Complex Network Metrology

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Abstract

In order to study some complex networks like the Internet, the Web, social networks or biological networks, one first has to explore them. This gives a partial and biased view of the real object, which is generally assumed to be representative of the whole. However, up to now nobody knows how and how much the measure influences the results.

Using the example of the Internet and a rough model of its exploration process, we show that the way a given complex network is explored may strongly influence the observed properties. This leads us to argue for the necessity of developing a science of metrology of complex networks. Its aim would be to study how the partial and biased view of a network relates to the properties of the whole network.

Introduction.

Some complex networks of high interest can only be known after an exploration process. This is in particular true for the Internet (interconnection of computers), the Web (links between pages), social networks (acquaintance relations for example), and biological networks (brain topology or protein interactions for example). There have been many studies published on these objects, see for instance [3, 6, 8, 10, 16, 17, 19, 21, 22, 23, 26]. Most of them rely on from partial views obtained using various, and often intricate, exploration methods. Until now, the approach generally used is to obtain views as large as possible and then assume that they are (representative of) the whole, see for instance [12, 15, 20, 28]. However, except in a few limited cases [12, 18, 27], nobody has any idea on the bias introduced by the partial exploration methods and the influence it may have on the results.

We show here that this bias may be very important, even under some very optimistic assumptions. Using the representative example of the Internet topology, we show how some natural models of the exploration process give very different views of a given network, which proves that the way one explores a complex network has a strong influence of the properties of the obtained view. We therefore insist on the necessity of developing a theory of complex network metrology Its aim would be to study how the partial and biased view of a network relates to the properties of the whole network.

Our global approach is the following: we consider a (known) network G, we simulate an exploration of this network to obtain a *view* G' of it, and then we compare the two objects.

The final aim is to deduce properties of G from properties of G'. In this communication, we only make a first step in the direction of this ambitious objective, but we will see that it is enough to prove its validity and relevance, which is our aim. In order to do this, we will first present the way the Internet topology is explored, then we will introduce very simple and natural models to simulate this and finally discuss the obtained results. Let us insist on the fact that this global approach is absolutely general, and may be applied to other cases (like the Web, social networks or biological networks) with benefit.

Exploring the Internet.

Many operators and administrations act on the Internet topology in a totally distributed way. There is no central decision on what is done on the Internet, and no central knowledge of its topology. And yet, it plays an important role in many contexts like the robustness of the network, see for instance [4].

There are various ways to retrieve some data on the Internet topology from publicly available data. They give a (partial) view of the global topology. Moreover, the available information is influenced by many parameters (economical, technical, political, etc.) which may introduce a *bias* in the sample we get. This is however the unique method one has to know this topology. This is what we call *exploring the Internet*.

There exist various methods and many heuristics to explore extensively the Internet. We will not enter in the details of these techniques here but will concentrate on one of the two main. This restriction is motivated both by the fact that very large explorations of the Internet have indeed been conducted using this method, see for instance [10, 12, 15, 20, 28], and that it is quite easy to model whereas other methods are much less precisely defined.

We will concentrate on the exploration of the Internet using only the `traceroute` tool. It is a simple program which, used from a *source* computer, gives the path followed by messages from this source to a *destination* computer on the Internet. This path is a set of nodes and links of the network, which can be seen as a (small) part of the Internet topology. Using this tool extensively, one can obtain large parts of the whole topology.

Notice that, in order to use `traceroute`, one has to run the program *on* the source computer. On the contrary, nothing specific is needed at the destination and so one can choose any destination. Therefore, if one uses `traceroute` to explore the Internet, the number of sources used is generally very limited (typically a few dozens) whereas the number of destinations may be huge (typically several hundreds of thousands), see for instance [12, 15, 28]. Notice also that, if one explores the Internet from one source, one cannot obtain a perfect view of the whole, even if it uses `traceroute` to every possible destination. Indeed, there are some links which will never be crossed by any message from the source. Moreover, due to bandwidth, knowledge and time limitations, one can never use `traceroute` to every possible destination. How many destinations should one consider? How many sources are needed? Up to now, no one has any idea of the answers to these questions, but we propose a step towards them below.

Modeling.

