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Complementation in spaces of symmetric tensor products and polynomials

by

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Abstract. For a locally convex space E we prove that the space of n-symmetric tensors is complemented in the space of $(n+1)$-symmetric tensors endowed with the projective topology. Applications and related results are also given.

In this paper we obtain complementation properties of spaces of continuous homogeneous polynomials on a locally convex space E that have, in particular, consequences in the study of the property $(BB)_{n,\mathrm{s}}$ recently introduced by Dineen [7].

As was pointed out by Ryan in his thesis [14], the completion of the space of n-symmetric tensors endowed with the projective topology (denoted by $\widehat{\bigotimes}_{n,\mathrm{s},\pi}E$) is a predual for the space $\mathcal{P}(^nE)$ of all n-homogeneous continuous polynomials on E. Using this we prove results for spaces of polynomials from results we obtain about symmetric tensors.

We prove that $\widehat{\bigotimes}_{n,\mathrm{s},\pi}E$ is a complemented subspace of $\widehat{\bigotimes}_{n+1,\mathrm{s},\pi}E$ and from this we show that, for locally convex spaces E and G, the space $\mathcal{P}(^nE;G)$ of all n-homogeneous continuous polynomials from E into G is a complemented subspace of $\mathcal{P}(^{n+1}E;G)$ when we endow these spaces with the strong dual topology. Moreover, the complementation of $\mathcal{P}(^nE;G)$ in $\mathcal{P}(^{n+1}E;G)$ for all the usual topologies on these spaces is obtained. From this it follows that the property $(BB)_{n,\mathrm{s}}$ on a locally convex space implies the property $(BB)_{m,\mathrm{s}}$ for $m=2,\ldots,n$.

We consider locally convex spaces over $\mathbb{K}=\mathbb{R}$ or $\mathbb{C}$. The notation concerning locally convex spaces and tensor products is standard ([12, 13, 11]). The family of continuous seminorms on the locally convex space E will be denoted by $\mathrm{cs}(E)$. In order to deal with polynomials we consider the symmetric tensor product ([14, 8]). We say that a tensor $\theta\in E\otimes E$ is *symmetric* if it has a representation $\theta=\frac{1}{2}\sum_{i=1}^{N}(a_i\otimes b_i+b_i\otimes a_i)$, where $N\in\mathbb{N}$ and

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$a_i, b_i \in E$ for each $i \in \{1, \ldots, N\}$. The symmetric tensors form a linear subspace of $E \otimes E$ that we denote by $E \otimes_s E$. $E \widehat{\otimes}_{s,\pi} E$ will denote the closure of $E \otimes_s E$ in the completion $E \widehat{\otimes}_\pi E$ of $E \otimes E$, where π denotes the projective topology on $E \otimes E$. In the same way we define an *n-symmetric tensor* as a tensor $\theta \in \bigotimes_n E = E \otimes \overset{n}{\ldots} \otimes E$ that admits a representation

$$\theta = \sum_{i=1}^{N} \frac{1}{n!} \sum_{\sigma \in \mathbb{P}_n} x_{\sigma(1)}^{(i)} \otimes x_{\sigma(2)}^{(i)} \otimes \ldots \otimes x_{\sigma(n)}^{(i)},$$

where $\mathbb{P}_n$ denotes the group of permutations of n elements and $x_1^{(i)}, \ldots, x_n^{(i)} \in E$ for each $i \in \{1, \ldots, N\}$. We let

$$\frac{1}{n!} \sum_{\sigma \in \mathbb{P}_n} x_{\sigma(1)} \otimes x_{\sigma(2)} \otimes \ldots \otimes x_{\sigma(n)} =: x_1 \otimes_s x_2 \otimes_s \ldots \otimes_s x_n.$$

Note that for $x_1 = \ldots = x_n = x$ we have

$$x \otimes_s \overset{n}{\ldots} \otimes_s x = x \otimes \overset{n}{\ldots} \otimes x =: \bigotimes_n x.$$

