

# Complex network analysis of the robustness of the Hanoi, Vietnam bus network

Tran, Vu Hieu; Cheong, Siew Ann; Bui, Ngoc Dung

2019

Tran, V. H., Cheong, S. A. & Bui, N. D. (2019). Complex network analysis of the robustness of the Hanoi, Vietnam bus network. *Journal of Systems Science and Complexity*, 32(5), 1251-1263. <https://dx.doi.org/10.1007/s11424-019-7431-x>

<https://hdl.handle.net/10356/150172>

<https://doi.org/10.1007/s11424-019-7431-x>

---

© 2019 The Editorial Office of JSSC & Springer-Verlag GmbH Germany. This is a post-peer-review, pre-copyedit version of an article published in *Journal of Systems Science and Complexity*. The final authenticated version is available online at:  
<http://dx.doi.org/10.1007/s11424-019-7431-x>.  
<https://www.springer.com/gp/open-access/publication-policies/self-archiving-policy>.

*Downloaded on 27 Aug 2022 23:12:26 SGT*

# COMPLEX NETWORK ANALYSIS OF THE ROBUSTNESS OF THE HANOI, VIETNAM BUS NETWORK

TRAN Vu Hieu · CHEONG Siew Ann · BUI Ngoc Dung

DOI:

Received: x x 20xx / Revised: x x 20xx

©The Editorial Office of JSSC & Springer-Verlag Berlin Heidelberg 2013

**Abstract** Many complex networks exist to facilitate the transport of material or information. In this capacity, we are often concerned with the continued flow of material or information when a fraction of the links in the complex network is disrupted. In other words, we are interested in the robustness of the complex network. In this paper, we survey measures of robustness like the average path length, the average clustering coefficient, the global efficiency, the size of largest cluster and use these to analyze the robustness of the bus network in Hanoi, Vietnam. We find that the bus network is robust against random failure but sensitive to targeted attack, in agreement with its scale-free character. By examining sharp drops in the average path length within the largest cluster of the Hanoi bus network under successive targeted attack, we identified five nodes whose loss lead to the fragmentation of the network into five or six disconnected clusters. These isolated clusters represent geographically the Central, Western, Southern, and Northwestern districts of Hanoi. Special considerations must therefore be given to these five nodes when planners wish to expand the bus network, or make it more robust.

**Keywords** Complex Network, Robustness, Hanoi Bus Network, Random Failure, Targeted Attack.

## 1 Introduction

The bus is a very important mode of public transportation. First of all, because of the huge number of bus commuters the bus fare per person is low [1]. Secondly, unlike airline systems that are heavily correlated with geographical constraints, or metro systems that connect some

---

TRAN Vu Hieu

*Faculty of Information Technology, University of Transport and Communications, Hanoi, Vietnam. Email: hieutu@utc.edu.vn.*

CHEONG Siew Ann

*Division of Physics and Applied Physics, School of Physical and Mathematical Sciences, Nanyang Technological University, 21 Nanyang Link, Singapore, 637371, Email: cheongsa@ntu.edu.sg.*

BUI Ngoc Dung (Corresponding author)

*Faculty of Information Technology, University of Transport and Communications, Hanoi, Vietnam. Email: دنبui@utc.edu.vn*

◊ *This paper was recommended for publication by Editor .*

parts of cities, bus systems can be easily expanded to all locations of a city or even whole cities or a country [1, 2]. Buses are even more important in many developing countries which do not have good rail systems. By creating bus lanes, using multiple vehicles with short headways, and designating stops further apart, part of the bus fleet can also be reconfigured into a Bus Rapid Transit (BRT) network very quickly and cheaply to meet the huge demands of the largest metropolitan areas [3]. Thus, bus and BRT network design has always attracted a lot of attention from transportation researchers.

In the traditional design research of transport systems in general and bus systems in specific, researchers mostly consider geography, connectivity, route speed, distance between nearby stations, route length, and ease of expansion as important factors [4–6]. Kocur and Hendrickson (1982) [7] highlighted the importance of bus transit system designs and showed mathematical formula which can be used to calculate and maximum revenue, reduce cost and also user benefit. There are also studies on bus route optimization and fleet management [8, 9]. Working with objective functions such as total system cost, operator profit, and social welfare, Chang and Schonfeld (1991) [8] proposed a multiple period model to construct the best route, in such a way that the service headways of the route can be improved over time. For example, for the same routes the headways can be differently optimized during peak, off-peak and night times. In contrast, Chien et al. [9] used a combination approach with Geographic Information Systems database that contains real-world information such as street geometry and traffic conditions to build a model for bus route optimization. Some other researchers use genetic algorithms in bus research [10–12]. Pattnaik, Mohan and Tom (1998) [11] proposed two genetic algorithm methods to design the bus route network, while Bielli et al. [10] and Shrivastava et al. [12] suggested using genetic algorithm for bus network optimization. However, results from these studies were limited since they focus mostly on developing theoretical models, algorithms and do not carry out comprehensive and detailed experiments on large scale transportation networks.

