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Competing Vertical Structures: Precommitment and Renegotiation[†]

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ABSTRACT

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We consider a model where two agents play a (normal form) game on behalf of two principals. We analyze the existence of *precommitment effects* through public announcement of contract, in a model where agency contracts, designed under *incomplete information* between principal and agent, can be *secretly renegotiated*. We show that the existence of precommitment effects depends both on the strategic complementarity of the agents' actions and on the direct effect of the opponent's action on each principal's welfare. In our model, the possibility of renegotiation is crucial for the existence of precommitment effects. Applications to the field of Industrial Organization are discussed.

Key Words: Principal-agent theory, agency contracts, renegotiation, precommitment effects, strategic complementarity.

JEL Classification number: 026, 611.

RESUME

Concurrence entre structures verticales: engagement et renégociation.

On considère un modèle où deux agents jouent un jeu (sous forme normale) pour le compte de deux principaux. On étudie l'existence d'*effets d'engagement* à travers la divulgation publique des contrats, dans un modèle où les contrats sont établis en situation d'*information incomplète* entre principal et agent, et peuvent être *secrètement renégociés*. On montre que l'existence d'effets d'engagement dépend, d'une part, de la complémentarité stratégique entre les actions des agents, et, d'autre part, de l'effet direct de l'action de l'adversaire sur les objectifs de chaque principal. Dans notre modèle, la possibilité de renégocier est centrale pour l'existence d'effets d'engagement. On discute enfin d'applications au domaine de l'économie industrielle.

Mots Clés: Théorie principal-agent, contrats, renégociation, effets d'engagement ou effets d'annonce, complémentarités stratégiques.

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I. INTRODUCTION

The strategic use of agency contracts is now well documented: they may be used as precommitment device vis-à-vis third parties. By publicly disclosing the contract that determines her agent's incentives, a principal can force her opponents to take her agent's actions as given, thereby gaining a first-mover advantage when desirable. We will say that *precommitment effects* exist when allowing public disclosure of agency contracts leads to a different outcome than if all contracts were kept secret.[1]

The literature on rivalrous agencies has successfully developed these ideas in situations where several agencies interact, usually focusing on perfect information situations (with specific forms of contracts).[2] A crucial assumption of this whole literature is that contracts, once publicly disclosed, cannot be secretly renegotiated. Indeed, in a perfect information setting, the possibility of secret renegotiation leads to no precommitment effects at all: whatever the public contract she signed, a principal will always have an incentive to propose her agent a new contract that implements the best response to the opponents' actions; the opponents should then anticipate this behavior and play as in the game with secret

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1 The mere fact of hiring an agent, as opposed to playing the game oneself, may also have precommitment effects, even though the content of the contract is not public: see Katz [1987] and Caillaud-Hermalin [1991]. Here we concentrate on situations where principals must hire agents to play on their behalf.

2 Economic applications include the effect of the separation of ownership and management in oligopoly models (Fershtman-Judd [1987a], Sklivas [1987]), the strategic aspect of vertical separation or of vertical restraints between competing wholesaler-retailer structures (Bonnano-Vickers [1988], Rey-Stiglitz [1987]), the use of tax or subsidy policy to help domestic firms in international competition (Brander-Spencer [1983] & [1985]). More game-theoretical analysis can be found in Fershtman-Judd-Kalai [1986], Katz [1987] and Fershtman-Judd [1987b]. The last two articles investigate situations of, respectively, incomplete and imperfect information within agencies.

contracts.

This paper analyzes the existence of precommitment effects when public contracts can always be *secretly renegotiated* and are designed under *incomplete information* between principal and agent. More precisely, we consider two competing agencies where agents have private information. We show that the existence of precommitment effects depends both on the strategic complementarity of the agents' actions and on the direct effect of the opponent's action on each principal's welfare.[3] Indeed we find that precommitment effects exist if actions are strategic substitutes and a lower opponent's action increases one principal's welfare, or if actions are strategic complements and a higher opponent's action increases one principal's welfare (as it is usually assumed, the higher an action the costlier it is). Precommitment effects do not exist in the two opposite cases. The results do not depend on any particular assumptions on the set of contracts allowed, provided an agent's action is verifiable for his principal.

The assumption that secret renegotiation is possible is essential to these results. In our model, the two agencies sign their public contracts simultaneously. Precommitment effects arise when renegotiation is allowed because each hierarchy has the opportunity to react to its opponent's public contract disclosure. A hierarchy benefits from the inability of the other to preclude renegotiation, but this gain is limited by the fact that the hierarchy can itself renegotiate: for the initial announcement to be credible, the publicly disclosed contract must make renegotiation too costly. The initial contract, signed under asymmetric information,

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3 The fact that the results depend on these two features is unsurprising since contracts constitute a form of strategic investment: see Bulow-Geanakoplos-Klemperer [1985], Fudenberg-Tirole [1984] and Tirole [1988] on the taxonomy of business strategies.

achieves this purpose by modifying the distribution of reservation utilities for the renegotiation process across the various types of agents. It endows good-type agents with contractual utility levels larger than in the optimal secret contract; this makes renegotiation to the optimal secret contract excessively costly since it would require to leave also high contractual utilities to bad-type agents. It follows that a hierarchy can only precommit to actions that are *costlier* than the ones implemented by the optimal secret contract. From this, the aforementioned results follow naturally.

Our results are particularly relevant for Industrial Organization. For example, if principals are producers and agents are salesmen, it is natural to assume that the salesmen's compensation contracts may be publicly disclosed but also may be secretly renegotiated. Likewise for shareholders and managers, or for wholesaler and retailers. Now in all these cases, our results are in line with previous contributions provided there are asymmetries of information within agencies, and, most importantly, that agents compete in a Cournot way: precommitment effects exist, yielding, under mild assumptions, a more competitive outcome than if no contract could ever be disclosed. If, however, agents compete in a Bertrand way, say in differentiated products, no precommitment effects arise: therefore, in contrast with the results of the literature cited in footnote 2, collusion is not facilitated by the possibility of disclosing agency contracts, if secret renegotiation cannot be prevented.

The paper is also relevant for International Economics. Principals can be different states or governmental agencies, and agents could be public firms or privately regulated firms that compete in a common market with no trade barriers. Therefore, the analysis could be of some use in addressing the problem of state monopoly in the European Market of 1992, and the legal framework that should be adopted by the European Agency.

To our knowledge, precommitment effects under incomplete information and renegotiation have only been previously considered by Dewatripont [1988]. There are however several major differences between Dewatripont's model and ours. First, Dewatripont [1988] considers the case of one hierarchy competing with a third party. When signing an initial contract, the hierarchy therefore benefits from a first-mover advantage due to the specification of the timing of the model. Precommitment effects are then maximal when no renegotiation can take place. Second, in Dewatripont [1988], initial contracts are signed ex-ante (before the agent receives his private information) while renegotiation occurs at the interim stage. In contrast, we will assume that both initial and renegotiated contracts are signed at the interim stage; so, renegotiation matters here, despite the fact that the information structure remains unchanged in the game.

The paper is organized as follows. Section 2 presents the general structure of the model and, in particular, the nature of contracts. Section 3 characterizes the outcome of the renegotiation stage. Section 4 (i and ii) develops the equilibrium analysis and contains our main results about existence of precommitment effects. Subsection 4iii contains more specific results and in particular, the complete characterization of renegotiation in the case of a binary information structure. Section 5 provides further results about the possibility of under- or overprovision of actions in equilibrium; it also performs some comparative statics in the framework of a simple example. Section 6 concludes with a few applications of our results.

II. THE GENERAL MODEL

The general situation we consider is summarized in the next figure :

	P^A			P^B	
Contract	$\downarrow$		market	$\downarrow$	Contract
	A	->	or	<- B	
			general game		

Two principals P^A and P^B each hire an agent A and B to act on their behalf on some market or more generally in some game. Each pair of principal-agent, called a hierarchy, may be interpreted as a simple model of a firm (shareholders/manager, producer/salesman, etc). The two firms interact in a market, competing in prices or in quantities for example. A hierarchy may also be composed of two firms within a vertical structure (producer/retailer, R&D/producer, etc).

Contracts are fully enforceable commitments by all parties that signed it. They are offered by principals on a take-it-or-leave-it basis. We do not impose any restriction on the form of contracts. So the principal can choose within a set of contracts that specify transfers between the principal and her agent on the basis of verifiable variables. These may or may not include the opponent's action and some public information about the opponent's contract. But we assume that *the action of an agent is verifiable for its own hierarchy*. Hence, contracts based on the action of the agent always constitute a possible agreement within a hierarchy.

Moreover a contract can be published in newspapers or presented on TV, so that contracts may be *publicly observable* if desired by all contractual parties. However, we want to emphasize that secret renegotiations are always possible: public disclosure of a contract has no legal force in forbidding any subsequent change of contract, if all contracting parties agree to tear up the (publicly disclosed) contract. Therefore, we consider a global game that allows *renegotiation*.

Stage 0: Public contracting: Each principal may either sign a public contract or not.[4]

Stage 1: Renegotiation: Secret renegotiation may occur in every hierarchy.

Stage 2: Market Game: Agents play a game in normal form, where agent A's (resp. B's) pure strategy is denoted by $a \in S^A=[0,\bar{a}]$ (resp. $b \in S^B=[0,\bar{b}]$).

This three stage game will be called game G. Stage 0 allows for the possiblity of precommitment through public disclosure of contracts. Stage 1 leaves the opportunity of secret renegotiation open. Stage 2 is mostly meant to model a one-shot simultaneous move game. We abstract from the possibility of renegotiation due to some sequentiality of moves in a general extensive form game, as considered in Dewatripont [1988] and in Caillaud-Jullien-Picard [1990a]. In what follows, we shall also consider a game G_0, in which no public contracting is allowed. This game coïncides with the subgame of G starting at stage 1 when no contract has been signed at stage 0.

The basic issue we want to address is whether "no public contracting" is an equilibrium of G or not. When, at any equilibrium, some public contract is signed at stage 0, we shall say that *precommitment effects exist*. In the opposite case, the equilibrium outcome of G_0 may be obtained as an equilibrium outcome of G. In this case, we shall also look for conditions that guarantee that any equilibrium outcome of G can be obtained as an equilibrium outcome of G_0.

