Complete Deterministic Linear Optics Bell State Analysis

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(Received 27 July 2005; published 17 May 2006)

We show how hyperentanglement allows us to deterministically distinguish between all four polarization Bell states of two photons. In this proof-of-principle experiment, we employ the intrinsic time-energy correlation of photon pairs generated with high temporal definition in addition to the polarization entanglement obtained from parametric down-conversion. For the identification, no nonlinear optical elements or auxiliary photons are needed. The new possibilities this complete Bell measurement offers are demonstrated by realizing an optimal dense coding protocol.

DOI: 10.1103/PhysRevLett.96.190501 PACS numbers: 03.67.Hk, 03.67.Mn, 42.50.Dv

Entanglement is the key resource of quantum information processing. Entangled qubits are required for quantum communication, most prominently for quantum teleportation [1] and quantum dense coding [2]. Nowadays, spontaneous parametric down-conversion [3,4] or, more recently, cold atoms [5] offer suitable sources of entangled photon pairs. But to fully unleash the power of these protocols and the complete and deterministic projection onto entangled states, the so-called Bell state analysis is required.

So far, only a partial Bell state analysis of photonic qubits could be performed in the implementations of quantum communication protocols [6,7]. In these experiments, two-photon interferometry is used in order to identify up to two of the four Bell states or, very recently, based on positive operator valued measurements, to observe probabilistically three Bell states [8]. The efficiency of these methods is 50%, which is the optimum when using linear optics only [9]. When including the nonlinearity of the detection process, it is, in principle, possible to construct deterministic quantum gates for photons, however, only with significantly increasing resources [10]. More recently, the addition of entangled photon pairs and conditioned detection turned out to enable probabilistic, but heralded, quantum logic with linear optics [11]. Proof-of-principle demonstrations of both methods have been realized [12,13] and a complete, but still probabilistic, Bell state analysis was achieved [14].

In this Letter, we demonstrate a method for the deterministic and complete Bell state analysis of polarization entangled photon pairs. The key element is to employ hyperentanglement between pairs of photons, which is obtained when embedding the states of the photonic qubits in a larger Hilbert space [15]. In particular, we use the additional time-bin entanglement originating in the process of parametric down-conversion pumped by a narrow-band pump [16]. After the first step of the Bell state analysis at a beam splitter [17], we are thus able to perform a second two-photon interference in an unbalanced Mach-Zehnder interferometer to identify all four Bell states [18]. This deterministic, linear optics method is applicable in all quantum communication protocols utilizing photon pairs entangled in multiple degrees of freedom [19]. Here we demonstrate its features by implementing quantum dense coding of two classical bits in a single qubit [2].

In the experiment, we used a noncollinear, degenerate type II spontaneous parametric down-conversion source of entangled photons [4]. Because of the high temporal coherence with which photon pairs are generated in the nonlinear β-barium borate (BBO) crystal, we can employ entanglement in the time degree of freedom in addition to the standard polarization entanglement. The coherence time of the pump laser, a cw Ar^+ laser running at 351 nm, was measured to be 17 ns. The intersection points of the ordinary and extraordinary light cones of 702 nm photons were imaged into single mode optical fibers to select clean spatial modes of equal wavelength. The output coupler of one of the single mode fibers was mounted on a translation stage to equalize the path lengths at the input of the Bell state analyzer (BSA). The coherence time of the down-conversion photons is determined by the collected bandwidth of about 4 nm and amounts in our case to approximately 300 fs. The state supplied at the output of the single mode fibers is $|\psi\rangle = (|HV\rangle + i|VH\rangle)/\sqrt{2}$, from which the four Bell states are obtained by locally manipulating the polarization (with a half-wave $\lambda/2$ or quarter-wave plate $\lambda/4$, respectively) of only one of the two particles. To characterize the polarization entangled states generated with this source, the polarization correlations in the H/V and $\pm 45°$ bases were measured. The visibilities of the correlation curves obtained for the four Bell states are all $\geq 98\%$ with an error of 0.2%.

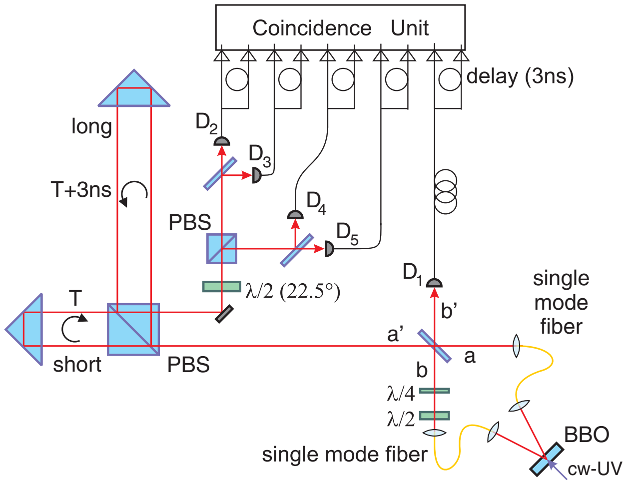

FIG. 1 (color online). Setup for a complete Bell state analysis (see text).

