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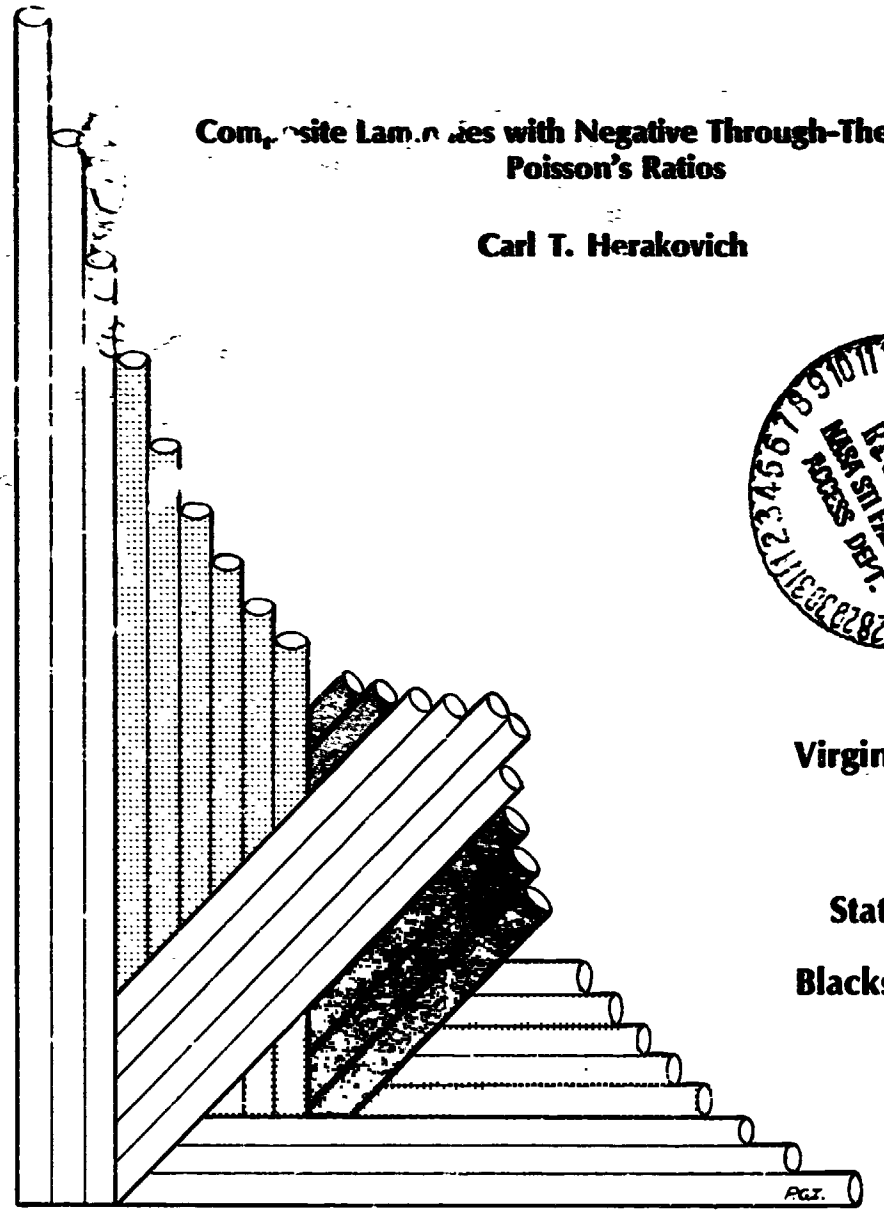

Composite Laminates with Negative Through-The-Thickness Poisson's Ratios

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Composite Laminates with Negative Through-the-Thickness Poisson's Ratios

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INTRODUCTION

The through-the-thickness effective poisson's ratio of a composite laminate can be an important property in the application and testing of composite materials. Thickness changes can influence the gripping force on test specimens and the measurement of transverse displacements. They have the potential to have a critical effect on structures requiring a high degree of dimensional stability such as optical benches, space telescopes and antennas, and joints. Further, thickness changes can be very important to the performance of bonded and bolted joints. However, little if any attention has been given to this laminate property. In fact, no references to the subject were found in the literature during the preparation of this manuscript.