These studies also do not address the question on how robust the bus system is, which clearly impact bus operations on the network. For example, we can ask what will happen to the connectivity of a bus system if a bus line is added or removed? In response to system-level questions like this, a new research direction using complex networks has emerged. For the study of public transport systems, complex networks have the following advantages. First of all, public transport network can be effectively visualized as a complex network [13]. Many topological representations such as L-space and P-space [14–16] can be used. For example, the degree distribution, average clustering coefficient, average shortest path aspects of a public transport system can be analyzed within the L-space representation, whereas the average transfer time can be analyzed within the P-space representation [17]. Secondly, within the vocabulary understood for complex networks there are many statistical physics concepts that can be applied to quantify transport systems [13, 16]. Already, a number of empirical studies have been conducted, for example, studying traffic flow on a weighted transport system [13, 18, 19], or the fact that many large public transport systems are scale-free networks [20]. So far complex network research of bus systems has been carried out in developing countries such as Brazil, India and China [1, 2, 21, 22]. These researches are starting to bear fruit for public transportation in such

developing countries, especially to increase the development in their cities. However, no such research has ever been done in Vietnam [14].

Since its city limits were expanded in 2008, Hanoi is the biggest city in Vietnam with a land area of  $3.345 \text{ km}^2$  (3.6 times larger after expansion) and total population of 7.327 million (twice the population before expansion) [23]. Hanoi is served by a bus system with 63 lines connecting 13 districts with 483 stations and 643 connections between stations. The bus system is especially important to Hanoi because its rail system is still in an early stage of development. Even so, the bus system caters only to 10% of the travel demand in Hanoi [23]. The fast expansion of the city and rapid increase of population require the Hanoi bus system to be upgraded faster, without compromising its stability. This is complicated by the many modifications of road systems and bus lines in Hanoi. Our research aimed therefore to carry out a comprehensive survey on the robustness of the Hanoi bus system, applying complex network methodologies to foresee and detect if any modification can reduce or improve the effectiveness of the bus system.

In this paper, the Hanoi bus network is abstracted as a complex network, the structure of which will be changed by random failure and targeted attack. We then introduce measures of robustness that we can calculate for the system, to show that the Hanoi bus network is robust against random failures, but sensitive towards targeted attacks. To tell this story, our paper is organized as follows. **In Section 2, we review the definition of a complex network, and its important properties. A comprehensive review of how to use these properties to measure robustness of complex network will also be given in Section 2. In Section 3, we simulate random failures and targeted attacks on the Hanoi bus system, and discuss what these results tell us about the robustness of the bus network. Finally, we conclude in Section 4.**

## 2 Robustness of Complex Networks

The study of complex networks is a growing research field related to biological, technological, and social systems. **Important questions that are being addressed include, for example, the ability of a complex network to synchronize [24], and the identification of important spreaders in a complex network [25].** A complex network can be presented as a graph  $G = (V, E)$ , where  $V$  is a collection of nodes (vertices) and  $E$  is a collection of edges (links). One way to represent a graph is in terms of its adjacency matrix  $A$ , whose matrix element is  $A_{ij} = 1$  if there is an edge between two nodes  $i$  and  $j$  then and  $A_{ij} = 0$  otherwise. From  $G$  or its adjacency matrix  $A$  we measure network properties such as its degree distribution, average path length, size of largest cluster, average size of isolated cluster, clustering coefficient, and global efficiency. The first and most important property of complex networks to survey is the degree distribution. For an undirected network, the degree of a node is the number of connections it has with other nodes. Over the entire network, the degree distribution  $P(k)$  is then the fraction of nodes having  $k$  connections [26].

Typically, to study a complex network's robustness, we remove a fraction of its nodes (and all edges they are connected to), and check if the network is still fully connected, or if it has fragmented into disconnected sub networks, the size of the largest cluster. This node removal

can be done randomly (error) or in a targeted fashion (attack). One of the most common criteria for choosing nodes to attack is to choose those with higher degrees first (degree-based attack). The network's ability to continue functioning when these damages occur is called its *robustness* [27, 28]. Surveys of random failures and targeted attacks based on structural properties are important for understanding the effects of these two types of damages to all kind of network systems, such as the internet, transportation, biology, social networks, and so on.

Previous studies have shown that the robustness of a complex network depends very much on its topology [29–31]. In a random network, each node has approximately the same degree, and is thus homogeneous [29]. The degree distribution of random networks follows a Poissonian distribution [26, 30, 31].

$$P(k) = \frac{e^{-z} z^k}{k!} \quad (1)$$

where  $z = \langle k \rangle$  is the average degree.

On the other extreme, many real complex networks such as the World Wide Web, the internet, air travel network, and biological systems are scale-free networks [31]. A scale-free network is heterogeneous, in that some nodes have very high degrees compare to other nodes in same networks [29]. Scale-free networks have a power-law degree distribution [26, 29–31].

$$P(k) \approx ck^{-\gamma} \quad (2)$$

where  $k$  runs from the minimum to the maximum degree of nodes, and the exponent  $\gamma$  may be different for different networks.

Besides the degree distribution, another property related to the robustness of a complex network is the average path length, obtained by averaging the shortest paths over all pair of nodes in the network [29, 33, 34]. The average path length of a network tells us how connected the network's nodes are. In real world complex networks, the average path length is very small [29, 34]. Researches also used other network properties, such as size of largest cluster ( $S$ ) and average size of isolated cluster ( $\langle S \rangle$ ), clustering coefficient ( $C$ ), characteristic path length ( $L$ ), and global efficiency ( $E$ ) to examine the robustness of a network [29–32, 35, 36].

