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ABSTRACT

In our model, two profit-maximizing sellers sell a homogeneous good to Bayesian, risk-neutral buyers in an online comparison shopping service. Buyers use a reputation system to update their beliefs about sellers. Buyers purchase from the seller that maximizes the buyer's expected utility from the purchase. We find that the seller's profit depends on the distribution of buyer beliefs. A degenerate distribution of beliefs implies either Bertrand competition or a monopolistic market. A non-degenerate distribution implies that both sellers can be profitable, if their reputations differ from each other. The seller with a higher reputation score receives a greater profit. If sellers are similar in every respect, the Bertrand equilibrium obtains. We test the theory with data from Pricegrabber using OLS and quantile regression. Controlling for different seller types, the evidence indicates that higher reputation scores may support price premiums.

Keywords: electronic commerce, comparison shopping, competition, reputation systems

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1. Introduction

The ease of comparing prices and product offerings increases efficiency in electronic markets. Designed for this purpose, comparison shopping services are electronic marketplaces that lower buyer search costs by gathering and distributing information about sellers[1]. The buyer that wishes to purchase some product uses the search engine of a comparison shopping website to receive a list of price quotes from the sellers that offer the desired product. As a result, the buyer can choose the most preferable offer from the available sellers in terms of price, delivery, payment and the seller's quality.

In the economic literature, comparison shopping services relate to information clearinghouse models. In Varian (1980), a fraction of buyers use the information clearinghouse, such as a newspaper, to locate the seller who sets the lowest price, whereas other buyers are evenly distributed among all sellers. As a result, buyer heterogeneity produces price dispersion. Baye and Morgan (2001) take information clearinghouse models to electronic markets. They suggest that an optimizing monopolistic operator of a comparison shopping service sets its fees for sellers high enough to induce some sellers to stay out of the service. In contrast, the fees for buyers are low enough to encourage full participation. In consequence, the prices are lower in the comparison shopping service than in the outside market, which encourages buyers to use the service. From the buyer's perspective, comparison shopping markets may provide considerable increase in consumer surplus because it mitigates the buyer's information costs and spurs competition among sellers. From the seller's perspective, they could lead to cut-throat price competition, because there is little room for product differentiation and free entry erases supernormal profits.

Despite the challenging market environment, empirical evidence shows that well-known e-commerce giants as well as less-known small firms participate in comparison shopping markets (Saastamoinen, 2008). Since the operators of comparison shopping websites

[1]These services are also known as a price comparison service or a price engine. See more details in http://en.wikipedia.org/wiki/Price_comparison_service.

often charge fees from the participating vendors, benefits from participation must exceed its costs for sellers. For a small firm, a comparison shopping market could bring visibility at low costs (Wan, 2006). Visibility is vital because buyers are aware of only a fraction of sellers in the market (Grover et al., 2006). To attract unaware buyers to their online stores, sellers have to advertise or organize promotional alliances with search engines (Latcovich & Howard, 2001; Filson, 2004). On the other hand, a firm must pursue an aggressive pricing strategy which restrains profitability. While incentives to participate in comparison shopping services are not obvious, some benefits from participation may exist. First, firms may organize periodical sales or inventory clear outs and occasionally win the bidding contest as suggested by Varian (1980). Second, as more buyers learn to use the search mechanisms of the Internet for commercial purposes, it is harder to maintain prices above the competitive level. Third, it gives an opportunity to monitor prices or issue a commitment to certain price level. Smith (2001) entertains a possibility that dominant sellers could use a comparison shopping service in collusion to maintain higher prices.

Price alone cannot explain competition in comparison shopping markets because the problems of asymmetric information and moral hazard are inherent in e-markets. The lack of direct contact between buyers and sellers raises concerns about opportunistic fraudulent behavior (Friedman et al. 2000). In markets of perfect and complete information, every action an agent takes and the agent's action history is observable to other agents rendering reputation irrelevant in such markets. Asymmetric information creates incentives to reputation building. Cabral (2005) defines reputation as "the situation when agents believe a particular agent to be something." This belief may be crucial for commercial transactions to take place. For this reason, seller reputations may play a large role in competition. To address this problem, many e-commerce marketplaces have introduced reputation systems which gather and distribute aggregated information from buyers about the past behavior of sellers (Resnick et al., 2000).

As the online business environment cultivates concerns over the trustworthiness of a trading partner, this may impede market entry because buyers trust the established firms more than newcomers. Economic benefits of reputation building may explain the proliferation of reputation systems. First, reputation can be viewed as an asset. In Klein and Leffler (1981) and Shapiro (1983), a firm invests in reputation by selling high quality products at loss initially but earning a price premium on the established reputation later. To be qualified as an asset implies that established reputations can be bought. As a result, Mailath and Samuelson (2001) show that a reputation may not be a good signal of quality because incompetent firms buy good reputations. Second, Klein and Leffler (1981) suggest that consumers view reputation as a protection for contractual obligation. A price premium from reputation induces a firm to maintain good quality because the profit stream from good quality products exceeds the gains from cheating. Hörner (2002) argues that this does not provide sufficient incentives to maintain good quality. Instead, competition provides such incentives by creating an outside option to buyers who can patronize the seller's rival, if they detect cheating on behalf of the seller.

The comparison shopping services with reputation systems may provide simultaneously a low-cost entry point to the market as well as insulation from price competition. Zhou (et al. 2008) present a model for online markets, in which they show that a reputation system can reduce asymmetric information in an online market and replicate the results of Shapiro (1983). An efficient reputation system provides incentives to fulfill contractual obligations. There must also be incentives to participate and report truthful feedback through the reputation system. Bakos and Dellarocas (2003) show that an online reputation system can be more efficient in enforcing desired behavior than a threat of litigation process.