We focus on situations of hidden knowledge between principals and

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4 We do not allow at this stage the principal to propose several contracts, ask the agent to choose one contract, and then disclose this contract. In other words, only the contract proposal by the principal is publicly observable. We can see the situation as one in which the principal uses different agents on several local markets and at various dates. The public contract is then a standard contract on which the principal is legally precommitted.

agents: each agent has initially some private information about a parameter θ^A or θ^B , of his (dis)utility of playing the market game (θ may possibly affect the global aggregate payoffs accruing to a hierarchy). θ^A and θ^B are assumed to be independently drawn according to the distribution:

$$\theta^A \in \{\theta^A_1, \ldots, \theta^A_n\} \qquad \text{with } \theta^A_1 < \theta^A_2 < \ldots < \theta^A_n \text{ and } n \geq 2,$$

$$h^A_i = \text{Prob}\{\ \theta^A = \theta^A_i\ \}\ ,\ H^A_i = \text{Prob}\{\ \theta^A \leq \theta^A_i\ \},\quad \text{for } i = 1, \ldots, n.$$

and similarly for θ^B_j, h^B_j, and H^B_j, for $j = 1,\ldots,m$ (we allow $m = 1$).

Given a tranfer t^A from P^A to A, and actions a and b chosen by the agents, payoffs are denoted Π^A for principal P^A, and W^A for agent A, with:

$$\Pi^A = \phi^A(a,b,\theta^A) - U^A$$

$$W^A = U^A + m^A(b,\theta^A)$$

where

$$U^A = t^A - u^A(a,\theta^A)$$

and similarly for Π^B, W^B and U^B. The agent's payoff W^A is thus separable between a *control utility* U^A, exhibiting risk neutrality w.r.t. revenue, and a term of pure externality due to the opponent's action.[5] When agents A and B do not participate, they obtain zero utility : in this case, $W^A = W^B = 0$. ϕ^A is the aggregate payoff of the hierarchy P^A-A, up to the externality term $m^A(b,\theta^A)$. We make the following assumptions:

A1: $\phi^A(.,.,.)$ is C^2, strictly concave in a, linear in b; for any b and θ^A_i,

$$0 < A^*_i(b) \equiv \underset{0\leq a\leq \bar{a}}{\text{Argmax}}\ \phi^A(a,b,\theta^A_i) < \bar{a}\ ,$$

similarly for B. Moreover, for all a, b, θ^A, θ^B, $\phi^A_b(a,b,\theta^A)\phi^B_a(a,b,\theta^B) > 0$.

A2: For all a, b, θ^A, θ^B, $0 < \dfrac{\phi^A_{ab}(a,b,\theta^A)}{\phi^A_{aa}(a,b,\theta^A)}\ \dfrac{\phi^B_{ba}(a,b,\theta^B)}{\phi^B_{bb}(a,b,\theta^B)} < 1.$

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5 This term is introduced to allow for games between agents such as Bertrand competition games.

A3: i) For all a and θ^A, $u^A(a,\theta^A) = \theta^A a$; similarly for B;

ii) For all a and b, $\phi^A_a(a,b,\theta^A)$ is decreasing in θ^A; similarly for B;

iii) $\frac{h^A_{i+1}}{\Delta^A_i H^A_i}$ is non-increasing in i, where $\Delta^A_i \equiv \theta^A_{i+1} - \theta^A_i$; similarly for B.

A4: i) For all a and b, $m^A(b,\theta^A) - \theta^A a$ is non-increasing in θ^A; similarly for B;

ii) For all a, b and θ^A, $\phi^A_b(a,b,\theta^A)\, m^A_b(b,\theta^A) \geq 0$; similarly for B.

A1 is technical. The linearity in the opponent's action,[6] and the symmetry of the effects of the opponent's action on a hierarchy's payoff are made for simplicity. A2 is standard from the I.O. literature: the full information game verifies the traditional stability/unicity condition, i.e. the product of the slopes of the reaction curves is between 0 and 1. It ensures that G_0 has a unique equilibrium in contract proposals. A3 is standard from contract theory: it ensures that, in G_0 or with type-independent reservation control utilities, the principal and the agent will (secretly) negotiate a *separating* contract. A3i) incorporates, in a simple linear form, the *sorting* (or *Spence-Mirrlees*) *condition.*[7] A3ii) is the corresponding *responsiveness* condition; responsiveness was characterized in Caillaud-Guesnerie-Rey-Tirole [1988] as requiring enough compatibility between the principal's and the agent's objectives so that the optimal contract under incomplete information be *separating*. A3iii) is the standard

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6 As will become clear later, many results can be extended to a more general setting. The linear framework allows reaction functions to depend only on the opponent's *expected* action, so that the analysis becomes actually one-dimensional.

7 The general results of the article could be extended to control utilities of the form $U^A = t^A - u(a,\theta^A)$, with $\partial_{a\theta} u > 0$, at the cost of an assumption of unicity of the continuation equilibrium payoffs for any subgame starting at stage 1.

monotone hazard rate property. A4 is slightly less conventional, since it relates to the unusual externality term: its goal is to limit difficulties that could arise from the effect of the externality term on the individually rational, or reservation, utility levels of the agent at different steps of contractual (re)negotiations. A4i) requires that a bad-type agent does not benefit more from the externality term than a good-type agent: this allows classical results in G_0, i.e. equilibrium contracts will exhibit "no distortion at the top" and "no informational rent at the bottom". A4ii) ensures that, whenever P^A benefits from a given change in the opponent's actions, A does, too.[8]

Example:

To fix ideas, let us consider the following example. Let principals be two producers with zero production costs. Agents are sellers; agent A bears a cost $\theta^A q^A$ when selling a quantity q^A ($\theta^B q^B$ for agent B). A contract specifies a transfer to the agent depending on the quantity sold q^A and the profit $p^A q^A$ (where p^A is the price). Therefore prices and quantities are verifiable. Let us consider two forms of market game:

Cournot competition with an homogeneous good:

Demand is $p^A = p^B = 1 - q^A - q^B$.

Actions are $a = q^A$ and $b = q^B$.

We then have: $m^A = m^B = 0$

$$\phi^A = a(1 - a - b) - \theta^A a \;;\; \phi^B = b(1 - a - b) - \theta^B b.$$

In the relevant domain of positive quantities and positive prices, assumptions A1-4 are satisfied.

Bertrand competition in differentiated products:

Demands are $q^A = 1 - p^A + \sigma p^B$ for A, $q^B = 1 - p^B + \sigma p^A$ for B, $0 < \sigma < 1$.

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8 This assumption could be relaxed at the cost of increasing complexity; it is moreover verified by all games we examined.

Define the action as follows: $a = 1 - p^A$ and $b = 1 - p^B$.

Then $q^A = a + \sigma - \sigma b$, $m^A = - \theta^A\sigma + \theta^A\sigma b$ and $\phi^A = (1 - a)(a + \sigma - \sigma b) - \theta^A a$, and similarly for hierarchy P^B-B. If we restrict attention to actions such that quantities and prices are positive, assumptions A1-4 hold. Our normalization implies that actions are the opposite of prices.

The remaining of this paper is devoted to the analysis of the equilibrium of our global game. Strategies are contract proposals, both at stage 0 and 1, by principals, and acceptance rules and actions by agents. Our equilibrium concept is *strong perfect Bayesian equilibrium* (hereafter equilibrium) as defined in Fudenberg-Tirole [1991]. We shall first examine the renegotiation at stage 1.

III. RENEGOTIATION

In this section, the outcome of stage 0 is considered as given, denoted $(\mathcal{C}_0^A, \mathcal{C}_0^B)$, with $\mathcal{C}_0^A = \emptyset$ or $\mathcal{C}_0^B = \emptyset$ if no contract has been signed by the corresponding hierarchy. At stage 1, each hierarchy renegotiates a contract, $\mathcal{C}^A$ and $\mathcal{C}^B$, possibly randomly. Then at stage 2, agent B will choose an action b, the probability distribution of which may depend on $\mathcal{C}_0^A$, $\mathcal{C}_0^B$, $\mathcal{C}^B$ and θ^B which constitute the information of agent B. It is clear that from the viewpoint of hierarchy P^A-A at stage 1, only the final probability distribution of B's action conditional on $\mathcal{C}_0^A$ and $\mathcal{C}_0^B$ matters, not $\mathcal{C}^B$ and θ^B (since they are not observed). This anticipated probability distribution on S^B is denoted by $\mu^B \in \Delta(S^B)$. Let b^e denote the average action of agent B:

$$b^e = \int b d\mu^B(b).$$

If at stage 1, a type-i agent A rejects P^A's offer, he obtains an expected payoff, denoted by $\underline{W}_i^A$, which may depend on $\mathcal{C}_0^A$, $\mathcal{C}_0^B$ and on the opponent's anticipated behavior μ^B. Let us then define the profile of *reservation control utilities* $\mathbf{V}^A \equiv (V_1^A, \ldots, V_n^A)$, by

$$V_i^A = \underline{W}_i^A - \int m^A(b,\theta_i^A)d\mu^B(b) \tag{1}$$

If no contract has been signed at stage 0 by P^A and A, then $\underline{W}_i^A = 0$. So

$$V_i^A = - \int m^A(b,\theta_i^A)d\mu^B(b). \tag{2}$$

Now it is clear that if, along some equilibrium path, $(\mathcal{C}_0^A,\mathcal{C}_0^B)$ occurs at stage 0 and P^A proposes $\mathcal{C}^A$ at stage 1, $\mathcal{C}^A$ must be a contract that maximizes P^A's payoff given μ^B and reservation control utilities $\mathbf{V}^A$ (in other words $\mathcal{C}_0^A$ and $\mathcal{C}_0^B$ only matter to the extent that they affect the reservation control utilities at the renegotiation stage). The object of this section is to solve this particular contracting problem. We shall omit superscripts A since only the renegotiation between P^A and A is considered.