Consider a node  $i$  of the network, and the  $k_i$  nodes it is connected to. Let  $G_i$  be the subgraph that contains these  $k_i$  nodes [30, 39]. If  $G_i$  is fully connected it can have the maximum number of edges  $k_i(k_i-1)/2$ . The clustering coefficient  $C_i$  of node  $i$ ,

$$C_i = \frac{2E_i}{k_i(k_i-1)} \quad (3)$$

is the ratio of number of edges  $E_i$  in graph  $G_i$  to the maximum number of edges. We then obtain the average clustering coefficient

$$C = \frac{1}{N} \sum_{i \in G} C_i \quad (4)$$

of the whole graph by averaging over  $C_i$  [29, 30], where  $N$  is number of nodes of graph  $G$ . For the graph  $G$ , the characteristic path length is [30, 37, 38]:

$$L(G) = \frac{1}{N(N-1)} \sum_{i \neq j \in G} d_{ij} \quad (5)$$

where  $d_{ij}$  is the shortest path length between two nodes  $i$  and  $j$ . By definition  $d_{ij} \geq 1$  and if  $d_{ij} = 1$  there is an direct edge connecting the two nodes. In principle  $L$  is well-defined if graph  $G$  is fully connected. If there is no path between nodes  $i$  and  $j$  then  $d_{ij} = +\infty$ , and as a result  $L$  is divergent [30]. But when damages happen, nodes and edges are removed, then some nodes can become disconnected and  $L$  will become divergent, and cannot be a measure of robustness.

In view of this, Latora and Marchiori suggested global efficiency ( $E$ ) as a new measure of robustness [30, 31, 38]. By modifying the adjacency matrix such that  $A_{ij} = 1$  when there is an edge between  $i$  and  $j$ , and  $A_{ij} = +\infty$  otherwise, then the distance of shortest path  $d_{ij}$  between node  $i$  and node  $j$  will automatically become  $d_{ij} = +\infty$  when there is no path between  $i$  and  $j$ . The authors also define  $E$  in terms of  $\frac{1}{d_{ij}}$ , which fixes the divergence problem of characteristic path length. With a further normalization by factor  $1/N(N-1)$ , the global efficiency.

$$E(G) = \frac{1}{N(N-1)} \sum_{i \neq j \in G} \frac{1}{d_{ij}} \quad (6)$$

is then always more than 0 and less than 1.

Using the above properties to survey the robustness of complex networks, many researchers arrived at the same general conclusion [29–32, 35, 36]: scale-free networks are extremely robust against failure but are vulnerable against attack (especially degree-based attack). In their experiments, we see how properties measuring robustness of the scale-free networks are stable under failure but change rapidly to reach their limits under attack. We now understand why this is so: scale-free networks are heterogeneous, and some nodes are really important. These important nodes have many connections compared to other nodes, so if the attack is based on removing important nodes first then the structures of the networks will be easily destroyed.

To arrive at these conclusions, both Albert's team [29] and Jeong's team used [32] the average path length as a measure of robustness for the complex network, as it depends sensitively on the interconnectedness of networks. Normally, well-functioning real-world networks with very many nodes (such as WWW, social networks) has small average path lengths, and if we remove nodes we can normally increase distances between nodes and as a result will increase the average path length [29]. In Albert's and his colleagues' research an artificial scale-free network with 10,000 nodes and 20,000 links was used. In addition, they also used two real complex networks,

the first is the topological map of the Internet with 6,209 nodes and 12,200 edges, while the second is a sample of the world-wide web (WWW) with 325,729 nodes and 1,498,353 edges [29]. Changes of the average path lengths of networks under attack and failure were compared. In the case of the scale-free network, when the percentage of removed nodes was increased from 0 to 5, the average path length under failure remained almost unchanged (at about 5), while the average path length under attack increased rapidly from 5 to 10 [29]. This result shows that scale-free networks are robust against failures but are vulnerable against attacks. This holds for the 10,000-node artificial scale-free network, as well as the two real complex networks. Jeong et al. [32] also used the average path length to survey the robustness of metabolic networks. Metabolic networks can be represented as graphs, in which the nodes are substrates and the edges are metabolic reactions [31]. The degree distribution of the metabolic network is a power law with  $\gamma = 2.2$  [32]. When 8% of the nodes (substrates) are randomly deleted the average path length remained unchanged, whereas when 8% of the nodes are deleted under attack the average path length increased by 400% [32].