In this paper, we present a theoretical model of interactions between buyers and sellers in a comparison shopping service with an integrated reputation system, and derive implications for competition. In addition, we test the model with empirical data from Pricegrabber, which is a popular online comparison shopping service. The paper proceeds as follows. In the second section, we present the theoretical model. In the third section, we test the model with least-squares regression (OLS) and quantile regression (QR). In the final section, we conclude the paper.

2. Model of Competition in Online Comparison Shopping Services

2.1 Buyers

Consider an electronic marketplace for a homogeneous good. The marketplace is a comparison shopping service with an integrated reputation system. A comparison shopping service is an electronic marketplace, where buyers receive simultaneously a list of price quotes for the desired product from all the sellers that participate in the comparison shopping service[2]. We assume that the use of the comparison shopping service is costless to buyers and sellers. A reputation system gathers and distributes information about a seller's past behavior. The buyers that have transacted with the seller report their experience through a feedback mechanism. After this, aggregated buyer feedback is made publicly visible. This feedback profile forms the seller's reputation in the marketplace. The reputation system of the comparison shopping service exists as long as the market exists, and all buyers elicit feedback after completed transactions.

The overall market for the good consists of a mass of buyers normalized to one. Buyers enter the market in cohorts, one cohort in each period of time. A cohort buys at one seller or multiple sellers in each period $t,\ t = 0,\dots,N$. Repeated purchases are possible. Buyers are utility maximizers that are concerned about the price of the good and the quality of service (Smith and Brynjolfsson, 2001). A fundamental distinction between types of goods was proposed by Nelson (1970) who categorizes them into search goods and experience goods. Price and/or quality comparisons precede consumption of search goods, whereas experience goods have to be consumed before their quality can be ascertained. As delivery often places a significant lag between purchase and consumption of a good, the entire transaction process could be considered as a good that has the characteristics of both good types. Comparison shopping services provide easy access to price information. However, the uncertainty over the overall purchase experience raises concerns about the seller's trustworthiness.

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[2] Few examples of such websites include www.bizrate.com, www.pricegrabber.com and shopping.yahoo.com.

The quality of a transaction with the seller is discernible to the buyer after the transaction has been concluded. For simplicity, the buyer rates the transaction as a success or a failure. Therefore, the reputation system is similar to the binary system presented in Dellarocas (2004). In consequence, the feedback takes two values: "good" (*G*) for a successful transaction and "bad" (*B*) for a failure[3]. As the *t*th buyer elicits feedback on the seller, the reputation system updates the seller's feedback profile by adding $G_t = G_{t-1} + 1$ if the feedback is good, or by adding $B_t = B_{t-1} + 1$ if the feedback is bad. The initial values before are $B_0 = G_0 = 0$. Hence, a seller's reputation in period *t* is the likelihood that a seller is good, which is given by the ratio

$$PR(Good)_t = \frac{G_t}{G_t + B_t} = \gamma_t \, . \tag{1}$$

Consequently, the likelihood that the seller is bad is $PR(Bad)_t = 1 - PR(Good)_t = 1 - \gamma_t$. In any given period, a seller's public feedback profile, which is visible to all subsequent buyers and rival sellers, shows the likelihood that the seller is good (γ_t) and the number of reviews the seller has received (*t*). The public feedback profile is a crude measure for a seller's reputation when *t* is small, but its precision increases as a more feedback is being accumulated. A consistent feedback profile could provide the same proof as repeat purchases to quality-conscious on the seller's commitment to maintain high quality service (Rao and Bergen, 1992). While switching one's identity easy on the Internet, a large value of *t* signals the seller's commitment to stay in the market under the same guise. As reputation building is a gradual, time-consuming process, a long market history implies greater costs of an identity switch to the seller.

If the seller's type is unknown to the buyer before a transaction, the buyer must assess the seller's trustworthiness from the available information. Each buyer has a private signal $\theta \in [0,1]$ (prior probability) on the seller's type. The seller's reputation can be interpreted

[3] A candidate parameterized distribution for estimation would be the Beta-distribution. Equation (1) coincides with the expected value of Beta trials.

as a buyer's belief of the seller's true type (Cabral, 2005). It is easy to imagine numerous factors that could contribute to θ. For example, previous transactions with the seller could completely override the public information (Smith and Brynjolfsson, 2001). The buyer assigns $\theta = 1$ (extremely favorable) or $\theta = 0$ (extremely unfavorable) depending on her previous experience[4]. New buyers may have lower values for θ in general, whereas experienced buyers are more trustful on sellers (or vice versa). Allowing for herd behavior, the buyer could also take cue from her immediate predecessor's opinion by setting a low or high value for θ to conform to the predecessors' reviews. In addition, a price may signal the seller's type (Doyle, 1990; Tirole, 1994). For now, we only assume that θ is distributed according to some distribution with density $f(\theta)$. Priors for all sellers are drawn from this distribution.

Infrequent purchases and a constant influx of new buyers into the market make the forming of seller reputations (Tirole, 1994). For this reason, an important piece of information is the public feedback profile provided by the previous buyers. This is an electronic counterpart to the word-of-mouth in the physical world (Resnick et al., 2000). We assume that the only communication mechanism between buyers is a reputation system, so other buyers' private signals are only observable through their feedback. Moreover, a seller cannot be a buyer which prevents manipulation of sellers' reputations.