By the *polarization formula*, any symmetric tensor can be written as a sum $\sum_{j=1}^{M} \varepsilon_j \bigotimes_n x_j$, with $M \in \mathbb{N}$, $x_j \in E$ and $\varepsilon_j \in \{-1, 1\}$ for each j when $\mathbb{K} = \mathbb{R}$; in the complex case we can assume $\varepsilon_j = 1$ for each j, so a symmetric tensor is written as $\sum_{j=1}^{M} \bigotimes_n x_j$. We define $\bigotimes_{n,s,\pi} E = E \otimes_{s,\pi} \overset{n}{\ldots} \otimes_{s,\pi} E$ as the linear space spanned by the symmetric n-tensors and the *projective topology* on this space as the topology induced by the projective topology on $\bigotimes_{n,\pi} E = E \otimes_\pi \overset{n}{\ldots} \otimes_\pi E$. A fundamental system of seminorms for the projective topology π on $\bigotimes_n E$ is $\{\bigotimes_n \alpha : \alpha \in \mathrm{cs}(E)\}$ ([13, 8]), where

$$(\bigotimes_n \alpha)(\theta) = \inf\Big\{\sum_{j=1}^{N} \alpha(x_j^{(1)}) \ldots \alpha(x_j^{(n)}) : \theta = \sum_{j=1}^{N} x_j^{(1)} \otimes \ldots \otimes x_j^{(n)}\Big\},$$

so the projective topology on $\bigotimes_{n,s} E$ can be defined by the restrictions of these seminorms. Another fundamental system of seminorms for the projective topology π on $\bigotimes_{n,s} E$ is $\{\bigotimes_{n,s} \alpha : \alpha \in \mathrm{cs}(E)\}$ ([14]), where

$$(\bigotimes_{n,s} \alpha)(\theta) = \inf\Big\{\sum_{j=1}^{N} \alpha(x_j)^n : \theta = \sum_{j=1}^{N} \varepsilon_j \bigotimes_n x_j\Big\}.$$

The completion of $\bigotimes_{n,s,\pi} E$ will be denoted by $\widehat{\bigotimes}_{n,s,\pi} E$.

If G is a locally convex space, we denote by $\mathcal{P}(^nE; G)$ the space of all continuous homogeneous polynomials of degree n from E into G. When $G = \mathbb{K}$ we shall write $\mathcal{P}(^nE; \mathbb{K}) = \mathcal{P}(^nE)$. For a polynomial $P \in \mathcal{P}(^nE; G)$ we write $\check{P}$ to denote the n-linear symmetric mapping satisfying $P(x) = \check{P}(x, \overset{n}{\ldots}, x)$ for every $x \in E$. The space $\mathcal{P}(^nE; G)$ can be identified with $\mathcal{L}(\widehat{\bigotimes}_{\pi,s} E; G)$ in the following way: given $P \in \mathcal{P}(^nE; G)$, we take $\dot{P} \in \mathcal{L}(\widehat{\bigotimes}_{\pi,s} E; G)$ defined, in the natural way, from the equality $\langle \bigotimes_n x, \dot{P} \rangle = P(x)$ ([14]).

We require the following two lemmata in the proof of the main result.

LEMMA 1. *Let x, y be linearly independent vectors in E. Then, given $n = 1, 2, \ldots,$ there exist $\lambda_1, \ldots, \lambda_{n+1} \in \mathbb{K}$ such that*

$$\bigotimes_n x = \sum_{k=1}^{n+1} \lambda_k \bigotimes_n (x + ky).$$

Proof. Let F be the 2-dimensional vector space spanned by x and y. By [14], Lemma 5.1, $\{x \otimes_s \overset{r}{\ldots} \otimes_s x \otimes_s y \otimes_s \overset{n-r}{\ldots} \otimes_s y : r = 0, \ldots, n\}$ is a basis of $\bigotimes_{n,s} F$. Since

$$\bigotimes_n (x + ky) = \sum_{i=0}^{n} \binom{n}{i} k^i x \otimes_s \overset{i}{\ldots} \otimes_s x \otimes_s y \otimes_s \overset{n-i}{\ldots} \otimes_s y$$

for each $k = 1, \ldots, n+1$, and

$$\det\left(\left((i+1)^j \binom{n}{j}\right)_{i,j=0}^{n}\right) \neq 0,$$

we conclude that $\{\bigotimes_n (x + ky) : k = 1, \ldots, n+1\}$ is a basis of $\bigotimes_{n,s} F$. In particular,

$$\bigotimes_n x = \sum_{k=1}^{n+1} \lambda_k \bigotimes_n (x + ky)$$

for some $\lambda_1, \ldots, \lambda_{n+1} \in \mathbb{K}$. ■

LEMMA 2. *Let E be a locally convex space, $\theta \in \bigotimes_{n,s} E$ and $\varphi \in E'$, $\varphi \neq 0$. Then there exists a representation $\theta = \sum_{i=1}^{N} \varepsilon_i \bigotimes_n x_i$ with $\varphi(x_i) \neq 0$ for each $i \in \{1, \ldots, N\}$.*