Negative in-plane poisson's ratios ν_{xy} have been reported previously [1] and negative average through-the-thickness poisson's ratios on the free edge of laminates have been measured [2]. These negative values of the free edge were shown to correlate well with finite element results. However, no attempt was made to study the general laminate behavior away from edges. As will be shown, there is a significant difference between the behavior in interior regions and that at the free edge. Negative poisson's ratios have also been reported for a small number of other materials [3].

It should be emphasized that the effective poisson's ratio is a laminate (or structural) property and not a material property. As will be demonstrated, it is a function of the internal stress state in the

laminate. Henceforth in this paper, the term poisson's ratio will be used to imply the effective laminate property.

PROBLEM FORMULATION

The magnitude of the through-the-thickness poisson's ratio ν_{xz} can be predicted quite easily through a combination of lamination theory and the appropriate three-dimensional constitutive equation. Consider a symmetric composite laminate under axial load N_x (Fig. 1). Lamination theory gives the midplane strains

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}^{-1} \begin{Bmatrix} N_x \\ 0 \\ 0 \end{Bmatrix} \tag{1}$$

where $A_{ij} = \sum_{k=1}^{N} \bar{Q}_{ij}^{k} t^k$, $\bar{Q}_{ij}^{k}$ are coefficients of the reduced stiffness matrix, t^k are the individual layer thicknesses and N is the number of plys. The through-the-thickness poisson's ratio is defined

$$\nu_{xz} = \frac{-\epsilon_z}{\epsilon_x} \tag{2}$$

where the strain ϵ_z equals the total transverse displacement W divided by the laminate thickness 2H. The total W displacement can be written

$$W = \int_{-H}^{H} \epsilon_z dz = \sum_{k=1}^{N} \epsilon_z^{\,k} t^k \tag{3}$$

where ε_z^k is the constant strain in the k^{th} layer. The strain ε_z^k in any layer can be determined using the three-dimensional constitutive equation for a monoclinic (off-axis) layer

$$\varepsilon_z^k = \bar{S}_{31}{}^k\sigma_x^k + \bar{S}_{32}{}^k\sigma_y^k + \bar{S}_{33}{}^k\sigma_z^k + \bar{S}_{36}{}^k\tau_{xy}^k \tag{4}$$

where the $\bar{S}_{ij}$ are the coefficients of the transformed 3-D compliance matrix.

Away from free edges, the laminate is in a state of plane stress and hence σ_z^k is zero. Thus, ε_z^k in each layer is

$$\varepsilon_z^k = \bar{S}_{31}{}^k\sigma_x^k + \bar{S}_{32}{}^k\sigma_y^k + \bar{S}_{36}{}^k\tau_{xy}^k \tag{5}$$

The stresses in each layer can be determined from two-dimensional constitutive equations and the inverse of A_{ij} as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}^k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}^k \begin{Bmatrix} A_{11}^{-1} \\ A_{12}^{-1} \\ A_{16}^{-1} \end{Bmatrix} N_x \tag{6}$$

Substituting (6) into (5) gives ε_z^k in each layer in terms of material and laminate properties and the applied load

$$\varepsilon_z^k = N_x[A_{11}^{-1}(\bar{S}_{31}^k\bar{Q}_{11}^k + \bar{S}_{32}^k\bar{Q}_{12}^k + \bar{S}_{36}^k\bar{Q}_{16}^k)$$

$$+ A_{12}^{-1}(\bar{S}_{31}^k\bar{Q}_{12}^k + \bar{S}_{32}^k\bar{Q}_{22}^k + \bar{S}_{36}^k\bar{Q}_{26}^k) \qquad (7)$$

$$+ A_{16}^{-1}(\bar{S}_{31}^k\bar{Q}_{16}^k + \bar{S}_{32}^k\bar{Q}_{26}^k + \bar{S}_{36}^k\bar{Q}_{66}^k)]$$