Buyers use the Bayes' rule to obtain the posterior probability $\mu(\theta, \gamma)$ for the seller's trustworthiness. The posterior probability obtained by Bayesian updating can be interpreted as the seller's reputation (Cabral, 2005). The reputation system provides the public feedback profile ($\gamma_{i,t}$) of Seller *i*, which is the evidence-based likelihood that the particular seller is good, for any period *t*. The buyer's prior probability that Seller *i* is good in period *t* is $\theta_{i,t}$. After the transaction is concluded, the buyer elicits feedback through the reputation system, which updates the seller's feedback profile. All

[4] Notice that the absolute certainty on the seller's type does not have any larger impact on subsequent buyers than uncertainty because only concluded transactions are registered. For this reason, only $\theta = 1$ is indirectly observable to other buyers, whereas $\theta = 0$ leads to the buyer abandoning the seller.

subsequent buyers benefit from the feedback given by their predecessors. In general, a buyer updates the posterior probability $\mu_{i,t}$ by

$$\mu_{i,t} = \frac{\theta_{i,t}\gamma_t}{\theta_{i,t}\gamma_t + (1-\theta_{i,t})(1-\gamma_t)} . \tag{2}$$

Notice that if no transaction takes place, the next buyer has only the first buyer's feedback at disposal. For this reason, the length of a ratings history may also provide important information about the seller's quality.

Suppose that buyers in the market are risk-neutral. They have identical valuations (w) for the homogeneous good. Let k denote the cost of an unsuccessful transaction, and $w > k$. Given the posterior probability the buyer's expected value V of the good is

$$V = \mu \cdot w + (1-\mu)(-k) . \tag{3}$$

Equation (3) can be simplified by dividing it with w, so the value of the good to the buyer is 1. To simplify the analysis, assume that $k = 0$. This could be interpreted as the third party, such as a credit card company, bearing the cost of misdemeanor, or the good being low in value. Let $v = \frac{V}{w}$. As a result, Equation (3) simplifies to

$$v = \mu . \tag{4}$$

The value of the good to the buyer depends only on the buyer's posterior probability that the seller's type is good. This is the buyer's reservation price for the good that is purchased from the specific seller.

Buyers seek to maximize their (expected) utility (u)

$$u = \mu - p , \tag{5}$$

where $p \in [0,1]$ is the set of prices. The utility is increasing in μ and decreasing in *p*. Clearly, transactions take place only if $\mu \geq p$. Moreover, Equation (5) implies that risk-neutral buyers buy from the seller that guarantees them the highest (expected) surplus.

Since μ implies that buyers also care about the level of service, we assume that there exists a price that buyers consider too low for a seller to provide sufficient service. We assume that this price is common knowledge. This sets a lower bound to the set of possible prices. Any price below the lower bound signals with certainty that the seller's type is bad. Klein and Leffler (1981) show that consumers can use price to judge the quality of a firm's products. Their model suggests that consumers are able to distinguish the situations in which the price is too low to produce quality products.

2.2 Sellers

Sellers are retailers in a vertical market structure. The good is produced by an upstream manufacturer. The upstream market is competitive and thus, the manufacturer's price p_w (the wholesale price) equals to the manufacturer's marginal cost. Sellers maximize their profit in the downstream market. There is no vertical integration between the upstream manufacturer and downstream retailers. Since the upstream market is competitive, the linear pricing contract in the vertical market structure is admissible and sellers take p_w as given (Tirole, 1994).

Sellers face two strategy variables. Strategy variables are choice variables that affect a seller's rivals profits or the payoffs accruing to buyers, or both (Doyle, 1990). First, the seller selects a retail price *p*. The choice is effectively constrained by the monopolist from below and the buyers' maximum willingness to pay ($\overline{\mu}$) from above. Thus, $p \in [p_w, \overline{\mu}]$. Second, the seller chooses a level of effort denoted by e after the buyer makes a purchase. For example, effort could be understood as effective customer service, measures that secure confidentiality in an electronic transaction, fast delivery, and so forth. Effort enters as a cost per unit sold in the seller's profit function. Let e be a continuous choice on $[0,\infty)$. The seller's profit function is then

$$\pi(p,e) = \lambda[p - p_w - e], \tag{6}$$

where $\lambda \in [0,1]$ is the fraction of buyers the seller receives (we define this measure later). There is a mass of buyers normalized to one in the market in each period. This assumption allows us to split the market between sellers if a buyer cohort receives an equal surplus from both of them.

The level of effort has an indirect impact on a seller's reputation. The probability of a successful transaction is increasing in the level of effort. For this reason, even good sellers occasionally disappoint buyers, but the seller's reputation score is a good approximation of the level of effort the seller has chosen in the past. By taking high effort, the seller increases the probability that the buyer has a positive experience with the seller and the resulting feedback is positive.

2.3 Market

For simplicity, Suppose that there are two sellers in the marketplace, Seller *h* and Seller *l*. In any period, a competitive price cannot be equal to the wholesale price $p = p_w$. If a seller sets its price equal to the wholesale price, it signals to the buyer that the seller chooses zero effort. The buyer concludes that the seller must be a bad seller and assigns $\theta = 0$. Since this leads to $v = \mu = 0$, the only possible price that a transaction could take place is $p = 0$. But this means that $0 = p < p_w$ and the seller loses money. Hence, the seller is always better off with $p > p_w$ and $p = p_w$ cannot be a Nash-equilibrium strategy. In consequence, the seller that chooses no effort has always an incentive to mimic the seller that selects a positive level of effort. Thus, a bad type can be signaled with certainty but a good type cannot.[5]

Consider now a one-period game when two sellers are identical in every respect. As a result, this is simply a repetition of the Bertrand equilibrium, in which sellers undercut each other by ε until the market price equals the marginal cost. The market equilibrium obtains at the competitive price

$$\underline{p} \equiv p_w - \underline{e} \tag{7}$$

where $\underline{e}$ is the minimum level of effort that enables a successful fulfillment of the transaction. This price yields the normal profit that includes the opportunity cost forgone in an alternative investment. Hence any price above $\underline{p}$ is a dominated strategy. However, any price below $\underline{p}$ is also a dominated strategy because it signals zero effort to buyers. Thus, the seller who sets $e = 0$ is always better off by setting $p \geq \underline{p}$. If the seller chooses $e > 0$, its expected profit is $\pi(\underline{p}, e) = \frac{1}{2}[\underline{p} - p_w - e] = 0$. If $e = 0$ is selected, this yields the profit $\pi(\underline{p}, 0) = \frac{1}{2}[\underline{p} - p_w] > 0$. The dominant strategy in the one-period game is to select $(\underline{p}, 0)$ which maximizes the seller's profit. However, rational buyers could expect this and conclude that the seller is bad and assign $\theta = 0$. As a result, the market unravels due to asymmetric information in the one-period game.