Proof. Take a representation $\theta = \sum_{j=1}^{R} \varepsilon_j \bigotimes_n x_j$ and $e \in E$ such that $\varphi(e) \neq 0$. We only have to consider $J = \{j \in \{1, \ldots, R\} : \varphi(x_j) = 0\}$ and apply Lemma 1 to each couple (x_j, e) with $j \in J$. ■

THEOREM 3. *The space $\bigotimes_{n,s,\pi} E$ is a complemented subspace of $\bigotimes_{n+1,s,\pi} E$, for each positive integer n.*

Proof. Fix $e \in E$ and $\varphi \in E'$ such that $\varphi(e) = 1$. Define a mapping j on the tensors $\bigotimes_n x$, $x \in E$, by

$$j(\bigotimes_n x) = \sum_{k=1}^{n+1} \binom{n+1}{k} (-1)^{k+1} \varphi(x)^{k-1} e \otimes_s \overset{k}{\cdots} \otimes_s e \otimes_s x \otimes_s \overset{n-k+1}{\cdots} \otimes_s x.$$

Easy computations give us the following formula that will be used later:

$$j(\bigotimes_n x)\varphi(x) = \bigotimes_{n+1} x - \bigotimes_{n+1} (x - \varphi(x)e).$$

Extend the mapping j by linearity to each element $\theta = \sum_{i=1}^N \varepsilon_i \bigotimes_n x_i \in \bigotimes_{n,s} E$ as follows: $j(\theta) = \sum_{i=1}^N \varepsilon_i j(\bigotimes_n x_i)$. The mapping $j : \bigotimes_{n,s} E \to \bigotimes_{n+1,s} E$ is well defined and linear.

Next we define a projection π from $\bigotimes_{n+1,s,\pi} E$ onto $\bigotimes_{n,s,\pi} E$ by

$$\pi\Big(\sum_{i=1}^N \varepsilon_i \bigotimes_{n+1} x_i\Big) = \sum_{i=1}^N \varepsilon_i \varphi(x_i) \bigotimes_n x_i.$$

The mapping π is a well defined linear mapping and we claim that $\pi \circ j = \mathrm{Id}_{\bigotimes_{n,s} E}$.

From the equality

$$\bigotimes_{n+1} x - \bigotimes_{n+1} (x - \varphi(x)e) = \varphi(x) j(\bigotimes_n x),$$

we get

$$\pi(\bigotimes_{n+1} x - \bigotimes_{n+1} (x - \varphi(x)e)) = \varphi(x)\pi(j(\bigotimes_n x)),$$

i.e.

$$\varphi(x) \bigotimes_n x - \varphi(x - \varphi(x)e) \bigotimes_n (x - \varphi(x)e) = \varphi(x)(\pi \circ j)(\bigotimes_n x).$$

Since $\varphi(x - \varphi(x)e) = 0$, for every $x \in E$, we have

$$\pi \circ j(\bigotimes_n x) = \bigotimes_n x$$

for $x \notin \ker \varphi$ and $\pi \circ j$ is the identity mapping over the tensors that can be written as $\bigotimes_n x$ with $x \notin \ker \varphi$. Using Lemma 2 we write each tensor $\theta \in \bigotimes_{n,s} E$ as a sum $\theta = \sum_{i=1}^N \varepsilon_i \bigotimes_n x_i$ with $\varphi(x_i) \neq 0$ for each $i \in \{1, \ldots, N\}$, and then

$$\pi \circ j = \mathrm{Id}_{\bigotimes_{n,s} E}.$$

Finally, we prove continuity of the mappings j and π.