Combining (3) and (7) gives the total W displacement

$$W = N_x [A_{11}^{-1} \sum_{k=1}^{N} (\bar{S}_{31}^k\bar{Q}_{11}^k + \bar{S}_{32}^k\bar{Q}_{12}^k + \bar{S}_{36}^k\bar{Q}_{16}^k)t^k$$

$$+ A_{12}^{-1} \sum_{k=1}^{N} (\bar{S}_{31}^k\bar{Q}_{12}^k + \bar{S}_{32}^k\bar{Q}_{22}^k + \bar{S}_{36}^k\bar{Q}_{22}^k)t^k \qquad (8)$$

$$+ A_{16}^{-1} \sum_{k=1}^{N} (\bar{S}_{31}^k\bar{Q}_{16}^k + \bar{S}_{32}^k\bar{\zeta}_{26}^k + \bar{S}_{36}^k\bar{Q}_{66}^k)t^k]$$

And dividing by the applied axial strain ε_x (Eqn. 1) and the thickness 2H provides the expression for the through-the-thickness poisson's ratio

$$\nu_{xz} = \frac{-1}{2HA_{11}^{-1}} [A_{11}^{-1} \sum_{k=1}^{N} (\bar{S}_{31}^k\bar{Q}_{11}^k + \bar{S}_{32}^k\bar{Q}_{12}^k + \bar{S}_{36}^k\bar{Q}_{16}^k)t^k$$

$$+ A_{12}^{-1} \sum_{k=1}^{N} (\bar{S}_{31}^k\bar{Q}_{12}^k + \bar{S}_{32}^k\bar{Q}_{22}^k + \bar{S}_{36}^k\bar{Q}_{26}^k)t^k \qquad (9)$$

$$+ A_{16}^{-1} \sum_{k=1}^{N} (\bar{S}_{31}^k\bar{Q}_{16}^k + \bar{S}_{32}^k\bar{Q}_{26}^k + \bar{S}_{36}^k\bar{Q}_{66}^k)t^k]$$

Defining

$$F_i = \sum_{k=1}^{N} (S_{31}Q_{i1} + S_{32}Q_{i2} + S_{36}Q_{i6}) \qquad (i = 1,2,6) \qquad (10)$$

we can write

$$\nu_{xz} = \frac{-1}{2HA_{11}^{-1}} \left[A_{11}^{-1}F_1 + A_{12}^{-1}F_2 + A_{16}^{-1}F_6\right] \tag{11}$$

A similar development for axial loading N_y gives the through-the-thickness poisson's ratio ν_{yz}

$$\nu_{yz} = \frac{-1}{2HA_{22}^{-1}} \left[A_{21}^{-1}F_1 + A_{22}^{-1}F_2 + A_{26}^{-1}F_6\right] \tag{12}$$

Equations (11 and (12) can be written in condensed notation

$$\begin{aligned} \nu_{xz} &= \frac{-1}{2HA_{11}^{-1}} \left[A_{1i}^{-1}F_i\right] \\ \nu_{yz} &= \frac{-1}{2HA_{22}^{-1}} \left[A_{2i}^{-1}F_i\right] \qquad (i = 1,2,6) \end{aligned} \tag{13}$$

where summation of the repeated subscript i is implied.

RESULTS

Typical results for T300/5208 graphite-epoxy are shown in Fig. 2. The figure shows a plot of the variation in ν_{xz} for unidirectional off-axis laminae and angle-ply laminates ($[\pm\theta]_s$) with fiber orientations ranging from 0 to 90 degrees. It can be seen that ν_{xz} varies radically with fiber orientation in angle-ply laminates, attaining a minimum negative value of -0.21 for a $[\pm 25]_s$ laminate compared to a maximum

positive value of 0.49 for a [90] laminate. The negative poisson's ratios for angle-ply laminates with fiber orientations between 15° and 40° is due to the high degree of normal-shear coupling (non-zero $\bar{Q}_{16}^{k}$) and the constraining influence of adjacent layers. The coupling results in high inplane shear stresses and a lateral poisson's ratio ν_{xy} greater than 1.0 [1]. A lateral poisson's ratio greater than one and a negative through-the-thickness poisson's ratio are physically consistent.

The through-the-thickness poisson's ratio for angle-ply laminates differs drastically from that for off-axis lamina (Fig. 2) much like the inplane poisson's ratio of the laminate differs from that of the lamina [1]. Both results show clearly that a laminated composite is a structure whose elastic constants cannot be determined by a simple rule of mixtures. In fact ν_{xz} is an even function of θ for unidirectional off-axis laminae and thus is positive for both plus and minus fiber orientations. And yet the combination of plus and minus theta layers has a negative poisson's ratio. This result is a clear indication that all interactions between constraining layers must be properly accounted for.