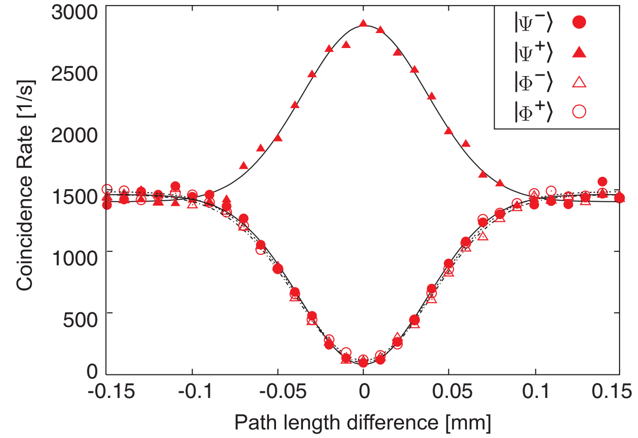

FIG. 2 (color online). Hong-Ou-Mandel interference on a beam splitter for all four Bell states. Experimental results (symbols) and fitted curves of the coincidence count rates as a function of the path length difference between the BBO crystal and the beam splitter for the two photons, corresponding to the time delay of the photons arriving at the beam splitter. The visibilities of the curves are between 92.4% and 95.3%.

The Bell state analyzer (Fig. 1) consists of three parts, discriminating stepwise all four Bell states by (i) two-photon interference at a beam splitter, (ii) polarization dependent delay, and (iii) polarization analysis in a rotated basis observing two-photon interference in temporally separated detection events.

In the first step, we distinguish $|\Psi^+\rangle$ from the other three Bell states [6,20]. The discrimination is based on the Hong-Ou-Mandel interference effect [21], relying on the indistinguishability of photons from a down-conversion source overlapped on a beam splitter (BS). It should be noted that we observe constructive interference at the beam splitter in the coincidence basis of the spatial output modes for the $|\Psi^+\rangle$ because the reflected components of horizontally and vertically polarized photons are measured to suffer a relative phase shift of approximately π for the beam splitter used, while the relative phase of the transmitted components remain unchanged. Hence, the Bell states transformed by the beam splitter read:

$$|\Phi^\pm\rangle \xrightarrow{BS} (|H_{a'}H_{a'}\rangle + |H_{b'}H_{b'}\rangle \mp |V_{a'}V_{a'}\rangle \mp |V_{b'}V_{b'}\rangle)/2,$$

$$|\Psi^-\rangle \xrightarrow{BS} (|H_{a'}V_{a'}\rangle + |H_{b'}V_{b'}\rangle + |V_{a'}H_{a'}\rangle + |V_{b'}H_{b'}\rangle)/2,$$

$$|\Psi^+\rangle \xrightarrow{BS} (|H_{a'}V_{b'}\rangle - |H_{b'}V_{a'}\rangle + |V_{a'}H_{b'}\rangle - |V_{b'}H_{a'}\rangle)/2,$$

with a' and b' denoting the spatial output modes as given in Fig. 1 [22]. Accordingly, for the $|\Psi^+\rangle$ input state we measure an increase in the coincidence rate of photons detected in modes a' and b', while for the remaining three Bell states the coincidence rate is observed to drop around zero path length difference (Fig. 2). Here the two photons always leave the beam splitter in the same spatial mode a' or b'. The width of the curves, 89 μm FWHM, determines the coherence time of the down-converted photons [21] to be 297 fs. For the following steps, the path length difference is set to zero such that, when detector D1 fires in coincidence with any of the other four detectors D2–D5, the input state is identified as $|\Psi^+\rangle$. An upper bound on the reliability of a correct identification of the $|\Psi^+\rangle$ is set by the 90% visibility of the Hong-Ou-Mandel interference [24]. The next two steps in the analysis, in the ideal case, should be implemented in both outputs of the beam splitter. For this proof-of-principle experiment, we use only one such device, which consequently analyzes all photon pairs exiting in the mode a'.

