Compensation of Depolarizing Effects in Electron Positron Storage Rings

by

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Abstract

It is shown that the depolarizing effects in the arc of an electron-positron storage ring can be minimized by minimizing certain Fourier-harmonics of the particle trajectories. In more detail it is shown that the strength of the depolarizing resonance $((g-2)/2).\gamma = n \pm Q_x$ is related to the harmonics of the closed orbit, the strength of the depolarizing resonance $((g-2)/2 \cdot \gamma = m \pm Q_z$ to the harmonics of the vertical betatron oscillations, and the strength of the depolarizing resonance $((g-2)/2) \cdot \gamma = i \pm Q_s$ to the harmonics of the closed orbit and the dispersion orbit. For all these depolarizing resonances compensation schemes are discussed.

1. Introduction

In the last decade the spin-polarization of electrons and positrons circulating in a storage ring was used in many different ways. The most important applications of the polarization was the measurement of the quark-1/2-spin at SPEAR /1/ and the precision measurements of the masses of the Y and the J/Ψ-resonances at VEPP, DORIS and CESR /2/.

Although the time constant for the build-up of the polarization into the vertical direction by the Sokolov-Ternov effect is approximately the same for all existing electron-positron machines, the depolarizing mechanisms become worse with increasing machine size /3/. From the experience at DESY it is known that it is relatively easy to have a high degree of polarization at DORIS (beam energies up to 5.6 GeV). It turned out to be much more difficult to have polarized beams at PETRA (beam energy up to ∿23 GeV). For the future storage rings (i.e. HERA, LEP, TRISTAN) no polarization can be expected unless they are designed and constructed for polarization. The most interesting results are expected when the particles have longitudinal polarization in the interaction region /4/. The spins have to be rotated after the arc, where they are vertical, into the longitudinal direction and after the interaction region back into the vertical direction by so-called spin-rotators. Even in an ideal machine these rotators have strong depolarizing effects. The reason for this depolarizing effect is the following. The particles of the beam are different in energy, momentum and position. The rotation into the longitudinal direction and back depends in general on these parameters. Therefore a vertically polarized beam is slightly depolarized after a pass through the rotator. The depolarization can be minimized when certain conditions, so called spin transparency conditions, are fulfilled /5/.

In this paper the spin transparency condition for the arc is discussed. In such a spin-transparency condition it must be taken into account that the spins are depolarized by the sequence of quadrupoles and bending magnets. It will be shown in this paper that spin-transparency in an imperfect machine (a real machine with all sorts of errors) can be approximately achieved when some Fourier-harmonics in the particle trajectories are minimized.

In more detail it is shown that

- the strength of the Q_s resonance is connected to harmonics in the closed orbit and the dispersion orbit
- the strength of the Q_x resonance to harmonics in the closed orbit
- the strength of the Q_z resonance to harmonics in the betatron-trajectories.

As a consequence three different corrections have to be applied to compensate the resonance effects and to make the arc spin-transparent:

- Eight correction dipoles can compensate the harmonics of the vertical closed orbit. This cure reduces the strength of the Q_x and the Q_s resonances. The scheme was both simulated and experimentally tested in the storage ring PETRA /6/.

- The vertical dispersion is in general strongly influenced by asymmetric beam bumps in the interaction regions. Moving these bumps in an intelligent manner the depolarizing effects caused by synchrotron resonances are reduced by reducing the strength of some harmonics of the dispersion.

- Resonances driven by vertical oscillations can be compensated with the help of eight quadrupoles. These quadrupoles are used in a similar way as the correction coils for the $Q_{x/s}$-resonance compensation.

Numerical calculations using the SLIM program /7/ demonstrate that the applied compensation optimizes the degree of polarization from less than 30 % up to more than 80 %. The calculations are done for the storage ring PETRA in an optics currently used as a luminosity optics.

2. The polarizing and the depolarizing effects

The polarization of an electron beam in a storage ring is built up by the so-called Sokolov-Ternov effect. The spin of a particle can flip when the particle emits synchrotron radiation in a magnetic field. The probability of a spin-flip in one direction is higher than the probability in the other. For electrons the polarization is built up in the direction opposite to the magnetic field /8/. The maximum degree of polarization in a plane, perfect storage ring is 92.4 %.

The second effect which changes the polarization is the continuous rotation of the spins in electromagnetic fields. This effect is described by an equation of motion, the Thomas-BMT equation /9/ :

$$\frac{d\vec{s}}{dt} = \vec{\Omega} \times \vec{s}$$

$$\vec{\Omega} = \frac{e}{m\,\gamma} \left((1+a\gamma)\ \vec{B}_{\perp} + (1+a)\ \vec{B}_{\parallel} \right) \qquad (1)$$

$\vec{s}$ ... spin vector

γa ... so-called spin-tune with:

  a ... anomalous magnetic moment of the electron

  γ ... gamma-factor

$\vec{B}_{\perp/\parallel}$ ... magnetic field parallel and orthogonal to the direction of motion (electric fields are omitted)

In the following it will be explained that depolarization occurs when the direction opposite to the deflecting field and the spin direction do not coincide. The deviation of the spins from this direction is caused by machine imperfections, vertically deflecting magnets, and longitudinal magnetic fields.

The first depolarizing mechanism is caused by a reduction of the effectiveness of the Sokolov-Ternov effect. To explain this it is assumed that an electron travels on the closed orbit. The closed orbit shall deviate from the ideal plane closed orbit due to vertically deflecting magnets and field and alignment errors of magnets.

The magnetic field along the closed orbit is described by

$$\vec{B}(s) = \vec{B}_0(s) + \vec{B}_{co}(s)$$

$\vec{B}_o$ is the magnetic field on the ideal orbit, $\vec{B}_{co}$ the additional field on the closed orbit. s is the length along the closed orbit. Equation (1) can be solved for $\vec{B}(s)$. The solution of this equation consists of one real vector $\vec{n}$ and two complex vectors $\vec{\eta}$ and $\vec{\eta}^*$. The complex vectors can be expressed by the real vectors $\vec{n}$, $\vec{l}$ and $\vec{m}$. (see fig. 1)

$$\vec{\eta} = \frac{1}{\sqrt{2}}\,(\vec{l} + i\,\vec{m})\quad e^{-i\phi} \qquad (2)$$

$$\vec{\eta}^* = \frac{1}{\sqrt{2}}\,(\vec{l} - i\,\vec{m})\quad e^{+i\phi}$$

$\phi = \gamma a \alpha$, ... where α is the angle by which the electron is deflected in the bending magnets.

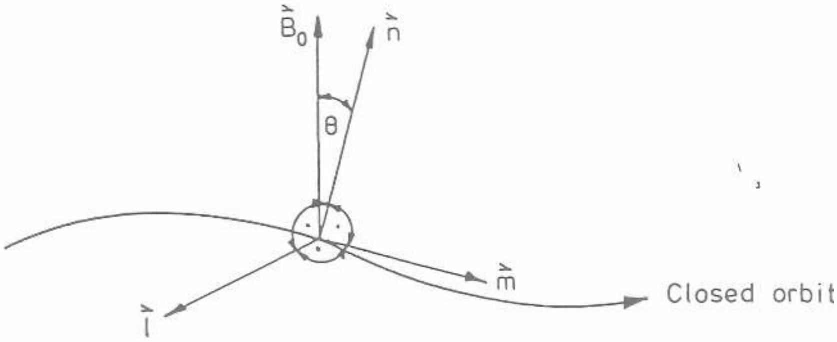

Fig. 1 Spin base vectors $\vec{l}$, $\vec{m}$ and $\vec{n}$

In general, the $\vec{n}$-axis and the direction of the magnetic field do not coincide. The polarization is built up into the direction of the transversal magnetic field but only the component along $\vec{n}(s)$ can survive.

The maximum degree of polarization is reduced to /10/:

$$P_{max} = 0.924 \frac{\oint \frac{ds}{|\rho|^3} \vec{e}_B \vec{n}}{\oint \frac{ds}{|\rho|^3} - \frac{2}{9} \oint \frac{\vec{n}\,\vec{e}_y}{|\rho|^3}\, ds} \qquad (3)$$

$\vec{e}_B$ ... unit vector in the magnetic field direction

$\vec{e}_y$ ... $\frac{\vec{v}}{|\vec{v}|}$; $\vec{v}$ ... velocity of the electrons

ρ ... bending radius.

The second depolarizing mechanism is caused by the emission of synchrotron radiation. After the emission of a photon the electron moves in a complicated way around the closed-orbit. The fields acting on the particle can be divided into two parts:

$$\vec{\Omega}(t) = \vec{\Omega}_{co}(t) + \vec{\omega}(t) \qquad (4)$$

$\vec{\Omega}_{co}$ describes the periodic field on the closed orbit and $\vec{\omega}$ describes the aperiodic perturbation. Due to radiation damping $\vec{\omega}$ (t) becomes small after several damping times. The spin of a particle, paralled to $\vec{n}$ before emission, points after the damping into the direction $\vec{n} + \delta\vec{s}$. The polarization is reduced proportional to $|\delta \vec{s}|^2$ due to the above mentioned fact that only the component along $\vec{n}$ can survive /11/. This depolarizing mechanism excites the depolarizing resonances and is the main limitation for the polarization in a storage ring.

