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CASE STUDY Open Access

Comparison of Weibull parameters computation methods and analytical estimation of wind turbine capacity factor using polynomial power curve model: case study of a wind farm

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Abstract

Introduction: Wind speed probability at a site has to be modeled for evaluating the energy generation potential of a wind farm. Analytical computation of wind turbine capacity factor at the planning stage of a wind farm is very crucial. Thus, the comparison of Weibull parameters estimation methods and computation of wind turbine capacity factor are the focus of this paper.

Case description: Soda wind farm used in this case study is located in the Jaisalmer district of western Rajasthan in India. Modeling of wind speed probability and power curve of wind turbines installed at Soda site were done for analytically estimating the capacity factor of wind turbine. Estimated capacity factors were then compared with the measured values of wind farm for validation of results.

Discussion and evaluation: Four numerical methods namely graphical, empirical, modified maximum likelihood, and energy pattern factor were used for month-wise Weibull parameters estimation at hub height of 65 m. Power curve of the wind turbine installed at site was modeled using eighth-degree polynomial. Coefficients of polynomial were calculated from the combined use of linear least square method and QR decomposition using Gram-Schmidt orthogonalization method.

Conclusions: Results show that the percentage error in annual capacity factor estimation using Weibull parameters estimated from graphical, empirical, modified maximum likelihood, and energy pattern factor methods were +9.98%, −1.59%, −1.22%, and −1.29%, respectively. Annual capacity factor that was estimated using the Weibull parameters calculated from modified maximum likelihood method matched best with the measured values. Graphical method gave the most erroneous results.

Keywords: Weibull parameters; Frequency distribution; Wind turbine; Capacity factor; Power curve

Background

Wind power of a site changes with the change in seasons and thus affects the capacity factor of wind turbines. Wind speed distribution at hub height has to be month-wise modeled for estimating the influence of atmospheric parameters on wind power. Wind speed probability modeling and estimation of wind turbine capacity factor for a site are investigated by many researchers. Jangamshetti & Rau (1999, 2001) used normalized power curves as a tool for identification of optimum wind turbine generator parameters. Rehman and Ahmad (2004) analyzed wind data for five coastal locations. Rocha et al. (2012) explained the analysis and comparison of seven numerical methods for finding the parameters for Weibull probability distribution. Jowder (2009) presented the statistical study of wind speed and power at various heights. EL-Shimy (2010) studied the problem of site matching of wind turbine generator through improved formulation of capacity factor. Huang and Wan (2011, 2012) determined a modular approach to enhance capacity factor computation of wind turbine generators. Albadi and El-Saadany (2009, 2010, 2012)

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proposed a novel method for estimating the capacity factor of variable speed wind turbines. Chang et al. (2003) investigated and compared monthly wind characteristics and monthly wind turbine characteristics for four meteorological stations with high winds. Chang and Tu (2007) analyzed monthly energy output and monthly capacity factor of a wind farm. Ditkovich et al. (2012) proposed a method for estimating capacity factor for stall and pitch-regulated wind turbines. Hu and Cheng (2007) presented a method for determining sites and wind turbine generator pairing.

This paper presents the month-wise graphical comparison between measured wind speed frequency and Weibull wind speed probabilities estimated using four numerical methods. It also uses a polynomial of eighth degree for modeling wind turbine power curve. A method for estimating the *n*th degree polynomial coefficients of wind turbine power curve with combined use of linear least square and QR decomposition using Gram-Schmidt orthogonalization through MATLAB is also presented. Coefficients of eighth-degree polynomial are used in the capacity factor estimation from generic model given by Albadi (2010). Estimated capacity factors are compared with the measured capacity factor of a wind turbine working at Soda site, for validation of results.

Case description

Details of the wind farm studied

Wind farm located at Soda site in the Thar desert region of western Rajasthan, India is selected for this study. It is in Jaisalmer district where May and June are hottest and January is the coldest month. Rainfall is very low and monsoon winds that bring rains in India bypass this region. Wind farm has twenty 1.25-MW capacity Suzlon-S66 turbines as shown in Figures 1 and 2. The total capacity of wind farm is 25 MW and turbines are having hub height of 65 m, cut-in speed v_c of 3 m/s, rated speed v_r of 14 m/s, and cut-off speed v_f of 22 m/s (http://www.suzlon.com/pdf/s66%20product%20brochure.pdf. Accessed 09 September 2014). Wind and meteorological data measurement mast of 65-m height at Soda wind farm is shown in Figure 3. Its specific position in the wind farm is marked in Figure 2.

Figure 1 Suzlon S-66 wind turbine of 1.25 MW at the wind farm.

Wind data modeling and analysis

Mean wind speed and standard deviation of grouped data are defined by Jangamshetti and Rau (1999), Manwell et al. (2009), and Bird (2003) as:

$$\bar{v} = \frac{\sum_{i=1}^{n} (f_m(v_i) \times v_i)}{\sum_{i=1}^{n} f_m(v_i)} \tag{1}$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} f_m(v_i) \cdot (v_i - \bar{v})^2}{\sum_{i=1}^{n} f_m(v_i)}} \tag{2}$$

where $\bar{v}$ is the mean wind speed in meter per second, σ is the standard deviation of wind speed in meter per second, v_i is the wind speed in meter per second at ith bin midpoint, $f_m(v_i)$ is the measured frequency of wind speed for ith bin, and n is the number of wind speed bins.

