

Complex network and fractal theory for the assessment of water distribution network resilience to pipe failures

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ABSTRACT

Water distribution networks (WDNs) must keep a proper level of service under a wide range of operational conditions, and, in particular, the analysis of their resilience to pipe failures is essential to improve their design and management. WDNs can be regarded as large sparse planar graphs showing fractal and complex network properties. In this paper, the relationship linking the geometrical and topological features of a WDN to its resilience to the failure of a pipe is investigated. Some innovative indices have been borrowed from fractal geometry and complex network theory to study WDNs. Considering all possible network configurations obtained by suppressing one link, the proposed indices are used to quantify the impact of pipe failure on the system's resilience. This approach aims to identify critical links, in terms of resilience, with the help of topological metrics only, and without recourse to hydraulic simulations, which require complex calibration processes and come with a computational burden. It is concluded that the proposed procedure, which has been successfully tested on two real WDNs located in southern Italy, can provide valuable information to water utilities about which pipes have a significant role in network performance, thus helping in their design, planning and management.

Key words | complex networks, fractal theory, resilience, topology, vulnerability, water distribution networks

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LIST OF SYMBOLS

APL	average path length	k_i	total number of connections of the i -th node
$APL_{(1/D)}$	inverse diameter-weighted average path length	l	pipe index
B_r	Bridge Ratio index	L_{TOT}	total pipes length [km]
C	number of total pipe failure combinations	m	total number of pipes
C^*	number of total pipe failure combinations that do not disconnect the network	m_d	number of pipes disconnecting the network
D_f	fractal dimension	n	total number of nodes
h^*	nodes design pressure [m]	n_n	total number of demand nodes
h_i	node hydraulic head [m]	n_R	total number of reservoirs
H_R	head of the j -th reservoir [m]	$N(r)$	number of square boxes of dimension r
i	node index	q_i	water demand at i -th node [m^3/s]
I_r	resilience index	Q_R	water flow entering through the R -th reservoir [m^3/s]
k	average node-degree	q	link density

r	dimension of square box
R	reservoirs index
WSN	water supply network
λ_2	algebraic connectivity

INTRODUCTION

Nowadays, improving the resilience of communities is one of the most important and universal societal issues. Different definitions of resilience are given in the literature, highlighting specific aspects based on the field of application (Holling 1973; Hamel & Välikangas 2003; Pickett *et al.* 2004; Folke 2006; Wieland & Wallenburg 2013).

Resilience is a typical feature of network systems where the redundancy in links, paths and loops allows systems to maintain and adapt their operational performance when some of their components fail or other unplanned and adverse conditions arise (Laprie 2005; Strigini 2012).

In relation to communal network infrastructure (e.g. telecommunications, transport, public services and power grids) which continuously require renovation (owing to either rising demand, changes in user profiles or age), it is possible to measure resilience in terms of energy, i.e. the system capacity to exploit the available energy with minimum dissipation in ordinary operational conditions, or in terms of the system redundancy, that allows the level of service to users to be maintained notwithstanding unplanned operational conditions including loss of nodes/links caused by failures or intentional damage (Cohen *et al.* 2001; Estrada 2006).

In relation to water distribution networks (WDNs), resilience is linked to the network geometry and topology based on the premise that a densely looped layout offers a system redundancies which are capable of overcoming operational perturbations (Mays 2000). These operational perturbations can range from variable water demand (Gargano & Pianese 2000), to breakages and leakage caused by deterioration of pipes and pumping units (Walski & Pelliccia 1982), to natural or man-made disasters (e.g. earthquakes, floods, vandalism) (Zohra *et al.* 2012). Furthermore, unplanned urban development without suitable design can lead to the modification of the topology of a WDN by simply adding

nodes and links in order to satisfy increased water demand. This can have unintended consequences for the system resilience.

The resilience of WDNs depends essentially on three aspects:

- (a) network geometry;
- (b) topological redundancy; and
- (c) energy redundancy.

