Complexity Results and Algorithms for Extension Enforcement in Abstract Argumentation*

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Abstract

Understanding the dynamics of argumentation frameworks (AFs) is important in the study of argumentation in AI. In this work, we focus on the so-called extension enforcement problem in abstract argumentation. We provide a nearly complete computational complexity map of fixed-argument extension enforcement under various major AF semantics, with results ranging from polynomial-time algorithms to completeness for the second-level of the polynomial hierarchy. Complementing the complexity results, we propose algorithms for NP-hard extension enforcement based on constrained optimization. Going beyond NP, we propose novel counterexample-guided abstraction refinement procedures for the second-level complete problems and present empirical results on a prototype system constituting the first approach to extension enforcement in its generality.

Introduction

Argumentation is a core topic in Artificial Intelligence (AI) (Bench-Capon and Dunne 2007), with applications in e.g. decision support (Amgoud and Prade 2009), legal reasoning (Bench-Capon, Prakken, and Sartor 2009), and multi-agent systems (McBurney, Parsons, and Rahwan 2012). Argumentation frameworks (AFs) (Dung 1995) provide the fundamental formal model for many approaches to argumentation in AI. Syntactically, AFs are directed graphs, where arguments are abstract entities represented by vertices. Conflicts among arguments are formalized as attacks, and represented with directed edges between arguments. Semantics of AFs—several of which have been proposed—specify criteria for arguments' acceptance resulting in sets of jointly acceptable arguments called *extensions*.

Argumentation is inherently a dynamic process. Recently, several works have focused on fundamental aspects of argumentation dynamics (Baumann 2012a; Baumann and Brewka 2015; Bisquert et al. 2013; Coste-Marquis et al. 2014a; 2014b; Delobelle, Konieczny, and Vesic 2015; Diller et al. 2015). In this work, we focus on *extension enforcement* (Baumann 2012b; Bisquert et al. 2013; Coste-Marquis et al. 2015), a specific form of AF dynamics with connections to belief revision, concerned with finding changes to a given AF in order to support a desired point of view, represented as a set of arguments, under pre-specified semantics.

While the complexity landscape of non-dynamic problems on AFs, including the credulous and skeptical reasoning tasks for a given fixed AF, is already well-established (Dunne and Wooldridge 2009), the complexity of extension enforcement under different semantics and problem variants has not been thoroughly studied until now. Furthermore, while several efficient systems for the NP-hard variants of non-dynamic problems are available (Cerutti et al. 2014; Cerutti, Giacomin, and Vallati 2014; Dvořák et al. 2014; Egly, Gaggl, and Woltran 2010; Nofal, Atkinson, and Dunne 2014), to our best knowledge the single existing system for extension enforcement was only recently proposed (Coste-Marquis et al. 2015), and currently supports extension enforcement only w.r.t. specific AF semantics (the *stable* semantics). This paper aims at bridging these gaps.

Our main contributions are the following.

- We provide a nearly complete computational complexity map of fixed-argument extension enforcement, where the task is to enforce a given extension by modifying the attack relation of a given AF. Our results cover nine standard AF semantics and both the so-called *strict* and *non-strict* variants of extension enforcement. For examples, we provide polynomial-time algorithms for strict enforcement under the admissible and stable semantics (the latter of which was in fact proposed to be solved using the NP-machinery of integer programming (IP) by Coste-Marquis et al. (2015)); show that most non-strict enforcement problems are NP-complete, along with strict enforcement under the complete and grounded semantics; and establish second-level completeness for strict enforcement under preferred and semi-stable semantics as well as for non-strict semi-stable and stage semantics.
- We propose algorithms for the NP-hard variants of the enforcement problems based on applying constraint optimization solvers. We detail maximum satisfiability (MaxSAT) encodings for the NP-complete problem variants, and, perhaps most interestingly, pro-

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pose novel counterexample-guided abstraction refinement (CEGAR) (Clarke et al. 2003; Clarke, Gupta, and Strichman 2004) procedures for the second-level Σ_2^P-complete variants using optimization solvers as functional NP oracles. We provide an overview of an empirical evaluation of a prototype system implementation that supports the considered extension enforcement variants.

While our main focus is fixed-argument extension enforcement, we also shortly discuss the *normal*, *strong*, and *weak* variants (Baumann 2012b) of enforcement.

Preliminaries

We recall concepts related to argumentation frameworks (Dung 1995), their semantics (Baroni, Caminada, and Giacomin 2011), and enforcement operators (Baumann 2012b; Coste-Marquis et al. 2015).

Definition 1. *An* argumentation framework (AF) *is a pair $F = (A, R)$ where A is a finite set of arguments and $R \subseteq A \times A$ is the attack relation. The pair $(a, b) \in R$ means that a attacks b. An argument $a \in A$ is* defended *(in F) by a set $S \subseteq A$ if, for each $b \in A$ such that $(b, a) \in R$, there exists a $c \in S$ such that $(c, b) \in R$.*

Example 1. *Let $F = (A, R)$ be an AF with $A = \{a, b, c, d\}$ and $R = \{(b, a), (b, c), (c, a), (c, d), (d, b)\}$. The corresponding graph representation is shown in Figure 1.*

Semantics for argumentation frameworks are defined through a function σ which assigns to each AF $F = (A, R)$ a set $\sigma(F) \subseteq 2^A$ of extensions. We consider for σ the functions $naive$, stb, adm, com, grd, prf, sem, and stg which stand for naive, stable, admissible, complete, grounded, preferred, semi-stable and stage extensions, respectively. These semantics are defined as follows.