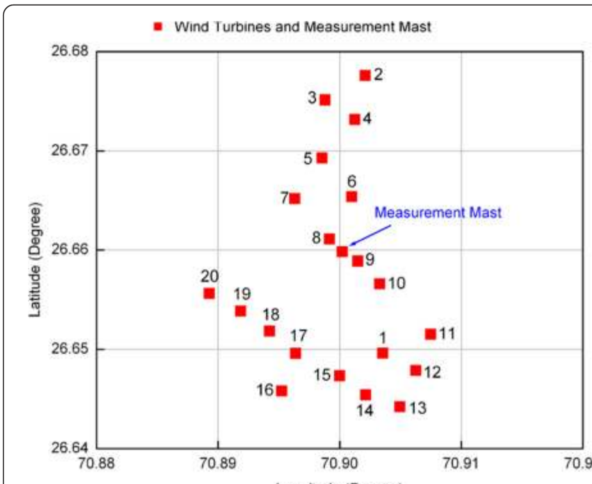

Figure 2 Locations of wind turbines and measurement mast in the 25-MW wind farm at Soda.

Figure 3 Measurement mast of 65-m height at Soda wind farm.

Weibull probability density function and its cumulative distribution function, used for describing the wind speed frequency distribution of a site, are defined by Masters (2004) as:

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1}\exp\left(-\left(\frac{v}{c}\right)^{k}\right) \quad (3)$$

$$F(v) = 1-\exp\left(-\left(\frac{v}{c}\right)^{k}\right) \quad (4)$$

where $f(v)$ is the Weibull wind speed probability density function at hub height, $F(v)$ is the Weibull cumulative distribution function, v is the wind speed in meter per second, k is the shape parameter at hub height, and c is the scale parameter at hub height.

Power available in the wind ($P_w(v)$) is expressed as $P_w(v) = 0.5\rho A v^3$, where ρ is the air density in kilogram per cubic meter, A is the rotor swept area in square meter, and v is the wind speed in meter per second. Wind power density (WPD) of a site that is based on Weibull distribution is defined by Jowder (2009), Huang and Wan (2012), and Chang et al. (2003) as:

$$\mathrm{WPD} = \int_0^{\infty} P_w(v)f(v)dv = 0.5\rho c^3\Gamma(1+3/k) \quad (5)$$

where Γ is a gamma function.

Root mean square error (RMSE) is based on the variation between measured and estimated values. RMSE of wind speed probability is defined by Rocha et al. (2012) and Bird (2003) as:

$$\mathrm{RMSE} = \sqrt{\left[\frac{1}{n}\sum_{i=1}^{n}\left(f_m(v_i)-f_c(v_i)\right)^2\right]} \quad (6)$$

where $f_m(v_i)$ is the measured wind speed frequency for ith bin, $f_c(v_i)$ is the estimated Weibull wind speed probability, v_i is the wind speed at ith bin midpoint, and n is the number of observations/bins. The percentage error between measured and estimated value is calculated using expression:

$$\text{Error}\ \% = \frac{\text{measured value}-\text{estimated value}}{\text{measured value}}\times 100. \quad (7)$$

Estimation of Weibull scale and shape parameters

Graphical method (GM) (Johnson 1978) uses Weibull cumulative distribution function and least square approximation for calculating the scale and shape parameters. Using Equation 4 and on taking twice the logarithm of each side, it becomes a form of straight line equation written as $y = ax + b$ where $y = \ln[-\ln(1-F(v))]$, $a = k$, $x = \ln(v)$, and $b = -k\ln(c)$. For n pairs of (x, y) where all summations are from 1 to n, the values of a and b are expressed as:

$$a = \frac{\sum xy - \frac{\sum x\sum y}{n}}{\sum x^2 - \frac{\left(\sum x\right)^2}{n}} \quad (8)$$

$$b = \bar{y} - a\bar{x} = \frac{1}{n}\sum y - \frac{a}{n}\sum x. \quad (9)$$

Shape and scale parameters are then expressed as $k = a$ and $c = \exp(-b/k)$.

Empirical method (EM) uses shape and scale parameter defined by Jangamshetti and Rau (1999) and Rocha et al. (2012) as:

$$k = (\sigma/\bar{v})^{-1.086} \quad (10)$$

$$c = \frac{\bar{v}}{\Gamma\left(1+\frac{1}{k}\right)}. \tag{11}$$

Modified maximum likelihood (MML) method uses frequency distribution of wind speed. Shape parameter is calculated by using numerical iterations and then scale parameter is obtained by solving equation explicitly. Value of shape parameter is around 2 for majority of sites and is a good initial estimate for iterative process. Shape and scale parameters are defined by Rocha et al. (2012) as:

$$k = \left[\frac{\sum_{i=1}^{n} v_i^k \ln(v_i) f(v_i)}{\sum_{i=1}^{n} v_i^k f(v_i)} - \frac{\sum_{i=1}^{n} \ln(v_i) f(v_i)}{f(v \geq 0)}\right]^{-1} \tag{12}$$

$$c = \left(\frac{1}{f(v\geq 0)} \sum_{i=1}^{n} {v_i}^k f(v_i)\right)^{\frac{1}{k}} \tag{13}$$

where v_i is the wind speed at ith bin midpoint, n is the number of bins, $f(v_i)$ is the frequency of wind speed occurrence in bin i, and $f(v \geq 0)$ is the probability of wind speed ≥ 0.

Energy pattern factor (EPF) is expressed as mean of the sum of cubes of all individual wind speed considered in a sample, divided by the cube of mean wind speed of sample (Centre for Wind Energy Technology 2011):

$$\mathrm{EPF} = \frac{1}{(\bar{v})^3} \times \left(\sum_{i=1}^{n} v_i^3/n\right) \tag{14}$$

where v_i is the wind speed in meter per second for ith observation, n is the number of wind speed samples, and $\bar{v}$ is the monthly mean wind speed. The monthly wind power density (WPD) is given by:

$$\mathrm{WPD} = 0.5\rho\left(\sum_{i=1}^{n} v_i^3/n\right) \tag{15}$$

where ρ is the monthly mean air density at hub height in kilogram per cubic meter. By substituting Equation 15 in Equation 14, EPF is expressed as:

$$\mathrm{EPF} = \frac{1}{(\bar{v})^3} \times \left(\frac{\mathrm{WPD}}{0.5 \times \rho}\right). \tag{16}$$

Shape parameter is calculated from EPF parameter using an expression defined by Rocha et al. (2012) as:

$$k = 1 + \frac{3.69}{(\mathrm{EPF})^2}. \tag{17}$$

Scale parameter is then calculated by using the expression given in Equation 11.