The network geometry deals with the layout and extent of the network and the physical dimensions of its links and nodes (Csanyi & Szendrői 2004). Topological redundancy is related to the presence of independent alternative paths (usually looped), which connect the source with demand nodes, along which water can flow to satisfy supply requirements during interruption or failure of links (Goulter & Coals 1986). Energy redundancy is related to both topological redundancy (i.e. more paths imply more pipes to convey flow, thus reducing energy losses) and to geometrical redundancy (i.e. the use of conduit diameters, larger than those strictly needed to satisfy the required level of service for the users in terms of nodal pressure and water demand, provides additional capacity to respond to unplanned service conditions).

As a result of these multi-dependent factors, there is no single established approach or method to assign and to quantify the resilience of a WDN. The factors which are crucial to a network's resilience include the choice of pipe materials, pipe diameters and the levels of topological and energy redundancy which can be assessed through an in-depth analysis of pipe lengths, topological layout and loop number.

In this regard, it is clear that the resilience depends on the connectivity of the network and on the location of each single element (e.g. pipes, pumps, valves) and also on the complex network geometry produced by traditional design criteria, i.e. placing looped pipes under nearly every street. These complex geometries and topologies require innovative approaches for the analysis and management of a WDN.

Several concepts from graph theory (Konig 1936), fractal geometry (Mandelbrot 1982), social network theory (Freeman 2005) and, more recently, complex network theory (Watts & Strogatz 1998; Barabasi & Albert 1999)

have been applied to better analyze WDNs. This has led to the development of mathematical approaches and operational procedures that use innovative metrics and algorithms to better understand and describe, and then optimise and control, complex WDNs. In fact, many water supply systems consisting of up to tens of thousands of nodes and hundreds of looped paths can be considered as complex networks (Boccaletti et al. 2006). Recently, the fractal theory (Kowalski et al. 2014) and topological metrics (Yazdani & Jeffrey 2010; Gutiérrez-Perez et al. 2013; Herrera et al. 2016; Di Nardo et al. 2016a) were also used to analyse global resilience of a WDN.

Because the WDN resilience depends on different aspects, in this paper, attention is mainly focused on using innovative metrics to characterise the relationship between geometrical, topological and energy redundancy, in order to better assess and measure the resilience of WDNs. Specifically, the relationship between energy resilience and average path length (APL), used as a surrogate for a network topological measure, is investigated. Further, the feasibility of identifying critical links with the help of topological metrics only, and without recourse to hydraulic simulations, which require complex calibration procedures and come with a computational burden, is investigated. The proposed procedure allows identification, *a priori*, of the pipes on which the attention and the maintenance efforts should be focused to preserve network resilience.

The effectiveness of the procedure is tested on two real, medium-sized distribution networks serving two towns located near the city of Naples (Italy), namely Parete and Villaricca (Di Nardo & Di Natale 2011).

METHODS AND METRICS

The following section briefly describes the geometrical, topological and energy metrics used in this paper. Their properties and how they can express and measure global network redundancy is also described hereinafter. The adopted combinatorial methodology to analyse the vulnerability of a network to pipe failures is then described. This procedure is aimed at identifying a possible relationship between WDN topology and its resilience.

Geometrical metrics

Fractal theory is used to analyse the network geometry to establish if a self-similar spatial distribution of the elements (nodes and pipes) can be identified in the WDN layout, and if this assumption can help in the understanding of the redundancy of a water system. As is well known, one of the first definitions of fractals was proposed by Mandelbrot (1982), according to which an object is fractal if it can be indefinitely divided into parts that are similar to the whole object. The similarity may be exact, with the parts being precise copies of the entirety, or it may be approximate, where the subsets are similar to the whole, but not identical to it, which is typical of real systems (Mandelbrot 1982). In fractal theory, the self-similarity can be expressed with the fractal dimension, which is a measure of the complexity of an object observed at increasingly small scales. This can also be interpreted as the degree to which the object fills the Euclidean space that contains it (Sagan 1994). In this respect, it is expected that the fractal dimension of a planar layout of a WDN, which quantifies the complexity of its multiple connections, can provide information useful to quantify its resilience. The fractal dimension can be computed in different ways. In this paper the box-counting method for 2D objects (Falconer 1990; Peitgen et al. 1992) developed in a MatLab[®] tool on the basis of the work of Minkowski–Bouligand, has been used to define the relation:

$$N(r) \sim r^{D_f} \quad (1)$$

where $N(r)$ is the number of square boxes of r dimension necessary to cover the planar layout of the network, and D_f is the fractal dimension, according to the following formulation:

$$D_f = \lim_{r \rightarrow \infty} \frac{\log N(r)}{\log r} \quad (2)$$

The self-similarity, and so the fractal nature of the spatial distribution of nodes and links, can be seen as an indicator of the low reachability of the system (Csanyi & Szendrői 2004), typical of spatial networks with strong geographical constraints, as opposite to small world networks (Boccaletti et al. 2006).