We want to simulate an exploration process. In order to do this, we first need a network to explore. There are several natural choices for this. One can for instance obtain the *real* topology of a large computer network provided by a firm. One can also use one of the various models proposed to generate random networks, for instance in [1, 2, 7, 14, 13, 24, 25, 30]. It has been shown recently that the Internet topology, like many other complex networks, has specific statistical properties, see for instance [10, 29]. However, in this paper we are mostly concerned by the *exploration process*. Therefore, we will choose the most simple and well known model of random networks [9, 5] to generate the topology to explore: the Erdös and Rényi random graph model. This model has two parameters: the number of nodes, n, and the probability of existence of any link, p. A network is then generated by considering that each possible pair of nodes is linked with probability p. This gives an expected number of links $m = p \cdot \frac{n \cdot (n-1)}{2}$. Notice that this model is not the more realistic one, but it is sufficient for the purpose of this paper.

The `traceroute` tool gives the path followed by messages from a source to a destination. Up to now, very little is known on the properties of such paths, see [11] and the references therein. For instance, one may suppose that the aim of network protocols is to deliver information efficiently, and so that the paths they follow are shortest paths (paths of minimal length). It is however known that this is not always the case, but no precise information is currently available on how they differ from shortest paths [11]. Moreover, there exist in general many shortest paths for a given pair of computers, and there is no *a priori* reason for `traceroute` to give one of them rather than another. Finally, the paths change during time but again very few is known on their dynamics.

In the current state of our knowledge, designing a realistic model of `traceroute` is therefore impossible. The assumption usually made is that `traceroute` always gives a shortest path, which will actually be sufficient for our current aim. We will also consider that, during the exploration process, one may use `traceroute` many times, which lead to the discovery of *all* the shortest paths between given sources and destinations.

We have a model to generate the network to explore, and some models for the `traceroute` tool. We now need a model for the exploration process itself. As already noticed, we will suppose that it only relies on `traceroute`. But this is not sufficient: we must say how we will choose sources and destinations, and how many of them we will consider. Our aim being to show that the exploration method may influence the obtained view of the actual network, we will consider several realistic models of the exploration. Again, we will only consider the simplest ones, which is sufficient for our purpose. Since it is the case in practice, we will suppose that the exploration process is based on one or a few sources, and uses many or all the possible destinations. Moreover, we will suppose that the sources and destinations are chosen randomly, which makes sense since the networks we explore are totally random (and so all the nodes play similar roles).

Let us insist on the fact that, to make a complete study of the influence of the exploration process on the view we obtain, one would actually have to consider many models, both for the network to explore, for the `traceroute` behaviour, and for the exploration

method. Therefore, one obtains several dozens of triples of models to consider, and for which experiments and comparisons should be conducted. However, this is not our aim here. We only want to show that the exploration method indeed influences the results. To achieve this, as we will see in the following, it is sufficient to consider a few simple cases.

Finally, the models we use in the following are very simple. The network to explore is produced by the classical random network model, which gives a network of n nodes where each link exists with probability p. We will always suppose that `traceroute` gives shortest paths, but we will consider both the case where it gives *one* shortest path and the case where it gives *all* of them. Finally, we will consider a varying number of sources and destinations from one to a few for the sources and many to all for the destinations, which reflects the values used in practice. We explained above why all these choices are reasonable considering our aim, but clearly many others would be relevant too.

All the values we will plot are averaged over 1 000 instances. The variance is in general neglectible (we plotted it in the case of Figure 2). The shortest path computations are done using breadthfirst search.

How much do we see?

We now consider a random network G in which each link exists with probability p. We will make explorations of G using the various models explained above. We first consider that we use only one source, chosen at random, and then consider the case with several sources. All the experiments are conducted with two models of `traceroute`, the USP model (where we discover a Unique Shortest Path between each pair of source and destination), and the ASP model (where we discover All the Shortest Paths for each pair). The plots are averaged over one thousand runs.

Unique source.

Let us denote by $G_u(x)$ the view of G obtained from a given source if we consider x random destinations, with the USP model for `traceroute`. Let $n_u(x)$ be the number of nodes of this view, and $m_u(x)$ its number of links. Similarly, we introduce $G_a(x)$, $n_a(x)$ and $m_a(x)$ the results obtained with the ASP model for `traceroute`. The plots of these functions, Figure 1, show how much of the network we obtain, both in terms of nodes and links, as a function of the number of destinations.

At various points, these plots fit well the intuition. First, when we consider very few destinations, we obtain a very small part of the network. Then, if the number of destination grows, we see more and more. Finally, we of course see all the nodes when we consider each of them as a destination.