In a machine without strong vertical deflections or longitudinal fields both depolarizing effects have a common cause. The spin is rotated in the arcs away from the direction of the bending field. The reason for this rotation is described in the following.

In the arcs the spins are subsequently rotated by the bending magnets and the quadrupoles .The bending field rotates the spin around the vertical (z-axis) with an angle $\gamma a \alpha$, the deflection angle times the spin tune (fig. 2).

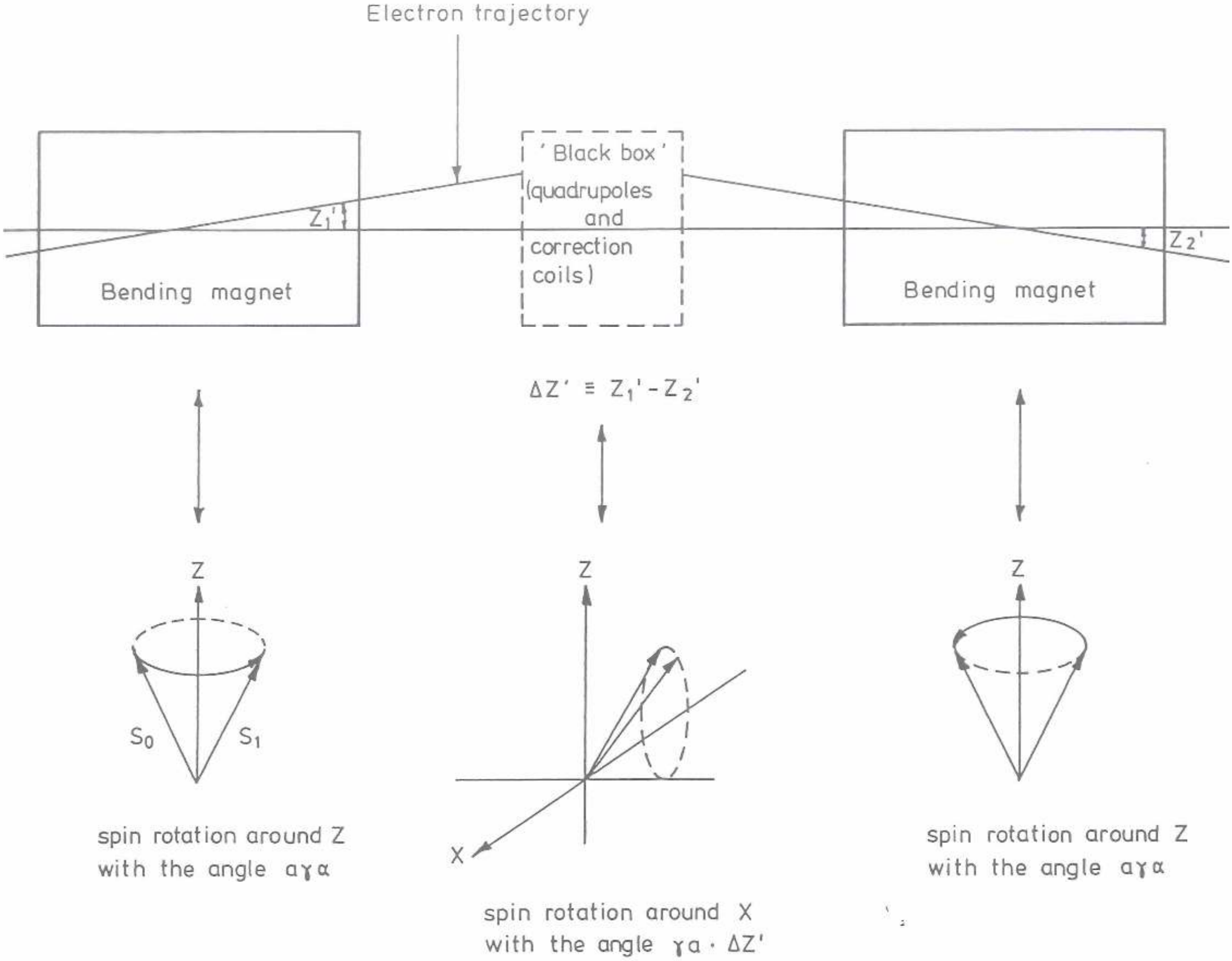

Fig. 2 Spin rotation in the arcs

The vertical projection of the spin remains constant. Between the bending magnets quadrupole fields and small radial fields (correction coils or errors of the quadrupoles) rotate the spin around the radial axis (x-axis).

These rotations change the projection of the spin on the vertical axis. The rotation angle is proportional to the change of the vertical slope of the particle (fig. 2)

In a storage ring the size of PETRA or HERA some hundred bending magnets are installed. Between all these bending magnets the spin is rotated by quadrupole fields. The individual rotations are small ($\sim$ some milliradians) but they can add up, depending on the path of the electron.

The basic idea in this paper is the following : A sequence of dipoles and quadrupoles rotate the spin. The rotations around the radial axis can only add if the rotations of the bending field and the "black box" (fig. 2) have a certain relation. It is only necessary to correct the dangerous Fourier-components of the rotations contributing to the relation. It will be shown that for the closed-orbit, the betatron, and the synchrotron oscillation similar correction schemes can be developed.

3. The Strength of the Depolarizing Resonances

3.0. General remarks on the influence of aperiodic perturbations on the degree of polarization

In the following the strength of the depolarizing resonances excited by photon emission is calculated.

The argumentation in this chapter is similar to the argumentation found in several papers, e.g. Yokoja /12/. Details can be found in these papers.
The BMT-equation for the spin of an electron in the aperiodic field $\vec{\omega}(t)$ must be solved (see eq. 4) :

$$\frac{d\vec{s}}{dt} = (\vec{\Omega}_{co}(t) + \vec{\omega}(t)) \times \vec{s} \qquad (5)$$

with $\vec{\Omega}_{co}$ ... periodic field on the closed orbit and $\vec{\omega}$ (t) ... aperiodic field

The equation can be solved by a perturbation approach:

$$\vec{s} = \vec{n} + \delta\vec{s}$$

$\vec{n}$ is the solution of the BMT-equation on the closed orbit (see chapter 2). The vector $\delta\vec{s}$ is combined by the eigenvectors of the BMT-equation (eq. 2) $\vec{n}, \vec{\eta}, \vec{\eta}^*$ with unknown coefficients :

$$\delta\vec{s} = a\vec{n} + \frac{1}{\sqrt{2}} b\vec{\eta} + \frac{1}{\sqrt{2}} b^*\vec{\eta}^*$$

If $\vec{\omega}(s)$ is small compared to $\vec{\Omega}_{co}$ a is equal to zero, b can be calculated as

$$b = i\sqrt{2} \int_{t=0}^{\infty} \vec{\omega}(t)\, \vec{\eta}^*(t) \quad dt \qquad (6)$$

$|\delta\vec{s}|^2$ is given by $\quad |\delta\vec{s}|^2 = bb^* \quad (t\to\infty) \qquad (7)$

3.1 The Strength of the Q_z-Resonance

After the emission of a photon the electron performs inter alia vertical betatron oscillations. To calculate the resonance strength b, $\vec{\omega}(t)$ is calculated first. Then it will be shown that the integral b (eq. 6) can be expressed as a product of a ring-periodic integral and a resonance factor. The reference frame is given in fig. 3.

We assume an electron moving on the (nominal) closed orbit with nominal energy. After the emission of a photon, the electron has lost the energy $\delta\varepsilon$. The emission of the electron changes the direction of the electron. Firstly, the recoil of the photon changes the direction by a small angle of the order $1/\gamma$. Secondly, if the vertical dispersion at the point of emission is not zero the electron starts to perform betatron oscillations around the off-energy closed orbit (fig. 4).

The distance of the electron from the nominal closed orbit is given by:

$$\delta z(t) = \delta z_\beta(t)\, e^{-t/\tau_z} + \delta\varepsilon \cos\varphi_s(t)\, D_z(s)\, e^{-t/\tau_s} \qquad (8)$$