Results for ν_{yz} of angle-ply laminates are shown in Fig. 3. They correspond to a reflection of the ν_{xz} results about the 45° orientation.

The underlying physics of the negative poisson's ratio phenomenon may be better understood through consideration of the plane stress expression for the through-th thickness strain in material principal coordinates

$$\epsilon_3 \equiv e_z = \frac{-\nu_{13}\sigma_1}{E_1} - \frac{\nu_{23}\sigma_2}{E_2} \tag{14}$$

From Eq. (14) it is evident that ϵ_z can be negative if either σ_1 or σ_2 is compressive and of sufficient magnitude. For angle-ply laminates, σ_1 is a constant tensile value and σ_2 is a constant compressive value for all layers (Fig. 4). In the fiber orientation range 15°-40°, the magnitude of σ_2 is sufficiently large to render ϵ_z positive for all layers of the laminate.

Figure 5 shows through-the-thickness poisson's ratios for $[0/\pm\theta]_s$ and $[0_2/\pm\theta]_s$ laminates compared to the results for the laminae and angle-ply laminates of Fig. 2. Clearly, negative through-the-thickness poisson's ratios are not limited to $[\pm\theta]_s$ angle-ply laminates. There is a wide variety of $[0_2/\pm\theta]_s$ and $[0/\pm\theta]_s$ laminates which have negative poisson's ratios ν_{xz}. These curves show the influence of fiber orientation as well as the influence of the percent 0° layers. They also show that there is a wide variety of laminates which can be tailored to have a zero through-the-thickness poisson's ratio.

The $[0_2/\pm 90]_s$ laminate is equivalent to the cross-ply laminate $[0_2/90_2]_s$. Its ν_{xz} is 0.413. Also shown for comparison is the through-the-thickness poisson's ratio for a quasi-isotropic $[0/\pm 45/90]_s$ laminates ($\nu_{xz} = 0.301$). It is interesting that the $[0/\pm 45/90]_s$ quasi-isotropic laminate has the same through-the-thickness poisson's ratio as the $[0/\pm 60]_s$ quasi-isotropic laminate. It should be noted that all laminate results are independent of stacking sequence for symmetric laminates.

As mentioned previously, the analysis presented is valid only away from free edges. The influence of edge effects is demonstrated in Fig. 6. Figure 6a shows the in-plane (v,w) displacements (not to scale) in a quarter section of a $[30_2/-30_2]_s$ laminate under tensile axial load N_x as obtained using finite elements [4]. (All finite element plots have been normalized for a uniform maximum deformation). Both undeformed and deformed shapes are indicated in the figure. It is evident that the laminate is thicker under load in interior regions away from the edge corresponding to a negative possion's ratio as predicted by the simplified analysis of Eqns. 1-13. It is also interesting to note that the opposite is true in the boundary layer along the free edge where the complex triaxial stress state results in a variable poisson's ratio which attains a maximum positive value at the free edge. It is important to note that the simplified analysis of Eqns. 1-13 and the elasticity solution of the finite element analysis give the same result in interior regions.

Unlike interior regions, the thickness changes in the boundary layer region are a function of the laminate stacking sequence. This is demonstrated more dramatically in Figs. 6b and 6c where the deformed shapes of two quasi-isotropic laminates of different stacking sequence are shown. Both laminates have identical thickness changes in interior regions, but entirely different deformed shapes in the boundary layer region. The $[0/-45/90/45]_s$ laminate has a positive ν_{xz} at the free edge, but the $[+45/0/90]_s$ laminate has a small negative ν_{xz} at the free edge. Obviously, the stacking sequence plays a dominant role in the

boundary layer region. These results are consistent with those presented in reference [2].

Another interesting feature of the results presented here is the influence of the fiber orientation in angle-ply laminates on the dilatation $e = \varepsilon_x + \varepsilon_y + \varepsilon_z$. The laminate dilatation can be positive or negative depending on fiber orientation. For example, a $[\pm 10]_s$ laminate has a positive dilatation $e = 0.32\varepsilon_x$ whereas a $[\pm 30]_s$ laminate has a negative dilatation $e = -0.32\varepsilon_x$. Thus it is possible to tailor the dilatation of a composite laminate over a range of values including zero. This compares to the positive dilatation of a typical metal ($\nu = 0.3$) of $e = 0.4\varepsilon_x$.