There are however a few remarkable facts. Both $n_u(x)$ and $n_a(x)$ grow rapidly and reach a critical point where they start a linear growth, but the initial growth of $n_a(x)$ is much more rapid than the one of $n_u(x)$. On the contrary, $m_u(x)$ and $m_a(x)$ grows linearly from the beginning, but the maximal values they reach, $m_u(n)$ and $m_a(n)$, remain

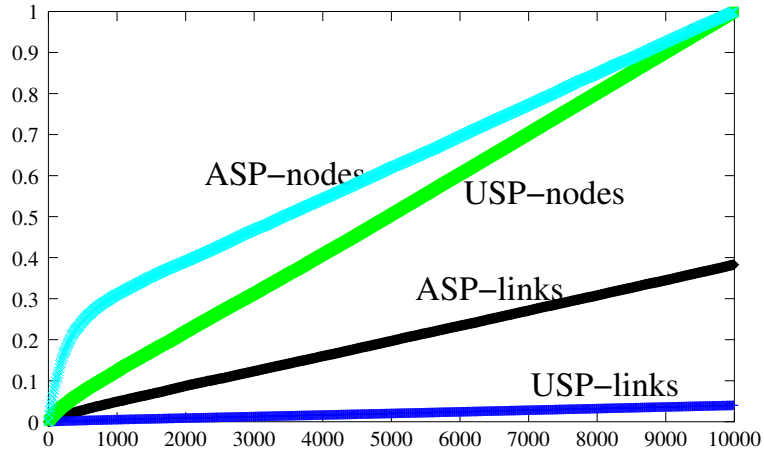

Figure 1: *Ratio of the total number of nodes and links discovered during an exploration, as a function of the number of destinations. These plots correspond to a random network with* $n = 10\,000$ *and* $p = 0.005$*, which gives an average degree in accordance with what is generaly assumed for the Internet topology.*

surprisingly low. It means that the exploration misses many links, even if we consider all the possible destinations, which indicates that the obtained view is very incomplete. This is even more surprising when we consider the optimistic case where *all* the shortest paths are discovered, and all the nodes are used as destinations.

These behaviours are similar for any values of n and p (the plots presented in Figure 1 always have the same shape). However, the maximal value reached by $m_u(x)$ and $m_a(x)$, *i.e.* the maximal proportion of discovered links, varies with the probability p of existence of any link. To know how p influences these values, let us study the proportion of links discovered using one source and all the possible destinations, as a function of p. They are plotted in Figure 2 for the two models of `traceroute` we consider.

The two plots have some properties in common which can be easily explained. First notice that below a certain value of p, the network is not connected (it is composed of many independent parts) [5]. Therefore, below this threshold, any exploration using a small number of sources will give a very small part of the whole. When the network becomes connected, it is almost a tree, in which there is a unique path from the source to each node. Therefore, the two exploration methods we consider discover almost all the links, which corresponds to the maximal values reached by the plots in Figure 2. On the opposite, when p is almost 1, then almost every possible link exists, and so almost every node is at distance 1 from the source. Therefore, the obtained view, both with the USP and with the ASP model, is almost a star. It therefore contains almost $n-1$ links, which, compared to the total number of links, almost $\frac{n\cdot(n-1)}{2}$, is negligible.

The plot for the USP model is easy to understand. Indeed, the exploration using this model gives a tree (it has no cycle), and therefore it contains exactly $n-1$ links if p is above $\frac{\log(n)}{n}$ since in this case the network is (almost surely) connected. The expected total number of links being itself $m = p \cdot \frac{n\cdot(n-1)}{2}$, the ratio between the number of links

