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COMPOSITE HIGG'S FIELDS AND FINITE SYMMETRY BREAKING IN GAUGE THEORIES*

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ABSTRACT

Gauge models in which the symmetry breaking is dynamical, realized by certain composite Higg's fields acquiring a vacuum expectation value, are more constrained than corresponding models in which the symmetry breaking is implemented by auxiliary elementary scalar representations present at the Lagrangian level. Thus, in the former case, physical quantities which would otherwise be free parameters become computable. We illustrate this notable fact for the interesting case of the electron-muon mass ratio in Weinberg's chiral SU(3) model.

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Multiplets of scalar fields play a central role in the current efforts to construct renormalizable models of the weak interactions based on a gauge principle.[1] It is in fact through the symmetry-violating vacuum expectation value (VEV) of such multiplets that otherwise massless vector mesons and fermions acquire mass and that the excess of symmetry present in the Lagrangian is prevented from being communicated to the solutions of the theory.

In model-building activities, it has become customary to think of these multiplets as elementary and, consequently, to associate with them sets of canonical fields in the Lagrangian whose couplings, among themselves and to the fermions, are regarded as more or less arbitrary. Following one's intuition, choices of different scalar representations can then be made to generate the desired pattern of VEV's and masses.

While this approach is certainly very useful in that it allows one to simply monitor the symmetry breaking, it is not aesthetically appealing in that it introduces a dependence on more parameters than one would expect in a fundamental theory. Furthermore, as we have been reminded recently[2], the presence of canonical scalar fields in the Lagrangian does not a priori appear to be a vital prerequisite for spontaneous symmetry breakdown. The role of elementary scalar fields in these phenomena could, in fact, be assumed by dynamical bound states, similar in spirit to the fermion pairs of Nambu and Jona-Lasinio.[3] The fundamental world-Lagrangian would then only involve elementary fermions and gauge fields.

Of course, whatever we gain by adopting this approach we pay for by giving up our controls on the theory (its solutions) and, short of acquiring a considerable amount of insight into non-perturbative effects, our general ability to perform even approximate calculations. What we gain, on the other hand, includes very important advantages such as being able to compute, at least in principle, quantities which would otherwise be free parameters. Some such quantities may actually be computed using conventional perturbative techniques if certain conditions, dependent upon truly non-perturbative effects, are met. Within certain assumptions about the structure of the VEV's of dynamical bound states (which assumptions are testable by non-perturbative calculation), it is thus possible to test a gauge theory in which the symmetry breaking is postulated to be of purely dynamical origin.

In the present note, we shall illustrate the above assertions with reference to one of the longstanding problems of particle physics, the calculation of the electron-muon mass ratio. It is well known that attempts to solve this problem within the context of gauge theories with elementary Higg's fields have been faced by very serious difficulties.[4] After rephrasing these difficulties in our own language, it will become clear how an approach using dynamical symmetry breaking avoids them and, thus, makes it possible to compute that ratio in models based on gauge groups for which we had previously been forced to regard the ratio in question as a free parameter. This is true, in particular,of Weinberg's SU(3)xSU(3) model,[5] in the context of which the suggestion that the electron- muon mass ratio in gauge theories should be computable was originally made , and obstacles in the way to its implementation were later pointed out.[4] It is in terms of that particular model that, to have something definite in mind, we shall present our arguments, in spite of the fact that our considerations will manifestly possess a wider range of applicability .

In gauge models with elementary Higg's fields, the mass of a lepton is usually obtained from the perturbative expansion of Fig. 1. The first and second terms on the r.h.s. of this equation represent zeroth order singlet and tadpole conributions, where, in the latter, the VEV (denoted by a cross) of the relevant scalar field is computed in the tree approximation from the scalar potential appearing in the Lagrangian. Subsequent terms in the expansion of Fig. 1 represent one-loop contributions from gauge boson and scalar exchanges and tadpole terms, while a sum over two-loops and higher order contributions is, also, implicitly understood.

As follows from general renormalization theory arguments[6] (partially verified by explicit calculations[7]), the divergent parts in the above expansion transform under the gauge group either trivially (as singlets) or as the zeroth order tadpole terms and can therefore be absorbed in a redefinition of the Lagrangian parameters associated with these quantities. Consequently, from this perturbative point of view we conclude that, if and only if the gauge group and the group representations are such as to prevent zeroth order singlet and tadpole terms from contributing to a certain lepton mass, will this mass (say, that of the electron) be "calculable" in terms of other physical quantities.