Definition 2. *Given an AF $F = (A, R)$, the* characteristic function *$\mathcal{F}_F : 2^A \to 2^A$ of F is defined as $\mathcal{F}_F(S) = \{x \in A \mid x \text{ is defended by } S\}$. Moreover, for a set $S \subseteq A$, we define the* range *of S as $S_R^+ = S \cup \{x \mid (y, x), y \in S\}$.*

Definition 3. *Let $F = (A, R)$ be an AF. A set $S \subseteq A$ is* conflict-free *(in F), if there are no $a, b \in S$, such that $(a, b) \in R$. We denote the collection of conflict-free sets of F by $cf(F)$. For a conflict-free set $S \in cf(F)$, it holds that*

- *$S \in naive(F)$ if there is no $T \in cf(F)$ with $S \subset T$;*
- *$S \in stb(F)$ if $S_R^+ = A$;*
- *$S \in adm(F)$ if $S \subseteq \mathcal{F}_F(S)$;*
- *$S \in com(F)$ if $S = \mathcal{F}_F(S)$;*
- *$S \in grd(F)$ if S is the least fixed-point of $\mathcal{F}_F$;*
- *$S \in prf(F)$ if $S \in adm(F)$ and there is no $T \in adm(F)$ with $S \subset T$;*
- *$S \in sem(F)$ if $S \in adm(F)$ and there is no $T \in adm(F)$ with $S_R^+ \subset T_R^+$;*
- *$S \in stg(F)$ if there is no $T \in cf(F)$ with $S_R^+ \subset T_R^+$.*

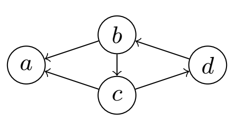
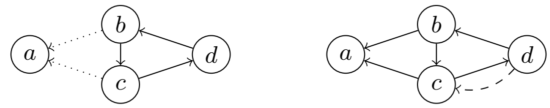


Figure 1: Example argumentation framework



(a) Strictly enforcing $\{a\}$ (b) Non-strictly enforcing $\{a\}$

Figure 2: Enforcement under complete semantics

For any AF F it holds that $cf(F) \supseteq adm(F) \supseteq com(F) \supseteq prf(F) \supseteq sem(F) \supseteq stb(F)$. We use the term σ-extension to refer to an extension under a semantics $\sigma \in \{naive, stb, adm, com, grd, prf, sem, stg\}$.

Extension enforcement is the problem of modifying the syntactic attack structure R of a given AF $F = (A, R)$ in a way that a given set T of arguments becomes (part of) an extension under a desired semantics σ in the modified AF. Several types of enforcement have been proposed (Baumann 2012b; Coste-Marquis et al. 2015), differing in the type of modifications allowed to the framework. We focus mainly on so-called fixed-argument extension enforcement (Coste-Marquis et al. 2015), where the set of arguments remains the same, but the attack structure may be modified arbitrarily. Later on we also discuss adaptation to enforcement under strong and weak expansions (Baumann 2012b). *Strict* enforcement requires that the given set T of arguments has to be a σ-extension, while in *non-strict* enforcement T is required to be part of a σ-extension. We denote the set of attack structures that strictly enforce T under σ for F by $enf_s^\sigma(F, T) = \{R' \mid F' = (A, R'),\ T \in \sigma(F')\}$, and by $enf_{ns}^\sigma(F, T) = \{R' \mid F' = (A, R'),\ \exists T' \in \sigma(F') : T' \supseteq T\}$ for non-strict enforcement. The number of changes of an enforcement is $|R \Delta R'| = |R \setminus R'| + |R' \setminus R|$, i.e., the symmetric difference of the attack structures R and R'. The following definition captures the optimization problem for extension enforcement, where $\arg\min$ is the standard notation for the set of domain elements that minimize a function.

> **Extension enforcement** ($x \in \{s, ns\}$)
> Input: AF $F = (A, R)$, $T \subseteq A$, and semantics σ.
> Task: Find an AF $F^* = (A, R^*)$ with
> $$R^* \in \operatorname*{arg\,min}_{R' \in enf_x^\sigma(F,T)} |R \Delta R'|.$$

Example 2. *In AF F from Example 1 we have $com(F) = \{\emptyset\}$. A way to strictly enforce $\{a\}$ as a complete extension is to remove attacks (b, a) and (c, a) (Figure 2a). Adding (d, c) makes $\{a, d\}$ a complete extension, and thus $\{a\}$ becomes non-strictly enforced (Figure 2b).*

In the decision problems for extension enforcement, we are given an AF $F = (A, R)$, a set $T \subseteq A$, and an integer $k \geq 0$, and are asked to decide if there is an $F' = (A, R')$ with $|R \Delta R'| \leq k$ that enforces T non-strictly (resp. strictly). For the complexity results, in addition to the standard complexity classes P, NP, and coNP, recall that the class Σ_2^P consists of problems which can be decided by a non-deterministic polynomial-time algorithm with access to an NP oracle.

Complexity Analysis

An overview of the complexity results of this paper is given in Table 1. We begin our analysis by considering non-strict enforcement. A basic observation is that, to enforce a set T under semantics σ, all attacks "inside" T need to be removed, since all considered semantics are based on conflict-free sets. For non-strict enforcement under conflict-free and naive semantics, this modification turns out to be optimal.