Polynomial model of power curve for pitch-regulated wind turbines

Relation between wind turbine electric power output ($P_e(v)$) and wind speed (v) for pitch regulated wind turbines are defined by Albadi (2010) as:

$$P_e(v) = P_r \times \begin{cases} 0, & (v < v_c \ or \ v > v_f) \\ P_{cinr}(v), & (v_c \leq v \leq v_r) \\ 1, & (v_r \leq v \leq v_f) \end{cases} \tag{18}$$

where P_r is the rated electrical power, and $P_{cinr}(v)$ is the turbine output power as a fraction of rated power between (including) cut-in wind speed v_c and rated wind speed v_r. v_f is cut-out wind speed.

There are many generic power curve models reported in the literature for representing the non-linear region between cut-in and rated wind speed of Figure 4. These models are not accurate as they do not fit the manufacturer's power curve data and only provide an approximate model of power curve that has errors. The approach used in this paper is to use a polynomial of eighth degree to model manufacturer wind turbine power curve data between cut-in and rated wind speed region.

A function is called polynomial of nth degree when it is expressed in the form as

$$P(v) = a_0 + a_1 v + a_2 v^2 + a_3 v^3 + \ldots + a_n v^n \tag{19}$$

where a_0, a_1, a_2, ..., a_n are the constant coefficients of polynomial function. The procedure of calculating coefficients of nth-degree polynomial by combined use of linear least square and matrix factorization methods through MATLAB are explained below.

Linear least square method

Consider given m sets of data (x_i, y_i) where $i = 1,.., m$ and the polynomial model that is fitted to data is of nth degree expressed as:

$$P(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_n x^n \tag{20}$$

where a_0, a_1, a_2, ..., a_n are the coefficients that are to be found out. The m sets of data and polynomial $P(x)$ are expressed in matrix form as $\boldsymbol{y} = \boldsymbol{X\alpha}$ where:

$$\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}, \tag{21}$$

$$\boldsymbol{X}_{(m,\ n+1)} = \begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_m & x_m^2 & \ldots & x_m^n \end{bmatrix}, \tag{22}$$

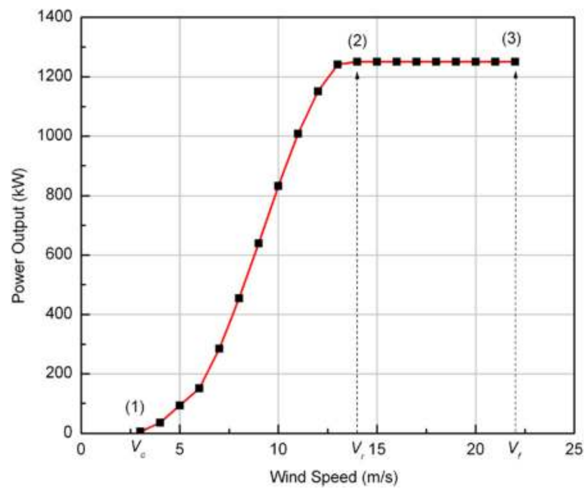

Figure 4 **Power curve of Suzlon S66 1.25-MW pitch-regulated wind turbine (Wind Power Program).**

$$\boldsymbol{\alpha} = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix}. \tag{23}$$

The coefficients $a_0, a_1, a_2, \ldots, a_n$, that best fit Equation 20 are found out by solving minimization problem, where the objective function S is given by Press et al. (2009) as:

$$S(\boldsymbol{\alpha}) = \sum_{i=1}^{m} \left[\boldsymbol{y}_i - \sum_{j=1}^{n+1} \boldsymbol{X}_{ij} \boldsymbol{\alpha}_j \right]^2 = \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\alpha}\|^2. \tag{24}$$

Normal equations of least square problem can be expressed in matrix notation as

$$\left(\boldsymbol{X}^T \boldsymbol{X}\right) \boldsymbol{\alpha} = \boldsymbol{X}^T \boldsymbol{y} \tag{25}$$

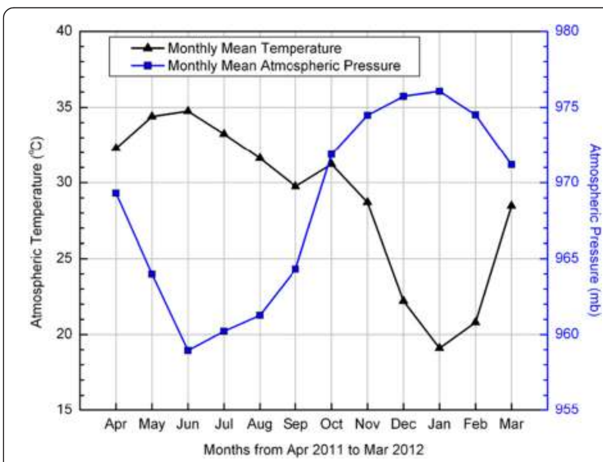

Figure 5 **Monthly mean temperature and monthly mean pressure measured at Soda.**