Choose $\alpha \in \mathrm{cs}(E)$ and $\theta \in \bigotimes_{n,s,\pi} E$. If $\sum_{i=1}^N \varepsilon_i \bigotimes_n x_i$ is a representation of θ, then

$$\begin{aligned}(\bigotimes_{n+1}\alpha)(j(\theta)) &= (\bigotimes_{n+1}\alpha)\Big(\sum_{i=1}^N \varepsilon_i \Big(\sum_{k=1}^{n+1} \binom{n+1}{k} (-1)^{k+1} \\ &\quad \times \varphi(x_i)^{k-1} e \otimes_s \overset{k}{\cdots} \otimes_s e \otimes_s x_i \otimes_s \overset{n-k+1}{\cdots} \otimes_s x_i\Big)\Big) \\ &\le \sum_{i=1}^N \Big(\sum_{k=1}^{n+1} \tbinom{n+1}{k} |\varphi(x_i)|^{k-1} \alpha(e)^k \alpha(x_i)^{n-k+1}\Big) \\ &\le \sum_{i=1}^N \Big(\sum_{k=1}^{n+1} \tbinom{n+1}{k} \alpha(e)^k\Big) (\sup\{\alpha, |\varphi|\}(x_i))^n \\ &= C \sum_{i=1}^N r(x_i)^n,\end{aligned}$$

where $r = \max\{\alpha, |\varphi|\} \in \mathrm{cs}(E)$ and $C = \sum_{k=1}^{n+1} \binom{n+1}{k} \alpha(e)^k$. Since this is true for each representation of θ, we have

$$(\bigotimes_{n+1}\alpha)(j(\theta)) \le C(\bigotimes_{n,s} r)(\theta)$$

and, consequently, the mapping j is continuous.

For $\theta = \sum_{i=1}^N \varepsilon_i \bigotimes_{n+1} x_i$ and $\alpha \in \mathrm{cs}(E)$,

$$\begin{aligned}(\bigotimes_n\alpha)(\pi(\theta)) &= (\bigotimes_n\alpha)\Big(\sum_{i=1}^N \varepsilon_i \varphi(x_i) \bigotimes_n x_i\Big) \le \sum_{i=1}^N (\bigotimes_n\alpha)(\varepsilon_i \varphi(x_i) \bigotimes_n x_i) \\ &\le \sum_{i=1}^N |\varphi(x_i)| \alpha(x_i)^n \le \sum_{i=1}^N r(x_i)^{n+1},\end{aligned}$$

with $r = \max\{\alpha, |\varphi|\} \in \mathrm{cs}(E)$. Taking the infimum over all representations of θ we have

$$(\bigotimes_n\alpha)(\pi(\theta)) \le (\bigotimes_{n+1,s} r)(\theta),$$

and the mapping π is continuous. ∎

The extension of the above theorem to the completed tensor product is obtained in the usual manner.

Corollary 4. *Let E be a locally convex space. Then for $n = 2, 3, \ldots$ and $k \in \mathbb{N}$, $1 \le k \le n$, $\widehat{\bigotimes}_{k,s,\pi} E$ is a complemented subspace of $\widehat{\bigotimes}_{n,s,\pi} E$.*

For locally convex spaces E and G we denote by β the topology, recently introduced in [7], induced on $\mathcal{P}(^nE; G) \simeq \mathcal{L}(\widehat{\bigotimes}_{n,s,\pi} E; G)$ by the topology β on $\mathcal{L}(\widehat{\bigotimes}_{n,s,\pi} E; G)$ of uniform convergence on bounded subsets of $\widehat{\bigotimes}_{n,s,\pi} E$. On $\mathcal{P}(^nE; G)$ we consider, apart from β, the following three natural topologies: the compact-open topology τ_0, the topology τ_b of uniform convergence on bounded subsets, and the Nachbin ported topology τ_ω ([5]). Corollary 4 and duality imply that $(\mathcal{P}(^mE),\tau)$ is a complemented subspace of $(\mathcal{P}(^nE),\tau)$ when $n\ge m$ and $\tau=\beta$ or τ_ω (recall that $(\mathcal{P}(^nE),\beta)\simeq(\widehat{\bigotimes}_{n,s,\pi}E)'_\beta$ and $(\mathcal{P}(^nE),\tau_\omega)\simeq(\widehat{\bigotimes}_{n,s,\pi}E)'_i$, the inductive dual). We extend this assertion in the following proposition.