Proposition 1. *Non-strict enforcement for conflict-free and naive semantics is in* P.

Proof. (sketch) Let $F = (A, R)$ be an AF and $T \subseteq A$ the set to be enforced. Define $F^* = (A, R^*)$ with $R^* = R \setminus (T \times T)$. Now $T \in cf(F^*)$ and thus there is a $T' \in naive(F^*)$ with $T \subseteq T'$. For any $R' \subseteq A \times A$ with $|R \Delta R'| < |R \Delta R^*|$ it holds that T and all supersets of T are not conflict-free in $F' = (A, R')$. Thus F^* is an optimal solution. □

For the remaining semantics, non-strict enforcement is presumably harder. This follows from the fact that it is computationally hard to check whether there is a superset of T that is a σ-extension of the input AF F.

Proposition 2. *Non-strict enforcement*

- *for admissible, complete, preferred, and stable semantics is* NP-*complete; and*
- *for semi-stable and stage semantics is* Σ_2^P*-complete.*

Proof. Hardness in all cases follows from a reduction from the credulous acceptance problem for the same semantics σ, where we have to decide whether an a is contained in one σ-extension of a given AF F. We reduce this problem to non-strict enforcement by defining $T = \{a\}$. Then T can be non-strictly enforced under σ with 0 changes iff a is credulously accepted. Complexity of credulous reasoning is analyzed in (Caminada, Carnielli, and Dunne 2012; Coste-Marquis, Devred, and Marquis 2005; Dimopoulos and Torres 1996; Dung 1995; Dvořák and Woltran 2010). Membership for all problems follows from a guess and check (verifying if a given set is admissible or stable can be checked in P; for semi-stable and stage this problem is in coNP). □

Coste-Marquis et al. (2015) established that the *union* of non-strict and strict extension enforcement under stable semantics is NP-hard. As a more fine-grained analysis, by Proposition 2 non-strict enforcement is in itself NP-complete; furthermore, in the following we will show that strict enforcement under stable semantics is in fact in P.

From the previous propositions it might appear that the main source of intractability does not originate from the modifications of the attack structure, but from (credulous) acceptance problems associated with the semantics under consideration. However, even for the computationally simple grounded semantics, non-strict enforcement turns out to be NP-complete. This suggests that for admissibility-based semantics the non-determinism introduced by changes in the attack structure is enough for NP-hardness.

Table 1: Complexity results for extension enforcement

σ	strict	non-strict
Conflict-free	in P	in P
Naive	in P	in P
Admissible	in P	NP-c
Stable	in P	NP-c
Complete	NP-c	NP-c
Grounded	NP-c	NP-c
Preferred	Σ_2^P-c	NP-c
Semi-stable	Σ_2^P-c	Σ_2^P-c
Stage	coNP hard and in Σ_2^P	Σ_2^P-c

Theorem 3. *Non-strict enforcement for grounded semantics is* NP-*complete.*

We move on to strict enforcement. Here we establish polynomial-time results for stable and admissible semantics.

Proposition 4. *Strict enforcement for conflict-free, naive, admissible, and stable semantics is in* P.

Proof. (sketch) Let $F = (A, R)$ be an AF, $T \subseteq A$, and $\sigma \in \{cf, naive, adm, stb\}$. For each σ, we define a polynomial-time computable $F^*_\sigma = (A, R^*_\sigma)$ that is an optimal solution to the strict enforcement problem under σ. We assume that $T \neq \emptyset$; otherwise the problem is trivial. Let $t_0 \in T$ be an arbitrary but fixed argument. For all considered semantics we have to remove conflicts inside T.

- $\sigma = cf$: let $R^*_{cf} = R \setminus (T \times T)$;
- $\sigma = naive$: add a self-attack to arguments $a \in A \setminus T$ where $T \cup \{a\}$ would be conflict-free otherwise, i.e. $R^*_{naive} = (R \setminus (T \times T)) \cup \{(a, a) \mid a \in A \setminus T, \nexists (b, a) \in R \text{ with } b \in T \cup \{a\}\}$;
- $\sigma = stb$: add attacks (t_0, a) with $a \in A \setminus T$ where a is not attacked by T, i.e. $R^*_{stb} = (R \setminus (T \times T)) \cup \{(t_0, a) \mid a \in A \setminus T, \nexists (t, a) \in R \text{ with } t \in T\}$;
- $\sigma = adm$: for each attack from $a \in A \setminus T$ to $t \in T$ that is not counterattacked by T add (t_0, a), i.e. $R^*_{adm} = (R \setminus (T \times T)) \cup \{(t_0, a) \mid a \in A \setminus T, \exists (a, t) \in R \text{ s.t. } t \in T \text{ and } \nexists (t', a) \in R \text{ s.t. } t' \in T\}$. □

In contrast to admissible semantics, strict enforcement for complete and grounded semantics is NP-complete. Intuitively, admissibility together with the fact that we must not defend arguments outside any desired set can be used for reducing satisfiability of Boolean formulas to strict enforcement under complete or grounded semantics.

Theorem 5. *Strict enforcement for complete and grounded semantics is* NP-*complete.*

For preferred and semi-stable semantics, we see a jump in complexity: the corresponding problems are in fact Σ_2^P-complete. Intuitively, in addition to the source of intractability that strict enforcement under complete semantics brings, one has to take into account that modifications to the attack structure might give rise to supersets of T that are admissible. Hardness can be proven by a reduction from satisfiability of quantified Boolean formulas.