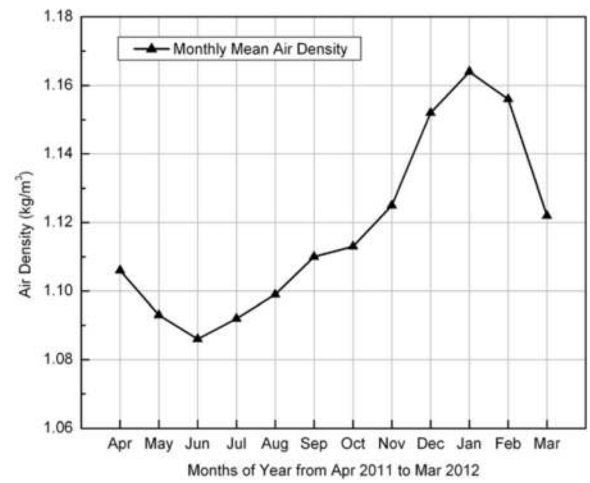

Figure 6 **Monthly mean air densities at Soda wind farm.**

where $\boldsymbol{X}^T$ is the transpose of matrix $\boldsymbol{X}$. The algebraic solution of Equation 24 is expressed (Demmel 1997) as

$$\boldsymbol{\alpha} = \left(\boldsymbol{X}^T \boldsymbol{X}\right)^{-1} \boldsymbol{X}^T \boldsymbol{y}. \tag{26}$$

Solution from normal equations can have round-off errors so QR decomposition of matrix $\boldsymbol{X}$ is done.

QR decomposition

QR decomposition is a matrix factorization method (Embree 2010). It states that for any $m \times n$ matrix $\boldsymbol{X}$ with $m \geq n$, there exists a unitary $m \times m$ matrix $\boldsymbol{Q}$ and an upper triangular $m \times n$ matrix $\boldsymbol{R}$ such that

$$\boldsymbol{X} = \boldsymbol{QR}. \tag{27}$$

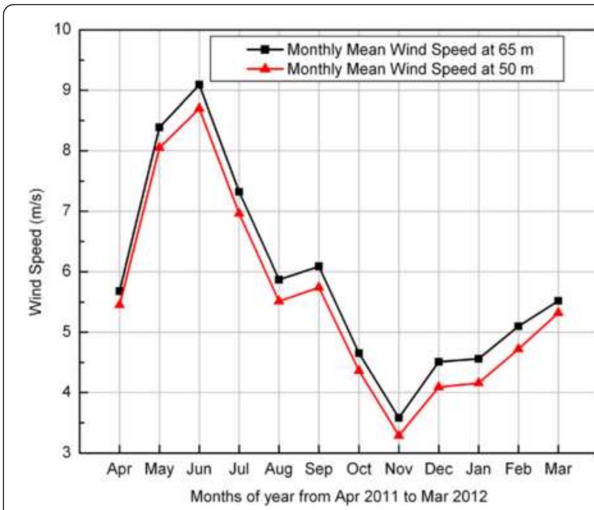

Figure 7 **Monthly mean wind speed measured at 65-m and 50-m heights.**

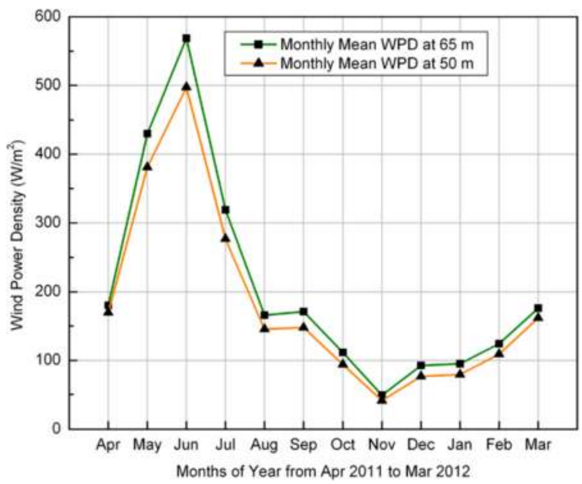

Figure 8 Monthly mean wind power density measured at 65-m and 50-m heights.

On substituting Equation 27 in Equation 26, the expression as explained by Demmel (1997) becomes:

$$\boldsymbol{\alpha} = \left(\boldsymbol{R}^T\boldsymbol{Q}^T\boldsymbol{Q}\boldsymbol{R}\right)^{-1}\boldsymbol{R}^T\boldsymbol{Q}^T\boldsymbol{y} = \left(\boldsymbol{R}^T\boldsymbol{R}\right)^{-1}\boldsymbol{R}^T\boldsymbol{Q}^T\boldsymbol{y} \tag{28}$$

$$\boldsymbol{\alpha} = \boldsymbol{R}^{-1}\boldsymbol{R}^{-T}\boldsymbol{R}^T\boldsymbol{Q}^T\boldsymbol{y} \tag{29}$$

$$\boldsymbol{\alpha} = \boldsymbol{R}^{-1}\boldsymbol{Q}^T\boldsymbol{y}\,. \tag{30}$$

On solving Equation 30, the required coefficients of polynomial Equation 20 are obtained. For computing QR decomposition of matrix $\boldsymbol{X}$, the MATLAB command used is (Embree 2010):

$$[\boldsymbol{Q},\boldsymbol{R}] = \mathrm{qr}(\boldsymbol{X}). \tag{31}$$

This application has a $m \times n$ matrix $\boldsymbol{X}$ with m much larger than n. So, the QR decomposition produces a $m \times m$ matrix $\boldsymbol{Q}$ that will require more storage than $\boldsymbol{X}$ (Embree 2010). Also, columns $n + 1, \ldots, m$ of $\boldsymbol{Q}$ are surplus as they multiply against zero entries of $\boldsymbol{R}$.

QR decomposition using Gram-Schmidt orthogonalization

It is one solution to the above mentioned concern. This procedure results in a *skinny QR decomposition*, $\boldsymbol{X} = \boldsymbol{Q}\boldsymbol{R}$, where $\boldsymbol{Q}$ is $m \times n$ matrix, $\boldsymbol{R}$ is a $n \times n$ matrix, and $\boldsymbol{Q}^*\boldsymbol{Q} = \boldsymbol{I}$. Here, $\boldsymbol{Q}^*$ is the conjugate transpose matrix and $\boldsymbol{I}$ is $n \times n$ identity matrix (Embree 2010). This algorithm can be easily computed in MATLAB using command:

$$[\boldsymbol{Q},\boldsymbol{R}] = \mathrm{qr}(\boldsymbol{X},\boldsymbol{0}). \tag{32}$$

If $m > n$, only the first n columns of $\boldsymbol{Q}$ and the first n rows of $\boldsymbol{R}$ are computed (http://in.mathworks.com/help/matlab/ref/qr.html. Accessed 09 September 2014). If $m \le n$, then, this is same as $[\boldsymbol{Q}, \boldsymbol{R}] = \mathrm{qr}(\boldsymbol{X})$.