Obviously, a market exists as long as buyers believe that good sellers that are committed to stay in the market with some positive probability exist. A multi-period game requires a device that signals the seller's commitment to quality to buyers. Dellarocas (2004) shows that a binary reputation system provides sufficient incentives for a seller to maintain good quality. In his model, cooperating sellers and cheating sellers both produce good quality and bad quality with positive probabilities. As a result, even good sellers, though not as

[5] A duopoly model can be easily expanded to comprise more sellers.

frequently as bad sellers, produce occasionally bad quality which is reported to buyers by the reputation system. Corresponding to their reputation profiles, sellers adjust their prices to maximize profit. We assume that sellers find it worthwhile to induce effort, which is reflected by their reputation profiles.

The distribution of buyer beliefs is crucial in determining seller profits. A degenerate distribution means that all buyer priors satisfy $\theta_i = \theta \in [0,1] \forall i$, which implies homogeneous buyer population. This has stark consequences on market structure. First, suppose that $\gamma_h > \gamma_l$. Then Seller *h* wins the price competition in every period. Since any $p < \underline{p}$ signals that the seller chooses zero effort, this is clearly a dominated strategy. Hence, Seller *h* can always offer a higher surplus to every buyer. Moreover, it can gradually increase the price such that $p \to 1$ without losing customers because of increasing buyer satisfaction. This gives a strong incentive to sustain $e > 0$ because cheating, if detected, undermines the seller's pricing power and lowers the future profits. Second, if the sellers are identical, which occurs when $\gamma_h = \gamma_l = \gamma$, the Bertrand equilibrium obtains. This could be a stable equilibrium. Suppose that a seller is tempted to select $e = 0$, "to cheat", because this could result in a short run profit provided that buyers do not detect cheating. Still, the seller cannot charge a higher price after a successful deviation because it shares γ with its rival. However, the probability that buyers detect cheating increases with $e = 0$. If the seller is caught cheating and the rival does not cheat, this gives the upper hand to the competitor with every period forward because its reputation score is higher. As a consequence, the cheating seller can be driven out of the market.

A non-degenerate distribution of private signals implies heterogeneity in buyer beliefs. Thus, some buyers are extremely pessimistic on sellers while others display extreme optimism. Assume that the distribution of buyer beliefs is independently distributed on $[\underline{\underline{\theta}}, \overline{\overline{\theta}}]$. Let $F(\theta)$ denote the proportion of buyers with prior beliefs of θ or less about the seller's type (since $F(\theta)$ is non-degenerate, there are at least two types of prior beliefs).

Buyer heterogeneity influences a seller's profitability. If the sellers are identical, that is $\gamma_h = \gamma_l = \gamma$, the usual argument of epsilon price cutting drives the price down to $\underline{p}$. Buyers with beliefs of $\underline{\theta} \in [\underline{\underline{\theta}}, \overline{\overline{\theta}}]$ or greater which satisfy $\mu(\underline{\theta}, \gamma) - \underline{p} = 0$ buy the good from either seller. As a result, the Bertrand equilibrium obtains in each period regardless of the buyer distribution. The sellers split the market and each seller earns zero profit.

Is there an incentive to choose zero effort in this setting? Obviously, selecting "no effort" yields $\pi(\underline{p}, 0) = \frac{1}{2}\int_{\underline{\theta}}^{\overline{\overline{\theta}}} [\underline{p} - p_w] dF(\theta) > 0$ at least once. If cheating is detected, the seller's rating score is lower in the next period (this occurs also if positive effort has resulted in a bad rating). Surprisingly, this may increase profits for *both* sellers. Suppose that $\gamma_{h,t} > \gamma_{l,t}$ in period *t*. Then $\mu_{h,t} > \mu_{l,t}$ for all $\theta \in [\underline{\underline{\theta}}, \overline{\overline{\theta}}]$ when $p_h = p_l = \underline{p}$ which means that all buyers place a higher value for Seller *h*'s offer. This gives an opportunity to Seller *h* to increase its price above $\underline{p}$ and make profit. Any $\mu_{h,t}(\underline{\theta}, \gamma_{h,t}) \leq p \leq \mu_{h,t}(\overline{\overline{\theta}}, \gamma_{h,t})$ dominates $\underline{p}$ because it yields higher profit than playing the Bertrand equilibrium. It is obvious that Seller *h* could force Seller *l* out of the market and still make profit by choosing the price p_h such that the consumer surplus when buyers buy from Seller *h*

$$\int_{\underline{\underline{\theta}}}^{\overline{\overline{\theta}}} [\mu_{h,t} - p_h] dF(\theta) > \int_{\underline{\underline{\theta}}}^{\overline{\overline{\theta}}} [\mu_{l,t} - \underline{p}] dF(\theta) . \qquad (8)$$

However, this choice may not maximize the profit of Seller *h*. Instead, an interval $[\underline{\theta}, \theta^*]$ which results in

$$\int_{\theta^*}^{\overline{\overline{\theta}}} [\mu_{l,t} - p_l^*] dF(\theta) > \int_{\underline{\theta}}^{\theta^*} [\mu_{h,t} - p_h^*] dF(\theta) \geq 0 , \qquad (9)$$

where $p_h^* > p_l^* \geq \underline{p}$ are the prices that solve the profit maximization problems, may yield higher profits for both sellers. Equation (9) states that Seller *h* may ignore the most optimistic buyers with $\theta > \theta^*$ and set prices high enough to sell the good to the buyers whose beliefs are drawn from $[\underline{\theta}, \theta^*]$. In essence, Seller *h* extracts surplus from the more distrustful buyers. As buyers maximize surplus, the most optimistic buyers purchase from Seller *l* which has to sell the good at a strictly lower price than Seller *h*.