In Figure 1, an example of two simple networks is shown, to illustrate the relationship between fractal dimension D_f – computed with the box counting method from $r = 16$ pixels to $r = 1$ pixel – and network topological resilience.

In particular, the example networks have the same number of nodes but different numbers of links, network A being less connected than network B. As reported in Figure 1, the value of D_f of network B, equal to 1.547, is higher than the D_f of network A (1.381), indicating that the less connected network also has less 2D Euclidean space covering. So, the fractal dimension can be used as a surrogate of topological resilience, measuring, at the same time, topological redundancy and space covering.

Topological metrics

In recent years, in order to describe network behaviour and to characterise some of their general properties, innovative

topological metrics have been proposed in the literature on complex networks theory (Boccaletti *et al.* 2006). Interesting applications to WDNs have confirmed their suitability for the investigation of vulnerability (Yazdani & Jeffrey 2010; Yazdani & Jeffrey 2011) and water network partitioning (Herrera *et al.* 2012; Di Nardo *et al.* 2015, 2016b, 2017).

The following metrics are used in this paper:

- (1) the link density, q , which expresses the ratio between the total number of network edges and the number of edges of a globally coupled network with the same number n of nodes, thus providing a measurement of network redundancy;
- (2) the average node-degree, k , which is the average of the degrees k_i of all nodes (with k_i being the total number of connections of the generic i -th node to other nodes), which expresses how nodes are interconnected, thus measuring the topological redundancy of a network;