For a fixed distribution μ^B and fixed reservation control utilities $\mathbf{V}$, principal P^A faces a standard adverse selection problem with type-dependent reservation utilities. We can therefore use the Revelation Principle to restrict attention to revelation mechanisms that offer a menu of pairs of actions and transfers $(\mathbf{a},\mathbf{t}) = \Big((a_1, \ldots, a_n),(t_1, \ldots, t_n)\Big)$, the pair (a_i,t_i) being choosen by the type-i agent. Equivalently, a contract is fully characterized by a menu $\mathcal{C} = (\mathbf{a},\mathbf{U})$, where $\mathbf{U} = (U_1,\ldots, U_n)$ is a profile of control utilities defined by $U_i = t_i - \theta_i a_i$, and that satisfies incentive compatibility:

$$U_i - U_j \geq (\theta_j - \theta_i)\, a_j \qquad \text{for all } i,\ j = 1, \ldots, n. \tag{3}$$

A contract $\mathcal{C}$ will be accepted by agent A at the renegotiation stage if, given the profile of reservation control utilities $\mathbf{V}$,

$$U_i \geq V_i \qquad \text{for all } i = 1, \ldots, n. \tag{4}$$

Using the linearity of ϕ in b, the principal's problem can be written as:

$$\underset{(\mathbf{a},\mathbf{U})}{\text{Max}} \quad \sum_{i=1}^{n} h_i \left\{ \phi(a_i,b^e,\theta_i) - U_i \right\} \tag{P}$$

such that (3) and (4) hold.

Given that reservation control utilities are type-dependent, we cannot reduce the set of incentive constraints to the set of *upward adjacent* incentive constraints. However, one can reduce it to the set of *all adjacent* incentive constraints.

Lemma 1: Assume A3i); for a given action profile **a**, there exists a utility profile **U** such that (**U**,**a**) satisfies the incentive constraints if and only if a_i is non-increasing in i. Moreover, the set of incentive constraints (3) is equivalent to the set of adjacent incentive constraints (5):

$$\text{for all } i = 1, 2, \ldots, n-1, \quad \begin{cases} U_i - U_{i+1} \geq \Delta_i . a_{i+1} \\ U_{i+1} - U_i \leq \Delta_i . a_i \end{cases} \tag{5}$$

Proof: see appendix.

The problem faced by the principal is thus to solve (P) where (3) is replaced by (5). The solution of (P) clearly exists under A1-A3 and is denoted by:

$\mathbf{A}(b^e,\mathbf{V}) = \{A_1(b^e,\mathbf{V}),\ldots,A_n(b^e,\mathbf{V})\}$, the vector of actions implemented,

$\mathbf{U}(b^e,\mathbf{V}) = \{U_1(b^e,\mathbf{V}),\ldots,U_n(b^e,\mathbf{V})\}$, the vector of control utilities.

$\Pi^A(b^e,\mathbf{V})$ will denote the optimal value of P^A's expected payoffs.

In this section we shall restrict our attention to the characteristics of the optimal contract that are of direct interest for the main results of the paper. Further material is provided in section V, where in particular we examine the possibility of "over-production". Notice that the knowledge of $\mathbf{A}(b^e,\mathbf{V})$ and $\mathbf{U}(b^e,\mathbf{V})$ is sufficient to derive the complete behavior of P^A and A at stage 1 and 2, since it specifies the actions chosen and the transfers, as a function of past contracts (signed at stage 0) and anticipated P^B and B's actions.

Proposition 1: Assume A1 and A3 hold. For any profile of reservation control utilities $\mathbf{V} \in \mathbb{R}^n$ and any opponent's expected action $b^e \in [0,\bar{b}]$,

$\mathbf{A}(b^e,\mathbf{V})$ is continuous with respect to b^e and $\mathbf{V}$ and

$\mathbf{A}(b^e,\mathbf{V}) \geq \mathbf{A}(b^e,0)$.

$\mathbf{A}(b^e,\mathbf{V}) = \mathbf{A}(b^e,0)$ if and only if $\forall i$, $U_i(b^e,0) + V_n \geq V_i$, in which case, $U_i(b^e,\mathbf{V}) = U_i(b^e,0) + V_n$.

Proof: see appendix.

This proposition contains the basic result for the remaining part of the paper: let us explain the intuition behind. The proposition provides a lower bound on the set of actions on which P^A is able to precommit through contracting, $\mathbf{A}(b^e,0)$, which is obtained for any type-independent reservation control utilities. $\mathbf{A}(b^e,0)$ is chosen so as to tradeoff a marginal increase of type-i's action, which induces a marginal increase of (full information) aggregate payoffs ϕ^A, with a marginal increase of all informational rents paid to types j, $j<i$, in excess of their reservation utilities. Clearly, when considering a type-dependent profile $\mathbf{V}$, the solution may be altered only if the reservation control utility of some type j is increased by such an amount that the previous solution would now violate individual rationality. In this case, P^A must pay type-j so much (for individual rationality reasons) that no increase of utility is needed to restore incentive compatibility if type-i's action is slightly increased, since the upward incentive compatibility constraint between i and j is slack. Therefore, the previous tradeoff tips more in favor of increasing type-i's action. Combine with the next proposition, Proposition 1 will allow us to identify situations where precommitment opportunities are effectively used in equilibrium.

Proposition 2: Assume A1-A3-A4i hold. If no contract has been signed at stage 0, and the opponent is expected to play according to μ^B, the optimal contract for principal P^A is $(\mathbf{A}(b^e,\mathbf{V}(\mu^B)),\mathbf{U}(b^e,\mathbf{V}(\mu^B)))$ where $\mathbf{V}(\mu^B)$ is given by (2). Moreover,

$$\mathbf{A}(b^e,\mathbf{V}(\mu^B)) = \mathbf{A}(b^e,\mathbf{0}) \quad \text{and}$$

$$\Pi^A(b^e,\mathbf{V}(\mu^B)) = \Pi^A(b^e,\mathbf{0}) - V_n(\mu^B).$$

Proof: see appendix.

This result states that, when no contract has been signed, the externality term does not affect the contract once adjusted so as to maintain the worst type (type n) at his reservation utility level. With Proposition 1, it implies that, when signing a public contract, P^A can only precommit to implement actions that are higher than the actions obtained when no public contract is signed.

In what follows we shall also use differentiability:

Corollary 1: Under A1-A3, if $\mathbf{A}(b^e,\mathbf{0})$ is interior in $[0,\bar{a}]^n$, $\Pi^A(.,.)$ is differentiable at any point $(b^e,\mathbf{V})$ such that $\mathbf{A}(b^e,\mathbf{V}) = \mathbf{A}(b^e,\mathbf{0})$.

Proof: see appendix.

What appears clearly is that P^A can gain a precommitment advantage by signing a public contract that modifies the profile of reservation utilities of the agent at the renegotiation stage 1. Her ability to precommit will thus be limited by the set of profiles that can be obtained out of a contract signed at stage 0. To conclude this section let us briefly examine this set.

Any contract must induce a profile that satisfies (5); therefore,

$$\text{for all } i = 1,\dots,n-2: \quad \bar{a} \geq \frac{V_i - V_{i+1}}{\Delta_i} \geq \frac{V_{i+1} - V_{i+2}}{\Delta_{i+1}} \geq 0 \tag{6}$$

We refer to a profile that verifies condition (6) as a *convex profile.*[9] Define a *non-contingent contract* as a contract that does not include any contingency on the opponent's public contract or observed action. Such a contract can always be summarized by a menu $(\mathbf{a},\mathbf{U}^A)$ for P^A-A. The possibility of signing non-contingent contracts is crucial, first, to derive the equilibrium of G_0, and secondly, to generate reservation control utilities at the renegotiation stage in a simple manner:[10]

Proposition 3: Under A1-A3, for any convex profile **V**, there exists a *non-contigent* contract such that, if signed at stage 0, the profile of reservation control utilities is **V** at stage 1.

Proof: Let **V** be a convex profile. Choose a n-uple of actions **a** such that:

$$a_i \geq \frac{V_i - V_{i+1}}{\Delta_i} \geq a_{i+1} \text{ for all } i = 1, \ldots, n.$$

Since actions a are verifiable for the hierarchy P^A-A, we can define a contract $\mathcal{C}$ that offers a choice to the agent between n pairs of actions and transfers (a_i, t_i), where $t_i \equiv V_i + \theta_i a_i$. $\mathcal{C}$ is non-contingent and satifies the requirement of the proposition.

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IV. EQUILIBRIUM ANALYSIS AND PRECOMMITMENT EFFECTS.

IV.1. Equilibrium with Unobserved Contracts and Classification of Games

We first analyze the equilibrium when no contract has been signed at stage 0 (game G_0). This situation serves as a benchmark: since contracts are not observed, there is actually only one strategic move in the game,

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9 The term "convex" can be understood by considering the continuous case.

10 The analysis could be adapted to treat specific cases where such contracts are not allowed (as for the case of franchise contracts with Bertrand competition for example).

stage 1 revelation mechanism, so that a hierarchy cannot affect its rival's future behavior.

Define:

$$A^0(b^e) = \sum_{i=1}^{n} h_i^A \, A_i(b^e,0) \tag{7}$$

$$B^0(a^e) = \sum_{j=1}^{m} h_j^B \, B_j(a^e,0) \tag{8}$$

An equilibrium of G_0 is obtained at expected actions a^0 and b^0 such that:

$$a^0 = A^0(b^0) \quad \text{and} \quad b^0 = B^0(a^0) \tag{9}$$

Equilibrium actions and payoffs are then given by proposition 2 and equation (2) applied to the corresponding distribution.

Proposition 4: Under A1-4, G_0 has a unique equilibrium outcome, corresponding to (a^0,b^0).

Proof: A1 and A2 imply that $A^0(b)$ and $B^0(a)$ are continuous and verify the stability condition: $0 \le \frac{\partial A^0(b)}{\partial b} \cdot \frac{\partial B^0(a)}{\partial a} < 1$, where derivatives are left- and right-derivatives. Therefore a^0 and b^0 that solve (9), are uniquely defined. Existence is trivial.