Since sellers know each other's reputation profiles and the updating mechanism of buyers, they can compute the lower bound for priors that yield a non-negative surplus to buyers. For example, with a choice of price p_h and the given reputation profile $\gamma_{h,t}$, the lower bound for the priors that yield non-negative surplus to buyers is

$$\underline{\theta}_h(p_h) = \frac{(1-\gamma_{h,t})p_h}{\gamma_{h,t} + p_h(1-2\gamma_{h,t})}. \tag{10}$$

We denote the absolute lower bound by $\underline{\theta}(\underline{p})$ which is the prior that induces a buyer to buy with the lowest possible price. The lower bound is increasing in p and decreasing in γ. In consequence, the minimum profit of Seller *h* is on the interval $[\underline{\theta}_h, \underline{\theta}_l]$ because Seller *l* cannot set any lower price that could expand its market. Thus, by raising the price a seller loses customers among the more pessimistic buyers but this increases revenue from the more optimistic buyers. Lowering the price attracts more customers from both ends of the buyer distribution but decreases the overall revenue per customer. As a result, the seller selects p^* that maximizes its profit. This may preclude some pessimistic buyers from the market because the decrease in the revenue per customer may more than offset the additional revenue from the increased market size.

The upper bound for Seller *h* is obtained by solving for θ in equation

$$\mu_h(\theta, \gamma_{h,t}) - p_h = \mu_l(\theta, \gamma_{l,t}) - p_l \tag{11}$$

which sets the surpluses equal between the two sellers. We assume that the seller with a better reputation receives buyers that are indifferent between the two sellers. Let $\overline{\theta}(p_h, p_l, \gamma_{h,t}, \gamma_{l,t})$ denote the solution to this problem[6]. By implicit differentiation of Equation (11), we notice that

$$\begin{aligned} \frac{d\overline{\theta}}{dp_h} &= \frac{1}{\frac{\partial \mu_h}{\partial \overline{\theta}} - \frac{\partial \mu_l}{\partial \overline{\theta}}} > 0 \\ \frac{d\overline{\theta}}{dp_l} &= -\frac{1}{\frac{\partial \mu_h}{\partial \overline{\theta}} - \frac{\partial \mu_l}{\partial \overline{\theta}}} < 0 \end{aligned} \tag{12}$$

because a marginal increase in μ_h is greater than in μ_l when θ increases[7]. Thus, the upper bound for Seller *h* increases if Seller *h* lowers its price and decreases if Seller *l* lowers its price. Also, let $\overline{\overline{\theta}}$ denote the absolute upper limit of the distribution of buyer priors.

In each period, Seller *h* sets the price $p_h^*(p_h \mid p_l = p_l^*)$ which maximizes its profit $\pi_h^*(p_h^*, e)$ given that Seller *l* sets the price that maximizes its profit. Seller *l* optimizes its profit in a similar manner. Seller *h* earns a price premium of $p_h^* = p_l^* + (\mu_h - \mu_l)$, in which $(\mu_h - \mu_l)$ is Seller *h*'s return on reputation. Thus, the profit of Seller *h* is

$$\int_{\underline{\theta}^*}^{\overline{\theta}^*} (p_h^* - p_w - e)dF(\theta), \tag{13}$$

where $\underline{\theta}^* = \theta(p_h^*, \gamma_{k,t})$ and $\overline{\theta}^* = \theta(p_h^*, p_l^*, \gamma_{h,t}, \gamma_{l,t})$. As a result, the buyer with a private signal θ receives surplus from the sellers according to

[6] The solution for this problem is available from the author upon request.
[7] See Appendix.

$$v_h - v_l \begin{cases} > 0, & \text{if } \theta > \underline{\theta}^* \\ = 0, & \text{if } \theta = \underline{\theta}^* \\ < 0, & \text{if } \theta < \underline{\theta}^* \end{cases} \quad \text{and } \theta \in [\underline{\theta}, \overline{\overline{\theta}}]. \tag{14}$$

The profit of Seller *l* is

$$\int_{\overline{\theta}^*}^{\overline{\overline{\theta}}} (p_l^* - p_w - e) dF(\theta), \tag{15}$$

and

$$\int_{\underline{\theta}^*}^{\overline{\theta}^*} (p_h^* - p_w - e) dF(\theta) \geq \int_{\overline{\theta}^*}^{\overline{\overline{\theta}}} (p_l^* - p_w - e) dF(\theta). \tag{16}$$

The optimal prices depend on the distribution of buyer beliefs and the seller's reputation. These support various market structures. A degenerate buyer distribution leads into a monopolistic market in which Seller *h* takes over the entire market extracting all consumer surplus. A non-degenerate buyer distribution may produce price dispersion. Seller *h* maximizes its profit from the buyer population whose priors are drawn from the interval $[\underline{\theta}^*, \overline{\theta}^*]$. Seller *l* receives the demand from the more optimistic buyers whose prior beliefs are drawn from the interval $(\overline{\theta}^*, \overline{\overline{\theta}}]$. Notice that if $\overline{\theta}^* = \overline{\overline{\theta}}$, Seller *h* takes over the entire market. Seller *h*'s profit is at least as high as Seller *l*'s profit, because Seller *h* can always undercut Seller *l* by a small amount and make profit. Profit-maximizing may also dictate that a part of the market, $[\underline{\underline{\theta}}, \underline{\theta}^*)$, may not be serviced because the additional revenue from the lower segment of the market does not offset the loss of income in the upper segment.