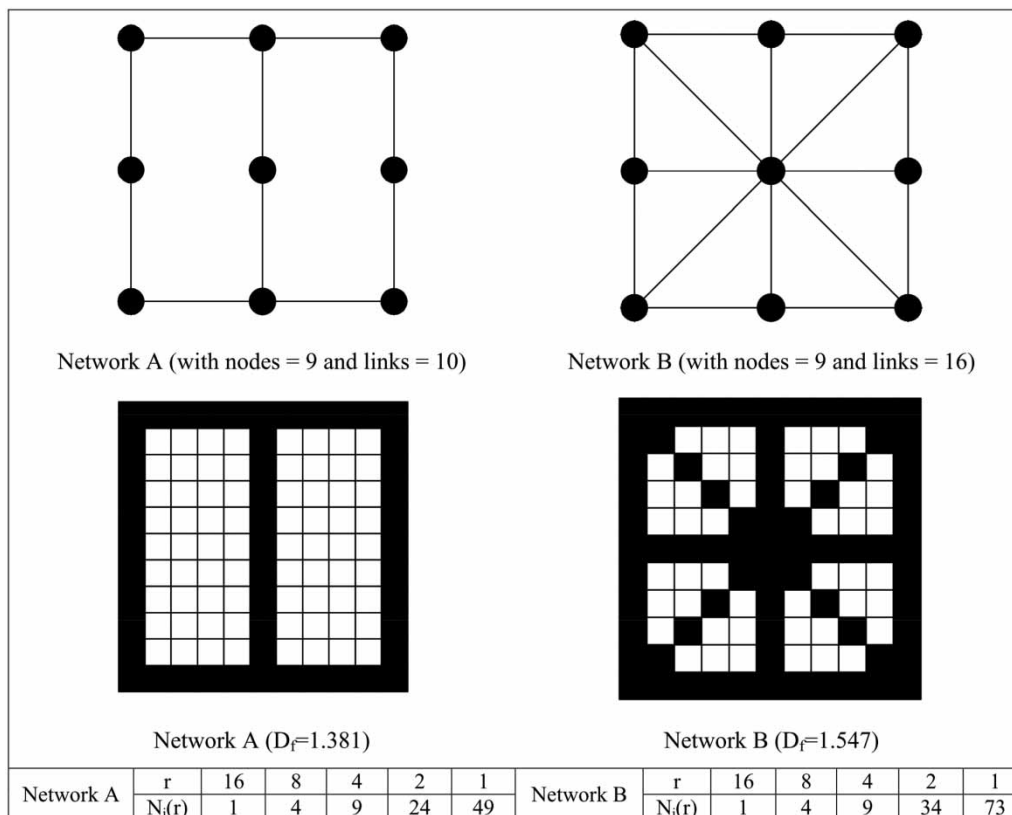


Figure 1 | Example of the box counting method to compute the fractal dimension D_f for two different networks.

- (3) the *APL*, which is the average number of steps along the shortest paths between all possible pairs of nodes in the network, which determines the average degree of separation between any pair of nodes (high values of *APL* indicate a much fragmented network (Guest & Namey 2014)); and
- (4) the algebraic connectivity, λ_2 , i.e. the second smallest eigenvalue of the Laplacian of the graph adjacency matrix (Fiedler, 1973), which quantifies the strength of network connections (a high value of algebraic connectivity indicates the tolerance of the network to link faults and, consequently, its resilience against efforts to cut it into parts (Yazdani & Jeffrey 2010)).

These indices are used generally to analyse both the redundancy and the resilience of a complex network (Yazdani & Jeffrey 2010).

It is important to highlight that, like many other real networks (i.e. ties between individuals in social networks, uneven fluxes in metabolic reaction pathways, different capabilities of transmitting electrical signals in neural networks, unequal traffic on the Internet (Boccaletti et al. 2006)), WDNs exhibit large heterogeneity in the intensity of the connections between nodes. In this regard, WDNs are better studied as weighted networks, in which each link has a numerical value measuring the 'strength of the connection'. Consequently, a weighted average path length, $APL_{1/D}$, is introduced, adopting the inverse of pipe diameter as weight, in order to take into account one of the most important hydraulic characteristics of the links between nodes. In this way, the shortest path between each pair of nodes is defined as the minimum sum of the inverse of the diameters D of the pipes linking them, in order to identify the path with minimum resistance to water flow.

Failure metrics

As reported in Creaco et al. (2016), failure metrics in WDN can be divided into two main categories: pressure head/energy related indices (Gessler & Walski 1985; Todini 2000; Prasad et al. 2003), and flow path related indices (Tanyimboh & Templeman 2000; Tanyimboh et al. 2011). Both categories of indices aim to measure the WDN

global behaviour under local failures of one or more components or unplanned changes.

In this paper two concepts are used to express WDN resilience to pipe failure, namely:

- (1) how much the network preserves the connectivity (flow path related index); and
- (2) how much the network preserves a pressure head surplus (pressure head/energy related index).

In the first case, when considering all the possible combinations of single pipe failure, some cases lead to network disconnection. The Bridge Ratio index is introduced to quantify this aspect,

$$B_r = \frac{m_d}{m} \quad (3)$$

as the ratio between the number of pipes, the failure of which leads to disconnection of the network, m_d (the value of the corresponding $APL_{1/D}$ tends to infinite) and the total number of pipes, m .

In the second case, a common method to measure the hydraulic resilience, which is the ability of a network to maintain supply under failure or unplanned conditions, was proposed by Todini (2000) in the form of the resilience index I_r , which is a measure of the available surplus of energy:

$$I_r = \frac{\sum_{i=1}^{n_n} q_i (h_i - h^*)}{\sum_{R=1}^{n_r} Q_R H_R - \sum_{i=1}^{n_n} q_i h^*} \quad (4)$$

where n_n is the number of demand nodes, n_r the number of reservoirs, q_i and h_i the demand and the head of the i -th node, Q_R and H_R the discharge and the pressure head of the generic R -th source point, and h^* the design pressure head of the network.

Combinatorial analysis of pipe failures

In order to evaluate the impact of pipe failures on network resilience, a combinatorial analysis of all possible network layouts, C , obtained by interrupting one pipe (Greco et al. 2012) was carried out. The hydraulic simulation of every configuration was carried out using Demand Driven Analysis

(Todini 2000) under peak conditions. The procedure consisted of the following steps:

1. Close a pipe l .
2. Evaluate $APL_{l/D}$.
3. Carry out network hydraulic simulation and evaluate the resilience index I_r .
4. Repeat the procedure, opening the pipe l and closing the pipe $l + 1$.
5. Stop the procedure when all possible network layouts C have been investigated.

