

COMPLETE MONOTONICITY PROPERTIES AND ASYMPTOTIC EXPANSIONS OF THE LOGARITHM OF THE GAMMA FUNCTION

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Abstract. We prove two conjectures of Chen concerning the complete monotonicity properties of some functions involving the gamma and polygamma functions. We prove asymptotic expansions of the logarithm of the gamma function in terms of the polygamma functions, and provide recurrence relations to calculate the coefficients of the asymptotic expansions. By using the results obtained, we derive recursive relations of the Bernoulli numbers.

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