Since reputation scores are public knowledge, both can use Equation (2) to compute reservation price paths for the known or expected distribution of buyer beliefs. Using Equation (5), sellers can experiment with prices that maximize their profits. If buyers place value on seller reputations, both firms may be able to sell their products in the market with supernormal profit and without collusion. Consequently, price dispersion may result from this because reputation provides pricing power. Since even good sellers receive bad reviews occasionally, prices may fluctuate as sellers adjust their prices to maximize profits in their reputation profiles.

The model explicitly shows that the distribution of buyers' private signals impacts sellers' profits. One could conjecture that new buyers might have lower priors which benefits more reputable sellers. As long as e-markets grow in size, which means that the share of new buyers in the market is steady or increasing, maintaining a good reputation is a profitable strategy. This offers rationale for well-known vendors to have presence in highly competitive comparison shopping services because their existing reputations may provide opportunities for premium pricing. However, a shift in distribution towards higher priors, which could happen when buyers become more experienced, may diminish the value of a good reputation in favor of more aggressive price competition. Moreover, this model offers an explanation for the observed price dispersion in online markets, which has been a finding in numerous studies (e.g. Brynjolfsson and Bakos (2000); Ancarani and Shankar (2004); Leiter and Warin (2007)).

3. Empirical Analysis

We test the theory of competition with reputations in a comparison shopping service with the data from two-seller markets listed in Pricegrabber, which is one of the most popular comparison shopping websites[8]. We use a portion of the data that was analyzed in Saastamoinen (2008). The sample data was obtained from Pricegrabber in May 2008. It consists of prices for various goods ranging from consumer electronics to auto parts. Pricegrabber has a reputation system which provides rating scores for each seller. A

[8] For more details about the website, see www.pricegrabber.com.

rating score, which ranges from 1 (the lowest) to 5 (the highest), is aggregated from buyer feedback. We approximate seller reputations with the rating scores.

To test the theory, it is important to control for different seller types. One way is to use the two seller packages offered by Pricegrabber as a controlling device. A merchant runs its own e-commerce websites and pays a click-through rate to Pricegrabber for buyers that are redirected to the merchant's website by Pricegrabber. A storefront pays a commission to Pricegrabber for each commercial transaction, but it does not run an own e-commerce website. Consequently, storefronts rely on the comparison shopping service as their only sales channel, while merchants use the service to lure in price-conscious buyers. Small sellers are likely to select the storefront package, whereas other sellers opt for the merchant package. The dummy variable *SF* denotes storefronts. In addition, we use the Internet Retailer's list of the largest e-commerce retailers to control for the largest sellers[9]. These are large companies whose brands may provide them some insulation from price competition in e-markets, because buyers view brands as a proxy for reliability (Smith, 2002). The dummy variable *TOP500* denotes large e-commerce vendors.

As the sample consist of two-seller markets, we calculate the difference in prices as

$$PDIF_k = p_k^{\max} - p_k^{\min}, \tag{17}$$

in which $p_k^{\max}$ ($p_k^{\min}$) is the maximum (minimum) price observed in the market *k*. Since $p_k^{\max} \geq p_k^{\min}$, it follows that $PDIF_k \geq 0$.

Due to diversity of product categories in the sample, product values vary considerably. For this reason, the pecuniary value of price differences may naturally be greater in expensive products than in relatively low priced products. To make price differences

[9] See www.internetretailer.com.

more comparable, we take a logarithmic transformation of $PDIF_k$. The logarithmic transformation must be defined as

$$LPDIF_k = Log(PDIF_k + 1) \tag{18}$$

because the difference between prices can be zero. Obviously, also $LPDIF_k \geq 0$.

We calculate also a difference between reputation scores. This is

$$RDIF_k = r_k^{\max} - r_k^{\min}, \tag{19}$$

in which $r_k^{\max}$ ($r_k^{\min}$) is the reputation score of the seller that sets the maximum (minimum) price observed in the market *k*. Taking a logarithmic transformation also from $RDIF_k$ provides a straightforward interpretation of regression coefficients as elasticities. The logarithmic transformation of requires scaling of $RDIF_k$. This is done by

$$LRDIF_k = Log(RDIF_k + |RDIF^{\min}| + 1) \tag{20}$$

where $|RDIF^{\min}|$ is the absolute value of the minimum of $RDIF_k$ in all *k* markets.

Descriptive statistics of the sample are shown in Table 1. They indicate that the mean (median) of price differences is greater in markets where storefronts and Top500-sellers operate, whereas the range of price differences is greater in all markets. In contrast, the mean (median) of differences in rating scores is higher in all the sample than in either control group. Storefronts and Top500-sellers did not overlap each other in this sample. Altogether, the control groups account for 15 per cent of the markets.

As a test hypothesis for regression analysis, we expect a positive relationship between $LPDIF_k$ and $LRDIF_k$. Higher prices should correlate with higher reputation scores. This correlation might emerge in the markets where storefronts are active because unlike other sellers, storefronts have a limited access or no access to the markets outside the comparison shopping service. As a consequence, storefront sales are mode dependent on the reputation-price tradeoff.

Table 1. Descriptive Statistics.

Statistic	***LPDIF***	***SF* LPDIF***	***TOP500* LPDIF***	***LRDIF***	***SF* LRDIF***	***TOP500* LRDIF***
Mean	0.913	2.352	1.675	1.625	0.102	1.506
Median	0.000	2.321	1.295	1.758	0.000	1.609
Maximum	8.355	6.512	7.963	2.169	2.158	2.015
Minimum	0.000	0.000	0.000	0.000	0.000	0.000
Std. Dev.	1.448	1.555	1.733	0.256	0.395	0.325
Obs.	1429	91	128	1429	91	128

We devise a log-linear regression model to test the theory. A log-linear functional form provides a convenient interpretation for estimates: a one unit increase in the difference between rating scores increases the difference between prices by $\hat{\beta} \cdot 100$ per cent. The regression equation is

$$\begin{aligned} LPDIF_k = C_k + SF_k + TOP500_k + \beta_1 RDIF_k + \beta_2 SF \cdot LRDIF_k \\ + \beta_3 TOP500_k \cdot RDIF_k + \varepsilon_k \end{aligned}. \qquad (21)$$