CONCLUDING REMARKS

It has been shown that the through-the-thickness poisson's ratios of graphite-epoxy angle-ply and related laminates exhibit a wide range of values including relatively large negative values for laminates having layers with high normal-shear coupling. Poisson's ratios which are negative in one plane, but greater than 1.0 in an orthogonal plane are unique to anisotropic, layered materials. It may be surprising to engineers and scientists unfamiliar with these materials that such properties are possible. Lateral poisson's ratios greater than one have been measured in the laboratory. To this authors knowledge, no attempt has been made to measure the through-the-thickness poisson's ratio away from edges.

Since the laminates studied here are symmetric and subjected to inplane loading, the results are independent of laminate stacking sequence (away from edges). Similarly unusual results are to be expected for other properties such as the through-the-thickness coefficients of thermal and hygroscopic expansion.

Finally, the ability to tailor materials to have negative or zero poisson's ratios and negative or zero dilatation presents some exciting possibilities for innovative uses of these materials.

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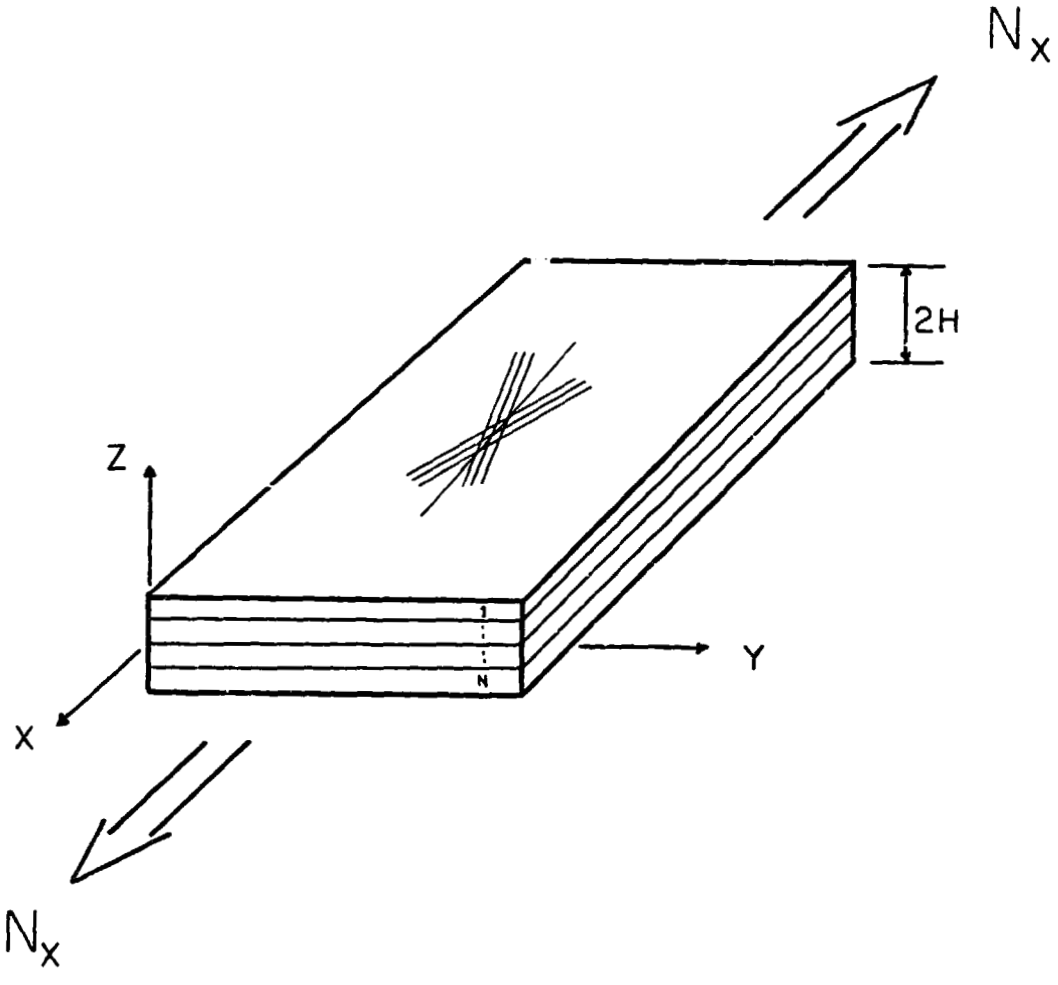