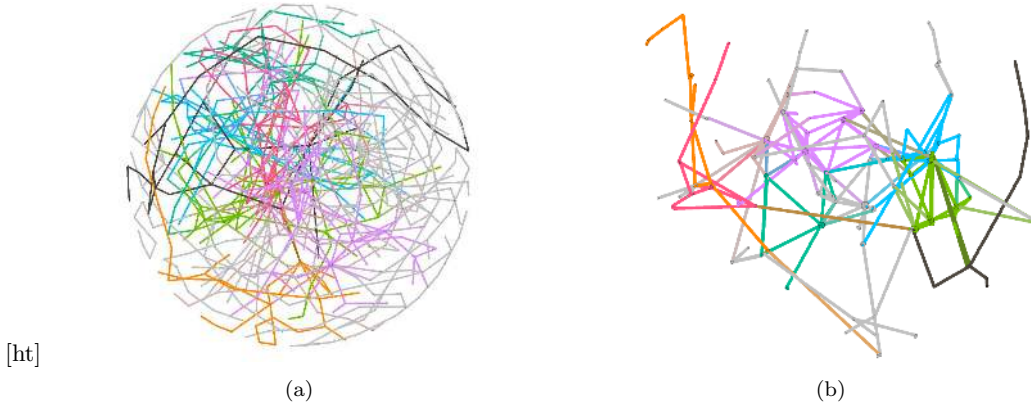
Albert's team as well as Sole and Montoya, examined the size of the largest cluster ( $S$ ) and the average size of isolated clusters ( $\langle s \rangle$ ) in their papers [29, 35, 39]. In their simulations, Albert and team found that when the percentage of removed nodes increased from 0% to 2.8% under failure, the value of  $S$  decreased slowly, whereas the value of  $\langle s \rangle$  remained nearly constant. On the other hand, the values of  $S$  and  $\langle s \rangle$  both changed dramatically when the percentage of removed nodes under attack was increased to 1.8%, indicating that the scale-free network is heavily damaged and broke into many small clusters [29]. The team applied the same methods to the maps of the Internet and WWW, and obtained similar results, i.e., the two scale-free networks are robust when large number of nodes are removed randomly, but broke up rapidly under targeted attack [29]. Measuring the robustness of ecological networks [35, 39], where node  $i$  can connect to other nodes if species  $i$  consumed (inward links) or is consumed (outward links) by these nodes [35], Sole and Montoya examined three ecological networks: Ythan Estuary, Silwood Park and Litte Rock Late. They showed that the first two networks are scale-free networks with  $\gamma = 1.04$  and  $\gamma = 1.13$ , with 134 and 154 nodes respectively. They found that under failure, the value of  $S$  declined gradually from 1, whereas the value of  $\langle s \rangle$  remained at 1, but under attack the value of  $S$  became 0 and the value of  $\langle s \rangle$  reached its peak when just 25% (Ythan Estuary network) and 7.5% (Silwood Park network) of nodes are removed [35, 39].

More recently, Crucitti and his team [30, 31] used clustering coefficient, characteristic path length, and global efficiency to test robustness of complex network. In Crucitti and his colleagues' experiment with the clustering coefficient ( $C$ ), an artificial scale-free network with 5,000 nodes and 10,000 links was used. They found that when just 2% of nodes were attacked  $C$  declined rapidly from 0.008 to 0.001, and if more than 2% of nodes were removed under attack then  $C$  was nearly zero. This same value of  $C$  under failure is obtained only when the percentage of removed nodes is already 80% [30]. In Crucitti and his colleague's experiment with global efficiency ( $E$ ), a scale-free network that has 2,000 nodes and 10,000 links was used. If 80% of the nodes were removed randomly then the scale-free network still has quite decent efficiency (0.15), but if only 15% of nodes were attacked then the efficiency was reduced by

half. If the percentage was increased to 35% then the network is almost completely destroyed [31]. Experiment with characteristic path length ( $L$ ) by Crucutti and his team [30, 31] shows similar results, but the results are not as clear as with global efficiency ( $E$ ). Therefore,  $E$  was considered to be a better property for surveying the robustness of complex networks than  $L$  [27, 30]. Sydney and his team [36] also measured a quantity called the elasticity to prove this conclusion.

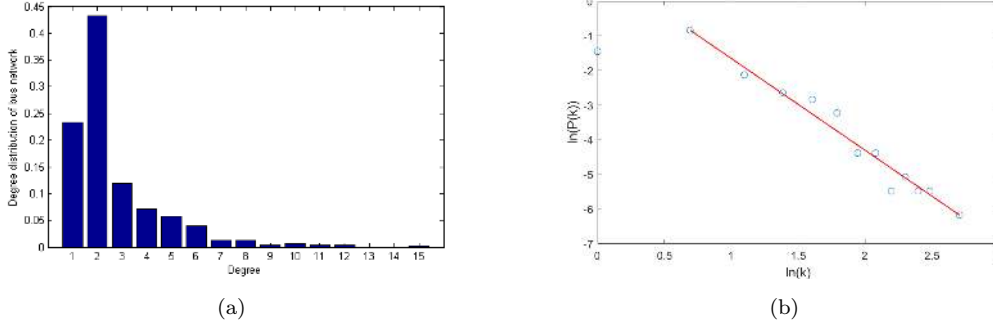
### 3 Results and Discussions

In this section, we will test the robustness of the Hanoi bus system using the complex-network properties mentioned in Section 2. First, we collected the data for the Hanoi bus network from <http://timbus.vn/fleets.aspx>. This is a graph that contains 483 nodes and 643 edges, where the nodes are bus stops, and the edges are direct bus connections between the bus stops. See Figure 1. The average degree per node of this network is  $\langle k_0 \rangle = 2.66$ , and the average path length is  $\langle L \rangle = 9.1$ . In our simulations, the Hanoi bus network is damaged by random failure as well as targeted attack. For targeted attacks, the nodes are ranked based on their degrees, with the more important nodes having higher degrees. Each time failure happens then some percentage of the nodes will be removed randomly, while each time an attack happens the same percentage of the most important nodes will be removed.