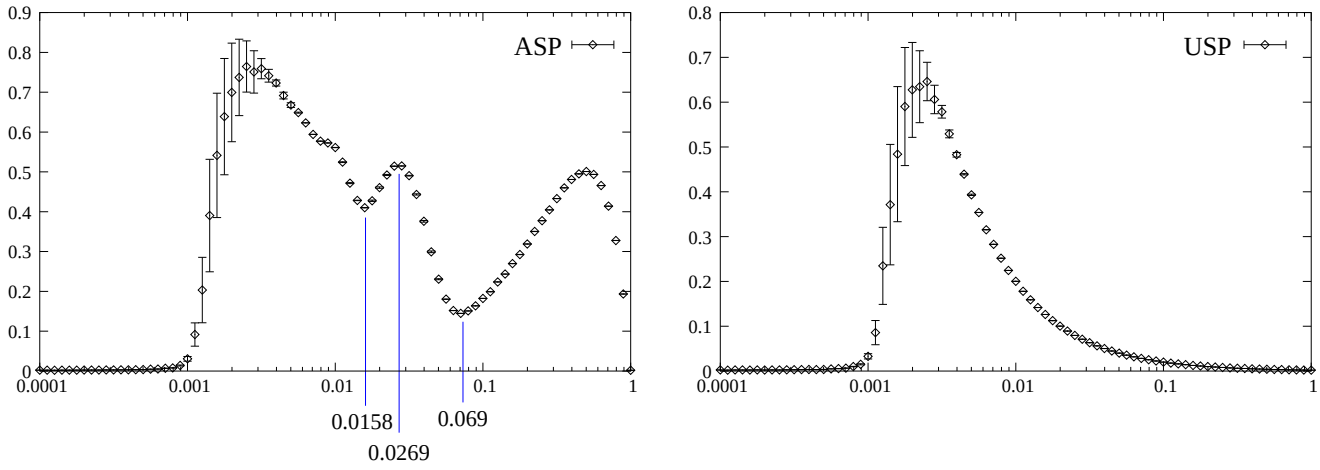

Figure 2: *Proportion of discovered links (one source, all the destinations) as a function of p for random graphs with $n = 1\,000$. Left:* ASP*; right:* USP*. The plots are the average over* $1\,000$ *instances, and the variance is displayed (it is neglectible everywhere except at the connectivity threshold). The plot obtained in the* ASP *case has a surprising shape, leading to the name* camel *plot.*

discovered during the exploration and the total number of links is then $\frac{n-1}{m} = \frac{2}{p \cdot n}$. When p grows, this ratio decays as $\frac{1}{p}$, which is confirmed by the simulation.

On the contrary, the irregular shape of the plot for the ASP model is very surprising: it has many peaks and valleys of high amplitude, which have no obvious interpretation. This is so surprising that we will name it the *camel* plot. There is however a natural explanation of this shape, which comes from specific properties of the exploration.

The *camel* plot.

Let us first characterize the links missed during the exploration. If a link is on a shortest path from the source to any other node then it is discovered, since *all* shortest paths to *all* nodes are discovered. Conversely, if a link is discovered during the exploration, it has to be on a shortest path. Therefore, we miss precisely the links which are on no shortest path from the source to any other node. These links are exactly the ones between nodes at equal distance from the source. In other words, the function plotted in Figure 2 is nothing but m minus the number of links between nodes equidistant from the source, over m.

Now let us consider the number of such links. To do this, we consider the distribution of the distances from the source. As shown in Figure 3, this distribution is centered around its mean value, which decays when p grows. This is not surprising, and notice that it has the same global shape independently of p. So, how can it help in understanding the *camel* plot? The point is that we have to consider the *discrete* distribution of the distances from the source, also displayed in Figure 3. Since distances are integers, these discrete distributions are the *actual* distributions. But when we consider a discrete distance distribution, two

cases may occur: the mean distance (or the distance for which the continuous distribution is maximal) can be close to an integer or it can be well centered between two integers. In the first case, almost all the nodes will be at this distance from the source, while in the second case almost half of them will be at some distance from the source and the other half at this distance plus one. These two cases are illustrated in Figure 3 (first case for $p = 0.0158$ and $p = 0.069$, second case for $p = 0.0263$). Recall that we miss the links between nodes at the same distance from the source. Therefore, when most nodes are at the same distance from the source, we miss many links, much more than in the other case. Since the average distance decays when p grows, there is an alternate series of such phases, which correspond to the peaks and valleys of the *camel* plot [1].

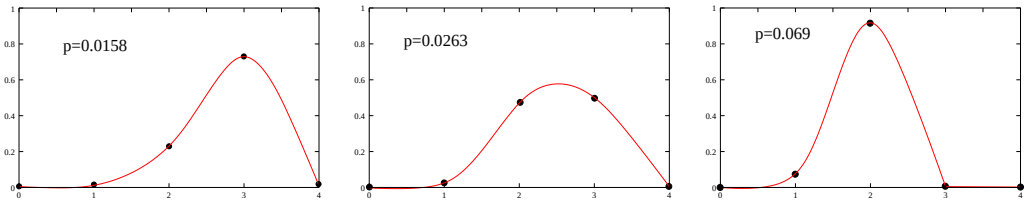

Figure 3: Distance distribution from the source for random networks ($n = 1\,000$ nodes) with various links densities p. The distribution is centered around the mean distance, which decays smoothly as p grows.