At the end of the procedure, the topological and energy resilience for all combinations were compared, in order to identify similar trends and so to establish a possible relationship which could help to identify, *a priori*, the worst single pipe failures, without carrying out hydraulic simulations.

It is worth highlighting that, in this study, the combinatorial analysis and hydraulic simulations to compute Todini's index were carried out only to find a possible relationship between weighted $APL_{l/D}$ and I_r . Clearly, thanks to the identified relationship, operators can identify, *a priori*, and without any hydraulic simulation, but only with the calculation of weighted $APL_{l/D}$, the pipes on which attention and maintenance efforts should be focused to best preserve network resilience.

CASE STUDY

The proposed procedure was tested on two real, medium-sized WDNs, serving the towns of Villaricca and Parete, both located near Naples (Italy), with about 30,000 and 11,000 inhabitants, respectively (Di Nardo & Di Natale 2011). Figure 2 shows the layout of the two WDNs, while Table 1 summarises their main characteristics.

The comparison of the two networks sketched in Figure 2 shows that, although the WDNs have roughly the same total pipe length (about 30 km), they have layouts which are significantly different both topologically and geometrically.

The geometrical characteristics of the two networks (spatial shape and extension, pipe orientations, etc.) are very different and, clearly, they cannot be described exhaustively with only a visual analysis or Euclidean geometry and therefore require more sophisticated metrics. In this regard, the fractal dimension of the two networks was evaluated using the box-counting method on image files plotted at the same scale, with a number of pixels equal to 6357×4490 (Villaricca) and 2649×1871 (Parete). In Figure 3, the log-log plot shows the number of boxes $N(r)$ of size r as a function of r for the two case studies. The linearity of the two curves (in log-log space) indicates that the networks exhibit some fractal behaviour, with a fractal dimension (i.e. the slope of the straight line in the log-log plot) equal

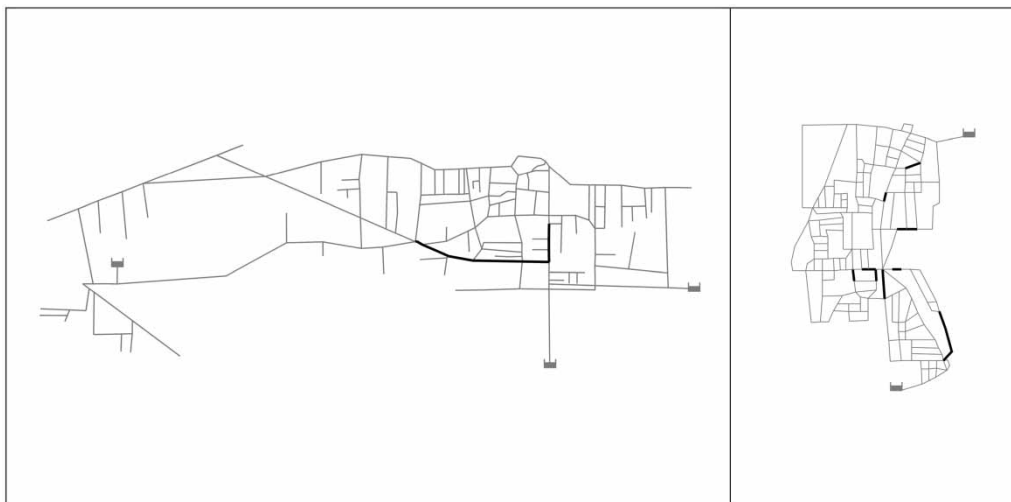
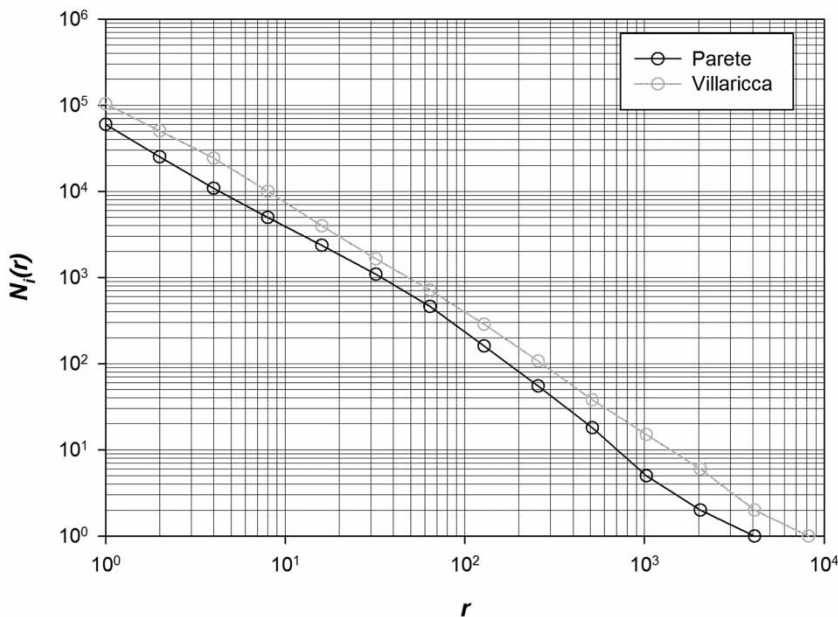