Analytical estimation of capacity factor

Capacity factor (CF) (Masters 2004) is defined as the ratio of average output power to rated output power over a certain period of time. Monthly capacity factor ($\mathrm{CF_m}$) is expressed as:

$$\mathrm{CF_m} = \frac{\text{monthly energy yield from wind turbine (kWh)}}{\text{rated power (kW)} \times \text{ total hours in particular month}} \tag{33}$$

and the annual capacity factor ($\mathrm{CF_a}$) is expressed as:

$$\mathrm{CF_a} = \frac{\text{annual energy yield from wind turbine(kWh)}}{\text{rated power (kW)} \times \text{ total hours in a year}}. \tag{34}$$

Capacity factor of a particular wind turbine at a site can be analytically estimated by using Weibull scale and shape

Table 1 Monthly Weibull parameters estimated from four numerical methods at hub height of 65 m

Months	Graphical method		Empirical method		Modified maximum likelihood method		Energy pattern factor method	
	k	*c (m/s)*	*k*	*c (m/s)*	*k*	*c (m/s)*	*k*	*c (m/s)*
Apr 2011	1.7438	5.6863	2.1545	6.4137	2.0761	6.3507	2.1681	6.4137
May 2011	2.3472	9.0708	3.3229	9.3500	3.2595	9.2646	3.0793	9.3845
Jun 2011	2.0217	9.7378	2.9515	10.1866	2.9708	10.1471	2.8994	10.1942
Jul 2011	2.0981	7.7550	2.7184	8.2294	2.6535	8.1868	2.6637	8.2351
Aug 2011	1.8876	5.8166	2.6121	6.6079	2.5513	6.5499	2.6569	6.6044
Sep 2011	2.1592	6.3457	3.0948	6.8103	3.0623	6.7587	2.9802	6.8218
Oct 2011	1.5196	4.5302	1.8902	5.2394	1.7922	5.1967	1.9311	5.2428
Nov 2011	1.5284	3.4343	1.9543	4.0376	1.8758	4.0250	2.0225	4.0404
Dec 2011	1.5917	4.4552	2.1348	5.0925	2.0331	5.0567	2.2038	5.0924
Jan 2012	1.6794	4.5200	2.1957	5.1489	2.1198	5.1213	2.2430	5.1484
Feb 2012	1.8906	5.1810	2.3963	5.7532	2.3343	5.7274	2.4081	5.7527
Mar 2012	1.7159	5.6665	2.0785	6.2319	2.0251	6.2152	2.0626	6.2314