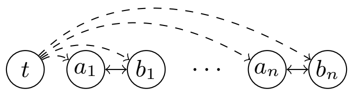


Figure 3: Strict enforcement under preferred semantics

Theorem 6. *Strict enforcement for preferred and semi-stable semantics is Σ_2^P-complete.*

As an illustrative example of when complete and preferred semantics differ w.r.t. strict enforcement, see Figure 3. Here strictly enforcing $\{t\}$ under complete semantics requires no modifications. In contrast, under preferred semantics each other argument requires one distinct modification.

Finally, for stage semantics we give straightforward bounds. Hardness follows from coNP-hardness of verifying if a set is a stage extension (Dvořák and Woltran 2010).

Corollary 7. *Strict enforcement for stage semantics is in Σ_2^P and* coNP *hard.*

We conjecture that strict enforcement for stage semantics is indeed Σ_2^P-complete; this is the only missing piece in the complexity map (recall Table 1) established in this paper.

Extension Enforcement via MaxSAT

In this section we present declarative encodings that can be used for solving extension enforcement optimally under the considered semantics. We employ maximum satisfiability (MaxSAT) as a well-suited declarative language. For non-strict enforcement under stable semantics, our encoding is essentially the same as the integer programming formulation presented by Coste-Marquis et al. (2015). Here we present MaxSAT encodings for the various semantics, as well as develop counterexample-guided abstraction refinement algorithms for solving the Σ_2^P-complete problem variables by applying the NP-encodings.

We recall the MaxSAT problem. For a variable x, there are two literals, x and $\neg x$. A clause is a disjunction ($\vee$) of literals. A truth assignment is a function from variables to $\{0, 1\}$. A clause c is satisfied by a truth assignment τ ($\tau(c) = 1$) if $\tau(x) = 1$ for a literal x in c, or $\tau(x) = 0$ for a literal $\neg x$ in c; otherwise τ does not satisfy c ($\tau(c) = 0$).

An instance $\varphi = (\varphi_h, \varphi_s)$ of the *Partial MaxSAT* problem consists of a set φ_h of *hard* clauses and a set φ_s of *soft* clauses. Any truth assignment τ that satisfies every clause in φ_h is a *solution* to φ. The *cost* of a solution τ to φ is $\text{COST}(\varphi, \tau) = \sum_{c \in \varphi_s} (1 - \tau(c))$, i.e., the number of soft clauses not satisfied by τ. A solution τ is *optimal* for φ if $\text{COST}(\varphi, \tau) \leq \text{COST}(\varphi, \tau')$ holds for any solution τ' to φ. Given φ, the Partial MaxSAT problem asks to find an optimal solution to φ. From here on, we refer to partial MaxSAT simply as MaxSAT.

We now present MaxSAT encodings for NP extension enforcement problems. Let $F = (A, R)$ be an AF and $T \subseteq A$ the set to be enforced under semantics σ. We use variables x_a and $r_{a,b}$ for $a, b \in A$ with the interpretation "$x_a = 1$ iff a is in an extension", and "$r_{a,b} = 1$ iff the attack (a, b) occurs in the modified AF".

For all semantics, the soft clauses are given by $\varphi_{soft}(F) = \bigwedge_{a,b \in A} r'_{a,b}$, where

$$r'_{a,b} \leftrightarrow \begin{cases} r_{a,b} & \text{if } (a,b) \in R \\ \neg r_{a,b} & \text{if } (a,b) \notin R, \end{cases}$$

i.e., a violated soft clause, contributing unit cost to the cost of a solution, implies that the corresponding attack has been modified (removed or added).

We now define clauses enforcing that the given set T must be part of a σ-extension for the modified AF encoded via the attack variables $r_{a,b}$. For non-strict enforcement (short-hand ns), we define $\varphi_{ns}(F, T) = \bigwedge_{a \in T} x_a$ which encodes that the given set must be part a σ-extension. For strict enforcement (short-hand s), there is no need to encode arguments' statuses as variables (their values are fixed), i.e., variables x_a are not required. For encoding the semantics, we adapt Boolean formulas from (Besnard and Doutre 2004), originally for non-dynamic problems, to extension enforcement. We start with non-strict enforcement. For conflict-free sets, if an attack between two arguments is present, then only one of them can be in a conflict-free set.

$$\varphi_{ns}^{cf}(F, T) = \varphi_{ns}(F, T) \wedge \bigwedge_{a,b \in A} \big(r_{a,b} \rightarrow (\neg x_a \vee \neg x_b)\big)$$

For admissible semantics (recall that non-strict enforcement for admissible, complete, and preferred semantics coincides), if a is in an admissible set and there is an attack on a, then a defender together with a defending attack must be assigned to 1.

$$\varphi_{ns}^{adm}(F, T) = \varphi_{ns}^{cf}(F, T) \wedge \bigwedge_{a,b \in A} \big((x_a \wedge r_{b,a}) \rightarrow \bigvee_{c \in A} (x_c \wedge r_{c,b})\big)$$

Stable semantics can be encoded in the following way. If an argument is not in the stable extension, an attacker in the set together with an attack in the new AF has to be found.