The first condition (absence of singlet contributions) is automatically enforced in chiral gauge theories, such as Weinberg's SU(3)xSU(3) model[5]. We remind the reader that in this model the leptons are arranged in a Konoponski-Mahmoud triplet (μ^+, ν, e^-) with left-handed and right-handed components transforming under the gauge group as a $(1,\bar{3})$ and (3,1), respectively. The only meson representation which couples to the leptons by the gauge-invariant Yukawa coupling $(h\bar{\psi}_L \phi \psi_R$ +H.C.) is a complex 3-by-3 elementary spinless matrix field transforming as a (3,3). Since the VEV of the latter is responsible for the zeroth order lepton masses, to satisfy our second condition that there be no zeroth order tadpole term contributing to the electron mass we must then insist that the meson field in the array which Yukawa-couples to the electron (ϕ_e) have, in the tree approximation, identically vanishing VEV.

Straightforward stability criteria imply in turn that, if this requirement is to be fulfilled, the effective Lagrangian cannot possess terms linear in ϕ_e, which may arise whenever ϕ is, in the language of Ref.8, "locked" to other meson representations (χ) also acquiring a non-vanishing VEV. The presence of such "locking terms" as counterterms in the Lagrangian is sometimes forced by renormalizability, i.e., by the need to absorb divergences in amplitudes involving both ϕ and χ which exist as a result of loop contributions present in some order of the gauge interaction.

Unfortunately, this is precisely what happens in the case of Weinberg's model. If the electron in this model is to acquire its mass (of order α times the muon mass) via radiative corrections involving the muon, it is necessary that there be a direct mass mixing between left-handed (W_L) and right-handed (W_R) gauge fields induced by some scalar meson representation (χ). But then there exist superficially divergent two-loop diagrams, considered by Georgi and Glashow[4] (see Fig. 2), with four external elementary meson legs including one ϕ_e whose renormalization indeed requires the introduction of the unwanted locking terms we referred to above. Thus, the choice of zeroth order VEV which gives the muon a mass while keeping the electron massless turns out to be inconsistent with simple stability criteria; the VEV of ϕ_e is incalculable and so is the electron-muon mass ratio.

While there appear to be ways to finesse the impasse presented by the diagram of Fig. 2 in models which still make use of elementary scalar representations,[4] it seems now unlikely that this goal can be achieved with an acceptable model, which is not too "ad hoc" and for which the electron-muon mass ratio does not depend on a number of artificial parameters.

Now let us consider the case in which there are no elementary scalar fields. Several authors have appealed to the possibility that the Schwinger mechanism or some variant may provide for the cancellation of the unwanted vector meson poles at zero momentum-squared. We do wish to assume that the theory exists, as these authors do implicitly. However, rather than miraculous pole cancellations, we only assume that the theory generates gauge group multiplets of scalar bound states. Given that assumption, it is necessary to follow the observation of Coleman and E. Weinberg,[9] namely that the true vacuum state is obtained by minimizing a potential functional in the effective action which is defined as a functional Legendre transform of the usual action.[10] The effective action is a functional of properly defined classical fields, but in addition to fields corresponding to the elementary fields in the starting Lagrangian, there are also fields corresponding to each of the bound states. The potential function to be minimized is expected to depend on the classical fields associated with the assumed scalar bound states. This dependence is calculable in principle, but we have no a priori knowledge as to the relative signs and magnitudes of the coefficients of the polynomial terms. (The potential is an infinite sum of polynomial terms by construction, see Ref.9).

If we assume that the minimum of the potential occurs at a non-zero VEV for some of the bound state fields, then we return to the structure of a theory with Higg's scalars, but without the embarassments of elementary scalars. The diagram in Fig. 2 still exists, but is finite and calculable due to the appearance at the vertices of convergence factors arising from the structure of the bound states. The structure of the fermion mass contributions is altered from that of Fig. 1 to that of Fig. 3, where we have replaced the classical fields associated with the scalar bound states by their VEV's everywhere in the effective action and abstracted the terms proportional to the product of the fermion field and its Dirac adjoint and containing no derivatives. In the lepton model considered here, the first term must still vanish as no mass scale can appear in it, and the third term likewise will not contribute by virtue of its group structure. We may refer to such structure since we have postulated that symmetry breaking arises only via VEV's, and so the polynomial terms in the effective action must themselves be gauge invariant.[11] Thus, the terms that contribute to the electron mass for instance, include single tadpole terms and also terms with three tadpoles such as is elaborated in Fig.4, namely terms of the structure proposed by Weinberg.