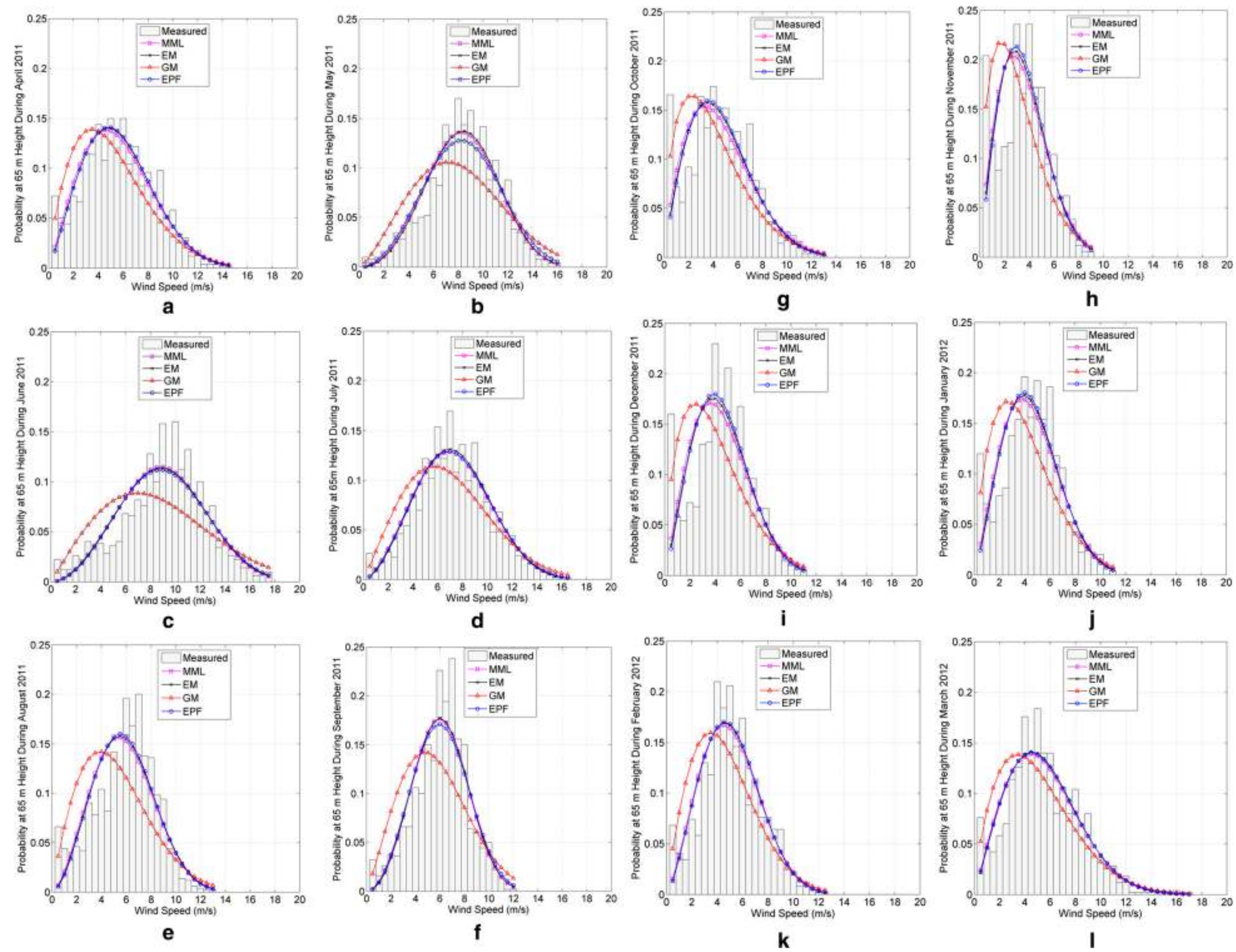

Figure 9 Comparison of estimated and measured wind speed probability at Soda for 65-m height (a–l).

parameters of site, wind turbine speed parameters, and coefficients of polynomial model for power curve in the expression defined by Albadi (2010) as:

$$\mathrm{CF} = -e^{-\left(v_f/c\right)^k} + \sum_{i=1}^{n}\left[a_i \times i \times \left(c^i/k\right) \times \Gamma(i/k) \times \left(\gamma\left(\left(v_r/c\right)^k, i/k\right) - \gamma\left(\left(v_c/c\right)^k, i/k\right)\right)\right] \tag{35}$$

where $\Gamma(a) =$ Gamma function $= \int_0^\infty t^{a-1}e^{-t}dt$, and $\gamma(u,a) =$ Incomplete gamma function $= [1/\Gamma(a)] \times \int_0^u t^{a-1}e^{-t}dt$.

Discussion and evaluation

Wind and meteorological data of Soda site for the duration from April 2011 to March 2012 were provided by the owner company of wind farm. Monthly mean atmospheric pressure and monthly mean temperature at Soda

Table 2 Comparison of RMSE of wind speed probability

Months	Monthly RMSE (graphical)	Monthly RMSE (empirical)	Monthly RMSE (MML)	Monthly RMSE (EPF)
Apr 2011	0.0461	0.0443	0.0437	0.0445
May 2011	0.0352	0.0399	0.0397	0.0377
Jun 2011	0.0329	0.0354	0.0357	0.0350
Jul 2011	0.0366	0.0387	0.0382	0.0381
Aug 2011	0.0529	0.0502	0.0501	0.0507
Sep 2011	0.0509	0.0530	0.0531	0.0517
Oct 2011	0.0557	0.0521	0.0511	0.0527
Nov 2011	0.0787	0.0737	0.0722	0.0751
Dec 2011	0.0635	0.0602	0.0593	0.0611
Jan 2012	0.0617	0.0583	0.0576	0.0590
Feb 2012	0.0530	0.0516	0.0510	0.0518
Mar 2012	0.0416	0.0402	0.0398	0.0401
Average RMSE	0.05073	0.04980	0.04929	0.04979

Table 3 Suzlon S66-1.25-MW wind turbine power curve data (Wind Power Program; I-Rivera et al. 2009)

Wind speed (m/s)	3	4	5	6	7	8	9	10	11	12	13	14
Power (kW)	5	35	93	151	285	454	639	832	1,008	1,152	1,241	1,250
Power (Normalized) y	0.004	0.028	0.0744	0.1208	0.228	0.3632	0.5112	0.6656	0.8064	0.9216	0.9928	1

are shown in Figure 5 and their 1-year average values are 968.49 mb (1 bar = 10^5 Pa) and 28.88°C, respectively. Monthly mean air density based on measured temperature and pressure data is shown in Figure 6 and its 1-year average value is 1.118 kg/m^3.

Monthly mean wind speed at 65-m and 50-m heights are shown in Figure 7 and their 1-year average values are 5.86 m/s and 5.53 m/s, respectively. Monthly mean wind power density at 65-m and 50-m heights are shown in Figure 8 and their 1-year average values are 206.87 W/m^2 and 181.71 W/m^2, respectively.

Estimation of monthly Weibull function parameters for Soda site

Table 1 shows the monthly Weibull parameters estimated from graphical, empirical, modified maximum likelihood, and energy pattern factor methods for Soda at height of 65 m.

Graphical comparison of measured and estimated wind speed probability

Figure 9a–l shows the month-wise wind speed probability at site. They are calculated from shape (k) and scale (c) parameters given in Table 1. Density histograms of month-wise measured wind speed frequency at hub height are also shown in each figure for comparison. A density histogram is a histogram that has been normalized, so it will integrate to one (Martinez and Martinez 2002).

It can be observed from Figure 9a–l that probability curves using graphical method are not fitting the measured wind speed frequency density histograms. Weibull probabilities calculated from empirical, modified maximum likelihood, and energy pattern factor methods are nearly similar and overlapping each other. They are also representing better fit with the density histograms of measured wind speed frequency.

Statistical analysis of four numerical methods

Table 2 gives the comparison of root mean square errors (RMSEs) of wind speed probabilities and is calculated using monthly Weibull parameters estimated from four methods at hub height. It is observed that modified maximum likelihood method has the lowest and graphical method has highest value for 1-year average monthly RMSE at Soda site. Thus, modified maximum likelihood method gives better results in calculating Weibull function parameters amongst the graphical, empirical, modified maximum likelihood, and energy pattern factor methods at Soda site. Empirical and EPF methods have almost the same monthly RMSE.