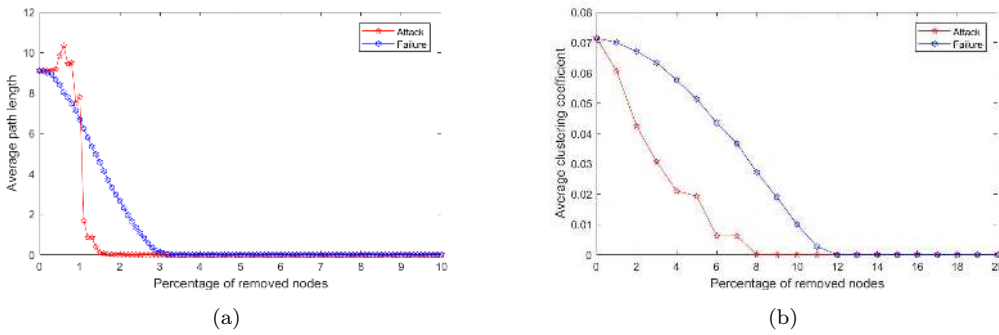
**Figure 1** The Hanoi bus network shown using two different layouts: (a) the OpenOrd layout, and (b) the Fruchterman-Reingold layout. In both pictures, a node's size is ranked based on its degree, whereas the color of a node is determined by which community it belongs to. A total of nine communities were recognized, using the algorithm of Blondel et al. [40].

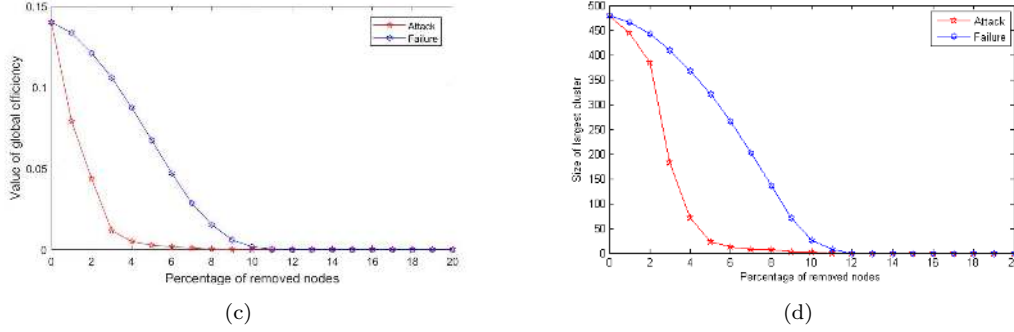
In Figure 2, we show the degree distribution of the Hanoi bus network. This network is heterogeneous: while 45% of the nodes have degree 2, we also find a few nodes with degrees larger than 9. When we plot this degree distribution on a log-log graph, it became apparent that the bus network is scale-free, with  $P(k) \approx ck^{-\gamma}$ , with  $\gamma = 2.6$ .



**Figure 2** (a) The degree distribution of the Hanoi bus network. (b) A log-log graph of this degree distribution, where the  $x$  axis presents  $\ln(k)$  and the  $y$  axis presents  $\ln(P(k))$ . The data points are shown as blue circles, and the best-fit red line has slope  $\gamma = -2.6$

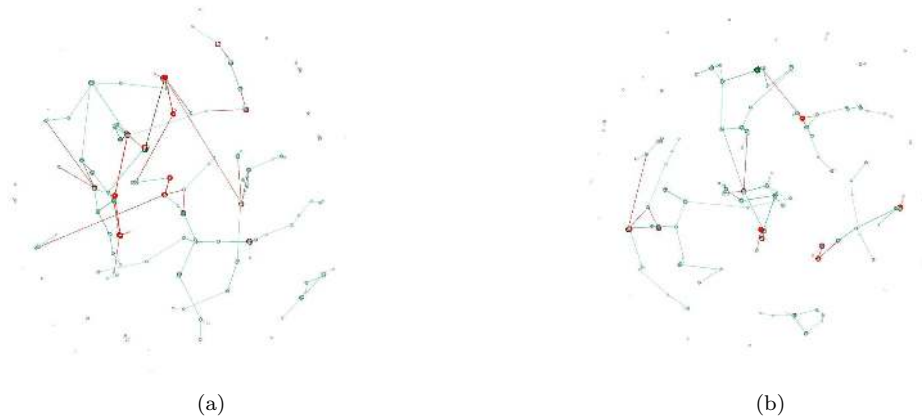
To test the robustness of the Hanoi bus system, we measure its average path length, clustering coefficient, size of largest cluster, and global efficiency under failure and attack. When the percentage of nodes removed is increased from 0% to 10%, we see from Figure 3 that as expected, the average clustering coefficient, the global efficiency, and the size of the largest cluster decreased monotonically. Also as expected, these quantities decrease more rapidly in attacks as compared to failures. In fact, the network collapsed when 8% to 10% of the most connected nodes are attacked. In contrast, when 10% of the nodes fail randomly, the values of the three quantities remain high, and only vanish when 12% of the nodes fail. According to previous research [41, 42], scale free networks with  $\gamma < 3$  are robust against random failures, but sensitive to attacks. Indeed, we see that the Hanoi bus network is sensitive to attack but is resilient to failure.