$$\varphi_{ns}^{stb}(F, T) = \varphi_{ns}^{cf}(F, T) \wedge \bigwedge_{a \in A} \big(\neg x_a \rightarrow \bigvee_{b \in A} (x_b \wedge r_{b,a})\big)$$

We move on to strict enforcement for conflict-free sets.

$$\varphi_{s}^{cf}(F, T) = \bigwedge_{a,b \in T} \neg r_{a,b},$$

i.e., strict enforcement simply consists of removing all attacks inside T. For admissible semantics, we need a defending counter-attack for each attack on set T.

$$\varphi_{s}^{adm}(F, T) = \varphi_{s}^{cf}(F, T) \wedge \bigwedge_{a \in T} \bigwedge_{b \in A \setminus T} \big(r_{b,a} \rightarrow \bigvee_{c \in T} r_{c,b}\big)$$

In the encoding for strict enforcement under complete semantics, we need to ensure that for each argument outside T there is an attack on it that is not defended against by T.

$$\varphi_{s}^{com}(F, T) = \varphi_{s}^{adm}(F, T) \wedge \bigwedge_{a \in A \setminus T} \bigvee_{b \in A} \big(r_{b,a} \wedge \bigwedge_{c \in T} \neg r_{c,b}\big)$$

In summary, for semantics σ, an optimal solution to the MaxSAT problem $\varphi = (\varphi_x^{\sigma}(F, T), \varphi_{soft})$ corresponds to an optimum solution to the strict enforcement problem (if $x = s$) or the non-strict enforcement problem (if $x = ns$).

Extension Enforcement Beyond NP

The second-level complexity of strict enforcement under preferred and semi-stable semantics, as well as non-strict enforcement under preferred, semi-stable, and stage semantics, hinders direct use of NP optimization solvers for these problems. However, the NP semantics, such as complete, overapproximate the preferred and semi-stable semantics, and conflict-free sets overapproximate stage semantics. This implies that the NP encodings can be used as base abstractions within a counterexample-guided abstraction refinement (CEGAR) approach to solving the second-level extension enforcement problems. As a general outline, in CEGAR an (over)abstraction of the set of solutions of interest is iteratively refined until an actual solution to the original problem instance is encountered. At each iteration, the abstraction is solved, typically using an NP oracle (such as a SAT solver). A thus obtained candidate solution is checked with another oracle call. If the oracle reports that the candidate is not an actual solution, a counterexample is obtained, and the abstraction is refined further based on the counterexample. This is repeated until no counterexamples are found, at which point the candidate solution is an actual solution.

Let $F = (A, R)$ be an AF and $T \subseteq A$ the set to be enforced under semantics $\sigma \in \{prf, sem, stg\}$. Let $x \in \{ns, s\}$ be the type of enforcement. Our procedures are presented in a unifying way as Algorithm 1. First, we select the "base" semantics χ for enforcement that acts as our first abstraction: conflict-free sets for stage and admissible, complete semantics otherwise. In the loop an optimal solution for non-strict or strict enforcement under semantics χ is computed by e.g. a MaxSAT or an IP solver, and represented by the truth assignment τ. We extract the AF $F' = (A, R')$, with $R' = \{(a, b) \mid a, b \in A, \tau(r_{a,b}) = 1\}$ from τ. We then check whether F' is also a solution to enforcement under semantics σ. For strict enforcement, we have to check whether $T \in \sigma(F')$ holds. For non-strict enforcement we check whether $T' \in \sigma(F')$ with $T' = \{a \in A \mid \tau(x_a) = 1\}$. We encode the base semantics as in (Besnard and Doutre 2004). Conflict-free sets are encoded by $\psi^{cf}(F') = \bigwedge_{(a,b)\in R'}(\neg x_a \vee \neg x_b)$, admissible semantics by $\psi^{adm}(F') = \psi^{cf}(F') \wedge \bigwedge_{(b,a)\in R'}(x_a \rightarrow (\bigvee_{(c,b)\in R} x_c))$, and complete semantics by $\psi^{com}(F') = \psi^{adm}(F') \wedge \bigwedge_{a\in A}((\bigwedge_{(b,a)\in R'}(\bigvee_{(c,b)\in R'} x_c)) \rightarrow x_a)$. We define the following shorthand for searching for a superset of a set $S \subseteq A$ in case of preferred semantics, or otherwise for a superset w.r.t. the range given by F'.

$$\alpha^\sigma(F', S) = \begin{cases} \bigwedge_{a\in S} x_a \wedge \bigvee_{a\in A\setminus S} x_a & \text{if } \sigma = prf \\ \bigwedge_{a\in S^+_{R'}} x^+_a \wedge \bigvee_{a\in A\setminus S^+_{R'}} x^+_a \wedge \beta(F') & \text{else.} \end{cases}$$

The formula $\beta(F') = \bigwedge_{a\in A}(x^+_a \leftrightarrow (x_a \vee \bigvee_{(b,a)\in R'} x_b))$ encodes the range w.r.t. F'. Now, the following formula verifies that F' is a solution to the (non-)strict enforcement under σ with $S = T$ if $x = s$ and $S = T'$ if $x = ns$.

