategrals can also be evaluated by substituting numerical integration for the residue series expansion. Due to the complexity of colving the integrals of Eq. 5 by the remidue

FIG. 18. Field Strength Versue Distance for WAVEHOP (corrected) and WAVEHOP (original) Comparison at $28.125 \mathrm{kHz}\left(\beta=0.5 \mathrm{~km}^{-1}, \mathrm{~h}^{\prime}=70 \mathrm{~km}\right)$.

method, the numerical integration equations and computer program corrections were developed and implemented into the WAVEHOP program. Thus, the correct field strengthe, as computed by WAVEHOP in this report, were obtained using the numerical integration method.

A corrected version of WAVEHOP where the residue series is again incorporated into the program is prewercly being prepared and will be available soon. ${ }^{2}$ This residue series program will be approximately three times fastes than the numerical integration version presently being used at the Corona Annex.

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[^0]:    ${ }^{1}$ Private communication with L. A. Berry of the Institure for Telecommunication Sciences and Aeronomy, Environmental Science Services Administration, Boulder, Colorado, dated May 1969.
    ${ }^{2}$ Private communication as cited in footnote 1, dated January 1970.

