

Complexes of Graph Homomorphisms

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Combinatorics, in particular graph theory, has a rich history of being a domain of successful applications of tools from other areas of mathematics, including topological methods. In this talk I will survey the study of the Hom-complexes, and the ways these can be used to obtain lower bounds for the chromatic numbers of graphs.

The structural theory will be developed and put in the historical context, culminating in the proof of the Lovasz Conjecture, which can be stated as follows:

For a graph G , such that the complex $\text{Hom}(C_{2r+1}, G)$ is k -connected for some integers r and k , $r > 0$, $k > -2$, we know that the chromatic number of G is at least $k + 4$.

Beyond the, more customary in this area, cohomology groups, the algebro-topological concepts involved are spectral sequences and Stiefel-Whitney characteristic classes.

I shall also describe how to use the spectral sequences to perform a complete calculation of the cohomology groups of the colorings of cycles.