δz_β ... betatron amplitude

φ_s ... synchrotron phase

D_z ... vertical dispersion, ringperiodic function $D_z(s+L)=D_z(s)$

$\tau_{z/s}$ ... damping time for vertical and longitudinal motion

s .... the length variable along the ring, s is regarded as a function of t

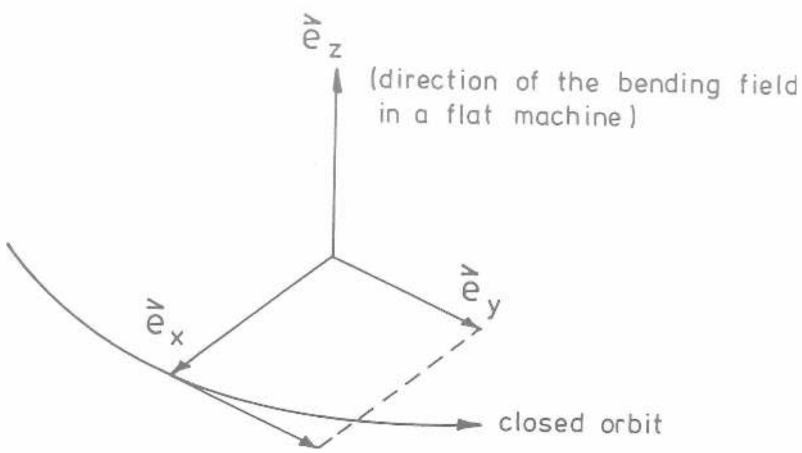

Fig. 3 Reference frame

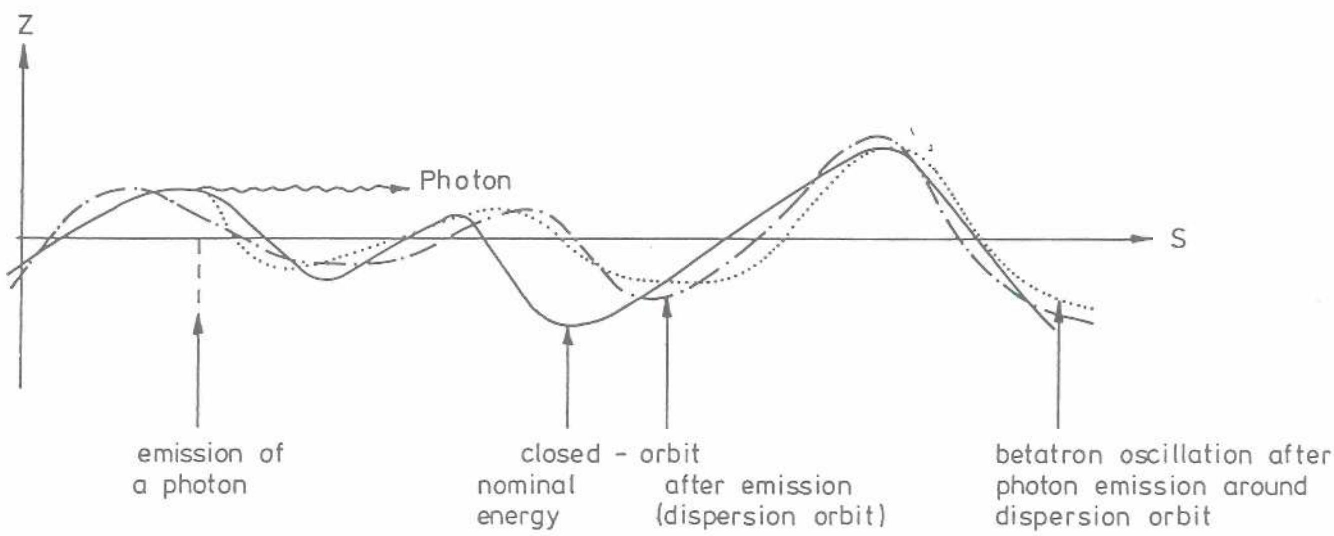

Fig. 4 Exitation of vertical oscillations

Calculation of $\vec{\omega}(t)$: due to vertical oscillations the electron experiences a radial magnetic field along its path. For a radial field $\vec{\omega}(t)$ is given by (see eq. (1)) :

$$\vec{\omega}(t) = \frac{e}{m\gamma} (1 + a\gamma) \quad B_x \vec{e}_x \qquad (9)$$

B_x is the product between the gradient and the position:

$$B_x(t) = \frac{E_o}{ec} \delta z(t) \, k(s)$$

E_o ... nominal energy of the electrons

k(s)... gradient field $\quad k(s) = \frac{ec}{E_o} \quad \frac{\delta B_x}{\delta z}$

B_x is inserted into $\vec{\omega}(t)$ using the expression for δz from eq. (8) :

$$\vec{\omega}(t) = C \, (1+a\gamma)k(s)\left\{\delta z_\beta(t) \, e^{-t/\tau_z} \, \vec{e}_x + \delta\varepsilon\cos\varphi_s(t) \, D_z(t) \, e^{-t/\tau_s}\right\} \; (11)$$

The following calculation for the strength of the Q_z-resonance takes into account only the first part in the brackets :

$$\vec{\omega}(t) = C \, (1+a\gamma) \, k(s) \quad \delta z_\beta(t) \, e^{-t/\tau_z} \, \vec{e}_x$$

The second term in the brackets leading to Q_s-resonances is discussed in the following chapter.

This expression for $\vec{\omega}(t)$ is inserted into b (eq. 6). It is shown in the appendix A that :

$$b = C \quad \frac{e^{-2\pi i(\gamma a \pm Q)}}{e^{-2\pi i}(\gamma a \pm Q_z)} \quad . \qquad (12)$$

$$\int_{S_o}^{S_o+L} e^{\pm i \varphi_z} \quad e^{i\phi} \quad \vec{e}_x(\vec{l}-i\vec{m}) \quad \sqrt{\beta_z} \quad k(s) \, ds$$

The depolarization is expressed by an integral around the ring including the optics-parameters φ and β and the $\vec{n}$, $\vec{l}$, $\vec{m}$-vectors. C is a constant including the optics-parameters at the emission point. From the denominator the conditions for resonance are obtained:

$$\gamma a = n \pm Q_z$$

3.2 The Strengths of the Q_s and Q_x-Resonances

In the last chapter the strength of the Q_z-resonance was calculated to :

$$b_z = C \frac{e^{-2\pi i(a\gamma \pm Q_z)}}{e^{-2\pi i(a\gamma \pm Q_z)} - 1} \int_{S_o}^{S_o+L} e^{\pm i\psi_z} e^{i\phi} \vec{e}_x(\vec{l}-i\vec{m}) \sqrt{\beta_z}\; k(s)\, ds \quad (13)$$

In the following the coupling between horizontal and vertical motions due to skew quads etc. are neglected. The influence of these effects on polarization is discussed in detail in a paper published 1972 by Derbenev and Kondratenko /14/. In this case of an uncoupled machine a similar calculation can be made for the horizontal betatron motion exciting the Q_x-resonances:

$$b_x = C \frac{e^{-2\pi i(a\gamma \pm Q_x)}}{e^{-2\pi i(a\gamma \pm Q_x)} - 1} \int_{S_o}^{S_o+L} e^{\pm i\psi_x} e^{i\phi} \vec{e}_z(\vec{l}-i\vec{m}) \sqrt{\beta_x}\; k(s)\, ds \quad (14)$$

The influence of the synchrotron resonances on the depolarization is given by 2 terms: a contribution from the vertical motion is explained in eq. (11):

$$\delta z_D = \delta\varepsilon \cos\psi_s(t)\, D_z(s)\, e^{-t/\tau_s}$$

and an analogous contribution from the horizontal motion :

$$\delta x_D = \delta\varepsilon \cos\psi_s(t)\, D_x(s)\, e^{-t/\tau_s}$$

The disturbing field $\vec{\omega}(t) = (\gamma a+1)(\vec{e}_x\, \delta z_D\, k(s) - \vec{e}_z \delta x_D\, k(s))$:

$$b_s = C \frac{e^{-2\pi i(a\gamma \pm Q_s)}}{e^{-2\pi i(a\gamma \pm Q_s)} - 1} \int_{S_o}^{S_o+L} e^{\pm i\psi_s} e^{i\phi} (\vec{e}_z D_x - \vec{e}_x D_z)(\vec{l}-i\vec{m})\; k(s)\, ds \quad (15)$$

These three expressions had been derived by Yokoya 1982 /12/.

The correction schemes suggested in the following reduce the strength of the resonances by reducing the value of the three integrals $b_{z/x/s}$.

4. Compensation of the Depolarizing Effects Caused by Closed Orbit Distortions

In the following the depolarizing effect caused by the deviation of the $\vec{n}$-vector $\delta\vec{n}$ from the vertical axis is discussed. It is shown that the deviation is driven by Fourier-components of the closed orbit. By a special orbit-correction scheme it is possible to reduce these Fourier-components, to reduce the deviation of the $\vec{n}$-vector, and to improve the degree of polarization. This cure reduces the strength of the Q_x and Q_z-resonances. By reduction of $\delta\vec{n}$ the vertical components of $\vec{l}$ and $\vec{m}$ get smaller. The product of $\vec{l},\vec{m}$ and $\vec{e}_z$ in eqs. (14) and (15) also becomes smaller.

The $\vec{n}$-vector is a solution of the BMT-equation:

$$\frac{d\vec{n}}{ds} = \frac{1}{c}(\vec{\Omega}_o + \delta\vec{\Omega}) \times \vec{n} \qquad (16)$$

$\vec{\Omega}_o$ contains the field on the ideal orbit, $\delta\vec{\Omega}$ the additional fields on the real closed orbit. Both quantities are ring-periodic.