Eighth-degree polynomial fit to wind turbine power curve data

Power curve data of Suzlon S66-1.25-MW pitch-regulated wind turbine (http://www.wind-power-program.com/download.htm. Accessed 09 September 2014; I-Rivera et al. 2009) between cut-in and rated wind speeds are shown in Table 3. Polynomial of eighth degree

$$P(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7 + a_8x^8 \tag{36}$$

is used to fit the data given in Table 3. Linear least square method and QR decomposition using Gram-Schmidt orthogonalization are used for calculating coefficients of polynomial using MATLAB. Coefficients of eighth-degree polynomial after calculations are in Table 4.

Figure 10 shows the eighth-degree polynomial curve and manufacturer's power curve data of Suzlon S66 wind turbine between cut-in and rated wind speeds. It can be observed that actual data and eighth-degree polynomial model both fit each other.

Measured data of wind turbine-9 at Soda

Various measured parameters of wind turbine-9 from April 2012 to March 2013 are given in Table 5. Wind turbine-9 data are used for comparison because measurement mast and turbine-9 are located near to each other as shown in Figure 2. So, it is a reasonably good assumption that turbine-9 and measurement mast will have the same wind availability.

Table 4 Coefficients of eighth-degree polynomial fit

Coefficients	Values
a_0	7.2789524
a_1	−9.0732954
a_2	4.6960724
a_3	−1.3208640
a_4	0.22157098
a_5	−0.0227409
a_6	0.0014020
a_7	−0.0000477
a_8	0.000000689

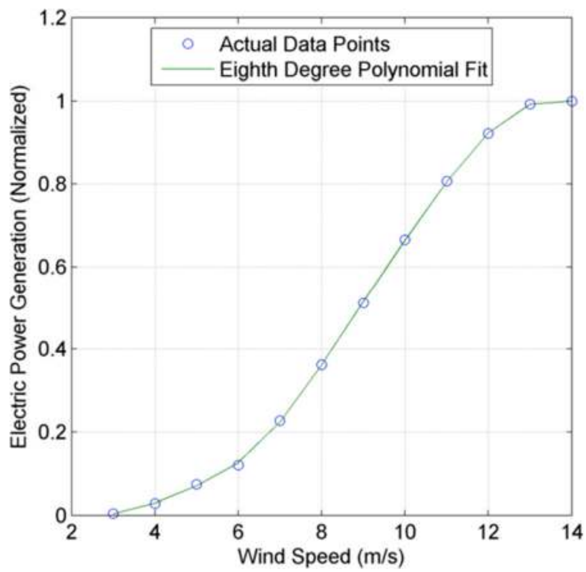

Figure 10 Eighth-degree polynomial fit to Suzlon S66 power curve between cut-in and rated wind speed.

Energy yield losses

Analytically, estimated values of monthly capacity factor are to be corrected for machine non-availability, grid non-availability, air density losses, and wake effect losses. The estimated monthly capacity factor values are multiplied by measured monthly machine availability and monthly grid availability given in Table 5 for adjusting the losses associated with machine non-availability and grid non-availability. The wake effect losses are assumed as 5% because the wind farm has turbines working in front of the other as shown in Figure 2 and so the estimated monthly capacity factor is multiplied by a factor of 0.95.

Table 5 Measured data of wind turbine-9 working at Soda wind farm

Months	Energy produced (kWh)	Capacity factor	Machine availability	Grid availability
Apr 2012	144,798	0.1609	0.9905	0.9826
May 2012	214,178	0.2303	0.9517	0.9516
Jun 2012	391,530	0.4350	0.8541	0.9813
Jul 2012	315,257	0.3390	0.8025	0.9606
Aug 2012	203,584	0.2189	0.977	0.9915
Sep 2012	94,305	0.1048	0.9965	0.9999
Oct 2012	44,503	0.0479	0.9701	0.993
Nov 2012	33,257	0.0370	0.9847	0.9932
Dec 2012	97,878	0.1052	0.9942	0.9952
Jan 2013	48,108	0.0517	1.0	0.9785
Feb 2013	94,453	0.1124	0.9891	0.992
Mar 2013	107,679	0.1158	0.9813	0.9784
Annual	1,789,530	0.1634	0.9576	0.9831

Table 6 Monthly mean air density and correction factor for density at Soda

Months	Monthly mean air density (kg/m^3)	Ratio of monthly mean air density and standard air density
Apr	1.106	0.903
May	1.093	0.892
Jun	1.086	0.887
Jul	1.092	0.891
Aug	1.099	0.897
Sep	1.11	0.906
Oct	1.113	0.909
Nov	1.125	0.918
Dec	1.152	0.940
Jan	1.164	0.950
Feb	1.156	0.944
Mar	1.122	0.916
Average	1.118	0.913

Suzlon S66 wind turbine has rated wind speed of 14 m/s. It is evident from Figure 7 that monthly mean wind speed at hub height is always less that 9.09 m/s during all the months. The Figure 9a–l shows that wind speed never reached 14 m/s during August to February months at Soda site. Moreover, the probability of wind speed occurrence at values equal to or more than 14 m/s during March to July period is very low. So, it can be concluded that wind turbines installed at the wind farm are operating below their rated wind speed for most of the time. Majority of the energy production is from ascending section of power curve, which is between cut-in and rated wind speed region. This conclusion is used in calculating the air density correction factor. Estimated monthly capacity factor is

Table 7 Monthly correction factors of estimated capacity factors

Months	Monthly correction factor
Apr	0.8348
May	0.7676
Jun	0.7059
Jul	0.6528
Aug	0.8256
Sep	0.8577
Oct	0.8315
Nov	0.8533
Dec	0.8839
Jan	0.8833
Feb	0.8796
Mar	0.8354
Average	0.8176