In the second step of the analysis, the $|\Psi^-\rangle$ state is distinguished from the remaining two Bell states. For $|\Psi^-\rangle$, the photons are always observed to have orthogonal polarizations in the H/V basis instead of parallel ones as for $|\Phi^\pm\rangle$. Thus, at a polarizing beam splitter only for $|\Psi^-\rangle$ the two photons split up into two paths. As their lengths differ, the states, initially encoded only in the polarization, are additionally marked in the time domain ($H \rightarrow H_e$; $V \rightarrow V_l$), with e (l) labeling early (late) arrival after passage along the short (long) arm. With a path length difference of 90 cm, we obtain a time delay of $\Delta t = 3$ ns, much longer than the detection-time resolution of about 1 ns. Thus, with a coincidence time window of 1.5 ns, coincidence events for which both photons travel the same path ($|\Phi^\pm\rangle$) and events for which they are detected temporally separated ($|\Psi^-\rangle$) can be distinguished. If two of the detectors D2–D5 click with a relative delay of approximately 3 ns, we therefore identify the event with the $|\Psi^-\rangle$ input state.

To allow a further processing of the $|\Phi^\pm\rangle$ states, in the third step the spatially delayed modes are overlapped to form an asymmetric Mach-Zehnder interferometer (Fig. 1). Because of the additional labeling, the $|\Phi^\pm\rangle$ states behind the interferometer are given by

$$|\Phi^\pm\rangle \rightarrow (|H_e H_e\rangle \pm e^{2i\phi}|V_l V_l\rangle)/\sqrt{2}, \quad (1)$$

with ϕ being the phase difference (modulo 2π) acquired within the asymmetric interferometer.

Note that the early or late labeling causes distinguishability if the path length difference results in a time difference (in our case, 3 ns) much longer than the coherence time of the photons ($<$ ps). Therefore, no first order interference occurs. However, since the creation time of the photon *pair* is uncertain within the coherence time of the pump laser (17 ns), the two terms in Eq. (1) are coherent with each other and, thus, form an entangled state [16]. This additional entanglement enables two-photon interference and the distinction of the $|\Phi^{\pm}\rangle$ states. Formally, the uncertainty of the creation time allows one to drop the time labeling. The orthogonality of the two states can be revealed by analysis in the $\pm 45°$ basis, where the two Bell states (for $\phi = 0$) are given by

$$|\Phi^+\rangle \rightarrow (|+45° + 45°\rangle + |-45° - 45°\rangle)/\sqrt{2},$$

$$|\Phi^-\rangle \rightarrow (|+45° - 45°\rangle + |-45° + 45°\rangle)/\sqrt{2}.$$

Thus, we finally obtain distinguishable detection events: In the case of the initial state being a $|\Phi^-\rangle$, the two photons are observed in different output modes of a polarization analyzer, whereas for $|\Phi^+\rangle$ they leave in the same mode. Interpreting the additional degree of freedom employed as an auxiliary qubit pair, formal similarities with Ref. [25] become obvious.

In the experiment, the analyzer is implemented by a half-wave plate oriented at 22.5° and a polarizing beam splitter (PBS) to achieve analysis in the $\pm 45°$ basis followed by beam splitters to enable (probabilistic) photon number resolution. For this configuration, coincidences of detection events in D2 or D3 with D4 or D5 identify $|\Phi^-\rangle$, while the $|\Phi^+\rangle$ is signaled by coincidence between detectors D2 and D3 or D4 and D5. To analyze the states $|\Psi^{\pm}\rangle$, where detection at times differing by the delay time of the asymmetric interferometer might occur, copies of the detection pulses were delayed electronically by 3 ns and fed, together with the originals, into a multicoincidence unit capable of registering any coincidences among all 9 input channels [26]. This finally allows the simultaneous analysis of all four Bell states.

For our particular setup, the detection probabilities for the four Bell states are not equal. First, for the purpose of demonstrating the proof of principle, the extended Bell state analyzer is used in only one of the two outputs of the usual beam splitter (mode a'). For this reason, the detection probability is 100% for $|\Psi^+\rangle$, but the other three states are detected only in 50% of all cases. A further reduction occurs for the states $|\Psi^-\rangle$ and $|\Phi^+\rangle$, where the two photons are absorbed by different detectors in only 75% and 50% of the cases, respectively. To maximize the detection probability, the extended Bell state analyzer has to be implemented in both beam splitter outputs (modes a' and b'), and true photon number resolving detectors will be needed.

Second, for $|\Psi^+\rangle$ one photon is detected right after the first beam splitter, while for the other Bell states both photons have to travel through the extended Bell state analyzer and will suffer loss at the various optical components. For better comparison, we therefore divided the measured count rates by the statistical detection probabilities and normalized the total detection probability of each input state to 100%.