With $|\delta\vec{\Omega}| << |\vec{\Omega}|$ the following solution is possible :

$$\vec{n} = \vec{n}_o + \delta\vec{n} \qquad (17)$$

where $\vec{n}_o$ is the $\vec{n}$-vector on the ideal closed orbit. It is shown in /6/ that the solution for $\delta\vec{n}$ can be written in the form :

$$|\delta\vec{n}(s)| = \frac{1/c^2}{2(1-\cos 2\pi\,\gamma a)} \left(\left\{ \int_s^{s+L} \delta\Omega_x \cos\phi\, ds \right\}^2 + \left\{ \int_s^{s+L} \delta\Omega_x \sin\phi\, ds \right\}^2 \right) \qquad (17)$$

with $\qquad \delta\vec{\Omega}_x(s) = \frac{e}{m\gamma}(1 + a\gamma)\, B_x(s)$

$\phi = a\gamma\,\alpha$ is defined in eq. (2)

As shown in fig. 2 the B_x-fields are located between the bending magnets. The integral $\int_s^{s+L} \ldots$ can be written as a sum :

$$\int_s^{s+L} \ldots = \sum_i \int_{s_i}^{s_{i+1}} \ldots \qquad (18)$$

s_i ... end of the bending magnet i

s_{i+1} ... beginning of the bending magnet i+1

α_i ... deflecting angle of the electrons after bending magnet i

$$|\delta\vec{n}| = \frac{(1+a\gamma)}{2(1-\cos 2\pi a\gamma)} \{(\sum_{i=1}^{N} \sin a\gamma\alpha_i \frac{e}{mc\gamma} \int_{s_i}^{s_{i+1}} B_x ds)^2 + (\sum_{i=1}^{N} \cos a\gamma\alpha_i \frac{e}{mc\gamma} \int_{s_i}^{s_{i+1}} B_x ds)^2\} \qquad (19)$$

The integral $\frac{e}{mc\gamma} \int_{s_i}^{s_{i+1}} B_x ds$ is known as the change of the angle of the closed orbit between two bending magnets $\Delta z'_i$ (see fig. 2).

With this definition $|\delta\vec{n}|$ becomes:

$$|\delta\vec{n}| = \frac{(1+\gamma a)}{2(1-\cos 2\pi\gamma a)} \{(\sum_{i=1}^{N} \sin a\gamma\alpha_i \, \Delta z'_i)^2 + (\sum_{i=1}^{N} \cos a\gamma\alpha_i \, \Delta z'_i)^2\} \qquad (20)$$

$\Delta z'$ can be expressed as a Fourier-sum

$$\Delta z' = \sum_{n=1}^{\infty} (a_n \cos\alpha n + b_n \sin\alpha n)$$

If $a\gamma$ has a half-integer value ($a\gamma = n+0.5$) $|\delta\vec{n}|$ is proportional to :

$$\frac{1}{(k-a\gamma)^2} (a_k^2 + b_k^2)$$

The Fourier-harmonics k=n and k=n+1 have the strongest influence on $|\delta\vec{n}|$.

Correction Scheme: The field errors in a storage ring are randomly distributed. With the help of correction dipoles the deviation of the closed orbit can be reduced to a mean value of approximately 1 mm. An example of the degree of polarization with such closed orbit deviations is given in fig. 5A.

In the storage ring PETRA a harmonic orbit correction scheme was successfully applied /6/. The scheme reduces the Fourier components next to the spin-tune. A vertical correction coil changes the orbit and therefore the amplitudes of all harmonics. The currents of 8 coils can be changed in such a way that only one amplitude of the four amplitudes next to the spin tune is changed. The four dangerous harmonics can be compensated successively.

For this method the ring symmetry is used in the following way. A machine with four identical quadrants is assumed (i.e. PETRA or HERA). Each quadrant is mirror-symmetric with respect to its middle axis (Fig. 6). The eight vertical correction dipoles are installed in the octants at symmetric positions. If dipole 1 is turned on the vertical closed orbit gets a kick $\delta_1 \cdot \delta_z$ becomes /6/ :

$$\delta z(s) = \frac{\sqrt{\beta_z(s)\,\beta_z(s_1)}}{2\pi \sin \pi Q_z} \cos\left(|\varphi_z(s) - \varphi_z(s_1)| - \pi Q_z\right) \delta_1 \quad (21)$$

s_1 ... position of the correction coil

From $\delta z(s)$ the rotation angles of the spin between the bending magnets $\gamma a\, \Delta z_i'$ are calculated. The Fourier amplitudes a_n and b_n for the n-th harmonic component are given by:

$$\begin{matrix} a_{n1} \\ b_{n1} \end{matrix} = \sum_{i=1}^{N} \begin{matrix} \cos \\ \sin \end{matrix} (n\alpha_i)\, \Delta z_1' (\alpha_i) \qquad (22)$$

The index $_{n1}$ relates to the n-th harmonics of the first correction coil. For the calculation of these amplitudes the computer-code FURIE was developed.