Table 8 Monthly capacity factors estimated using four numerical methods

Months	Estimated CF_m (GM)	Estimated CF_m (EM)	Estimated CF_m (MML)	Estimated CF_m (EPF)
Apr	0.1598	0.1919	0.1905	0.1913
May	0.4094	0.4465	0.4373	0.4465
Jun	0.4467	0.5149	0.5122	0.5139
Jul	0.3039	0.3363	0.3329	0.3373
Aug	0.1600	0.1910	0.1880	0.1894
Sep	0.1862	0.1969	0.1930	0.2003
Oct	0.1001	0.1189	0.1222	0.1167
Nov	0.0415	0.0476	0.0501	0.0453
Dec	0.0896	0.0966	0.0991	0.0937
Jan	0.0870	0.0976	0.0991	0.0957
Feb	0.1149	0.1308	0.1312	0.1303
Mar	0.1604	0.1810	0.1824	0.1818

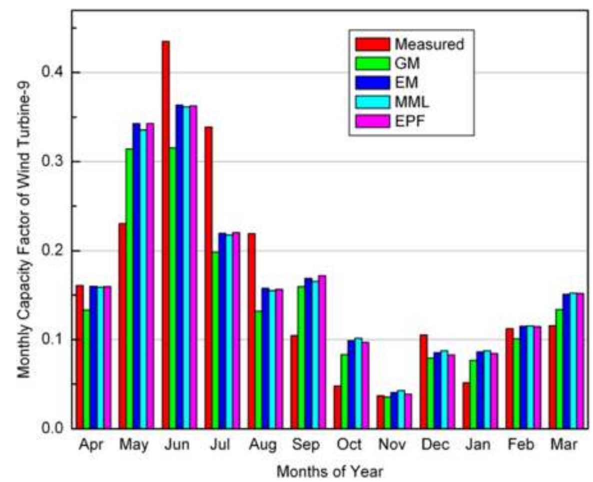

Figure 11 Comparison of measured and corrected estimated monthly capacity factors of wind turbine.

corrected by multiplying it with the ratio of monthly mean air density at site to standard air density of 1.225 kg/m^3. The values of ratio are given in Table 6 (Hau 2006). This correction process also takes care of the differences in air density between summer (May, June) and winter (December, January) seasons.

Comparison of measured and corrected estimated capacity factors

Monthly correction factors by considering machine non-availability, grid non-availability, air density losses, and wake effect losses are given in Table 7. Table 8 shows the estimated monthly capacity factor values. They are calculated using Equation 35 and data given in Tables 1 and 4. Table 9 shows the corrected monthly capacity factors. Corrected monthly capacity factors are obtained by multiplying the estimated monthly capacitor factors given in Table 8 with the monthly correction factors given in Table 7. It is to be noted that measured wind speed frequency distribution data are from April 2011 to March 2012 whereas measured wind turbine-9 energy production data are from April 2012 to March 2013. Comparison between the measured and corrected values of capacity factor are done assuming that the wind profile of a site does not change significantly from 1 year to another year.

Table 9 Corrected capacity factors estimated using four numerical methods

Months	Corrected CF_m (GM)	Corrected CF_m (EM)	Corrected CF_m (MML)	Corrected CF_m (EPF)
Apr	0.1334	0.1602	0.1590	0.1597
May	0.3143	0.3428	0.3357	0.3428
Jun	0.3153	0.3635	0.3615	0.3627
Jul	0.1984	0.2195	0.2173	0.2202
Aug	0.1321	0.1577	0.1552	0.1564
Sep	0.1597	0.1689	0.1655	0.1718
Oct	0.0832	0.0989	0.1016	0.0970
Nov	0.0354	0.0406	0.0427	0.0387
Dec	0.0792	0.0854	0.0876	0.0828
Jan	0.0768	0.0862	0.0875	0.0845
Feb	0.1011	0.1151	0.1154	0.1146
Mar	0.1340	0.1512	0.1524	0.1519

Figure 11 shows the graphical comparison of measured and corrected monthly capacity factor values given in Tables 5 and 9, respectively.

Corrected monthly capacity factors shown in Table 9 does not give a comprehensible result, as monthly wind profile may vary from 1 year to another year. So, corrected annual capacity factor of wind turbine are calculated and then the percentage errors between the measured and corrected values of annual capacity factor are obtained as shown in Table 10. It is observed that percentage error in annual capacity factor computation by using Weibull parameters estimated from MML method is −1.22%. It is the lowest in comparison to graphical, empirical, and energy pattern factor methods. Graphical method gave the most erroneous results.

Table 10 Comparison between corrected annual capacity factors along with percentage error

Methods of estimating Weibull parameters	Corrected annual capacity factor	Percentage error (comparing with wind turbine-9 measured annual CF of 0.1634) (%)
GM	0.1471	+9.98
EM	0.1660	−1.59
MML	0.1654	−1.22
EPF	0.1655	−1.29

Conclusions

This paper analyzed wind characteristics, Weibull wind speed distribution using four numerical methods, eighth-degree polynomial modeling of wind turbine power curve, and capacity factor estimation of wind turbines at Soda site in the desert region of western Rajasthan in India. The percentage error in annual capacity factor estimation using Weibull parameters estimated from graphical, empirical, modified maximum likelihood, and energy pattern factor methods were +9.98%, −1.59%, −1.22%, and −1.29%, respectively. Annual capacity factors calculated using Weibull parameters estimated from modified maximum likelihood method matched the measured values best and the graphical method gave the most erroneous results. Wind power density is highest in June and lowest in November with measured values of 568.45 W/m^2 and 49.03 W/m^2, respectively. It shows a large variation due to change in monthly weather conditions.

Abbreviations

WPD: wind power density; RMSE: root mean square error; GM: graphical method; EM: empirical method; MML: modified maximum likelihood; EPF: energy pattern factor; CF: capacity factor.

Competing interests

The authors declare that they have no competing interests.

Authors' contribution

BKS carried out the data acquisition, analysis, and interpretation and drafted the manuscript. KVSR contributed in the conception and designing of case study, data analysis, and critical review of the manuscript. Both authors read and approved the final manuscript.

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