The results from OLS-regression with White Heteroskedasticity Consistent Estimates (HSCE) are reported in Table 2. Excluding the dummy variables for the controlled groups, all estimates prove statistically significant. OLS estimates for the regression constant and dummy constants indicate that the difference between prices is higher in markets where storefronts and Top500-sellers are active than in all markets. The impact of an increase in the difference between rating scores varies. The general effect is negative (-0.815). This implies that a one per cent increase in the difference between rating scores decreases the difference between prices by -0.8 per cent. In contrast, the control groups display a positive dependency. Together with $\hat{\beta}_1$**,** the estimates for storefronts (1.931) correspond to 1.1 per cent increase in the price difference. $\hat{\beta}_1$ overshadows the positive coefficient of Top500-sellers (0.655) yielding a mild decrease of -0.2 per cent when the difference between rating scores increases by a one per cent. Hence, the evidence suggests that reputation has an impact on a seller's pricing especially in the markets where storefronts are active.

Table 2. OLS Estimates for $LPDIF_k$ with HSCE.

Coefficient	***C***	***SF***	***TOP500***	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$
Value	**2.059*** (0.243)**	-1.495 (1.102)	-0.143 (0.557)	**-0.815*** (0.142)**	**1.931*** (0.697)**	**0.655* (0.368)**
Regression Statistics	Adjusted R^2	0.117	F-Statistic	38.920***	Obs.	1429
*** p-value<0.01; ** p-value<0.05; * p-value<0.1; Standard errors in parentheses.						

The histogram of OLS residuals and the Jarque-Bera test for normality in Figure 1 indicate the distribution of residuals is not normal. Since the value of a good may be important in consumer's decision-making, we test the robustness of the results with quantile regression (QR). QR provides information about how changes in covariates impact in different points of the distribution of the response variable. Estimates are obtained for 0.25, 0.5 and 0.75 quantiles. The results of quantile process are presented in Table 3. QR estimates for 0.5 quantile corresponds to the median, which is a semiparametric alternative to OLS. Therefore, it is interesting to see that the estimates for 0.5 quantile do not agree with OLS. The intercept decreases in magnitude to 0.155. Also, the dummy for Top500-sellers becomes positive and statistically significant at 2.246. All estimates for storefronts are statistically insignificant. The estimates for all markets is -0.085 and for Top500-sellers -0.793. These correspond to -0.1 per cent and -0.9 per cent decreases in the difference between prices when the difference between ratings increases by a one percent. Thus, the median regression does not provide empirical evidence for the test hypothesis.

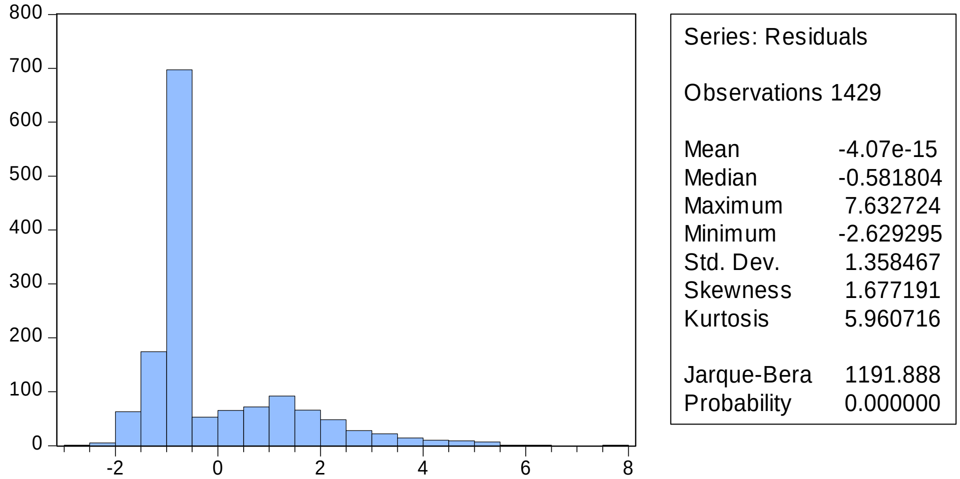

Figure 1. Histogram and Normality Test of OLS Residuals.

The tails of the distribution of the dependent variable provide some empirical evidence for the theory. In the lower tail (0.25 quantile), the only statistically significant estimates are for Top500-sellers. *TOP500* is 1.953 and $\hat{\beta}_3$ is -1.214 which indicates that the marginal effect is -1.2 per cent negative. In the upper tail (0.75 quantile), on the other hand, all estimates are statistically significant. The intercept and dummies are range from -2.618 to 5.119. Overall, there is a difference between prices that ranges from 5.1 percent to 2.5 per cent. The marginal effect for the general population is negative (-2.535) which implies -2.5 per cent decrease. In contrast, the estimates for the control groups are positive. The estimated coefficients for storefronts (3.120) and Top500-sellers (2.847) correspond to 0.6 per cent and 0.3 per cent, respectively, increases in the difference between prices. These results imply that a reputation may enable price premiums especially when buyers purchase more valuable goods. As a conclusion, QR estimates provide mixed evidence about positive dependency between $LPDIF_k$ and $LRDIF_k$.

These two regressions provide empirical evidence that reputation scores may explain price differences in electronic markets. The results from OLS give some support to the theory that a positive relationship between prices and reputation scores results from sellers' profit-maximizing behavior. While QR does not agree with OLS entirely, it implies that seller reputations are more important in the upper tail of the distribution of the dependent variable. Intuitively, this is hardly surprising because the pecuniary value of price differences is greater among more valuable goods. A greater value of a good implies potential for a greater financial loss to the buyer. Therefore, a seller whose reputation score is higher than its rival may charge higher prices in a market of zero search costs for price information.