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Starting from an initial situation with no public contract, P^A may consider whether to offer a public contract or not. If she offers a public contract, she can only induce her agent to increase his action. This will in turn induce P^B to increase or decrease B's expected actions depending on whether expected actions are strategic complements or strategic substitutes in the game G_0. This leads naturally to a first classification of games (see e.g. Fudenberg-Tirole [1984]):

<u>Classification S</u>

SS (Strategic Substitute): $\phi^A_{ab} < 0$ and $\phi^B_{ba} < 0$

SC (Strategic Complement): $\phi^A_{ab} > 0$ and $\phi^B_{ba} > 0$

We can cross this first criterion with a second one related to how each principal would like her opponent to move her action:

Classification E

PE (Positive Effect): $\phi_b^A > 0$ and $\phi_a^B > 0$

NE (Negative Effect): $\phi_b^A < 0$ and $\phi_a^B < 0$

For example, the model of Cournot competition described in Section II leads to SS and NE, while the model of Bertrand competition leads to SC and NE.

IV.2. Precommitment Effects, General Results.

The intuition for the existence of precommitment effects is the following. Suppose that P^B-B have not signed a public contract. In this case, B will choose the action $B^0(a^e)$. Let $\mathcal{C}_0^A$ be the optimal non-contingent contract at the equilibrium of G_0. Suppose that we start from the equilibrium of G_0 and that P^A proposes a public contract $\mathcal{C}$ close to $\mathcal{C}_0^A$ but leading to actions slightly above $A^0(b^0)$. Then the direct effect of $\mathcal{C}$ on P^A's payoff is second order since $\mathcal{C}_0^A$ is optimal given b^0. $\mathcal{C}$ has an indirect effect through the variation of B's expected action. The sign of this variation is the sign of the slope of the average reaction curve of the opponent hierarchy, which, in turn, is the sign of the slope of full information reaction curves, i.e. the sign of $-\phi_{ab}^B/\phi_{bb}^B$ or of ϕ_{ab}^B. The sign of the effect on P^A's payoff is the sign of $\phi_{ab}^B\phi_b^A$, obtained by crossing the classifications S and E. Therefore we find two cases where precommitment effects exist:

Theorem 1 : Assume A1-4 and that $A(b^0,0)$ and $B(a^0,0)$ are interior in $[0,\bar{a}]^n$ and $[0,\bar{b}]^m$. If SS and NE, or SC and PE, hold, then precommitment effects exist: not signing a public contract at stage 0 is not an equilibrium of G.

Proof: see appendix. The proof shows that S and E could take a local, much weaker form.

There also exist situations where precommitment effects may not exist, i.e. such that the equilibrium outcome of G_0 (with unobserved contracts) is also an equilibrium outcome of G. The reasoning is very similar to the previous one. Contracts have two purposes: they have "internal" purposes to overcome asymmetric information problems between principals and agents, and "external" purposes to affect the opponent's renegotiation and action play. Internal purposes are maximized with unobserved equilibrium contracts, since these neglect any precommitment effects. Now if external purposes can only be worsen by using other contracts, principals will sign contracts that maximize internal concerns only. And this is the case if committing to larger actions, on average, can only affect the average (equilibrium) opponent's action in the wrong direction, i.e. decrease it whereas PE holds or increase it whereas NE holds.

Theorem 2 : Assume A1-4. If SS and PE, or SC and NE, hold, then, not signing a contract at stage 0 is an equilibrium of G. Under SC and NE, it is a pareto dominant equilibrium.

Proof: see appendix.

The theorems deserve some comments. They are stated in a weak form. Theorem 1 does not exclude the possibility that the actions and transfers of the equilibrium of G_0 be obtained as the outcome of some equilibrium of G. It just implies that, if this is the case, such an equilibrium cannot be sustained by simple (non-contingent) contracts: both firms must sign *complex public contracts*. The possibility of such an outcome stems from the fact that we do not prevent contracts from depending on each other through cross-contingencies. Katz [1987] analysis of games with cross-contingent contracts when no renegotiation is allowed showed that some kind of "Folk Theorem" results could appear, that may allow the equilibrium allocation of G_0 to be sustainable as an equilibrium of G. Basically, the intuition is that cross-contingent contracts allow to precommit to out-of-equilibrium-path suboptimal strategies that hurt the opponent and that are only triggered by deviations in the design of the opponent's contractual clauses: these strategies act as retaliation in case one party deviates from the equilibrium. Allowing for renegotiation reduces but does not remove completely this possibility.[11]

Similarly Theorem 2 does not say that the equilibrium allocation of G_0 is the unique equilibrium allocation of G. This is due to the same reason. Notice however that the Pareto optimality of the "no public contract" equilibrium outcome, along with its simplicity in terms of strategies, provides a good case for this equilibrium in the case SC-NE.

These points deserve careful studies and should be the object of further analysis. Because of the self reference problem emphasized by Katz [1987], such an analysis requires some restrictions on the nature of allowed contracts, and therefore depends on the specific situation that one is whishing to study. Therefore at the level of generality of this paper, we can only point simple situations of common interest where the results can be refined.

IV.3 Specific Results

It is clear that a hierarchy which is not subject to asymmetric information cannot use public contracts as a precommitment device. Indeed

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[11] We do not know however, if using renegotiable cross-contingent contracts actually leads to the sustainability of the equilibrium of G_0 as an equilibrium outcome of G. We just could not prove that it does not.

whatever the initial public contract, the hierarchy will always renegotiate so as to choose the action that maximizes the full information aggregate payoff. In particular this removes completely the possibility for this hierarchy to include cross-contingencies in its public contract in a sensible way and therefore leaves the opponent free to use all the precommitment possibilities. We then obtain:

Theorem 3: Assume A1-4 hold and that *m=1*, then:

i) if the equilibrium actions of G_0 are interior in $[0,\bar{a}]^n \times [0,\bar{b}]$, under SS-NE and SC-PE, in any equilibrium of G, P^A signs a public contract at stage 0 and A's equilibrium expected action differs from a^0;

ii) under SS-PE and SC-NE, all equilibrium outcomes of G coincide with the equilibrium outcome of G_0 in terms of final actions and transfers.

Proof: see appendix.

A second possibility to remove the difficulties linked to cross-contingencies is to restrict our attention to games where only non-contingent contracts can be signed. These games are of particular interest because of their practical relevance: in many situations the opponents' behavior may be known but not verifiable, and even if it is verifiable, cross-contingencies may be forbidden by antitrust authorities.

To refine the results, one has to guarantee unicity of the continuation equilibrium after a public contract has been signed (this is developed in the proof of Theorem 4). Unfortunately, it turns out that, in the general case, the slope of the expected reaction curve, when a hierarchy is commited to a public contract at the renegotiation stage, may vary in a non trivial way (it may even change sign). This leads us to the following definitions.

Definition: Define the expected reaction functions:

$$a(b,\mathbf{V}^A) = \sum_{i=1}^{n} h_i^A A_i(b,\mathbf{V}^A)$$

and $b(a,\mathbf{V}^B)$ similarly.

The game G is <u>R-stable</u> if, for all profiles $(\mathbf{V}^A,\mathbf{V}^B)$ and a^*,b^* such that $a^* = a(b^*,\mathbf{V}^A)$ and $b^* = b(a^*,\mathbf{V}^B)$:

$0 < \frac{\partial a(b^*,\mathbf{V}^A)}{\partial b}.\frac{\partial b(a^*,\mathbf{V}^B)}{\partial a} < 1,$

for all i, $A_i(.,\mathbf{V}^A)$ and $a(.,\mathbf{V}^A)$ are comonotonic,

and for all j, $B_j(.,\mathbf{V}^B)$ and $b(.,\mathbf{V}^B)$ are comonotonic.

R-stability extends the basic stability property of the full information game to any possible continuation equilibrium. The comonotonicity assumption is here to take care of the effect of the externality term m on the agent's reservation utility.

Theorem 4: Assume that A1-4 hold, m ≥ 2, that G is R-stable and that only non-contingent contracts are allowed; let a^* and b^* be expected actions obtained at some equilibrium of G then:

i) if the equilibrium actions are interior in $[0,\bar{a}]^n x[0,\bar{b}]^m$, under SS-NE or SC-PE: $a^* \neq A^0(b^*)$ <u>and</u> $b^* \neq B^0(a^*)$;

ii) under SS-PE and SC-NE: $a^* = a^0$ and $b^* = b^0$.

Proof: see appendix.

<u>The case n=m=2</u>

To illustrate the relevance of Theorem 4, let us consider the case $n = m = 2$. This case is fully developed in Caillaud-Jullien-Picard [1990b] so that we shall only provide the results.

We shall consider as in section III the renegotiated contract offered by P^A. Remember that $A_i^*(.)$ denotes the full information best responses and that the action implemented for type 2 when $\mathbf{V}^A = \mathbf{0} = (0,0)$ is given by:

$$A_2(b,0) = \underset{a}{\text{Argmax}} \{ h_2^A\phi^A(a,b,\theta_2^A) - h_1^A\Delta_1^A a \}$$

The action implemented for type 1 when $\mathbf{V}^A = (V_1,0)$ with V_1 large, is:

$$\bar{A}_1(b) = \underset{a}{\text{Argmax}} \{ h_1^A\phi^A(a,b,\theta_1^A) + h_2^A\Delta_1^A a \}$$

This is the action that makes type 2 indifferent between $\overset{*}{A}_2(b)$ and $\bar{A}_1(b)$.

Finally let $X \equiv (V_1^A - V_2^A) / \Delta_1^A$. The actions implemented by P^A at the renegotiation stage are then functions of X only (only the difference between reservation control utilities matters). They are given in Table 1.

X	$A_2(b^e,0)$ $\overset{*}{A}_2(b^e)$ $\overset{*}{A}_1(b^e)$ $\bar{A}_1(b^e)$				
a_2	$A_2(b^e,0)$	X	$\overset{*}{A}_2(b^e)$	$\overset{*}{A}_2(b^e)$	$\overset{*}{A}_2(b^e)$
a_1	$\overset{*}{A}_1(b^e)$	$\overset{*}{A}_1(b^e)$	$\overset{*}{A}_1(b^e)$	X	$\bar{A}_1(b^e)$
U_2	V_2^A	V_2^A	V_2^A	V_2^A	$V_1-\Delta_1^A\bar{A}_1(b^e)$
U_1	$V_2^A+\Delta_1^A A_2(b^e,0)$	V_1^A	V_1^A	V_1^A	V_1^A

Table 1

Table 1 should be read as follows. The horizontal axis illustrates variations of X, the level of precommitted rent differential in control utilities, for which four critical values are outlined. These critical levels determine five distincts regions within which the optimal contract is characterized.