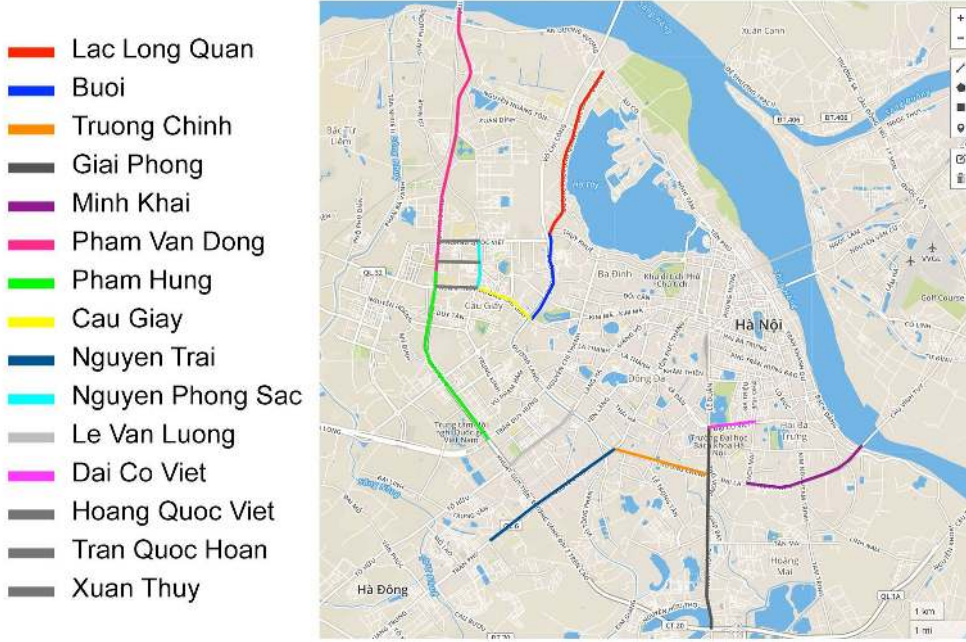
**Figure 3** Changes in the Hanoi bus system's (a) average path length, (b) average clustering coefficient, (c) global efficiency, and (d) size of largest cluster with random failure (blue line) and targeted attack (red line) when the percentage of nodes removed increase from 0% to 10% in (a) and from 0% to 20% in (b), (c), (d). The failure curve is averaged over 100 random trials.

More interestingly, the average path length decreases monotonically under random failure instead of increasing monotonically as expected. To understand why this is so, we look instead at the average path length of the Hanoi bus network under targeted attack. Here, we see that the average path length *increases* rapidly from 9.1 to 10.35 when the percentage of nodes removed increase from 0% to 0.8%, which is the loss of network robustness that we expected to see. When more high-degree nodes are attacked, the average path length *drops* abruptly before increasing again. This happens twice, until the percentage of nodes attacked reaches 1.1%, where the average path length plunges from 7.8 to 1.7. These sharp drops in the average path length are associated with the largest cluster suddenly losing a significant fraction of its nodes, because of the removal of high-degree nodes that are at the same time bridging nodes. In particular, the nodes removed going from 1.0% nodes removed to 1.1% nodes removed in the network are especially important, as they caused the Hanoi bus network to fragment into much smaller clusters (see Figure 4).



**Figure 4** (a) The Hanoi bus network after 1.0% of nodes are attacked, and (b) the Hanoi bus network after 1.1% of the nodes are attacked

The nodes that were removed when 1.1% nodes were attacked are Doc La, Chua Boc, Lac Long Quan, Buoi, and Truong Chinh. At this stage all five nodes have degree  $k = 5$ . The original degrees of these nodes are also  $k = 5$ , except for Buoi, which has degree  $k = 6$ . After 1.1% of nodes were attacked the average path length of the Hanoi bus network decreased sharply from 7.8 to 1.7. The Hanoi bus networks before 1.0% and 1.1% of nodes are attacked are shown in figure 4. In this figure, red dots are nodes with the highest degree in the network, green nodes are normal nodes, red lines are edges incident on nodes with the highest degree and if these red lines are deleted, the connected components of the graph becomes disconnected. The picture we have (see Figure 5) then is Lac Long Quan (red line), Buoi (blue line), Truong Chinh (orange line), combined with previous attacks Giai Phong (black line), Minh Khai (purple line), Pham Van Dong (pink line), Pham Hung (green line), Cau Giay (yellow line), Nguyen Trai (navy line), Nguyen Phong Sac (aqua line), Le Van Luong (silver line), Dai Co Viet (magenta line) with respective degrees are 15, 12, 12, 10, 10, 9, 6, 6, 5 will isolated the center of Hanoi from the West and South of Hanoi. The loss of Lac Long Quan (red line) and Buoi (blue line) will also create isolated areas in the Northwest of Hanoi bordering these two lines, and also Cau Giay (yellow line), Nguyen Phong Sac (aqua line), Pham Van Dong (pink line), Pham Hung (green line). This area is not totally isolated because there are three parallel roads (three small gray lines) that connect to Nguyen Phong Sac (aqua line) and Pham Van Dong (pink line). However, buses on these three parallel roads only can go to Nguyen Phong Sac and Pham Van Dong, and no bus can get out from this area. Therefore, when seeking to improve the robustness of the Hanoi bus network, by adding additional lines to the network, these vital nodes must be taken into consideration.