$$\Gamma^\sigma(\tau) = \psi^\chi(F') \wedge \alpha^\sigma(F', S)$$

In words, we search for a superset of T (T') that is a χ-extension. If this formula is satisfiable, then there is a counterexample witnessing that T (T') is not a σ-extension in F'. For the refinement step, we define the shorthand $\gamma(\tau) = \bigwedge_{(a,b)\in R'} r_{a,b}, \wedge \bigwedge_{\{a,b\}\subseteq A,(a,b)\notin R'} \neg r_{a,b}$. We refine the current abstraction ψ by $\textsc{Refine}(\tau, s) = \neg\gamma(\tau)$ in the strict case which rules out R', and in the non-strict case we additionally rule out all χ-extensions the range of which is a subset of the range of T' by $\textsc{Refine}(\tau, ns) = \neg\gamma(\tau) \vee \bigvee_{a\in A\setminus T'^+_{R'}} x^+_a$. In this case, we define range variables dependent on the attack variables via the shorthand $\delta(A) = \bigwedge_{a\in A}(x^+_a \leftrightarrow (x_a \vee \bigvee_{b\in A}(r_{b,a} \wedge x_b)))$ (Line 3).

Algorithm 1 solves strict enforcement for $\{prf, sem, stg\}$ and non-strict enforcement for $\{sem, stg\}$ optimally, as at each iteration the current abstraction is solved optimally.

Algorithm 1 Enforcement for $\sigma \in \{prf, sem, stg\}$ with $x = s$ or $\sigma \in \{sem, stg\}$ with $x = ns$.

1: **if** $\sigma \in \{prf, sem\}$ **then** $\chi \leftarrow com$ **else** $\chi \leftarrow cf$
2: $\psi \leftarrow \varphi^\chi_x(F, T)$
3: **if** $x = ns$ **then** $\psi \leftarrow \psi \wedge \delta(A)$
4: **while** true **do**
5: $\quad (c, \tau) \leftarrow \textsc{MaxSAT}(\psi, \varphi_{soft})$
6: $\quad result \leftarrow \text{SAT}(\Gamma^\sigma(\tau))$
7: $\quad$ **if** $result = unsatisfiable$ **then**
8: $\quad\quad$ return (c, τ)
9: $\quad$ **else**
10: $\quad\quad \psi \leftarrow \psi \wedge \textsc{Refine}(\tau, x)$

Experiments

We present empirical results on a prototype system implementation (available at `http://cs.helsinki.fi/group/coreo/pakota/`) for extension enforcement, which supports the considered NP-hard strict and non-strict extension enforcement variants, and allows for using both MaxSAT solvers as well as IP solvers via standard translation of MaxSAT into IP (Ansótegui and Gabàs 2013).

We generated enforcement instances as follows. Let $|A| = 25, 50, \ldots$ denote the number of arguments in the AF to be generated, and $|T|$ the size of subset $T \subseteq A$ of arguments to be enforced. For a fixed edge probability p, we sampled directed graphs by independently picking an edge to the AF with probability p (but disallowing self-attacks). For each $|A|$ and $p \in \{0.05, 0.1, 0.2, 0.3\}$, we sampled five directed graphs. For each AF, we picked uniformly at random five sets of arguments $T \subset A$ to be enforced for each $|T|/|A| \in \{0.05, 0.1, 0.2, 0.3\}$. For each number of arguments $|A|$, this gave 400 enforcement problem instances.

We used the OpenWBO (Martins, Manquinho, and Lynce 2014) MaxSAT solver—among the solvers in the 2015 MaxSAT Evaluation on Partial MaxSAT—and the CPLEX IP solver. The experiments were run on 2.83-GHz Intel Xeon E5440 quad-core machines with 32-GB memory and Debian GNU/Linux 8 using a per-instance timeout of 900 seconds.

We present results for a choice of four major AF semantics: the NP-complete enforcement problems of strict complete and non-strict admissible and stable, as well as the Σ^P_2-complete strict preferred. As the only system available for

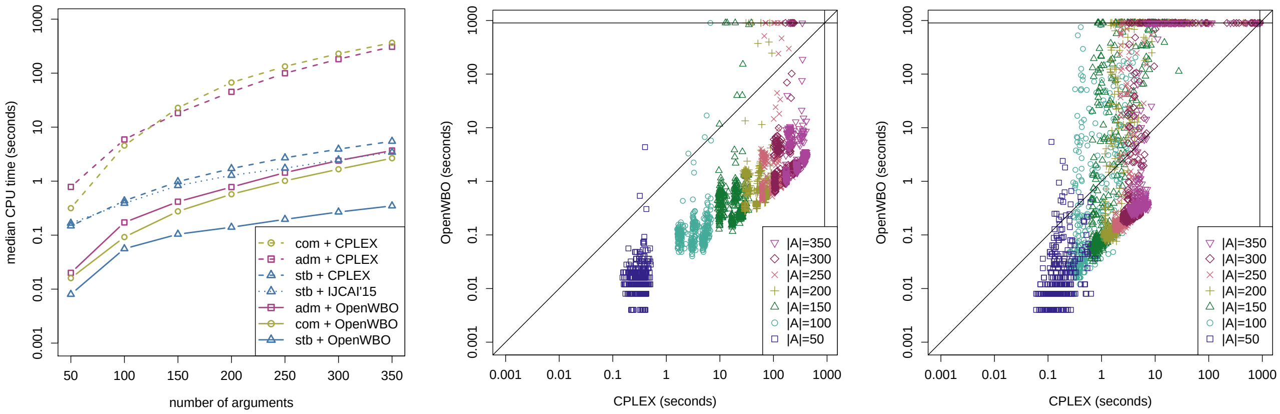


Figure 4: Median runtimes (left); OpenWBO v CPLEX strict complete (middle), non-strict stable (right).