Figure 2 | Sketches of the WDNs of Villaricca (left) and Parete (right) highlighting, with bold lines, the 10 most important pipes (in terms of $APL_{l/D}$). Note that the two networks are plotted at the same scale.

Table 1 | Main characteristics of the WDNs of Villaricca and Parete

WDN	Nodes	Links	Sources	Total length	Link density	Average node-degree	Average path length	Algebraic connectivity	Todini's resilience
	n [-]	m [-]	n_R [-]	L_{TOT} [km]	q [%]	k [-]	APL [-]	λ_2 [-]	I_r [-]
Villaricca	196	241	3	34.71	1.26	2.42	11.29	0.006	0.442
Parete	182	282	2	32.44	1.67	3.05	8.80	0.021	0.351

**Figure 3** | Power-law relationship between the number of boxes, N , and the size of the box, r , in the box-counting method used to estimate the fractal dimension of the two WDNs.

to $D_f = 1.348$ for Villaricca and $D_f = 1.484$ for Parete. In both cases, the smallest size of box-counting corresponds to the dimension of a single pixel (about 79.4×79.4 cm in real scale). The slope of the straight line has been evaluated in the range $10 < r < 10^3$. The lower bound and the upper bound of such a range are close, respectively, to the minimum and maximum length of pipes. In other terms, with $r = 10$ a box can include at most one pipe, while with $r = 10^3$ a box can contain a set of network pipes including the pipe with the maximum length.

As indicated in Figure 3, the fractal dimension D_f for Parete is larger than that of Villaricca in the range $10 < r < 10^3$. This result confirms some intuitive observations from visual analysis of system layouts, namely that the nodes of the network of Parete are more interconnected in

all directions than those of Villaricca. However, the fractal dimension allows these geometrical differences to be quantified. A relationship exists between geometrical and topological features, as reported in Csanyi & Szendrői (2004). Indeed, the APL in a fractal system is proportional to the number of nodes n through the inverse of D_f , $APL \sim n^{1/D_f}$. Thus, the estimated fractal dimensions are consistent with the values of APL , which for the network of Parete is lower than for Villaricca.

However, the fractal dimension does not catch all network characteristics. The other computed topological metrics confirmed the higher topological resilience of the Parete WDN compared to Villaricca. In fact, the Parete WDN exhibits higher values of q , k and λ_2 and a lower value of APL , and thus greater connectivity, and so it is

expected a higher strength against efforts to cut it into parts.

Further interesting results can be found from the combinatorial analysis obtained by considering all the possible combinations C of a single pipe failure, with $C = 241$ for Villaricca and $C = 282$ for Parete.

The Bridge Ratio indices for the two WDNs were evaluated giving $B_r = 0.245$ for Villaricca and $B_r = 0.007$ for Parete which confirmed that the Parete network is more resilient than that of Villaricca which suffers more disconnections.