FIG.1. LAMINATE UNDER AXIAL LOAD

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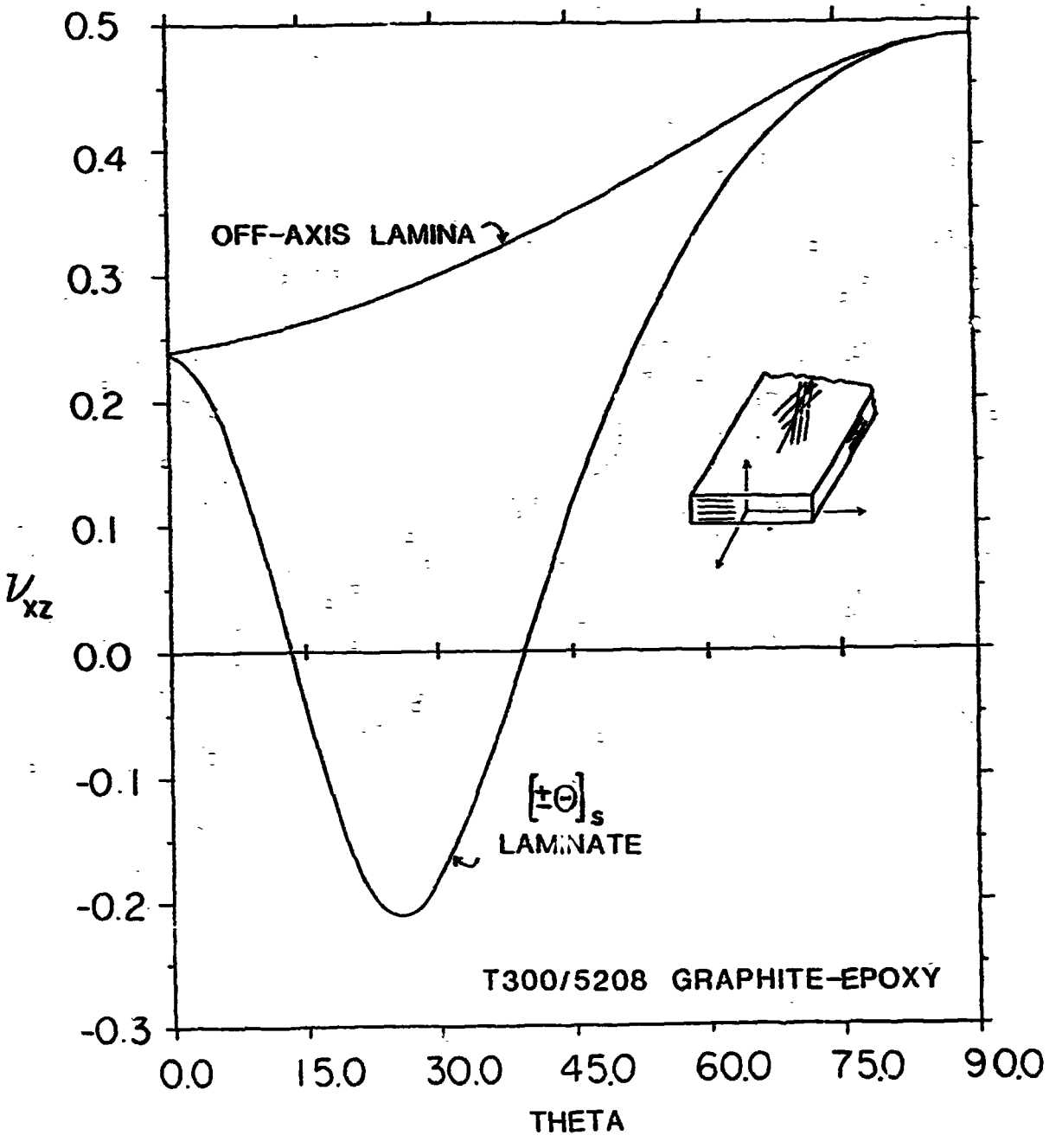