comparison, we consider the recently proposed IP-based approach to non-strict stable by Coste-Marquis et al. (2015) using CPLEX. The performance of CPLEX on our encoding and the approach of Coste-Marquis et al. for non-strict stable (Fig. 4 left) essentially coincide also empirically, corroborating the fact that the encodings are essentially the same. Interestingly, the relative performance of OpenWBO and CPLEX varies noticeably depending on the combination of (non)strictness and the semantics; CPLEX dominates on non-strict stable, while OpenWBO is better on strict complete; compare Fig. 4 middle and right. While OpenWBO tends to produce more timeouts, the median runtimes (Fig. 4 left) of OpenWBO are noticeably lower than those of CPLEX. For the challenging Σ_2^P-complete problem for strict preferred, our prototype implementation of the proposed CEGAR approach, using OpenWBO and complete as the base abstraction, already performs well, solving instances with 200 arguments and beyond (Fig. 5).

Other Extension Enforcement Variants

Finally, we shortly discuss other variants of extension enforcement, namely, under normal, strong, and weak expansions. Baumann and Brewka (2010) consider enforcement under so-called expansions of an AF $F = (A, R)$, which result in AF $F' = (A \cup A', R \cup R')$ with new arguments A' and new attacks R' s.t. $A \cap A' = R \cap R' = \emptyset$ and one of A' or R' is non-empty. An expansion is normal if for each $(a, b) \in R'$ we have $a \in A'$ or $b \in A'$; strong if $a \notin A$; and weak if $b \notin A$. The tasks for enforcement under these variants are the same as for strict and non-strict, with the additional requirement that the enforcing AF is to be a normal, strong, or weak expansion. Considering these variants, the proof of Proposition 2 implies the following for a fixed A'.

Corollary 8. *Non-strict enforcement under normal, strong, or weak expansions*

- *for admissible, complete, preferred, and stable semantics is* NP*-complete; and*
- *for semi-stable and stage semantics is* Σ_2^P*-complete.*

As pointed out by Coste-Marquis et al. (2015), enforcement under expansions can be encoded via additional hard constraints for a *fixed* set A' of additional arguments; the same holds for the MaxSAT encodings. Furthermore, our CEGAR algorithm (Algorithm 1) with adapted hard constraints can also be applied for semi-stable and stage semantics under normal, strong, or weak expansions. Also, our approach allows for further enforcement variants, e.g., any combination of (i) imposing other constraints on the way the attack structure may be changed, e.g., utilizing hard unit clauses to state that certain attacks must not be removed, (ii) attaching weights to attacks and searching for weight-minimum changes, or (iii) removal of arguments (by removing the argument and all attacks the argument is involved in). However, we note that it is not clear under which condition the cost of optimal solutions is preserved when restricting these problems by considering a fixed set A' (e.g. a singleton set). In fact, the following example shows that the costs of optimal solutions—and the sets of optimal solutions—do

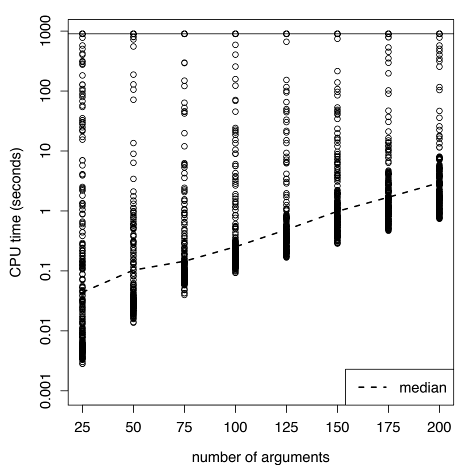


Figure 5: CEGAR on strict preferred

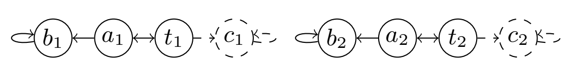


Figure 6: Enforcement under weak expansions

not in general coincide. Consider strictly enforcing $\{t_1, t_2\}$ under semi-stable semantics. Weak expansion in the AF in Figure 6 requires two new arguments (for each argument t_1 and t_2 we need a new argument to extend their range); restricting A' to singleton sets would not yield any solutions.

Conclusions

We presented both new complexity results and novel algorithms, based on a declarative optimization approach, for several variants of fixed-argument extension enforcement. As the main contributions, on the theoretical side we presented a nearly complete computational complexity map of the considered problem variants. Complementing the theoretical analysis, we proposed algorithms for the variants, ranging from polytime results to procedures going beyond NP for the second-level complete problem variants. Further improving the efficiency of the approach both via understanding what makes enforcement instances hard and via employing optimization solvers incrementally with the CEGAR approach, as well as extensions to other types of argumentation dynamics, are important aspects of future work.

References

Amgoud, L., and Prade, H. 2009. Using arguments for making and explaining decisions. *Artif. Intell.* 173(3-4):413–436.

Ansótegui, C., and Gabàs, J. 2013. Solving (weighted) partial MaxSAT with ILP. In *Proc. CPAIOR*, volume 7874 of *LNCS*, 403–409. Springer.

Baroni, P.; Caminada, M.; and Giacomin, M. 2011. An introduction to argumentation semantics. *Knowl. Eng. Rev.* 26(4):365–410.