The different metrics used in this study were analyzed to better understand their potential relationship with resilience to pipe failures. In particular, the link density and the average node-degree, although they are measures of network topological redundancy, undergo the same change after the failure of any single pipe; the spectral connectivity, that quantifies the strength of network connections, shows little sensitivity to single pipe failures. Conversely, the APL results were highly sensitive, suggesting further investigation of the possible relationship between the weighted APL and Todini's energy resilience would be of value.

So, a comparison was undertaken between the weighted average path length $APL_{1/D}$ and the Todini

resilience index I_r , only for the layout combinations that do not lead to disconnection of the graph (respectively $C^* = 182$ for Villaricca and $C^* = 280$ for Parete). These results are plotted in Figures 4 and 5. Failures which disconnect the networks have been considered as the worst cases, but also as trivial solutions.

The ascending ordered values of $APL_{1/D}$ for Villaricca and Parete are plotted in Figure 4. The x -axis shows the percentage of the total number of pipes. The plot of Villaricca appears shorter, because about 24% of the pipe failures lead to disconnection of the network, which causes the $APL_{1/D}$ to have an infinite value. Conversely, only 0.7% of pipe failures cause disconnection of the network of Parete. With the same order of the layouts used in Figure 4, the values of I_r are plotted in Figure 5.

From the two figures, it looks clear that the Parete WDN is more resilient than that of Villaricca. In fact, in the first case, about 70% of the failed-pipe layouts do not affect the original values of $APL_{1/D}$ of the network, while for Villaricca significant variations of $APL_{1/D}$ occur for nearly 40% of the layouts. Furthermore, although in both cases about 30% of the combinations hardly worsen the energy resilience I_r , it is clear that the energy redundancy of the network of Villaricca results in greater sensitivity to pipe

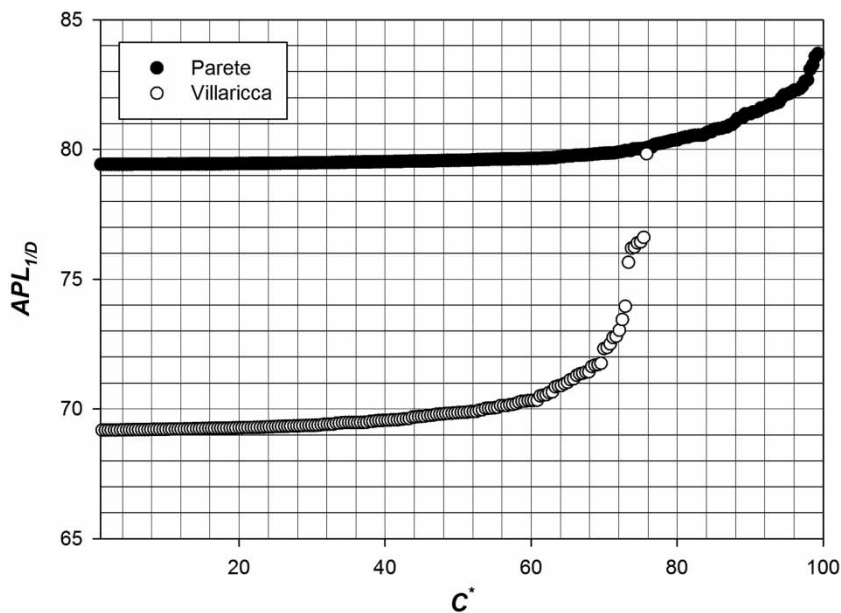


Figure 4 | Ascending-ordered APL , weighted according to the inverse of pipe diameter, $APL_{1/D}$, of all layout combinations C^* resulting from the failure of a single pipe, for the water distribution networks for Parete and Villaricca.