Now it is clear from the slopes of $\overset{*}{A}_1(b)$, $A_2(b,0)$ and $\bar{A}_1(b)$, that the R-stability condition is verified. Proposition 5 follows straightforwardly.

Proposition 5: If $n = m = 2$, then G is R-stable.

Table 1 gives a first insight on the general form of renegotiated contracts. The more "convex" the profile $\mathbf{V}^A$ (i.e. the larger X), the higher the actions implemented. For a given action b of the opponent, actions remain in some domain, with a lower bound obtained when $\mathbf{V}^A=0$ and an upper bound reached when $\mathbf{V}^A$ is extremely convex. Finally notice that for X large enough, actions are above the efficient actions so that the standard suboptimality result is reversed. We shall generalize these results for the general case.

V. THE DEGREE OF PRECOMMITMENT

In this section we shall refine the analysis of renegotiated contracts so as to provide bounds on the amount of precommitment that a hierarchy can obtain. It will appear that when precommitment effects are taken into account, the standard underproduction result in adverse selection problems is no longer valid: principal P^A may induce agent A to overproduce. We shall also develop a specific example to illustrate the effects of various parameters on the amount of precommitment that may be expected in equilibrium.

V.1 The Form of Renegotiated Contracts

As already mentioned, the problem faced by the principal at stage 1 is an adverse selection problem with type-dependent reservation utilities. This subsection is devoted to some properties of this problem when the profile of reservation utilities is "convex" (satisfies (6)).

Denote $\mathbf{A}^*(b)$ the vector of full information best responses: given A1, $A_i^*(b)$ is decreasing with i and therefore $\mathbf{A}^*(b)$ is implementable. In other words, there exists some profile $\mathbf{V}^*(b)$ such that $\mathbf{A}(b,\mathbf{V}^*(b)) = \mathbf{A}^*(b)$. Indeed, $\mathbf{V}^*(b)$ can be chosen such that:

$$\Delta_i^A\, A_i^*(b) \geq V_i^*(b) - V_{i+1}^*(b) \geq \Delta_i^A\, A_{i+1}^*(b)$$

When, at stage 1, the profile of reservation control utilities is flat, i.e. with type-independent reservation utilities, the rent differentials are such that: $0 = V_i - V_{i+1} < \Delta_i^A A_{i+1}^*(b)$, and the principal is lead to induce smaller actions than under full information: this is the classical result of underproduction. Proposition 6i below extends this result to a class of type-dependent profiles of reservation utilities.

Conversely, when the rent differentials are so high that $\Delta_i^A A_i^*(b) < V_i - V_{i+1}$, the principal is lead to induce higher actions than under full information: there is overproduction. In this case the traditional ranking of types from "good" types to "bad" types is reversed (agents with low marginal costs have a high reservation utility level, and therefore turn out to be of high costs). Proposition 6ii below proves this result for a class of highly convex profiles of reservation utilities. For this purpose, we need first to modify slightly the monotone hazard rate property:[12]

A3iv): $\dfrac{h_i^A}{\Delta_i^A (1-H_i^A)}$ is nondecreasing with i, and similarly for B.

Second, assuming A3iv and looking at the first order conditions for program (P') (see appendix, proof of Proposition 1), it is easy to check that for $\mathbf{V}^A = (V_1, 0, \ldots, 0)$ and V_1 large enough, the actions implemented are independent of V_1. We denote $\bar{\mathbf{A}}(b)$ the vector of implemented actions for V_1 large enough, defined by:

$\bar{A}_n(b) = A_n^*(b)$ and

for all $i < n$, if $h_i^A \phi_a(\bar{a}, b, \theta_i^A) + (1-H_i^A)\Delta_i^A \geq 0$, $\bar{A}_i(b) = \bar{a}$

otherwise, $h_i^A \phi_a(\bar{A}_i(b), b, \theta_i^A) = -(1-H_i^A)\Delta_i^A$.

A3iv ensures that $\bar{A}_i(b)$ is non-increasing in i. We are now in a position to

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12 With a continuum of types, distibuted according to cumulative $H(\theta)$ and density $h(\theta)$, A3iii corresponds to the traditional condition h/H nonincreasing, while A3iv corresponds to $h/(1-H)$ nondecreasing. This condition was already used to study an adverse selection problem with type-dependent reservation utilities by Lewis-Sappington [1989]; their emphasis was different from ours, however, since they focused on concave profiles.

prove:

Proposition 6: Assume A1-A3 hold. For all b and $\mathbf{V}$:

$$\mathbf{A}(b,0) \le \mathbf{A}(b,\mathbf{V}) \le \bar{\mathbf{A}}(b).$$

i) if $\mathbf{V}$ is convex and for all i, $V_i - V_{i+1} < \Delta_i^A A_{i+1}^*(b)$, $\mathbf{A}(b,\mathbf{V}) < \mathbf{A}^*(b)$;

ii) if $\mathbf{V}$ is convex and for all i, $\Delta_i^A A_i^*(b) < V_i - V_{i+1}$, $\mathbf{A}(b,\mathbf{V}) > \mathbf{A}^*(b)$.

Proof: see appendix.

The proposition provides conditions under which we can guarantee that underproduction or overproduction arises. If the profile of reservation utilities is convex enough, overproduction arises. An immediate consequence of this proposition is that overproduction may obtain in equilibrium although the basic structure of the problem always leads to underproduction if public contracts are not allowed. This will be developed in the next subsection.

The proposition also provides some information on what happens if the timing of the game is modified so that θ^A and θ^B are not known by the corresponding agents at stage 0, but are learnt by the agents between stage 0 and 1. If contracts signed at stage 0 are kept secret, in equilibrium, actions $\mathbf{A}^*(b^e)$ and $\mathbf{B}^*(a^e)$ are implemented and stage 1 is redundant. If contracts signed at stage 0 can be public, the principal can use a public contract to increase or to decrease the agent's actions compared to the full information best response. Therefore precommitment effects will always arise. However one should notice that this conclusion requires that the game be artificially contrived. We have to allow hierarchies to sign public contracts at an ex-ante stage but at the same time forbid them to renegotiate at this ex-ante stage. Indeed if hierarchies are allowed to renegotiate at an ex-ante stage, it is clear that they will use this possibility to renegotiate the optimal contract $\mathbf{A}^*(b^e)$ or $\mathbf{B}^*(a^e)$, and no precommitment effects would arise.

V.2 An Example for Comparative Statics.

As mentioned above, principal P^A can use public contract to precommit to any action between $A(b^e,0)$ and $\bar{A}(b^e)$. The extent to which P^A will use this strategic possibility depends on both the cost and the benefit of precommitment. Clearly the benefit depends on the slope of the reaction curve of P^B-B. The cost depends on incentive costs which in turn depend on the degree of the asymmetry of information. We shall illustrate this on a simple example.

The example is the reduced form of a Cournot competition game between two producer-salesman hierarchies. Agent B's type is known so that $m = 1$ and Theorem 3 applies. As mentioned above, hierarchy P^B-B will always implement the first best action $B^*(a^e)$, assumed to be:

$$B^*(a) = 1 + \sigma - \sigma a \ .$$

Agent A may be of 2 types: $\theta_1 = 1 - \delta$ or $\theta_2 = 1 + \delta$, with $h_1 = h_2 = 1/2$. The aggregate profit of the hierarchy P^A-A is given by:

$$\phi^A(a,b,\theta_1) = (1 + \gamma - \gamma b - \frac{a}{2})a + \delta a$$

$$\phi^A(a,b,\theta_2) = (1 + \gamma - \gamma b - \frac{a}{2})a - \delta a$$

The game is normalized so that when $\delta = 0$, the equilibrium is at $a=b=1$. Two parameters will be determinant: δ which measures the degree of uncertainty, and $\rho \equiv \sigma\gamma$ which measures the benefit from increasing a. The stability condition amounts to: $0 < \rho < 1$.

Given that $m = 1$, there is no loss of generality in restricting attention to non-contingent contracts. Skipping all tedious computations,[13] we shall only provide some results about the optimal degree of precommitment

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[13] An Appendix, containing the complete treatment of this example, is available upon request from the authors.

in the case where: $0 \leq \rho < 1 - 1/\sqrt{3}$. Moreover we shall use the parameter r, defined as $r \equiv \frac{\rho}{2-\rho}$, which is an increasing function of ρ.

In this model three continuation equilibria are of particular interest. First, the equilibrium of G_0 is characterized by a^0, agent A's expected action: $a^0 = 1 - \frac{1+r}{1-r}\delta$. Second, if agent A's type were known by principal P^A (but not P^B or B), P^A would always implement the vector of actions $A^*(b^e)$ and the equilibrium would be characterized by $a^* = 1$. Third, suppose P^A signs a public contract at stage 0 with $V=(V_1,0)$, where V_1 is very large; the continuation equilibrium would then be characterized by: $a^1 = 1 + \frac{1+r}{1-r}\delta$. Now, by publicly disclosing a contract, P^A can induce any equilibrium expected action a between a^0 and a^1. When $a < a^*$, P^A induces underproduction whereas when $a > a^*$, P^A induces overproduction from agent A; in contrast, with non-public contract, P^A can only induce underproduction and equilibrium expected action a^0. The maximum degree of precommitment is obtained at a^1. We shall moreover restrict attention to the case $\delta \leq \alpha(r) \equiv \frac{1-r}{3-r}$, for which all actions emerging in all the equilibria aforementioned are positive.