Baumann, R., and Brewka, G. 2010. Expanding argumentation frameworks: Enforcing and monotonicity results. In *Proc. COMMA*, volume 216 of *FAIA*, 75–86. IOS Press.

Baumann, R., and Brewka, G. 2015. AGM meets abstract argumentation: Expansion and revision for Dung frameworks. In *Proc. IJCAI*, 2734–2740. AAAI Press.

Baumann, R. 2012a. Normal and strong expansion equivalence for argumentation frameworks. *Artif. Intell.* 193:18–44.

Baumann, R. 2012b. What does it take to enforce an argument? minimal change in abstract argumentation. In *Proc. ECAI*, volume 242 of *FAIA*, 127–132. IOS Press.

Bench-Capon, T. J. M., and Dunne, P. E. 2007. Argumentation in artificial intelligence. *Artif. Intell.* 171(10-15):619–641.

Bench-Capon, T. J. M.; Prakken, H.; and Sartor, G. 2009. Argumentation in legal reasoning. In *Argumentation in Artificial Intelligence*. Springer. 363–382.

Besnard, P., and Doutre, S. 2004. Checking the acceptability of a set of arguments. In *Proc. NMR*, 59–64.

Bisquert, P.; Cayrol, C.; de Saint-Cyr, F. D.; and Lagasquie-Schiex, M. 2013. Enforcement in argumentation is a kind of update. In *Proc. SUM*, volume 8078 of *LNCS*, 30–43. Springer.

Caminada, M. W. A.; Carnielli, W. A.; and Dunne, P. E. 2012. Semi-stable semantics. *J. Logic Comput.* 22(5):1207–1254.

Cerutti, F.; Dunne, P. E.; Giacomin, M.; and Vallati, M. 2014. Computing preferred extensions in abstract argumentation: A SAT-based approach. In *TAFA 2013 Revised Selected Papers*, volume 8306 of *LNCS*, 176–193. Springer.

Cerutti, F.; Giacomin, M.; and Vallati, M. 2014. ArgSemSAT: Solving argumentation problems using SAT. In *Proc. COMMA*, volume 266 of *FAIA*, 455–456. IOS Press.

Clarke, E. M.; Grumberg, O.; Jha, S.; Lu, Y.; and Veith, H. 2003. Counterexample-guided abstraction refinement for symbolic model checking. *J. ACM* 50(5):752–794.

Clarke, E. M.; Gupta, A.; and Strichman, O. 2004. SAT-based counterexample-guided abstraction refinement. *IEEE TCAD* 23(7):1113–1123.

Coste-Marquis, S.; Konieczny, S.; Mailly, J.; and Marquis, P. 2014a. On the revision of argumentation systems: Minimal change of arguments statuses. In *Proc. KR*. AAAI Press.

Coste-Marquis, S.; Konieczny, S.; Mailly, J.; and Marquis, P. 2014b. A translation-based approach for revision of argumentation frameworks. In *Proc. JELIA*, volume 8761 of *LNCS*, 397–411. Springer.

Coste-Marquis, S.; Konieczny, S.; Mailly, J.; and Marquis, P. 2015. Extension enforcement in abstract argumentation as an optimization problem. In *Proc. IJCAI*, 2876–2882. AAAI Press.

Coste-Marquis, S.; Devred, C.; and Marquis, P. 2005. Symmetric argumentation frameworks. In *Proc. ECSQARU*, volume 3571 of *LNCS*, 317–328. Springer.

Delobelle, J.; Konieczny, S.; and Vesic, S. 2015. On the aggregation of argumentation frameworks. In *Proc. IJCAI*, 2911–2917. AAAI Press.

Diller, M.; Haret, A.; Linsbichler, T.; Rümmele, S.; and Woltran, S. 2015. An extension-based approach to belief revision in abstract argumentation. In *Proc. IJCAI*, 2926–2932. AAAI Press.

Dimopoulos, Y., and Torres, A. 1996. Graph theoretical structures in logic programs and default theories. *Theor. Comput. Sci.* 170(1-2):209–244.

Dung, P. M. 1995. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artif. Intell.* 77(2):321–358.

Dunne, P. E., and Wooldridge, M. 2009. Complexity of abstract argumentation. In Simari, G., and Rahwan, I., eds., *Argumentation in Artificial Intelligence*. Springer. 85–104.

Dvořák, W., and Woltran, S. 2010. Complexity of semi-stable and stage semantics in argumentation frameworks. *Inform. Process. Lett.* 110(11):425–430.

Dvořák, W.; Järvisalo, M.; Wallner, J. P.; and Woltran, S. 2014. Complexity-sensitive decision procedures for abstract argumentation. *Artif. Intell.* 206:53–78.

Egly, U.; Gaggl, S. A.; and Woltran, S. 2010. Answer-set programming encodings for argumentation frameworks. *Argument and Computation* 1(2):147–177.

Martins, R.; Manquinho, V. M.; and Lynce, I. 2014. Open-WBO: A modular MaxSAT solver,. In *Proc. SAT*, volume 8561 of *LNCS*, 438–445. Springer.

McBurney, P.; Parsons, S.; and Rahwan, I., eds. 2012. *ArgMAS 2011 Revised Selected Papers*, volume 7543 of *LNCS*. Springer.

Nofal, S.; Atkinson, K.; and Dunne, P. E. 2014. Algorithms for decision problems in argument systems under preferred semantics. *Artif. Intell.* 207:23–51.