Note that all of the fermion masses (that of the muon as well as that of the electron in Weinberg's model) are computable as all of the terms in Fig. 3 are finite (unrenormalized). However, given the present day poverty of non-perturbative calculational techniques, this seems an impossible task practically. Indeed it would require non-perturbative calculation at least of the bound states and of the potential functional that generates the VEV's. On the other hand, if we are more modest in our goals, guided by our experience with elementary scalars we can assume the existence of, and an approximate form for the VEV's and examine the resulting theory for self-consistency (iteratively) and implications. In particular this may enable us to calculate the electron-muon mass ratio approximately within the assumptions, though not the electron and muon masses separately.

Specifically, in Weinberg's chiral SU(3) model, we can reinstate in form the viability of his original conjecture as to the dominant contribution to the electron mass. In fact, Fig. 4 is just that contribution with composite scalars and form factors replacing elementary scalars and vertices. We have already argued that the first and third terms in Fig. 3 do not contribute to the electron mass in this model. Thus, for the contribution represented by Fig. 4 to be dominant we must assume that the second term and the implicit terms of Fig. 3 are, for the electron, in comparison, negligibly small. This implies, in particular, that the one-tadpole term for the electron must be sufficiently small (say, at least one order of magnitude smaller than α) in the scale set by the corresponding tadpole term for the muon.

To determine the self-consistency of this assumption we must consider terms in the effective action that are linear in ϕ_e and involve ϕ_μ (as well as any other fields), since these provide the kind of functional coupling that would tend to generate a non-vanishing VEV for ϕ_e once $\langle\phi_\mu\rangle\neq 0$, and argue that these terms are at least one order of magnitude smaller than α. This we expect to be true, indeed, in the case of contributions coming from diagrams with the topological structure of Fig. 2 because of the following reasons: a) these diagrams involve one explicit power of α originating in the vector meson-fermion vertices; b) the effective scalar-fermion coupling constants should be very small, if the appearance of parity and strangeness violation effects at intolerable levels in the hadron sector is to be avoided[12].

Thus we are led to the following approximate expression for the electron-muon mass ratio in a chiral SU(3) model with composite Higg's fields:

$$(m_e/m_\mu) \simeq (\alpha/2\pi)\left|\sin 2\theta \ \ln (M_1^2/M_2^2)\right| , \qquad (1)$$

where M_1, M_2 are the masses of the physical (diagonal) gauge bosons and θ is the mixing angle relating these bosons to W_L and W_R. Eq. (1) assumes that the mass scale characterizing the fall-off in momentum space of the form factors in Fig. 4 is sufficiently large so as not to appreciably affect the value of the loop integral. Needless to say, if this condition turned out not to be true, the electron-muon mass ratio would, from our point of view, regain its long-standing status as a mystery[13].

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References

1. For a review see, for example, E.S. Abers and B.W. Lee, "Lectures on Gauge Theories", Stony Brook preprint No. 73-20 (1973).

2. R. Jackiw and K. Johnson, MIT preprint No. 348; J. Cornwall and R. Norton, UCLA preprint 73/TEP 78.

3. Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961).

4. H. Georgi and S. L. Glashow, Phys. Rev. D7, 2457 (1973), and D6, 2977 (1972).

5. S. Weinberg, Phys. Rev. D5, 1962 (1972).

6. S. Weinberg, Phys. Rev. Letters, 29, 388 (1972).

7. See, for example, R. N. Mohapatra and P. Vinciarelli, Phys. Rev. D8, 481 (1973).

8. S. Weinberg, Phys. Rev. Letters, 29, 1698 (1972).

9. S. Coleman and E. Weinberg, Phys. Rev. D7, 1888 (1973).

10. J. Rzewuski, Field Theory, Vol. 2 (Revised English Edition, Hafner, New York, 1969).

11. This is true for all terms not involving vector gauge mesons or derivatives since such terms are not sensitive to the local character of the gauge group.

12. R. N. Mohapatra, J. Pati, and P. Vinciarelli, "Canonical Estimation of Weak Radiative Corrections in Unified Gauge Theories and Selection Rules", Phys. Rev., to be

published.

13. For an alternative point of view, see P. Vinciarelli "Non-perturbative Calculation of the Electron-muon Mass Ratio in Gauge Theories", SLAC-PUB-1352, November 1973.

FIGURE CAPTIONS

1. Loop expansion for fermion self-energies.
2. Superficially divergent Feynman diagram leading to the loss of the zeroth order masslessness of the electron in Weinberg's SU(3)xSU(3) model with elementary Higg's fields.
3. Tadpole expansion for fermion self-energies.
4. Diagram for the electron mass.