The measured coincidence rates between detectors D2 and D4 for the $|\Phi^{\pm}\rangle$ states at the input of the BSA are shown in Fig. 3 as the path length or the phase ϕ [Eq. (1)] of the asymmetric interferometer is varied. The observed rates are clearly anticorrelated; the visibility of the curves is about 92%. We attribute the reduction of this value to vibrations and acoustic noise in the interferometer. From this measurement, we determine a zero position with $\phi = 0$ for the path length difference. At this position, finally, we obtain the detection probabilities 88.8%, 84.0%, 81.1%, and 83.0% for a correct and simultaneous identification of the respective Bell states $|\Psi^-\rangle$, $|\Psi^+\rangle$, $|\Phi^-\rangle$, and $|\Phi^+\rangle$ (Fig. 4). The background of wrongly identified coincidence events is mainly due to imperfections in the Hong-Ou-Mandel interference and in the asymmetric interferometer.

To demonstrate the capability of the Bell state analyzer, quantum dense coding was performed [6]. The sender Alice and the receiver of the message Bob are each supplied with one photon of an entangled pair from the down-conversion source. Alice can encode one of *four* possible messages associated with the four Bell states by locally manipulating her qubit and forwarding it to Bob, who then performs the complete Bell measurement on both photons to decode the message. The achievable channel capacity can be deduced from the transfer matrix given by the detection probabilities in Fig. 4. We obtain a value of 1.18 ± 0.03 bit, which corresponds to the mutual informa-

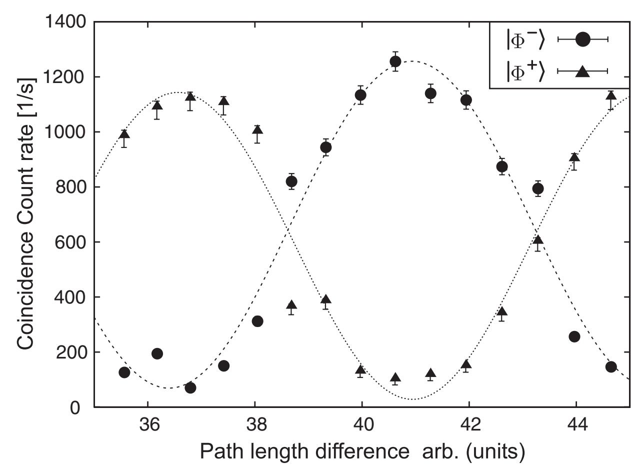

FIG. 3. Coincidence rate between detectors D2 and D4 as a function of the path length difference between the two interferometer arms. This corresponds to the relative phase between orthogonal polarizations, when preparing either $|\Phi^+\rangle$ or $|\Phi^-\rangle$ at the input of the BSA.

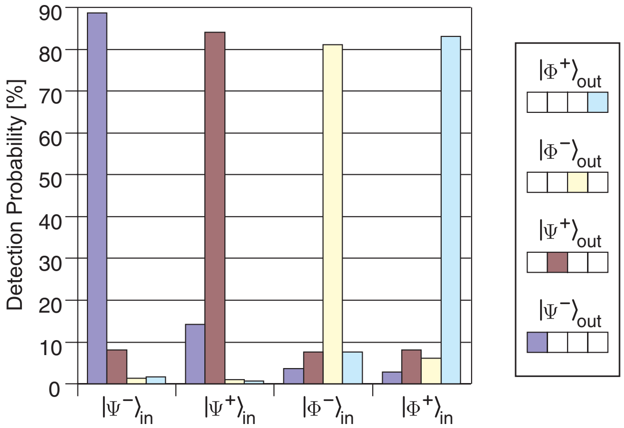

FIG. 4 (color online). Results of the complete Bell state analysis showing the relative detection probability for the various Bell states.

tion shared by Alice and Bob. Since the noise on the channel is not isotropic, the channel capacity still can be increased by appropriate coding.

In summary, we have demonstrated the feasibility of deterministic and simultaneous identification of all four polarization Bell states with probabilities between 81%–89%. This was possible with only linear optical elements by utilizing hyperentanglement in a higher dimensional Hilbert space. Although we have made use of neither auxiliary photons nor conditional measurements, our results do not contradict the no-go theorem for perfect linear optical Bell state analysis [9]. The method demonstrated here rather circumvents these restrictions by making use of the intrinsic time-energy entanglement of the down-conversion photons in addition to their polarization correlations. The complete Bell state analyzer was employed in a first demonstration of quantum dense coding using all four Bell states yielding a channel capacity of 1.18 ± 0.03 bit per photon. It now can be also used in many more two-photon quantum communication protocols, for example, to enhance the rate of the ping-pong quantum cryptography [27] or to serve as a truly efficient judge in novel quantum game protocols [28].

This work was supported by the Deutsche Forschungsgemeinschaft and the European Commission through the EU Projects RamboQ and QAP as well as by the Bavarian Forschungsstiftung. C. S. acknowledges support by the Generalitat de Catalunya and the European Regional Development Fund.

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