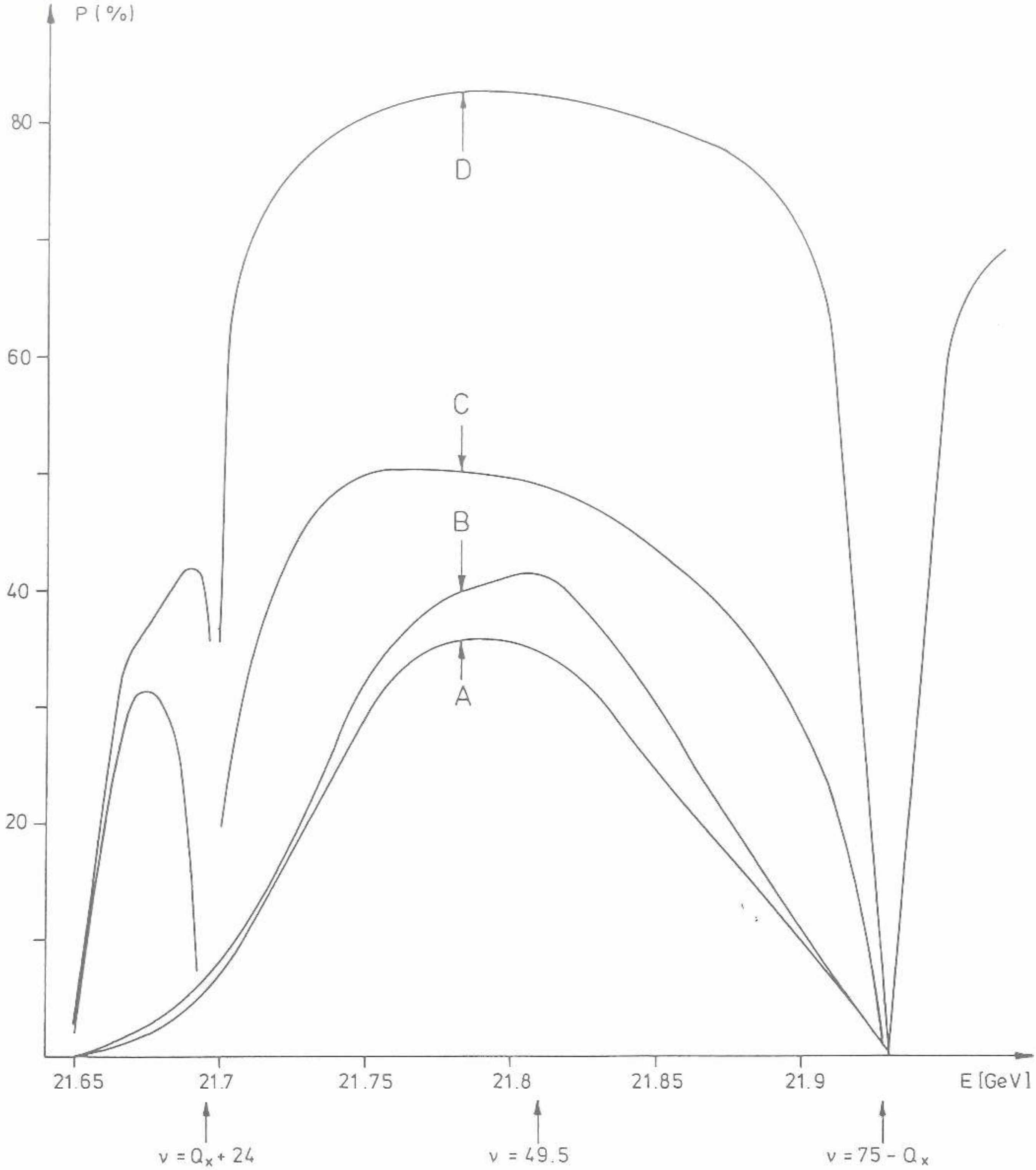

Fig. 5 Compensation of the Q_s and Q_x - resonances

A - without correction

B - closed orbit corrected along straight sections

C - dangerous Fourier components of the closed orbit corrected

D - both corrections were applied

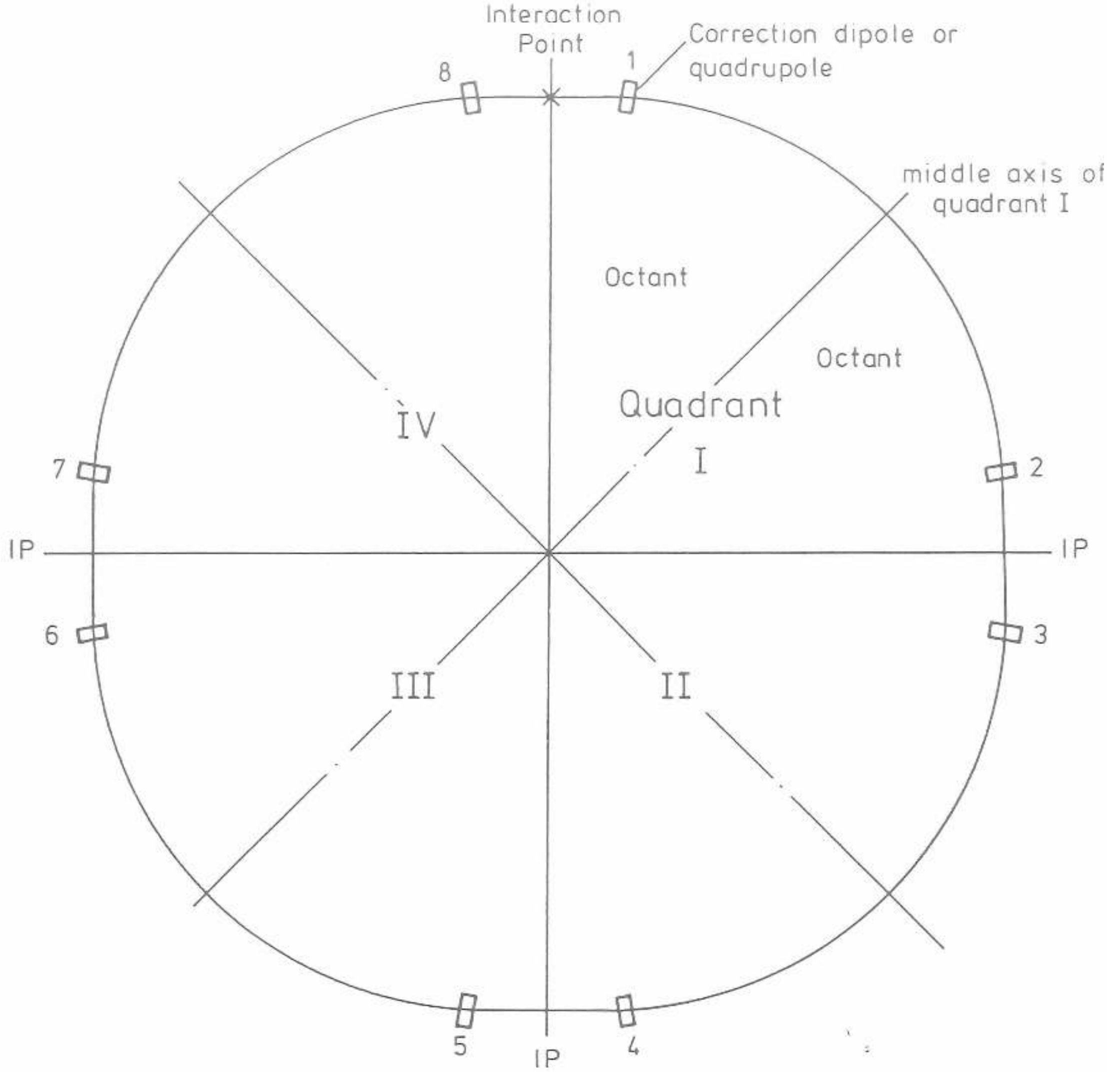

Fig. 6 Correction elements at PETRA

The amplitudes for the other coils are related to the amplitude of the first coil by symmetry conditions, i.e. for the second coil we obtain

$$\delta z_2(\alpha_i) = \delta z_1 \left(\frac{2\pi}{4} - \alpha_i\right) \qquad (23)$$

and for a_{n2}, b_{n2}:

$$\begin{matrix} a_{n2} \\ b_{n2} \end{matrix} = \sum_{i=1}^{N} \begin{matrix} \cos \\ \sin \end{matrix} (n\,\alpha_i) \quad \Delta z_1' \left(\frac{\pi}{2} - \alpha_i\right) \qquad (24)$$

substituting $\alpha_i \rightarrow -\alpha_i + \frac{\pi}{2}$ the relation between the amplitudes of the second and the first coil is :

$$\begin{aligned} a_{n2} &= \cos \frac{n\,\pi}{2}\, a_{n1} + \sin \frac{n\,\pi}{2}\, b_{n1} \\ b_{n2} &= \sin \frac{n\,\pi}{2}\, a_{n1} - \cos \frac{n\,\pi}{2}\, b_{n1} \end{aligned} \qquad (25)$$

The Fourier amplitudes of the other coils are calculated in a similar way. (see table 1, appendix B).

If the currents of the 8 coils are changed in the way indicated in table 2 (appendix B) only one of 8 amplitudes is changed. E.g. if the currents are

$$I_1 = I \qquad I_5 = -I$$

$$I_2 = +\, a_3/b_3 I \qquad I_6 = -a_3/b_3 I$$

$$I_3 = -a_3/b_3 I \qquad I_7 = a_3/b_3 I$$

$$I_4 = -I \qquad I_8 = I$$

only the amplitude a_{4n+1} is changed, all other seven amplitudes remain constant.

The deviation of the $\vec{n}$-vector from the vertical can be reduced by this method by a factor in the order of 10. The closed orbit itself changes only slightly. The improvement of the degree of polarization using this method is shown in fig. 5C.

5. Compensation of the Depolarizing Effects Caused by the Vertical Dispersion

The vertical dispersion is for the most part produced by closed orbit deviations in the strong interaction quadrupoles. The vertical dispersion contributes to the strength of the Q_s-resonance. The dispersion can be reduced by correcting the closed orbit in the interaction region in the following way:

With correction dipoles it is possible to minimize the deviation of the vertical closed orbit along the interaction region without changing the orbit in the arcs and, as a consequence, the $\vec{n}$ axis. This correction reduces the vertical dispersion and improves the degree of polarization (fig. 5B).

Beside this evident method a more sophisticated method will be derived in the following by compensating the dangerous Fourier-harmonics of the dispersion.

The depolarization strength of the Q_s-resonance driven by vertical dispersion is :

$$b_s(\text{vert. dispersion}) \sim \int_{s}^{s+L} e^{i\psi_s} e^{i\phi} \vec{e}_x D_z (\vec{\ell}+i\vec{m}) k(s)\, ds$$

(This is the second part of eq. (15)).

Assuming $\vec{n} \approx \vec{e}_z$:

$$b_s(\text{vert. dispersion}) \sim \int_{s_o}^{s_o+L} e^{i\phi} D_z k(s)\, ds$$

(It is assumed that the synchrotron phase changes only slightly during one turn.)

$D_z \cdot k$ is proportional to the radial field on the dispersion orbit. The further calculation is similar to the calculation of the deviation of the $\vec{n}$-vector. (compare eq. (17)).

The integral has to be taken over the radial field $D_z k(s)$ instead of the radial field $\delta\Omega_x$. Instead of correcting the Fourier components of the closed orbit the Fourier components of D_z' near the spin tune γa have to be minimized :

$$b_s(\text{vert. dispersion}) \approx \left(\sum_{i=1}^{N} \sin\gamma a \alpha_i \Delta D'_{zi} \right)^2 + \left(\sum_{i=1}^{N} \cos\gamma a \alpha_i \Delta D'_{zi} \right)^2$$

$\Delta D'_{zi}$ ... $D'_{zi+1} - D'_{z\,i}$

This can be done by the help of 8 beam bumps at symmetric positions. The bumps have to be moved in a similar way as the 8 dipole coils for the correction of the Fourier amplitudes of the closed orbit.

6. Compensation of the Q_z-Resonance

Different from $Q_{x/s}$-resonances which exist only in an imperfect machine several Q_z-resonances can be seen even in an ideal storage ring. They are excited by the recoil of the emitted photons. In an imperfect machine with a nonvanishing vertical dispersion the oscillations and therefore the Q_z-resonance are much more stronger (see chapter 3.1).

To compensate the Q_z-resonances the Fourier components of the betatron-trajectories have to be changed. This can only be done by changing the gradient fields of the quadrupoles. The following argumentation is divided into two parts. Firstly, the resonance strength is calculated from the Fourier components of the betatron trajectories. Secondly, it is shown how the resonances can be compensated by 8 quadrupoles at symmetric positions of the ring.

In a flat machine with small distortions eq. (13) becomes

$$b_z(s) = C \frac{e^{2\pi i(\gamma a \pm Q_z)}}{e^{-2\pi i(\gamma a \pm Q_z)} - 1} \int_s^{s+L} e^{\pm i \psi_z} e^{i\phi} \sqrt{\beta_z}\, k(s)\, ds \qquad (29)$$

There exist two different types of resonances:

$$\gamma a = n + Q_z \quad (+\text{ type})$$
$$\gamma a = n - Q_z \quad (-\text{ type}) \tag{30}$$

leading to two different integrals I_+ and I_- in the expression for b_z.

On the resonance $\phi = \gamma a \alpha = (n \pm Q_z)\alpha$ is fullfilled.

