

COMPLEXITY IN NUMERICAL OPTIMIZATION

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Editor

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Preface

Computational complexity, originated from the interactions between computer science and numerical optimization, is one of the major theories that have revolutionized the approach to solving optimization problems and to analyzing their intrinsic difficulty. The main focus of complexity is the study of whether existing algorithms are efficient for the solution of problems, and which problems are likely to be tractable. The quest for developing efficient algorithms leads also to elegant general approaches for solving optimization problems, and reveals surprising connections among problems and their solutions.

This book is a collection of articles on recent complexity developments in numerical optimization. The topics covered include complexity of approximation algorithms, new polynomial time algorithms for convex quadratic minimization, interior point algorithms, complexity issues regarding test generation of NP-hard problems, complexity of scheduling problems, min-max, fractional combinatorial optimization, fixed point computations, and network flow problems.

The collection of articles provide a broad spectrum of the direction in which research is going and help to elucidate the nature of computational complexity in optimization. The book will be a valuable source of information to faculty, students and researchers in numerical optimization and related areas.

I would like to take the opportunity to thank the authors of the papers, the anonymous referees, and the publisher for helping me to produce this beautiful book.

Panos M. Pardalos
University of Florida
April 1993

“ He who does not expect the unexpected will not detect it” - Heraclitus

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