PROPOSITION 5. *Let E and G be locally convex spaces and let $m\in\mathbb{N}$. Then $(\mathcal{P}(^mE;G),\tau)$ is a complemented subspace of $(\mathcal{P}(^nE;G),\tau)$ for $\tau=\tau_0,\tau_b,\beta$ or τ_ω and $n\ge m$.*

Proof. It is enough to consider $n=m+1$.

Fix $e\in E$ and $\varphi\in E'$ such that $\varphi(e)=1$. Consider the mappings j and π in Theorem 3, and define

$$J:\mathcal{L}(\textstyle\bigotimes_{m,s}E;G)\simeq\mathcal{P}(^mE;G)\to\mathcal{L}(\bigotimes_{m+1,s}E;G)\simeq\mathcal{P}(^{m+1}E;G),$$
$$\dot P\mapsto\dot P\circ\pi,$$

and

$$\varPi:\mathcal{L}(\textstyle\bigotimes_{m+1,s}E;G)\simeq\mathcal{P}(^{m+1}E;G)\to\mathcal{L}(\bigotimes_{m,s}E;G)\simeq\mathcal{P}(^mE;G),$$
$$\dot Q\mapsto\dot Q\circ j.$$

Since $\varPi\circ J=\mathrm{Id}_{\mathcal{P}(^mE;G)}$, we only have to prove that J and $\varPi$ are continuous for the different topologies we are considering. We begin with the compact-open topology τ_0. The topology induced on $\mathcal{L}(\bigotimes_{m,s}E;G)$ $(m\in\mathbb{N})$ by the compact-open topology τ_0 on $\mathcal{P}(^mE;G)$ through the algebraic isomorphism $\mathcal{L}(\bigotimes_{m,s}E;G)\simeq\mathcal{P}(^mE;G)$ is generated by the seminorms

$$\mathcal{L}(\textstyle\bigotimes_{m,s}E;G)\ni\dot P\mapsto\|\dot P\|_{\gamma,\bigotimes_mK}=\sup_{x\in K}\gamma(\dot P(\bigotimes_mx)),$$

where $\gamma\in\mathrm{cs}(G)$, K is a compact subset of E and $\bigotimes_mK=\{\bigotimes_mx:x\in K\}$.

Hence

$$\begin{aligned}\|J(\dot P)\|_{\gamma,\bigotimes_{m+1}K}&=\|\dot P\circ\pi\|_{\gamma,\bigotimes_{m+1}K}=\sup_{x\in K}\gamma(\dot P\circ\pi(\textstyle\bigotimes_{m+1}x))\\&=\sup_{x\in K}\gamma(\dot P(\varphi(x)\textstyle\bigotimes_mx))=\sup_{x\in K}\gamma(\varphi(x)\dot P(\bigotimes_mx))\\&\le|\varphi|_{\gamma,K}\sup_{x\in K}\gamma(\dot P(\textstyle\bigotimes_mx))=|\varphi|_{\gamma,K}\|\dot P\|_{\gamma,\bigotimes_mK}\end{aligned}$$

for every $P\in\mathcal{P}(^mE;G)$, which implies that the mapping J is continuous for τ_0.

On the other hand,

$$\begin{aligned}&\|\varPi(\dot Q)\|_{\gamma,\bigotimes_mK}\\&\quad=\|\dot Q\circ j\|_{\gamma,\bigotimes_mK}=\sup_{x\in K}\gamma(\dot Q(j(\textstyle\bigotimes_mx)))\\&\quad=\sup_{x\in K}\gamma\Big(\sum_{k=1}^{m+1}\binom{m+1}{k}(-1)^{k+1}\varphi(x)^{k-1}\check Q(e,\overset{k}{\smile\ldots},e,x,\overset{m-k+1}{\smile\ldots},x)\Big)\\&\quad\le\Big(\sum_{k=1}^{m+1}\binom{m+1}{k}|\varphi|_{\gamma,K}^{k-1}\Big)|\check Q|_{\gamma,(K\cup\{e\})^{m+1}}=C\|\dot Q\|_{\gamma,\bigotimes_{m+1}K'},\end{aligned}$$

where

$$C=\Big(\sum_{k=1}^{m+1}\binom{m+1}{k}|\varphi|_{\gamma,K}^{k-1}\Big)\frac{(m+1)^{m+1}}{(m+1)!}\quad\text{and}\quad K'=K\cup\{e\},$$

so the mapping $\varPi$ is continuous for τ_0 (to get the last inequality the polarization formula has been used).