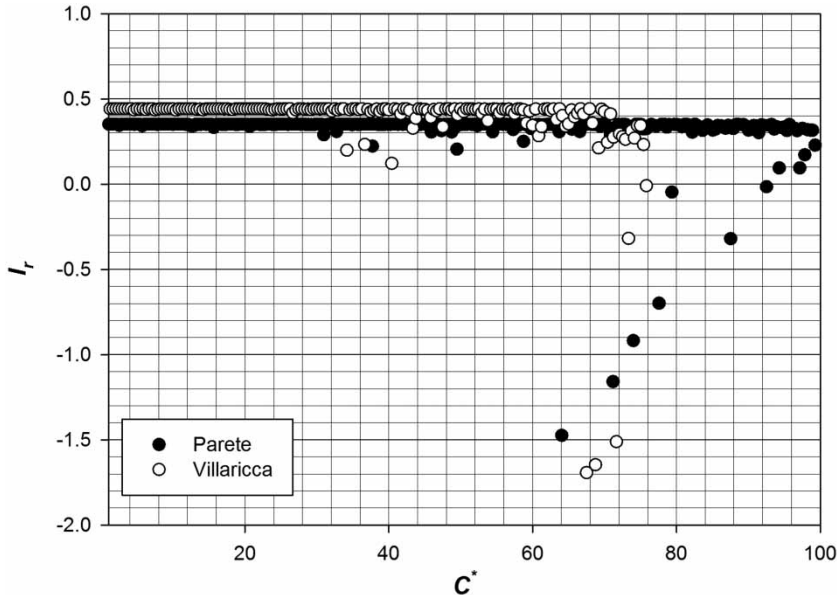


Figure 5 | Energy resilience I_r of all layout combinations C^* resulting from the failure of a single pipe, for the water distribution networks for Parete and Villaricca. The layout combinations are ordered as in Figure 4.

failures. This confirms the indications given by the various metrics reported in Table 1.

In the right part of the graphs, the performances are significantly worsened from both a topological and energy point of view, showing an interesting relationship between $APL_{1/D}$ and I_r . Therefore, a preliminary identification of the pipe failures, which are crucial for the hydraulic performance of the network, can be achieved using only topological information. Such a result is not trivial, as some of these pipe failures do not imply disconnection of the network (easy to compute). It is therefore possible to identify, without time-consuming hydraulic simulations, those pipes on which maintenance efforts should be concentrated to reduce costs and to maintain resilience.

Finally, in Figure 2, the 10 most important pipes in terms of the $APL_{1/D}$ for each network are highlighted in bold lines. This visualization allows better understanding of the effectiveness of the proposed procedure because, while some important pipes are closer to reservoirs, as expected, others are located far from the sources. Furthermore, Figure 2 shows that, without hydraulic simulations, but only using a topological metric such as $APL_{1/D}$, it is possible to recognize, besides some important pipes, also the most critical paths (formed by consecutive pipes) for the water

distribution system, thus providing valuable management information to water utilities.

CONCLUSIONS

This paper presents a novel procedure to assess the resilience of a water distribution network to failure of a single pipe based on the use of fractal and topological metrics. The effectiveness of the proposed methodology was tested on two real, medium-sized, WDNs. The fractal analysis indicated that the geometrical shape of the planar layout of both networks met the formal conditions of fractal sets. This result was expected, as WDNs are geographically constrained, which implies that, as connections occur mainly between immediately neighbouring nodes, water flow may be significantly affected by the interruption of single links. The relationships between geometrical and topological metrics and between topological metrics and energy resilience were also investigated. To this aim, the APL , weighted with the inverse of pipe diameter, was introduced. The results of a combinatorial analysis of all possible single pipe failures showed that this novel metric easily allowed identification, without hydraulic simulations, of the most

important pipes and crucial paths (composed of more pipes), the failure of which causes a significant decline of both the topological and hydraulic resilience of the network. The obtained relationship between topology and energy resilience provides water utilities with an effective and easy tool to improve WDN management.

This result represents a novelty compared to other studies on the global resilience of WDNs carried out by complex network metrics. In fact, the weighted topological metric adopted in this paper provides useful information, allowing the direct identification of pipes most likely affecting the energy resilience and, consequently, the hydraulic performance and the level of service offered to users. Further studies will investigate this relationship using other geometrically weighted topological metrics, in order to identify the hydraulic characteristics of pipes most suitable to be used as a proxy for network resilience. In addition, the possibility of incorporating the dependence of resilience on the probability of failure of a pipe (i.e. related to pipe material, age, etc.) will be investigated. Finally, the procedure will be tested on larger WDNs to further validate the results.

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