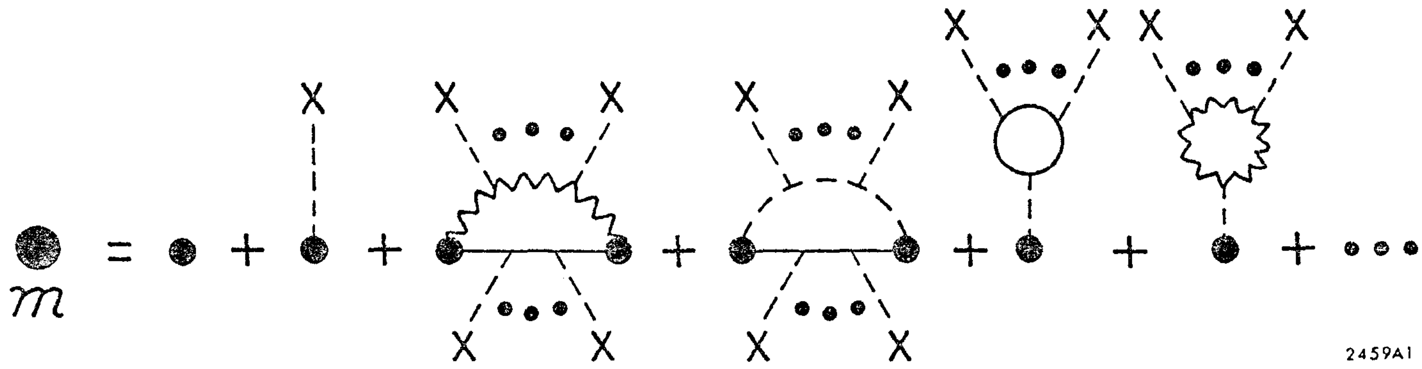

Fig. 1

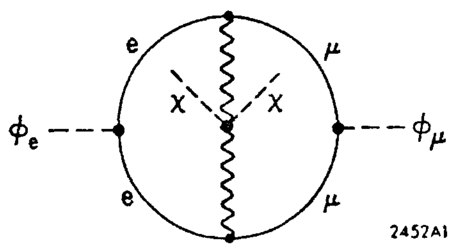

Fig. 2

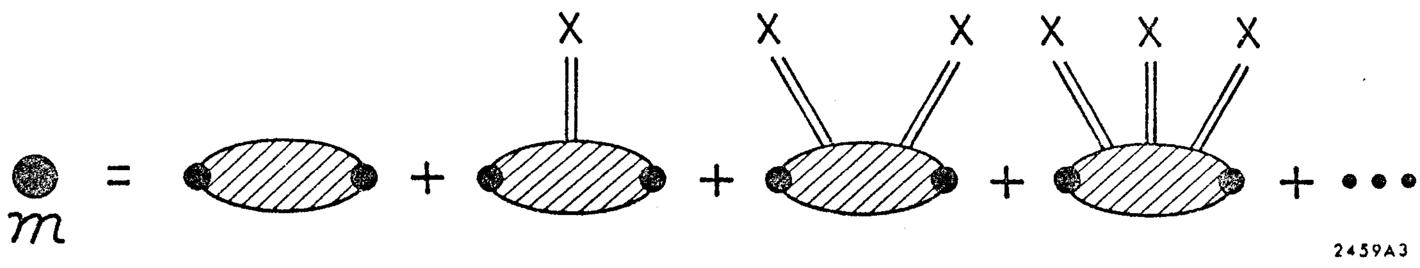

Fig. 3

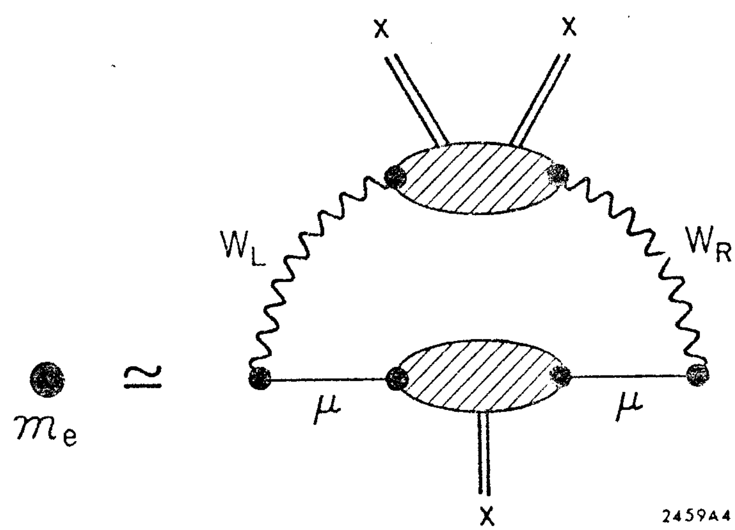

Fig. 4