There is no conceptual difficulty in finding that, in this example, the equilibrium of game G is characterized as follows:

- for $0 \leq \delta < \delta(r) \equiv \dfrac{2r(1-r)}{1-2r-r^2}$, P^A signs a public contract inducing a reservation utility profile $\mathbf{V} = \left(1 + \delta\,\frac{3-r}{1-r}\ ;\ 0\right)$ and leading to expected action $a = a^1 = 1 + \delta\,\frac{1+r}{1-r}$;
- for $\delta(r) \leq \delta < r$, P^A signs a public contract such that $\mathbf{V} = \left(1 - \delta\ ;\ 0\right)$, leading to expected actions $a = a^* = 1$;
- for $r \leq \delta < \alpha(r)$, P^A signs a public contract inducing a reservation utility profile $\mathbf{V} = \left(1 - \delta - \dfrac{2(\delta-r)}{1-2r-r^2}\ ;\ 0\right)$ and leading to expected action $a = 1 - \dfrac{1+r}{1-2r-r^2}(\delta - r)$.

One can first show that the equilibrium expected action a is increasing with r, or with ρ, whereas the equilibrium action with non-public contract is decreasing. The larger ρ, the larger the amount of precommitment acquired through public contracting. This is of no surprise since ρ measures the desirability of precommitment.

The effect of δ, the degree of the asymmetry of information, is more ambiguous. In equilibrium, when δ is small P^A induces a^1, i.e. induces the maximal precommitment advantage; for intermediate values of δ, P^A induces a^*, while for large δ, P^A merely induces underproduction as in the non-public contract case, but to a lesser extent. Figure 1 illustrates these findings.

<<INSERT FIGURE 1 HERE>>

Two effects are at work: first, a higher degree of uncertainty enhances the principal's ability to precommit as witnessed by the fact that a^1 increases with δ (indeed, uncertainty is necessary for the existence of precommitment); second, the incentive cost increases with the degree of uncertainty δ, making precommitment more costly. Both effects act in opposite directions. For δ small enough, the incentive costs are low, but the ability to precommit is also limited: the principal uses all her precommitment possibilities. At some point, the incentive cost becomes too large and maximal precommitment is not justified anymore.[14] For intermediate δ, the solution is a corner solution at a^*, but since a^0 decreases with δ, the degree of precommitment (measured by the difference $a - a^0$) still increases with δ, the effect still being an increase in the ability to

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[14] The problem is non-convexe, because inducing an action a slightly above a^* requires a finite increase in incentive cost compared to a slightly below a^*, since a^* is obtained for the range $A_1(b^*) \geq X \geq A_2(b^*)$.

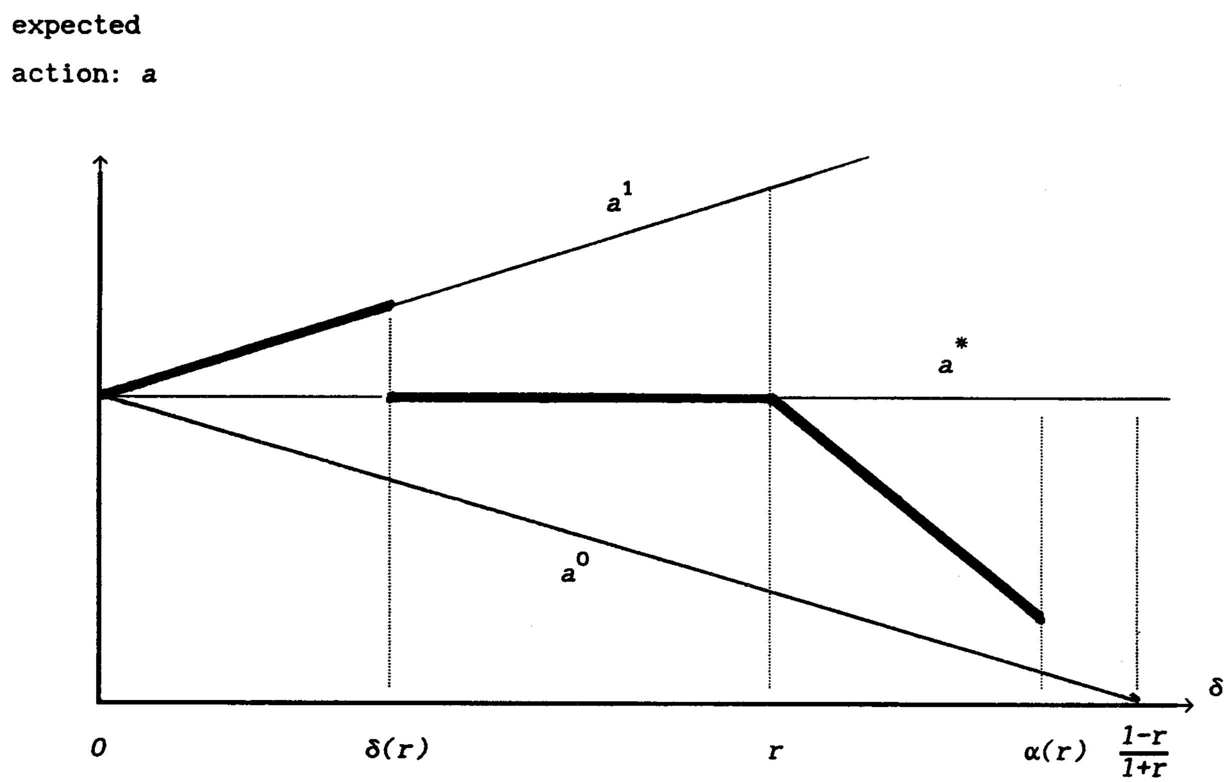

Figure 1: the degree of precommitment

precommit. For δ large, the effect due to the incentive cost dominates, and the degree of precommitment decreases with the degree of uncertainty. Therefore, in the setting of this particular example, the first effect (ability to precommit) dominates for small uncertainty while the second effect (incentive cost) dominates for large uncertainty.

VI. APPLICATIONS

Since the general and specific results we have presented may still look abstract, we provide in this concluding section applications of our results to various cases, without any formal model.

Producer-Salesman

Suppose principals are producers and agents are salesmen, as in the example of section II. As already mentioned in the case of Cournot competition, the situation corresponds to SS-NE, therefore precommitment effects should arise. In this situation, strategic announcements of salesmen compensation schemes should be crucial. In the case of Bertrand competition however, the situation corresponds to SC-NE (remember that with our normalization a higher price corresponds to a lower action). In this case, no precommitment effects should occur and strategic announcement of salesmen compensation schemes should not be observed.

Cost Reducing Investment

Two competing firms, P^A and P^B, can contract with an R&D or consulting firms, A and B, so as to reduce their marginal cost by an amount a and b respectively. In this context, it is natural to assume that the contract between a producer and an R&D firm can only bear on the R&D firm's investment or equivalently on the cost reduction obtained, so that contracts are non-contingent. With substitutable goods, if P^A induces an increase in the amount of cost reduction from a to $a+da$, P^B's production will be reduced, whatever the type of competition involved. As the marginal gain for P^B from reducing her marginal cost is proportional to her production, this will reduce P^B's incentive to reduce cost. We are thus in the case SS-NE: both firms should sign a public R&D contract in equilibrium.

<u>Advertising</u>

Two competing producers contract with publicists with unknown cost for a level of advertising *a* or *b*. Firms sell an homogeneous product and advertisement bears on the product only (see e.g. the recent campaigns on French TV for butter or sugar). Firms' profits depend on the total amount of advertising *a+b*. In this SS-PE case, there will be no precommitment effect (notice however that in the situation described a joint project would be more natural).

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APPENDIX

Proof of Lemma 1:

For any i and j, such that $i < j$, if **(U,a)** satisfies (3), then,

$$(\theta_j - \theta_i)\, a_j \leq U_i - U_j \leq (\theta_j - \theta_i)\, a_i$$

This implies $a_i \geq a_j$. Moreover, for $j = i + 1$, one gets (5).

Next, suppose (5) holds for **(U,a)**. Then, first, a_i is necessarily non-increasing in i, and, second, for $j < i - 1$,

$$U_i - U_j = \sum_{h=j}^{i-1} (U_{h+1} - U_h) \geq - \sum_{h=j}^{i-1} \Delta_h . a_h \geq - \sum_{h=j}^{i-1} \Delta_h . a_j = - (\theta_i - \theta_j)\, a_j$$

and similarly for $j > i + 1$. Then, (5) is equivalent to (3).

Finally, suppose a_i is non-increasing in i. Fix U_n and define recursively for $i < n$, $U_i = U_{i+1} + \Delta_i . a_{i+1}$. Since $a_{i+1} \leq a_i$, the profile of utilities so defined is such that **(U,a)** satisfies (5). Then, (3) holds.

■

Proof of proposition 1:

The principal P^A solves the programme:

$$\text{Max} \sum_{i=1}^{n} h_i \left\{ \phi_i(a_i, b^e, \theta_i) - U_i \right\} \qquad (P')$$

such that

$$U_i \geq V_i \ , \ i = 1, \ldots, n \qquad [\delta_i]$$

$$U_i - U_{i+1} \geq \Delta_i . a_{i+1} \ , \ i = 1, \ldots, n-1 \qquad [\beta_i]$$

$$U_{i+1} - U_i \geq - \Delta_i . a_i \ , \ i = 1, \ldots, n-1 \qquad [\alpha_i]$$

$$0 \leq a_i \leq \bar{a} \ , \ i = 1, \ldots, n \qquad [\eta_i], \ [\xi_i]$$

(P') is a concave maximization programme with linear constraints. The continuity of the solution $(\mathbf{A}(b^e,\mathbf{V}),\mathbf{U}(b^e,\mathbf{V}))$ follows from the maximum principle. The necessary and sufficient first order conditions are:

$$h_i\, \phi_a(a_i, b^e, \theta_i) = - \alpha_i \Delta_i + \beta_{i-1} \Delta_{i-1} + \xi_i - \eta_i \quad , \ i = 1, \ldots, n$$

$$H_i = \beta_i - \alpha_i + \sum_{j=1}^{i} \delta_i \quad , \ i = 1, \ldots, n,$$

$$\xi_i \geq 0 \ ; \ \xi_i > 0 \Rightarrow a_i = \bar{a} \quad , \ i = 1, \ldots, n,$$

$$\eta_i \geq 0 \ ; \ \eta_i > 0 \Rightarrow a_i = 0 \quad , \ i = 1, \ldots, n,$$

where we let $\beta_0 = \alpha_n = \beta_n = 0$.