These first results clearly show that even very simple properties like the ratio of discovered links cannot be easily derived from a partial view of the network. Indeed, the efficiency of the exploration method varies a lot with network properties like density of links, and, more surprisingly, small variations in these properties may have a strong impact on the exploration significance.

Several sources.

Until now, we have restricted ourselves to explorations using only one source. However, in practical cases, one generally uses several, but few, sources. We investigate here how this may influence the quality of the view we obtain. Again, we only concentrate on the ratio of the total number of discovered links, which previous remarks have shown to be essential.

Figure 4 shows the evolution of this ratio when the number of sources is increasing. Let us first consider the two topmost plots, which correspond to the cases where we use all the possible destinations. As expected, the quality of the view grows rapidly with the number of sources, and one may even be surprised by the rapidity of this growth. Despite our model of Internet exploration is very rough, one may consider this plot as good news since it indicates that one does not need many sources to obtain accurate views of the

[1] We checked this by computing the distance distributions of graphs and then the number of links between two nodes at the same distance from the source. The obtained results fit exactly the *camel* plot.

network. This is important since it is very difficult (and never done) to use many sources in practice.

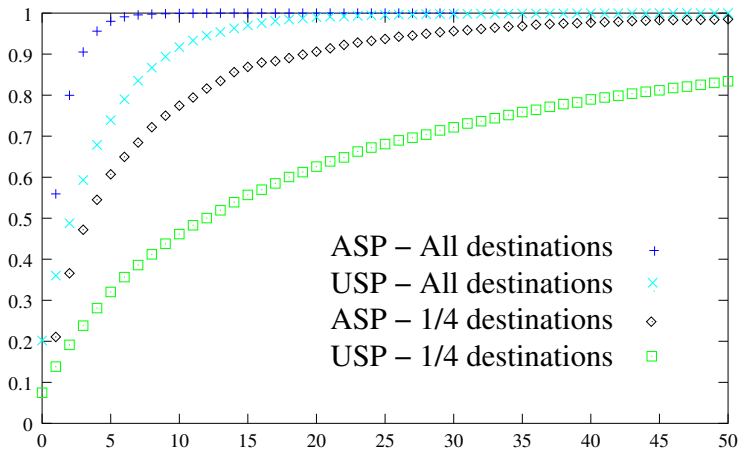

Figure 4: *Variation of the amount of discovered links as a function of the number of sources, in two cases: if all the nodes are destinations, and if only a quarter of them are. This plot corresponds to* $n = 2000$ *and* $p = 0.005$*, which leads to the conclusion that* $50 = 2.5\%$ *of the nodes should be used as sources. This is much more than usually done for the Internet.*

However, the assumption that *all* the nodes of the network serve as destinations is very rough. It is difficult to give an estimation of the number of nodes which actually contribute as destinations, but we can for instance suppose that only a quarter of them do, which is already huge. We then obtain the two other plots of Figure 4. Whereas the previous ones made us relatively optimistic, these ones show that quite a lot of sources are necessary to obtain an accurate view of the whole.

All these experiments cannot lead to conclusions concerning the exploration of the Internet itself. They show however that very reasonable hypothesis (in the limited state of our current knowledge) on the exploration process lead to qualitatively different results, which gives an evidence of the importance of taking it into account.

Conclusion.

In this communication, we considered the simplest possible question concerning the quality of a network view obtained by an exploration of a real network: the amount of the total number of nodes and links we obtain. Making natural variations on the way we model the Internet exploration, we show that this amount varies a lot and is very difficult to estimate.

Other properties, like the degree distribution or the clustering, are also biased by the exploration process. Moreover, as discussed, many models are possible for the exploration process, and we presented only the few simplest ones here. However, the results we have

presented are representative of what happens in all other cases and are sufficient for our purpose. This, added to their simplicity, is why we chose them to illustrate our arguments.

Let us insist once more on the fact that the results presented here do not provide any information on the Internet topology itself. They do not even give any information on how much, and how, the known results on the Internet topology are biased by the partial exploration process. Instead, they give evidences for the fact that this bias exists and may be very important. This fact is very general and can be proved in a similar fashion for the Web graph, various social or biological networks, and other complex networks.

We therefore argue that there is a need for the development of a new area of scientific activity, focused on complex network metrology. Results in this area are highly needed as they would make it possible to give rigorous results on a variety of complex networks which can not be studied directly. We suspect that this is actually the case of most complex networks, ranging from social to biological networks, including computer networks.

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