Table 3. Quantile Regression Estimates for $LPDIF_k$.

Coefficient Quantile	***C***	***SF***	***TOP500***	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$
0.25	0.000	-1.506	**1.954*****	0.000	1.530	-1.214***
	(0.087)	(1.717)	**(0.572)**	(0.051)	(1.018)	(0.364)
0.50	**0.155***	2.136	**2.426*****	**-0.085***	0.104	-0.793**
	(0.083)	(4.026)	**(0.445)**	**(0.048)**	(2.486)	(0.369)
0.75	**5.119*****	**-2.618***	**-2.538*****	**-2.535*****	**3.120*****	**2.847*****
	(0.794)	**(1.472)**	**(0.822)**	**(0.534)**	**(0.943)**	**(0.580)**
Regression Statistics (0.50)	Adjusted R^2	0.095	Quasi-LR Statistic	625.237***	Obs.	1429

*** p-value<0.01; ** p-value<0.05; * p-value<0.1; Standard errors in parentheses.

4. Conclusion

In this paper, we present a model of duopoly competition for a comparison shopping service which has an integrated reputation system. Electronic marketplaces that provide comparison shopping services have become widespread in retail e-commerce. These services reduce buyer's search costs by providing price quotes from several sellers for the buyer's benefit. Since buyers may feel that risks of an e-commerce transaction are greater than in a conventional commercial transaction, e-marketplaces have introduced reputation systems to reduce the risks of asymmetric information.

We assume Bayesian buyers with heterogeneous beliefs about seller types. Buyers use the reputation system to update their beliefs about seller types. A profit-maximizing seller takes into account buyer beliefs, its rating score and its rival's reputation score in its pricing decision. We find that sellers may earn supernormal profits as returns to their reputations. The seller that has a better reputation earns higher profit than its rival. If sellers are identical, competition erases supernormal profits. Since even good sellers occasionally disappoint buyers, sellers adjust their prices to maximize profits in their current reputation profiles. For this reason, market prices are likely to fluctuate and price dispersion emerges.

We test the theory with the price and rating score data from Pricegrabber, which is a popular comparison shopping website. We find evidence that there is a positive dependency between prices and reputation scores. This is especially evident among sellers whose only sales channel is the comparison shopping service. Moreover, well-known e-commerce sellers may be able to leverage their existing reputations and charge price premiums. Quantile regression reveals that this may be especially true among more valuable goods where the buyer's pecuniary risks are higher.

In conclusion, this paper proposes a theory and evidence why a good reputation could be a valuable asset in e-commerce. For this reason, a seller may find it profitable to keep consumer satisfaction at a high level at least initially to gain competitive advantage later. As a direction for future research, it would be interesting to find out, how much weight consumers actually place on sellers' reputation profiles in their purchase decisions. Also, a detailed view of which actions taken by the seller increase consumer satisfaction and lead to higher rating scores would provide valuable information to e-commerce vendors.

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Appendix

To show that the condition in (12) holds, it must be that $\frac{\partial \mu_h}{\partial \bar{\theta}} \geq \frac{\partial \mu_l}{\partial \bar{\theta}}$. First, use Equation (2) for both types and differentiate in respect of $\bar{\theta}$. We obtain

$$\begin{aligned} &\frac{\gamma_h}{\bar{\theta}\gamma_h + (1-\bar{\theta})(1-\gamma_h)} - \frac{\bar{\theta}\gamma_h(2\gamma_h - 1)}{\left[\bar{\theta}\gamma_h + (1-\bar{\theta})(1-\gamma_h)\right]^2} \geq \\ &\frac{\gamma_l}{\bar{\theta}\gamma_l + (1-\bar{\theta})(1-\gamma_l)} - \frac{\bar{\theta}\gamma_l(2\gamma_l - 1)}{\left[\bar{\theta}\gamma_l + (1-\bar{\theta})(1-\gamma_l)\right]^2} \end{aligned} . \tag{A1}$$

It can be shown that $\frac{\gamma_h}{\bar{\theta}\gamma_h + (1-\bar{\theta})(1-\gamma_h)} \geq \frac{\bar{\theta}\gamma_h(2\gamma_h - 1)}{\left[\bar{\theta}\gamma_h + (1-\bar{\theta})(1-\gamma_h)\right]^2}$ because manipulating the inequality yields $\frac{1}{\bar{\theta}}(1-\gamma_h) \geq 0$, which is true because $\gamma_h \in [0,1]$. Since the right-hand side of (A1) is analogous to the left-hand side, we can rearrange Equation (A1) to

$$\begin{aligned} &\frac{\gamma_h}{\bar{\theta}\gamma_h + (1-\bar{\theta})(1-\gamma_h)} - \frac{\gamma_l}{\bar{\theta}\gamma_l + (1-\bar{\theta})(1-\gamma_l)} \geq \\ &\frac{\bar{\theta}\gamma_h(2\gamma_h - 1)}{\left[\bar{\theta}\gamma_h + (1-\bar{\theta})(1-\gamma_h)\right]^2} - \frac{\bar{\theta}\gamma_l(2\gamma_l - 1)}{\left[\bar{\theta}\gamma_l + (1-\bar{\theta})(1-\gamma_l)\right]^2} \end{aligned} . \tag{A2}$$

Consider now the left-hand side of Equation (A2). For equation to hold, it must be that

$$\frac{\gamma_h}{\bar{\theta}\gamma_h + (1-\bar{\theta})(1-\gamma_h)} \geq \frac{\gamma_l}{\bar{\theta}\gamma_l + (1-\bar{\theta})(1-\gamma_l)} . \tag{A3}$$

Equation (A3) reduces to

$$\gamma_h > \gamma_l , \tag{A4}$$

which holds with strict inequality because $\gamma_h > \gamma_l$ by definition.