Therefore the integrals are given by :

$$I_+ = \int_s^{s+L} e^{in\alpha} e^{-i(\psi_z - Q_z\alpha)} \sqrt{\beta_z}\, k(s)\, ds \tag{31}$$

$$I_- = \int_s^{s+L} e^{in\alpha} e^{+i(\psi_z - Q_z\alpha)} \sqrt{\beta_z}\, k(s)\, ds$$

$\psi_z - Q_z \cdot \alpha$, $\sqrt{\beta}$ and $k(s)$ are ring-periodic. The strength of the resonances is proportional to the square of the integral in eq. (31):

$$I^2_{+/-} = (a_1 \pm b_2)^2 + (a_2 \pm b_1)^2 \quad \text{for} \quad \begin{matrix} \gamma a = n - Q_z \\ \gamma a = n + Q_z \end{matrix} \tag{32}$$

The coefficients are

$$\begin{matrix} a_1 \\ b_1 \end{matrix} = \int_s^{s+L} \begin{matrix} \sin \\ \cos \end{matrix} (n\alpha) \quad (\cos\psi_z \cos Q_z\alpha + \sin\psi_z \sin Q_z\alpha)\sqrt{\beta_z}\; k(s)\; ds$$

$$\begin{matrix} a_2 \\ b_2 \end{matrix} = \int_s^{s+L} \begin{matrix} \sin \\ \cos \end{matrix} (n\alpha) \quad (\cos\psi_z \sin Q_z\alpha - \sin\psi_z \cos Q_z\alpha)\sqrt{\beta_z}\; k(s)\; ds \tag{33}$$

These coefficients are calculated by the program FURIE using the equation:

$$\int_{s_i}^{s_{i+1}} \begin{matrix} \cos\psi_z \\ \sin\psi_z \end{matrix} \quad \sqrt{\beta_z}\; k(s)\; ds \approx \begin{matrix} \Delta z'_c \\ \Delta z'_s \end{matrix} = \begin{matrix} z'_{c_{i+1}} - z'_{c_i} \\ z'_{s_{i+1}} - z'_{s_i} \end{matrix} \tag{34}$$

The integration is performed from the end of bending magnet i to the next one.

$\Delta z'_{s/c}$ is the change in the angle of the sine- and cosine-like betatron trajectory between the two bending magnets (fig. 2).

The coefficients a and b can be written as a sum:

$$\begin{matrix} a_1 \\ b_1 \\ a_2 \\ b_2 \end{matrix} = \sum_{i=1}^{N} \begin{matrix} \sin \\ \cos \end{matrix} n\alpha_i (\cos Q_z\alpha_i \, \Delta z'_{ci} + \sin Q_z \alpha_i \Delta z'_{si}) \atop \begin{matrix} \sin \\ \cos \end{matrix} n\alpha_i (\sin Q_z\alpha_i \, \Delta z'_{ci} - \cos Q_z \alpha_i \Delta z'_{si}) \qquad (35)$$

<u>Machine Without Gradient Errors</u>

For a symmetric machine without gradient errors only the resonances $\gamma a = P \cdot n \cdot Q_z$ occur. (P is the periodicity, for PETRA and HERA P = 4).

All the other Q_z-resonances do not appear due to the fact that the integrals (eq. 31) are zero for $n \neq P\,k$.

Fig. 7 shows the Q_z-resonances at PETRA in the energy region between 14 - 16 GeV, calculated by the SLIM-program /7/. The relative strength of the resonances is calculated by a Fourier analysis using eq. (32). The strength of these resonances depends only on the optics for the ideal machine.

<u>Machine With Gradient Errors</u>

In a non-ideal machine the symmetry gets lost due to the gradient errors of the magnetic fields. All Q_z-resonances occur. Field errors also change the symmetry but the influence on the strength of the Q_z-resonances is very small.

Fig. 8 shows the degree of polarization as a function of the beam energy for the resonances $\gamma a = Q_z + 26$ and $\gamma a = 73 - Q_z$. The calculation is per-

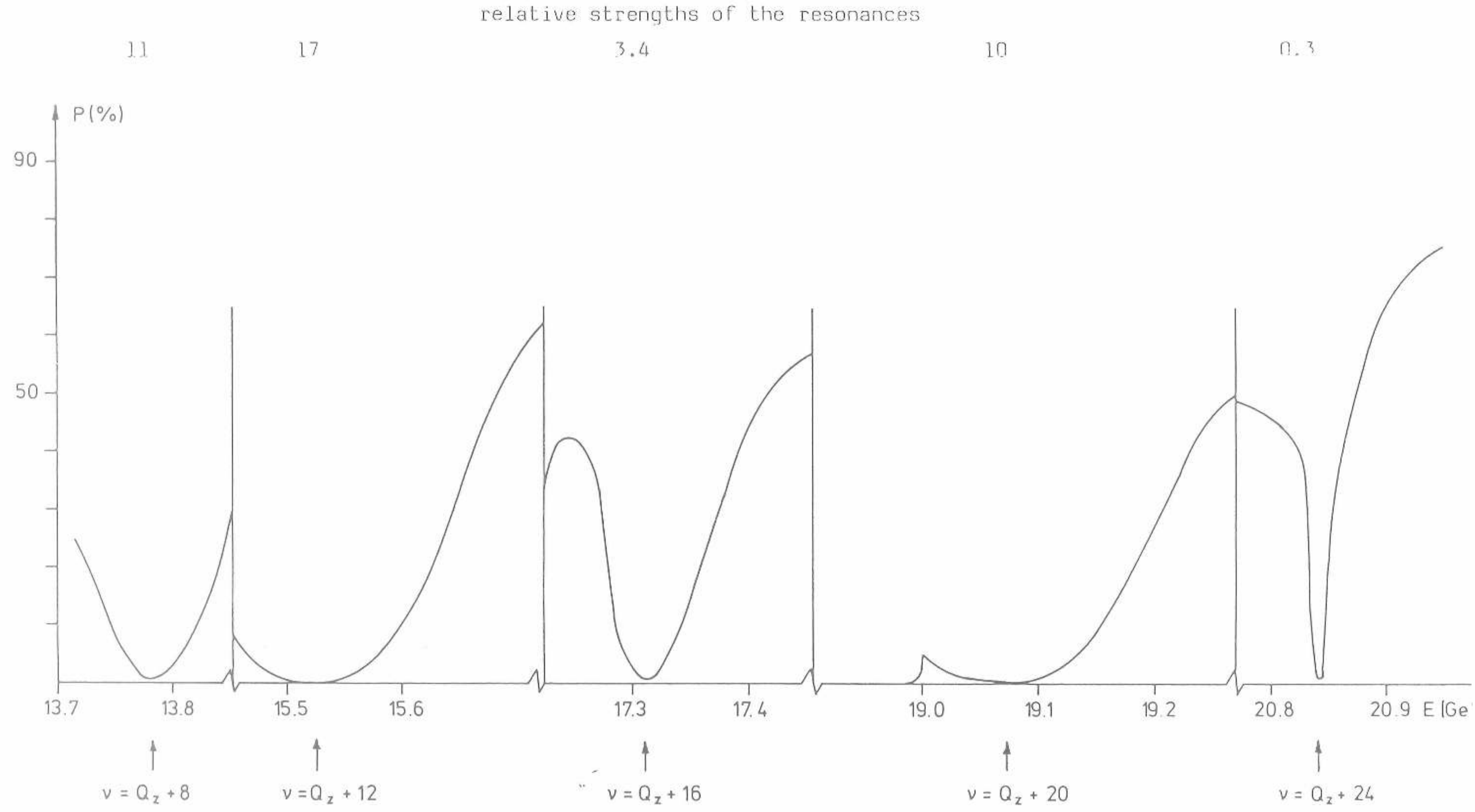

Fig. 7 Depolarizing Resonances $\nu = Q_z + 4n$ for PETRA in the energy region 13 - 21 GeV

formed with randomly distributed gradient and field errors. The so-called MEB-optics was used (β_z = 8 cm at the interaction points). The strength of the gradient errors is chosen in such a way that the β_z-function differs by about 10 % at symmetric points.

Curve A: The same random distributed field errors as in the calculation of fig. 5a are assumed.

Curve B: The correction schemes for the correction of the Q_s and Q_x-resonances were applied. After the correction the degree of polarization is mostly limited by Q_z-resonances.

One possibility to change the influence of the resonances is a change of the Q_z- value. During the operation of the machine this can be done only to a limited extend.

Another possibility is the reduction of the depolarizing strength of the resonances. In the following it is shown that this can be done with the help of 8 quadrupole magnets.

Correction scheme:

If an additional quadrupole field $\delta k(s)$ is switched on the beam optics changes. If $\delta k(s) << k$ a linear approach is possible:

$$\beta = \beta_o + \delta\beta$$

$$\varphi_z = \varphi_{zo} + \varphi_z$$

The betatron trajectories of the particles are given by

$$z_o(s) = A\sqrt{\beta_z}\ \sin(\varphi_z + \delta)$$

δ has an arbitrary value, A is a constant

The betatron trajectories in the distorted optics are

$$z(s) = A\sqrt{\beta_o + \delta\beta}\ \sin(\varphi_z + \delta + \delta\varphi_z)$$

For small perturbations we get

$$z(s) = z_o + \delta z$$

with

$$\delta z = \left(\frac{1}{2}\frac{\delta\beta}{\sqrt{\beta_o}} \sin(\psi_{z_o}+\delta) + \sqrt{\beta_o}\,\delta\psi_z \cos(\psi_{z_o}+\delta)\right)\cdot\alpha$$

If more than one additional quadrupole field is switched on, the effects of the different quadrupoles on the betatron trajectories adds linearly. The Fourier amplitudes of the trajectories also add up linearly.

In the following the correction of the amplitudes for the 4n+2 harmonics component is demonstrated. The numerical calculations are performed for the resonance $\gamma a = 26 + Q_z$ (and for $\gamma a = 73 - Q_z$).