Assume first that $\mathbf{V} = 0$. With type-independent reservation utilities and given A1-A3, it is well known that the optimal contract $(\mathbf{A}(b^e,0),\mathbf{U}(b^e,0))$ maximizes the principal's expected payoffs subject to the *upward* adjacent incentive constraints and $U_n \geq 0$:

$$A_1(b^e,0) = A_1^*(b^e)$$

and for $i > 1$, if $h_i\ \phi_a(0,b^e,\theta_i) > H_{i-1}\Delta_{i-1}$, $A_i(b^e,0)$ solves

$$h_i\ \phi_a(A_i(b^e,0),b^e,\theta_i) = H_{i-1}\Delta_{i-1};$$

otherwise, $A_i(b^e,0) = 0$.

$$U_n(b^e,0) = 0, \text{ and for } i < n, \quad U_i(b^e,0) = \sum_{j=i}^{n-1} \Delta_j . A_{j+1}(b^e,0)$$

This implies that: $A_i(b^e,0) \geq A_{i+1}(b^e,0)$ and $U_i(b^e,0) \geq 0$, for all $i < n$.

Consider now an arbitrary profile $\mathbf{V}$. Since $\alpha_1 \geq 0$ and $\beta_0 = 0$, we have

$$A_1(b^e,\mathbf{V}) \geq A_1(b^e,0).$$

Suppose $A_j(b^e,\mathbf{V}) \geq A_j(b^e,0)$ for all $j < i$ but $A_i(b^e,\mathbf{V}) < A_i(b^e,0)$. Then, the first order condition for type i and the concavity of ϕ in a imply that $\beta_{i-1} > \mathrm{H}_{i-1}$; then it follows,

$$\frac{U_{i-1}(b^e,\mathbf{V}) - U_i(b^e,\mathbf{V})}{\Delta_{i-1}} = A_i(b^e,\mathbf{V}) < A_i(b^e,0) \leq A_{i-1}(b^e,0) \leq A_{i-1}(b^e,\mathbf{V})$$

Consider the following deviation: $da_i = \varepsilon$, and for $j < i$, $dU_j = \varepsilon\Delta_{i-1}$. This deviation is compatible with all incentive constraints, and yields a payoff variation of

$$d\Pi = \left\{ h_i\phi_a(A_i(b^e,\mathbf{V}),b^e,\theta_i) - H_{i-1}\Delta_{i-1} \right\} \varepsilon > 0$$

which is impossible. By induction, then, $A_i(b^e,\mathbf{V}) \geq A_i(b^e,0)$ for all i.

Finally, for fixed $\mathbf{V}$ and b^e,

$$\underset{(U_1,\ldots,U_{n-1})}{\text{Min}} \quad \sum_{i=1}^{n-1} h_i U_i$$

$$\text{such that} \quad U_i - U_{i+1} \geq \Delta_i . A_{i+1}(b^e,0) \ , \ i = 1, \ldots, n-1$$

$$U_{i+1} - U_i \geq - \Delta_i . A_i(b^e,0) \ , \ i = 1, \ldots, n-1$$

is clearly uniquely solved by $(U_1(b^e,0), \ldots, U_{n-1}(b^e,0))$, hence the last part of the proposition.

Proof of proposition 2:

The first part follows from the definitions. From A4i, $V(\mu^B)$, given by (2), is such that, for any i,

$$V_i(\mu^B) + \theta^A_i A_{i+1}(b^e,0) \le V_{i+1}(\mu^B) + \theta^A_{i+1} A_{i+1}(b^e,0).$$

Equivalently, for any i,

$$V_{i+1}(\mu^B) + \Delta^A_i A_{i+1}(b^e,0) \ge V_i(\mu^B),$$

or,

$$\begin{aligned} V_n(\mu^B) + U_i(b^e,0) - V_i(\mu^B) &\ge V_n(\mu^B) + U_{i+1}(b^e,0) - V_{i+1}(\mu^B) \\ &\ge V_n(\mu^B) + U_n(b^e,0) - V_n(\mu^B) = 0. \end{aligned}$$

Then: $U_i(b^e,V(\mu^B)) = V_n(\mu^B) + U_i(b^e,0)$, and the second part of Proposition 2 follows from Proposition 1.

■

Proof of corollary:

Differentiability in b follows from the envelope theorem. $\Pi^A(.,.)$ is differentiable in V whenever the multipliers in programme (P) are uniquely defined. If $A(b^e,V) = A(b^e,0)$ and $A(b^e,0)$ is interior, the profile is strictly separating and therefore in programme (P), for all i, $\alpha_i\beta_i = 0$. Moreover, from the first order conditions, $\beta_{n-1} = H_{n-1}$ and therefore $\alpha_{n-1} = 0$; and proceeding backwards, $\beta_i = H_i$ and $\alpha_i = 0$, for all i. This implies:

$$\delta_i = 0 = \frac{\partial \Pi^A}{\partial U_i} \text{ for all } i < n \quad\text{and}\quad \delta_n = 1 = \frac{\partial \Pi^A}{\partial U_n}$$

■

Proof of Theorem 1:

If no firm signs a contract at stage 0, principals' payoffs are respectively $\Pi^A(b^0,U^A(b^0,0))$ and $\Pi^B(a^0,U^B(a^0,0))$. Consider the strategy for P^A that consists in signing a contract at stage 0 associated with an utility profile V^A, such that:

V^A_n equals the control utility obtained by a type-n agent in G_0

$V^A_i - V^A_n = U^A_i(b^0,0) + \varepsilon\Delta_{n-1}$ for $i = 1, \dots, n-1$, with $\varepsilon > 0$. This profile is convex and therefore can be obtained through a non-contingent contract (Proposition 3). The ensuing continuation equilibrium is characterized by (a^e, b^e), with $b^e = B^0(a^e)$. From Proposition 1, for any contract offered by P^A in stage 1,

$$\mathbf{a} = \mathbf{A}(b^e, \mathbf{V}^A) \geq \mathbf{A}(b^e, \mathbf{0}).$$

Then: $$\sum_{i=1}^{n} h_i A_i(b^e, \mathbf{V}^A) \geq A^0(b^e).$$

This implies that $a^e \geq a^0$ (by A2 which implies that $A_b^0(b)B_a^0(a) < 1$). Consequently, one also has: $A_n(b^e, 0) \geq A_n(b^0, 0)$. Take ε small enough so that $A_n^*(b^e) \geq A_n(b^0, 0) + \varepsilon$.

Claim 1: If $A_n(b^0, 0) + \varepsilon \geq A_n(b^e, 0)$, then any contract offered in equilibrium at stage 1 by P^A must be such that:

$$a_n = A_n(b^0, 0) + \varepsilon$$

$$a_i = A_i(b^e, 0) \quad \text{for } i < n.$$

Proof of the claim: With the profile **a** defined in the claim, consider the profile of utilities $\mathbf{u}^A$ defined by:

$$u_n^A = V_n^A$$

$$u_{n-1}^A - u_n^A = \Delta_{n-1} a_n = U_{n-1}^A(b^0, 0) + \varepsilon\Delta_{n-1}$$

$$u_i^A = \sum_{j=i}^{n-2} \Delta_j a_{j+1} + u_{n-1}^A.$$

Now, under SS or SC, $A_n(b^e, 0) \geq A_n(b^0, 0)$ implies that for all $i < n$,

$$A_i(b^e, 0) \geq A_i(b^0, 0).$$

Then,

$$u_i^A \geq \sum_{j=i}^{n-2} \Delta_j A_{j+1}(b^0, 0) + u_{n-1}^A = U_i^A(b^0, 0) + \varepsilon\Delta_{n-1} + V_n^A = V_i^A.$$

Therefore the contract $(\mathbf{a}, \mathbf{u}^A)$ is feasible. Consider now the necessary and sufficient first order conditions of (P') with the multipliers: $\alpha_i = \eta_i = \xi_i = 0$ for all i, and $\beta_i = H_i$ and $\delta_i = 0$ for all $i < n-1$. With:

$$h_n \phi_a^A(a_n, b^e, \theta_n^A) = \beta_{n-1}\Delta_{n-1}$$

$$H_{n-1} = \beta_{n-1} + \delta_{n-1}$$

$$1 = \delta_n + \delta_{n-1},$$

the contract (a,u^A) will be optimal if: $0 \le \beta_{n-1} \le H_{n-1}$, that is, if

$$A^*_n(b^e) \ge a_n \ge A_n(b^e,0)$$

which is the case.

□

Claim 2: $\frac{da^e}{d\varepsilon} > 0$

Proof of the claim: If the inequality in claim 1 holds, it follows that, in equilibrium,

$$a^e = A^0(b^e) + h_n\left[A_n(b^0,0) + \varepsilon - A_n(b^e,0) \right]$$

$$b^e = B^0(a^e).$$

Given the interiority assumption, total differentiation is valid and at $\varepsilon=0$:

$$\frac{da^e}{d\varepsilon} = \frac{h_n}{1 - \frac{dB^0}{da}(a^0)\left[\frac{dA^0}{db}(b^0) - h_n \frac{\partial A_n}{\partial b}(b^0,0)\right]} > 0$$

If, on the other hand, the inequality in claim 1 does not hold, then, for ε arbitrarily small,

$$A_n(b^e,0) > A_n(b^0,0) + \varepsilon.$$

Define implicitly $a(\varepsilon)$ as the solution of:

$$A_n(B^0(a),0) = A_n(b^0,0) + \varepsilon.$$

Then $a^e > a(\varepsilon)$ and, at $\varepsilon = 0$:

$$\frac{da}{d\varepsilon}(0) = \frac{1}{\frac{\partial A_n}{\partial b}(b^0,0)\ \frac{dB^0}{da}(a^0)} > 0$$

Therefore, at $\varepsilon = 0$, $\frac{da^e}{d\varepsilon} > 0$.