**Figure 5** The map of Hanoi when 1.1% nodes of bus network are attacked. Map data ©Mapbox ©OpenStreetMap.

Finally, let us compare our results to existing transportation network studies in Brazil, China, India, Poland [1, 2, 15, 16, 21, 22]. Most of these previous studies are concerned with the statistical properties of the networks, showing that they have scale-free degree distributions. The only exception is Zhang et al. [43], who in 2011 tested the robustness of the Shanghai subway network. Like us, they found the bus network robust against random failures but fragile against targeted attacks. Our novel contribution lies in us going beyond statistical descriptions, to interpret telling features in the average path length and identify special nodes whose removal under sustained targeted attacks can completely destroy the network.

## 4 Conclusions

In this paper we carried out a comprehensive review of measures of a complex network's robustness, in particular how scale-free networks are robust against failure, but sensitive to targeted attacks. As our case study, we analyzed the Hanoi bus network with 483 nodes (bus stops) and 643 edges (road links), and found that it is a scale-free network. We then simulated random failure and highest-degree-based attack on the Hanoi bus network, and found the average clustering coefficient, global efficiency, and size of the largest cluster decreasing monotonically with the number of nodes failed or attacked. This decrease is more dramatic in targeted attack than it is in random failure, suggested that the Hanoi bus network is robust against failures but sensitive to attacks. We also found sharp drops in the average path length

within the largest cluster when its high-degree nodes are attacked. Based on the sharpest drop that occurs when 1.1% of the nodes are attacked, we identified five important nodes whose losses result in the fragmentation of the Hanoi bus network into five or six clusters of moderate size. Special considerations must be given to these five nodes when planners expand the Hanoi bus network, or try to make it more robust.

**Acknowledgements** We would like to thank BSc. Hoang Thanh Tung, Tomo app Vietnam, for bus network data collection.

### References

- [1] Chatterjee A and Ramadurai G, Scaling laws in Chennai bus network, *arXiv preprint arXiv:1508.03504*, 2015.
- [2] Chatterjee A, Manohar M and Ramadurai G, Statistical analysis of bus networks in India, *PloS one*, 2016, **11(12)**: 1-16.
- [3] Levinson H S, Zimmerman S, Clinger J and Rutherford G S, Bus rapid transit: An overview, *Journal of Public Transportation*, 2002, **5(2)**: 1-30.
- [4] Ceder A, Designing Public Transport Networks and Routes, *Advanced Modeling for Transit Operations and Service Planning*, 2002, 59-91.
- [5] Ceder A and Wilson N H, Bus network design, *Transportation Research Part B: Methodological*, 1986, **20(4)**: 331-344.
- [6] Vuchic V R, *Urban transit: operations, planning, and economics*, John Wiley and Sons, 2005.
- [7] Kocur G, and Hendrickson C, Design of local bus service with demand equilibration, *Transportation Science*, 1982, **16(2)**: 149-170.
- [8] Chang S K and Schonfeld P M, Multiple period optimization of bus transit systems, *Transportation Research Part B: Methodological*, 1991, **25(6)**: 453-478.
- [9] Chien S I, Dimitrijevic B V and Spasovic L N, Optimization of bus route planning in urban commuter networks, *Journal of Public Transportation*, 2003, **6(1)**: 53-79.
- [10] Bielli M, Caramia M and Carotenuto P, Genetic algorithms in bus network optimization, *Transportation Research Part C: Emerging Technologies*, 2002, **10(1)**: 19-34.
- [11] Pattnaik S, Mohan S and Tom V, Urban bus transit route network design using genetic algorithm, *Journal of Transportation Engineering*, 1998, **124(4)**: 368-375.
- [12] Shrivastava P and O'Mahony M, A model for development of optimized feeder routes and coordinated schedules? A genetic algorithms approach, *Transport Policy*, 2006, **13(5)**: 413-425.
- [13] Lin J and Ban Y, Complex network topology of transportation systems, *Transport Reviews*, 2013, **33(6)**: 658-685.
- [14] de Regt R, von Ferber C, Holovatch Y and Lebovka M, Public transportation in UK viewed as a complex network, *arXiv preprint arXiv:1705.07266*, 2017
- [15] Sienkiewicz J and Holyst J A, Public transport systems in Poland: From Bialystok to Zielona Gora by bus and tram using universal statistics of complex networks, *arXiv preprint physics/0503099*, 2005, **1171-1178**.