A gradient error is caused by changing the strength of the quadrupole Q1. The harmonic amplitudes $a_1(Q1)$, $b_1(Q1)$, $a_2(Q1)$ and $b_2(Q1)$ of the betatron trajectories caused by the change of the quadrupole strength are calculated by using eq. 36 with the program FURIE.

The strength of the resonance $\gamma a = n \pm Q_z$ is defined by $(a_1 \pm b_2)^2 + (a_2 \mp b_1)^2 = A^2 + B^2$.

For the change of the first quadrupole A and B are given by :

$$A(Q1) = a_1(Q1) \pm b_1(Q1)\ , \quad B(Q1) = a_2(Q1) \mp b_1(Q1)$$

The amplitudes of the 4n+2 harmonics caused by the other seven quadrupoles at symmetric positions in the octants are related to the amplitudes of the first quadrupole due to symmetry conditions. The amplitudes can be found in table 3, appendix B. The currents for the quadrupoles are changed in such a way that only one amplitude is changed. These currents are used to compensate A and B for the 26 and the 73 harmonics.

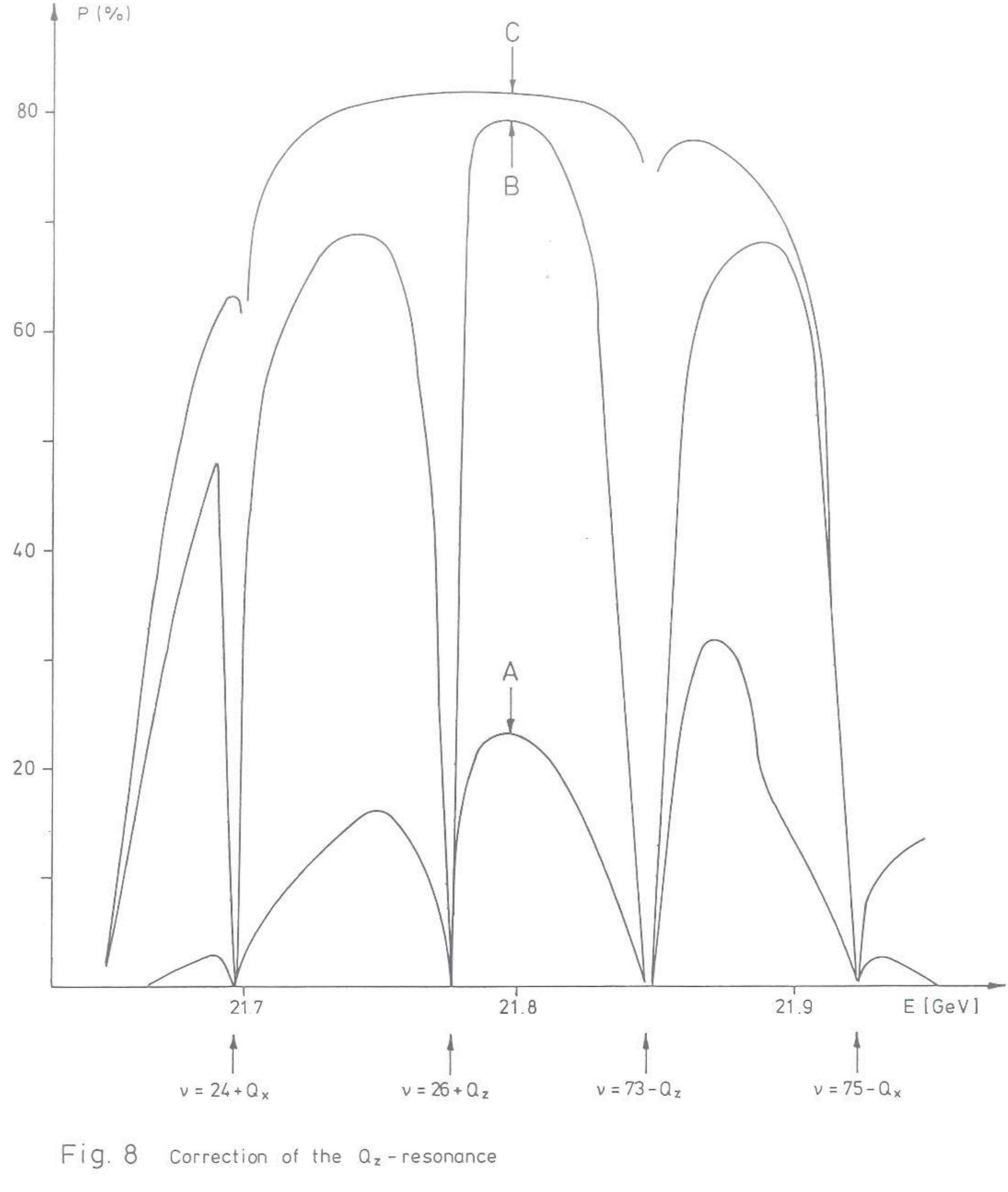

Fig. 8 Correction of the Q_z - resonance

A - dipole field errors and gradient field errors

B - after correction of the orbit harmonics and the dispersion

C - after an additional correction of the Q_z-resonance

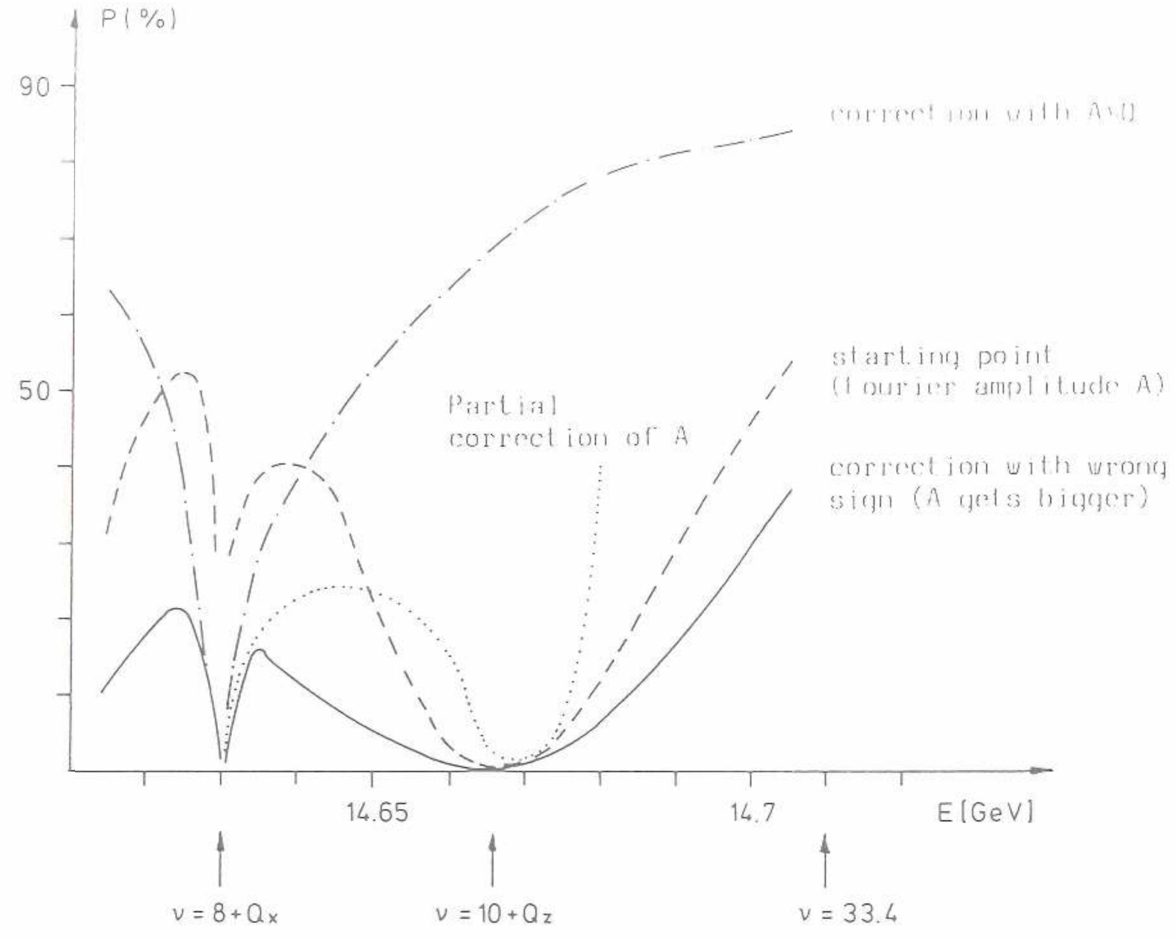

Fig. 9 Correction of the Q_z - resonance with different strengths

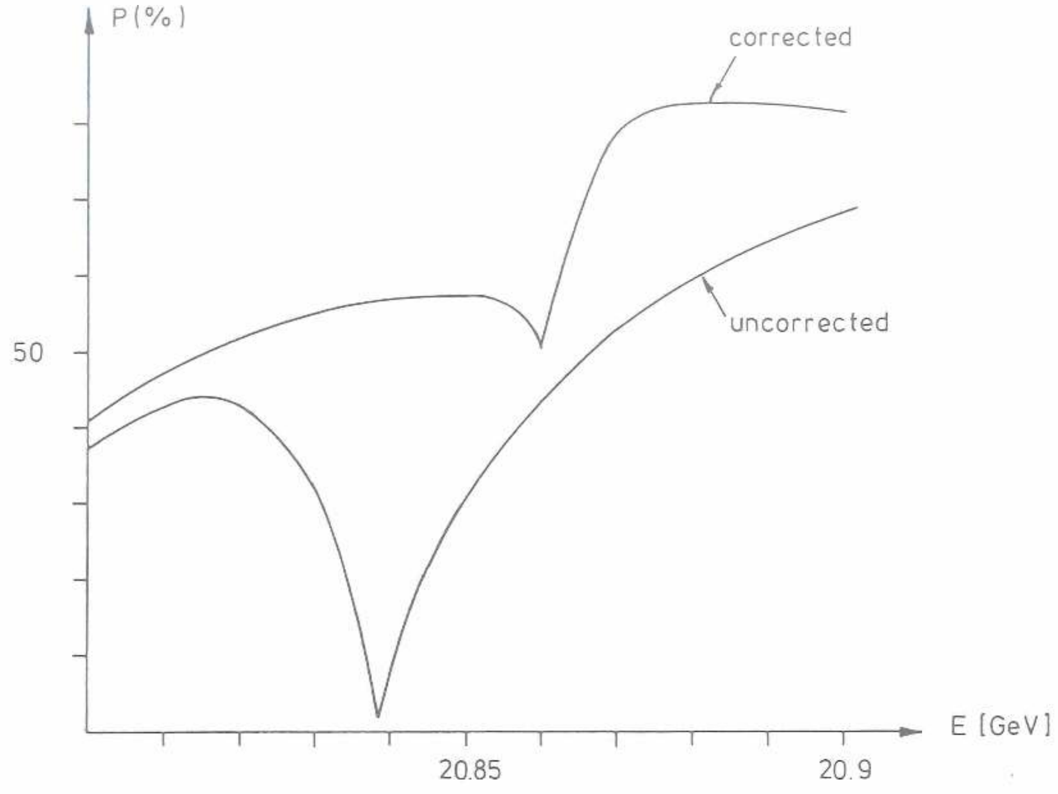

Fig. 10 Correction of the resonance $\gamma a = Q_z + 24$

The improvement in the degree of polarization after applying this method on the distorted ME8-optics is shown in fig. 8. Curve C shows the degree of polarization after applying all three correction schemes.

Fig. 9 shows the degree of polarization in an optics where only the A-amplitude has a nonzero value. The strength of a set of quadrupoles was changed step by step. The depolarizing resonance vanishes when A is cancelled.

In a machine with gradient errors all Q_z-resonances are found. The strongest are still the resonances $\gamma a = Pn \pm Q_z$. Their strength differs by more than a factor of 10 (see fig. 7). The working point of the machine should be chosen to be as far as possible away from these resonances.

The influence of the weaker 4 -resonances can be compensated by the harmonic correction. If the currents of the 8 quadrupoles are changed by (I,I,I,I,I,I,I,I) or (I,-I,I,-I,I,-I,I,-I) the resonance strength changes. This was done for the correction of the resonance $\gamma a = 24+Q_z$ (see fig. 10).

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APPENDIX A: Calculation of the depolarizing strength b

The aperiodic field $\vec{\omega}(t)$ is given by $\vec{\omega}(t) = C\,(1+a\gamma)\,k\,(s).\delta z_{\beta} e^{-t/\tau_z} \vec{e}_x$ with the betatron amplitude /13/

$$\delta z_{\beta}(t) = \sqrt{\beta_z(s)}\,(C \cos\psi_z + S \sin\psi_z) \qquad \text{(A1)}$$

$$C = -\frac{1}{\sqrt{\beta_{zo}}}\ \delta\varepsilon\ D_{zo}$$

$$S = -\delta\varepsilon\,\left(\frac{D_{zo}\alpha_o}{\sqrt{\beta_{zo}}} + \beta_{zo}\,D'_{zo} - \delta\gamma\right)$$

β, α are the optics functions, the subscript $_o$ indicates the point of emission. $\delta\gamma$ is the kick from the recoil of the photon $\psi_z(s)$... betatron phase with $\psi_{zo} = 0$.

