COMPLETE MONOTONICITY OF THE LOGARITHMIC MEAN

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Abstract. In the paper, the logarithmic mean is proved to be completely monotonic and an open problem about the logarithmically complete monotonicity of the extended mean values is posed. As two remarks, some errors appeared in [7, 21, 34] are corrected.

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