□

P^A's expected profits are given by $\Pi^A(b^e,u^A)$, so that, using (the proof of) Corollary 1, for $\varepsilon = 0$:

$$\frac{d\Pi^A}{d\varepsilon} = \left\{ \sum_{j=1}^{n} \frac{\partial \phi^A}{\partial b}(A_j(b^0,0),b^0,\theta^A_j) \right\} \frac{dB^0}{da}\,\frac{da^e}{d\varepsilon} > 0.$$

Finally, assumption A4ii guarantees that the deviation is also profitable for the agent (m^A increases), and therefore the new contract is accepted.

■

Proof of Theorem 2:

Suppose P^B does not sign a contract and P^A signs a public contract at stage 0. Let μ_0^B and μ^B denote respectively the probability distribution of B's action in the equilibrium of G_0 and in the continuation equilibrium of G if P^A signs a public contract. Let also π^A and π_0^A denote P^A's profits in G after proposing a public contract and in G_0 respectively. One has:

$$\pi^A \leq \Pi^A(b^e,U^A(b^e,0)) + \int m^A(b,\theta_n^A)\, d\mu^B(b),$$

since the RHS is the maximum profit P^A can ever expect given μ^B.

Assume SS holds. Then $a^e \geq a^0$ and then μ_0^B dominates μ^B in the sense of first order stochastic dominance. Now under PE and using A4ii, this implies: $\pi^A \leq \pi_0^A$.

The proof under SC and NE, using A4ii, is similar. Hence the first part of the theorem.

Assume now SC and NE. Consider an equilibrium where both principals sign a contract at stage 0, with principals' payoffs (π^A,π^B) and average actions (a^e,b^e). From Proposition 1, $a^e \geq A^0(b^e)$ and $b^e \geq B^0(a^e)$; therefore $a^e \geq a^0$ and $b^e \geq b^0$. μ^B dominates μ_0^B and μ^A dominates μ_0^A in the sense of first order stochastic dominance. Therefore $\pi^A \leq \pi_0^A$ and $\pi^B \leq \pi_0^B$.

■

Proof of Theorem 3:

If $m=1$, P^B will always induce the full information optimal choice of actions $B^*(a)$; thus, P^B and B cannot sign any relevant public contract at stage 0. Under SS-NE and SC-PE, the proof of Theorem 1 can be replicated with $B^*(.)$ in stead of $B^0(.)$. Part i) follows. Under SS-PE and SC-NE, the proof of Theorem 2 clearly shows that P^A can obtain the maximum payoff by not signing any public contract. If P^A signs a public contract that leads to $a^e \neq a^0$, the inequalities in the proof of Theorem 2 are strict (whether SS-PE or SC-NE hold) and this would be a dominated move. Part ii) follows.

■

Proof of Theorem 4:

Consider the cases SS-NE and SC-PE. Suppose that $a^* = A^0(b^*)$ and $b^* \neq b^0$, and that hierarchies have signed public contracts leading to profiles of reservation control utilities $\mathbf{V}^A$ and $\mathbf{V}^B$ (the same proof is true if P^A and A have not signed a public contract). The proof of Theorem 1 can be applied to show that P^A would sign a public contract so as to increase A's action. One just has to replace a^0, b^0 by a^*, b^*, and $B^0(a)$ by $b(a,\mathbf{V}^B)$. R-stability prevents that a small deviation from $\mathbf{V}^A$ triggers a discontinuous jump to a new continuation equilibrium, since it implies the unicity of this equilibrium. The deviation is still profitable for agent A (and therefore accepted) since the type-j agent B's action moves toward the same direction as the agent B's expected action (this is where the monotonicity part of R-stability is required).

Consider now the case SC-NE. Suppose that $a^* > A^0(b^*)$. Let P^A decide not to sign a public contract. The new equilibrium is obtained at:

$$a = A^0(b) \quad \text{and} \quad b = b(a,\mathbf{V}^B).$$

All we have to show is that $b < b^*$. This is where R-stability is required. If $b \geq b^*$, then the curve $b(a,\mathbf{V}^B)$ should cross at some point the curve $A^0(b)$ from below and would then contradict the R-stability condition at $\mathbf{V}^A = 0$. Therefore $b < b^*$. P^A has thus reduced its incentive costs and reduced B's expected action, which altogether would increase P^A's payoff.

The proof is similar for the case SS-PE.

∎

Proof of proposition 6:

Let us recall first, from A1, (the proof of) Proposition 1 and the definition of $\bar{\mathbf{A}}(b)$, that:

$$\mathbf{A}(b,\mathbf{0}) < \mathbf{A}^*(b) < \bar{\mathbf{A}}(b)$$

and even more precisely,

for all $i > 1$, $A_i(b,0) < A_i^*(b)$ and $A_1(b,0) = A_1^*(b)$;

for all $i < n$, $\bar{A}_i(b) > A_i^*(b)$ and $\bar{A}_n(b) = A_n^*(b)$.

From Proposition 1, it also comes: $\mathbf{A}(b,\mathbf{0}) \le \mathbf{A}(b,\mathbf{V})$. Mimicking the proof of Proposition 1 with the induction argument going from n to 1 and deviations consisting of $da_i = -\varepsilon$ and, for $j > i$, $dU_j = \Delta_i^A\varepsilon$, it is straightforward to prove that: $\mathbf{A}(b,\mathbf{V}) \le \bar{\mathbf{A}}(b)$.

Part i): Consider a convex profile $\mathbf{V}$ satisfying the condition in the text. Consider also the relaxed program (P^u), deduced from (P') by deleting the downward adjacent incentive constraints:

$$\max_{(\mathbf{a},\mathbf{U})} \sum_{i=1}^{n} h_i^A\left(\phi^A(a_i,b,\theta_i^A) - U_i\right)$$

$$\text{such that} \quad U_i \ge V_i \ , \ i = 1, \ldots, n \qquad [\delta_i]$$

$$U_i - U_{i+1} \ge \Delta_i^A a_{i+1} \ , \ i = 1, \ldots, n-1 \qquad [\beta_i]$$

$$0 \le a_i \le \bar{a} \ , \ i = 1, \ldots, n \qquad [\eta_i], \ [\xi_i]$$

The first order necessary and sufficient conditions are:

$$a_1 = A_1^*(b),$$

$$h_i^A\phi_a^A(a_i,b,\theta_i^A) = \beta_{i-1}\Delta_{i-1}^A + \xi_i - \eta_i \ , \ i = 2, \ldots, n,$$

$$h_1^A = \beta_1 + \delta_1 \ ,$$

$$h_i^A = \beta_i - \beta_{i-1} + \delta_i \ , \ i = 2, \ldots, n-1,$$

$$h_n^A = \delta_n - \beta_{n-1},$$

$$\xi_i \ge 0; \ \xi_i > 0 \Rightarrow a_i = \bar{a} \ , \ i = 2, \ldots, n,$$

$$\eta_i \ge 0; \ \eta_i > 0 \Rightarrow a_i = 0 \ , \ i = 2, \ldots, n.$$

It is clear that for all i, $\xi_i = 0$, $a_i = 0$ or $a_i \le A_i^*(b)$. Assume that for some i, $a_i = A_i^*(b)$. Then $\beta_{i-1} = 0$ and $\delta_{i-1} = h_{i-1}^A + \beta_{i-2} > 0$. Therefore, $U_{i-1} = V_{i-1}$. But then,

$$U_{i-1} - U_i \le V_{i-1} - V_i < \Delta_{i-1}^A A_i^*(b) = \Delta_{i-1}^A a_i$$

which would violate the incentive constraint. Therefore, for all $i > 1$, $a_i < A_i^*(b)$ and for all i, $\beta_i > 0$ and then $U_i - U_{i+1} = \Delta_i^A a_{i+1}$.

Finally let us prove that the solution to (P^u) is solution to (P'). Since the upward incentive constraints are all binding, it suffices to prove that for all i, $a_i \ge a_{i+1}$. Suppose that for some i, $a_i < a_{i+1}$.

Claim: $U_i > V_i$.

Proof of the Claim: Since **V** is convex, it satisfies (6), or, rearranging:

$$\Delta_i^A V_{i-1} + \Delta_{i-1}^A V_{i+1} \geq (\Delta_i^A + \Delta_{i-1}^A)\, V_i$$

Using the individual rationality constraints, it comes:

$$\Delta_i^A U_{i-1} + \Delta_{i-1}^A U_{i+1} \geq (\Delta_i^A + \Delta_{i-1}^A)\, V_i$$

Replacing the expressions of U_{i-1} and U_{i+1} obtained by the upward incentive constraints, one finds:

$$(\Delta_i^A + \Delta_{i-1}^A)\, U_i + \Delta_i^A \Delta_{i-1}^A (a_i - a_{i+1}) \geq (\Delta_i^A + \Delta_{i-1}^A)\, V_i$$

and therefore, $U_i > V_i$.

□

The claim implies that $\beta_i = h_i + \beta_{i-1}$. Now, using A3ii, the concavity of ϕ^A in a, and the first order conditions of (P^u), it comes:

$$\frac{\beta_{i-1}\Delta_{i-1}^A}{h_i} \geq \phi_a^A(a_i, b, \theta_i^A) > \phi_a^A(a_i, b, \theta_{i+1}^A) > \phi_a^A(a_{i+1}, b, \theta_{i+1}^A) = \frac{\beta_i \Delta_i^A}{h_{i+1}}$$

Using A3iii, and rearranging, this implies:

$$H_i \beta_{i-1} > H_{i-1} \beta_i$$

and therefore: $\beta_{i-1} > H_{i-1}$, which is impossible, since the first order condition can be rewritten as: $H_{i-1} = \beta_{i-1} + \sum_{j=1}^{i-1} \delta_j$. Therefore, $a_i \geq a_{i+1}$, and the solution to (P^u) is the solution to (P'), i.e. is **A**$(b,$**V**$)$.

Part ii): This proof exactly mimics the proof of part i) of the proposition; so we only sketch the main steps. One considers the program (P^d), deduced from (P') by deleting the upward incentive constraints; the solution is proved to be such that: for all $i < n$, $a_i > A_i^*(b)$. Then, using A3iv, this solution is proved to satisfy also the upward incentive constraints, and is therefore equal to **A**$(b,$**V**$)$.

■