The same idea, working with bounded sets instead of compact sets, proves that those two mappings are also continuous for the corresponding topologies τ_b.

To prove the continuity of J and $\varPi$ for the τ_ω topology, there is no loss of generality in assuming that G is normed. We then have

$$(\mathcal{P}(^mE;G),\tau_\omega)=\varinjlim_{\alpha\in\mathrm{cs}(E)}(\mathcal{P}(^mE_\alpha;G),\tau_b).$$

For $\alpha\in\mathrm{cs}(E)$ with $\alpha\ge|\varphi|$, the mappings $J_\alpha:\mathcal{P}(^mE_\alpha;G)\to\mathcal{P}(^{m+1}E_\alpha;G)$ and $\varPi_\alpha:\mathcal{P}(^{m+1}E_\alpha;G)\to\mathcal{P}(^mE_\alpha;G)$, defined by J and $\varPi$ above, show that $(\mathcal{P}(^mE_\alpha;G),\tau_b)$ is complemented in $(\mathcal{P}(^{m+1}E_\alpha;G),\tau_b)$.

The continuity of J and $\varPi$ is a direct consequence of the commutativity of the following diagrams:

$$\begin{array}{ccc}\mathcal{P}(^mE_\alpha;G)&\longrightarrow&\mathcal{P}(^mE;G)\\ {\scriptstyle J_\alpha}\downarrow&&\downarrow{\scriptstyle J}\\ \mathcal{P}(^{m+1}E_\alpha;G)&\longrightarrow&\mathcal{P}(^{m+1}E;G)\end{array}\qquad\begin{array}{ccc}\mathcal{P}(^{m+1}E_\alpha;G)&\longrightarrow&\mathcal{P}(^{m+1}E;G)\\ {\scriptstyle \varPi_\alpha}\downarrow&&\downarrow{\scriptstyle \varPi}\\ \mathcal{P}(^mE_\alpha;G)&\longrightarrow&\mathcal{P}(^mE;G)\end{array}\qquad\blacksquare$$

Remark 6. The above proposition is due to Aron and Schottenloher ([3]) for E Banach and $G=\mathbb{C}$ using a different technique.

For a Fréchet–Montel space E, $\tau_0=\tau_\omega$ on $\mathcal{P}(^nE)$ if and only if E has property $(BB)_{n,s}$. This property has been introduced by Dineen ([7]) as an n-fold version of property (BB) introduced by Taskinen ([15]) in relation with the "Problème des topologies" of Grothendieck ([10]).

A locally convex space E has property $(BB)_{n,s}$ for $n=2,3,\ldots$ if for every bounded subset B in $\widehat{\bigotimes}_{n,s,\pi}E$ there is a bounded subset C in E such that B is contained in the closed convex hull of $\bigotimes_{n,s} C = \{\bigotimes_n x : x \in C\}$. For several classes of Fréchet–Montel spaces E, $\tau_0 = \tau_\omega$ on $\mathcal{P}(^nE)$ ([1, 6, 9, 4, 7]) but Ansemil and Taskinen ([2]) gave an example of a Fréchet–Montel space E such that $\tau_0 \neq \tau_\omega$ on $\mathcal{P}(^2E)$. The following corollaries, which are consequences of Corollary 4, give new information about property $(BB)_{n,s}$.

COROLLARY 7. *If for a given* $n \in \mathbb{N}$, $\tau_b = \beta$ *in* $\mathcal{P}(^nE; G)$, *then* $\tau_b = \beta$ *on* $\mathcal{P}(^mE; G)$ *for every* m *with* $1 \leq m \leq n$.

COROLLARY 8. *If given* $n \in \mathbb{N}$, $n \geq 2$, E *has the* $(BB)_{n,s}$ *property*, *then* E *has the* $(BB)_{m,s}$ *property for each positive integer* m, $2 \leq m \leq n$.

Proof. It is enough to note that for $k \in \mathbb{N}$, $k \geq 2$, E has property $(BB)_{k,s}$ if and only if $\tau_b = \beta$ in $\mathcal{P}(^kE)$ ([7]). ■

This corollary simplifies the hypothesis in some theorems; see [6], for instance.

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