δz_{β} can be written as a complex function:

$$\delta z_{\beta} = c\ \left(e^{-i\psi_z} e^{i\psi_o} - e^{i\psi_z} e^{-i\psi_o}\right) \sqrt{\beta_z(s)} \qquad \text{(A2)}$$

The constants c and ψ_o contain the optics parameters at the point of emission.

Combining δz_{β} with $\vec{\omega}$, b (eq. 6) becomes

$$b = c \int_{t=o}^{\infty} \left(e^{+i\psi_z} e^{i\psi_o} - e^{i\psi_z} e^{-i\psi_o}\right) \vec{e}_x\,(\vec{l} - i\vec{m})\,e^{+i\phi} \sqrt{\beta_z(s)}\ k\,(s)\ e^{-t/\tau_z} dt \qquad \text{(A3)}$$

The integral can be written as:

$$\int_{t=o}^{\infty} \ldots = \int_{t=o}^{T_o} \ldots + \int_{t=T_o}^{\infty} \qquad T_o \text{ ... revolution time.}$$

The integrand includes the ring periodic functions $\vec{e}_x$, $\vec{l}$, $\vec{m}$, β_z and k. The betatron phase ψ_z and the spin phase ϕ can be written in a quasi periodic way

$$\psi_z(t-T_o) = \psi_z(t) - Q_z$$
$$\phi\,(t-T_o) = \phi(t) - 2\pi\gamma a \qquad \text{(A5)}$$

With a transformation of the variable $t \rightarrow t - T_o$ using eq. A4 the integral $\int_{t=T_o}^{\infty} \ldots$ is

$$\int_{t=T_o}^{\infty} = \frac{1}{e^{-2\pi i(\gamma a \pm Q_z)}} \int_{t=o}^{\infty} \ldots \qquad (A6)$$

This expression is inserted into eq. A4. After a simple transformation equation (12) is obtained.

Appendix B:

Table 1

Fourier amplitudes of 8 correction coils at symmetric positions.

The currents of the first coil are denoted with a_o, b_o etc. It is shown in the table how the currents of the other coils are related to the amplitudes of the first coil.

Fourier component / coil in octant	a(4n)	b(4n)	a(4n+1)	b(4n+1)	a(4n+2)	b(4n+2)	a(4n+3)	b(4n+3)
1	a_o	b_o	a_1	b_1	a_2	b_2	a_3	b_3
2	a_o	$-b_o$	b_1	a_1	$-a_2$	b_2	$-b_3$	$-a_3$
3	a_0	b_0	$-b_1$	a_1	$-a_2$	$-b_2$	b_3	$-a_3$
4	a_0	$-b_0$	$-a_1$	b_1	a_2	$-b_2$	$-a_3$	b_3
5	a_0	b_0	$-a_1$	$-b_1$	a_2	b_2	$-a_3$	$-b_3$
6	a_0	$-b_0$	$-b_1$	$-a_1$	$-a_2$	b_2	b_3	a_3
7	a_0	b_0	b_1	$-a_1$	$-a_2$	$-b_2$	$-b_3$	a_3
8	a_0	$-b_0$	a_1	$-b_1$	a_2	$-b_2$	a_3	$-b_3$

Table 2:

A correction scheme were only one of eight Fourier coefficients is influenced. The magnitudes are defined in the previous table.

	A(4N)	B(4N)	A(4N+1)	B(4N+1)	A(4N+2)	B(4N+2)	A(4N+3)	B(4N+3)
1	I	I	I	I	I	I	I	I
2	I	-I	$a_3/b_3 I$	$b_3/a_3 I$	-I	I	$-a_1/b_1 I$	$-b_1/a_1 I$
3	I	I	$-a_3/b_3 I$	$b_3/a_3 I$	-I	-I	$a_1/b_1 I$	$-b_1/a_1 I$
4	I	-I	-I	I	I	-I	-I	I
5	I	I	-I	-I	I	I	-I	-I
6	I	-I	$-a_3/b_3 I$	$-b_3/a_3 I$	-I	I	$a_1/b_1 I$	$b_1/a_1 I$
7	I	I	$a_3/b_3 I$	$-b_3/a_3 I$	-I	-I	$-a_1/b_1 I$	$b_1/a_1 I$
8	I	-I	I	-I	I	-I	I	-I

Table 3:

Compensation of Fourier components of the betatron motion by quadrupoles at symmetric positions.

The current change of the first quadrupole is denoted with A, B, C etc.

quadrupole in octant	Fourier component for $a\gamma = n-Q_z$ a_{4n+1}	b_{4n+1}	for $a\gamma = n+Q_z$ a_{4n+1}	b_{4n+1}	for $a\gamma = n-Q_z$ a_{4n+2}	b_{4n+2}	for $a\gamma = n+Q_z$ a_{4n+2}	b_{4n+2}
1	A	B	C	-D	E	F	G	H
2	B	A	D	-C	-E	F	-G	-H
3	-B	A	-D	-C	-E	-F	-G	-H
4	-A	B	-C	-D	E	-F	G	H
5	-A	-B	-C	D	E	F	G	H
6	-B	-A	-D	C	-E	F	-G	-H
7	B	-A	D	C	-E	-F	-G	-H
8	A	-B	C	D	E	-F	G	H

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