- [16] Sienkiewicz J and Ho?yst J A, Statistical analysis of 22 public transport networks in Poland, *Physical Review E*, 2005, **72(4)**: 1-11.
- [17] Zhang H, Zhao P, Gao J and Yao X, The analysis of the properties of bus network topology in beijing basing on complex networks, *Mathematical Problems in Engineering*, 2013.
- [18] Lu H and Shi Y, Complexity of public transport networks, *Tsinghua Science and Technology*, 2007, **12(2)**: 204-213.
- [19] Soh H, Lim S, Zhang T, Fu X, Lee G K K, Hung T G G, Di P, Prakasam S and Wong L, Weighted complex network analysis of travel routes on the Singapore public transportation system, *Physica A: Statistical Mechanics and its Applications*, 2010, **389(24)**: 5852-5863.
- [20] Von Ferber C, Holovatch Y and Palchykov V, Scaling in public transport networks, *arXiv preprint cond-mat/0501296*, 2005.
- [21] De Bona A, Fonseca K, Rosa M, Luders R and Delgado M, Analysis of Public Bus Transportation of a Brazilian City Based on the Theory of Complex Networks Using the P-Space, *Mathematical Problems in Engineering*, 2016.
- [22] Zhang H, Structural Analysis of Bus Networks Using Indicators of Graph Theory and Complex Network Theory, *The Open Civil Engineering Journal*, 2017, **11(1)**: 92-100.
- [23] Trinh T T and Van Trinh C, Promoting Public Transport for Sustainable Urban Development in Hanoi, *Proceedings of the Eastern Asia Society for Transportation Studies*, 2015.
- [24] Li K, Small M, Wang K and Fu X, Three structural properties reflecting the synchronizability of complex networks, *Physical Review E*, 2009, **79(6)**: 067201-067204.
- [25] Wang J, Hou X, Li K and Ding Y, A novel weight neighborhood centrality algorithm for identifying influential spreaders in complex networks, *Physica A: Statistical Mechanics and its Applications*, 2017, **475**: 88-105.
- [26] Cohen R and Havlin S, *Complex networks: structure, robustness and function*, Cambridge University Press, 2010.
- [27] Boccaletti S, Latora V, Moreno Y, Chavez M and Hwang D, Complex networks: Structure and dynamics, *Physics reports*, 2006, **424(4)**: 175-308.
- [28] Ellens W and Kooij R E, Graph measures and network robustness, *arXiv preprint arXiv:1311.5064*, 2013.
- [29] Albert R, Jeong H and Barabasi A, Error and attack tolerance of complex networks, *Nature*, 2000, **406(6794)**: 378-832.
- [30] Crucitti P, Latora V, Marchiori M and Rapisarda A, Efficiency of scale-free networks: error and attack tolerance, *Physica A: Statistical Mechanics and its Applications*, 2003, **320**: 622-642.
- [31] Crucitti P, Latora V, Marchiori M and Rapisarda A, Error and attack tolerance of complex networks, *Physica A: Statistical Mechanics and its Applications*, 2004, **340(1)**: 388-394.
- [32] Jeong H, Tombor B, Albert R, Oltvai Z N and Barabasi A, The large-scale organization of metabolic networks, *Nature*, 2000, **407(6804)**: 651-654.
- [33] Broder A, Kumar R, Maghoul F, Raghavan P, Rajagopalan S, Stata R, Tomkins A and Wiener J, Graph structure in the web, *Computer networks*, 2000, **33(1)**: 309-320.
- [34] Wang X F and Chen G, Complex networks: small-world, scale-free and beyond, *IEEE circuits and systems magazine*, 2003, **3(1)**: 6-20.
- [35] Sole R V and Montoya J M, Complexity and fragility in ecological networks, *Proceedings Biological sciences*, 2001, **268(1480)**: 2039-2045.
- [36] Sydney A, Scoglio C, Schumm P and Kooij R E, Elasticity: topological characterization of

- robustness in complex networks, *Proceedings of the 3rd International Conference on Bio-Inspired Models of Network, Information and Computing Systems ICST (Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering)*, 2008, 19.
- [37] Latora V and Marchiori M, Is the Boston subway a small-world network?, *Physica A: Statistical Mechanics and its Applications*, 2002, **314(1)**: 109-113.
- [38] Latora V and Marchiori M, Efficient behavior of small-world networks, *Physical Review Letters*, 2001, **87(19)**: 198701.
- [39] Albert R and Barabasi A, Statistical mechanics of complex networks, *Reviews of modern physics*, 2002, **74(1)**: 47-101.
- [40] Blondel V D, Guillaume J, Lambiotte R and Lefebvre E, Fast unfolding of communities in large networks, *Journal of statistical mechanics: theory and experiment*, 2008, **2008(10)**: 10008.
- [41] Cohen R, Erez K, Ben-Avraham D and Havlin S, Breakdown of the Internet under intentional attack, *Physical Review Letters*, 2001, **86(16)**: 3682-3685.
- [42] Paul G, Tanizawa T, Havlin S and Stanley H E, Optimization of robustness of complex networks, *The European Physical Journal B-Condensed Matter and Complex Systems*, 2004, **38(2)**: 187-191.
- [43] Zhang J, Xu X, Hong L, Wang S and Fei Q, Networked analysis of the Shanghai subway network, in China, *Physica A: Statistical Mechanics and its Applications*, 2011, **390(23)**: 4562-4570.