FIG.2. ν_{xz} FOR ANGLE-PLY AND OFF-AXIS LAMINA

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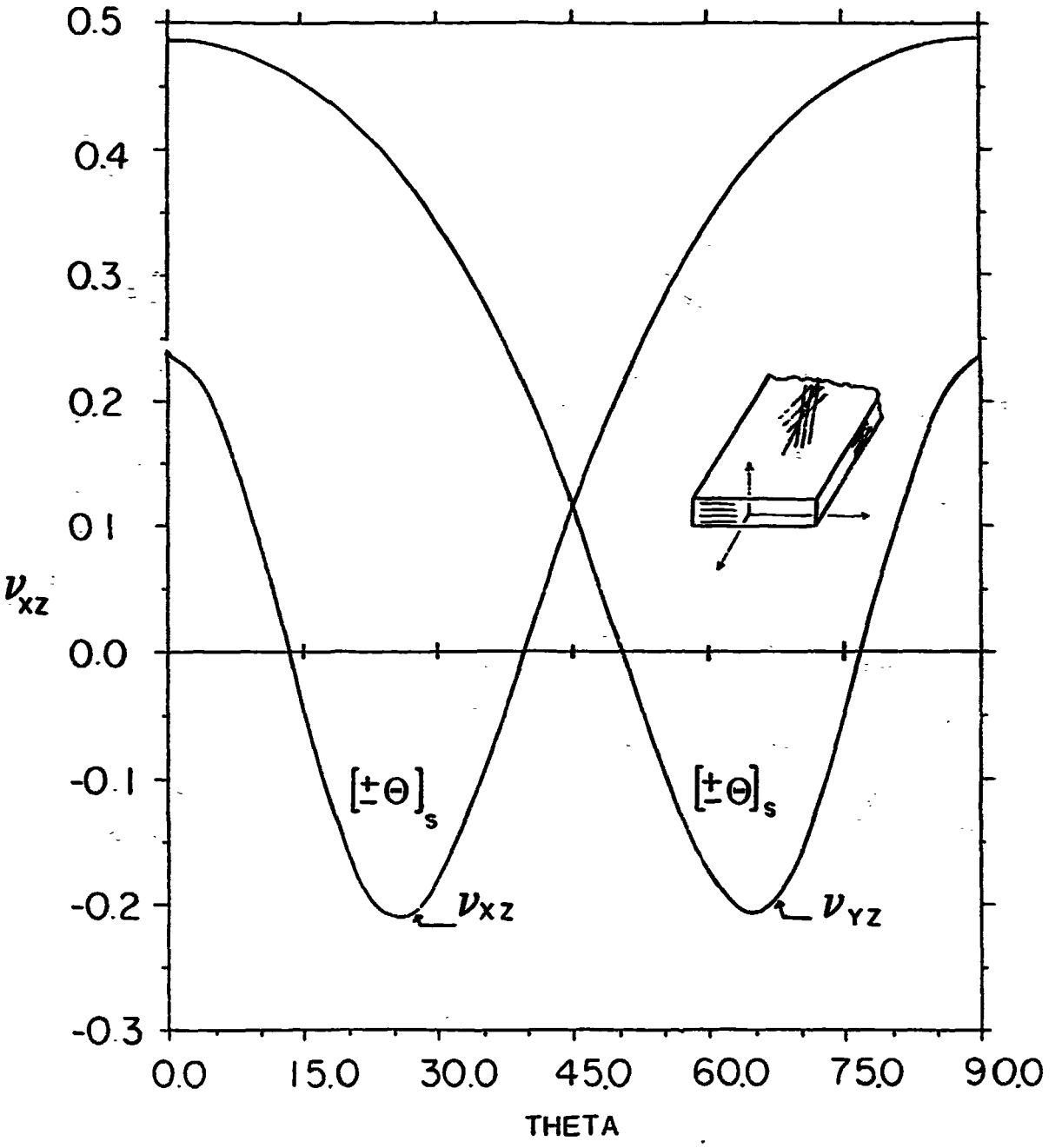

FIG.3. ν_{xz} AND ν_{yz} FOR ANGLE-PLY LAMINATES

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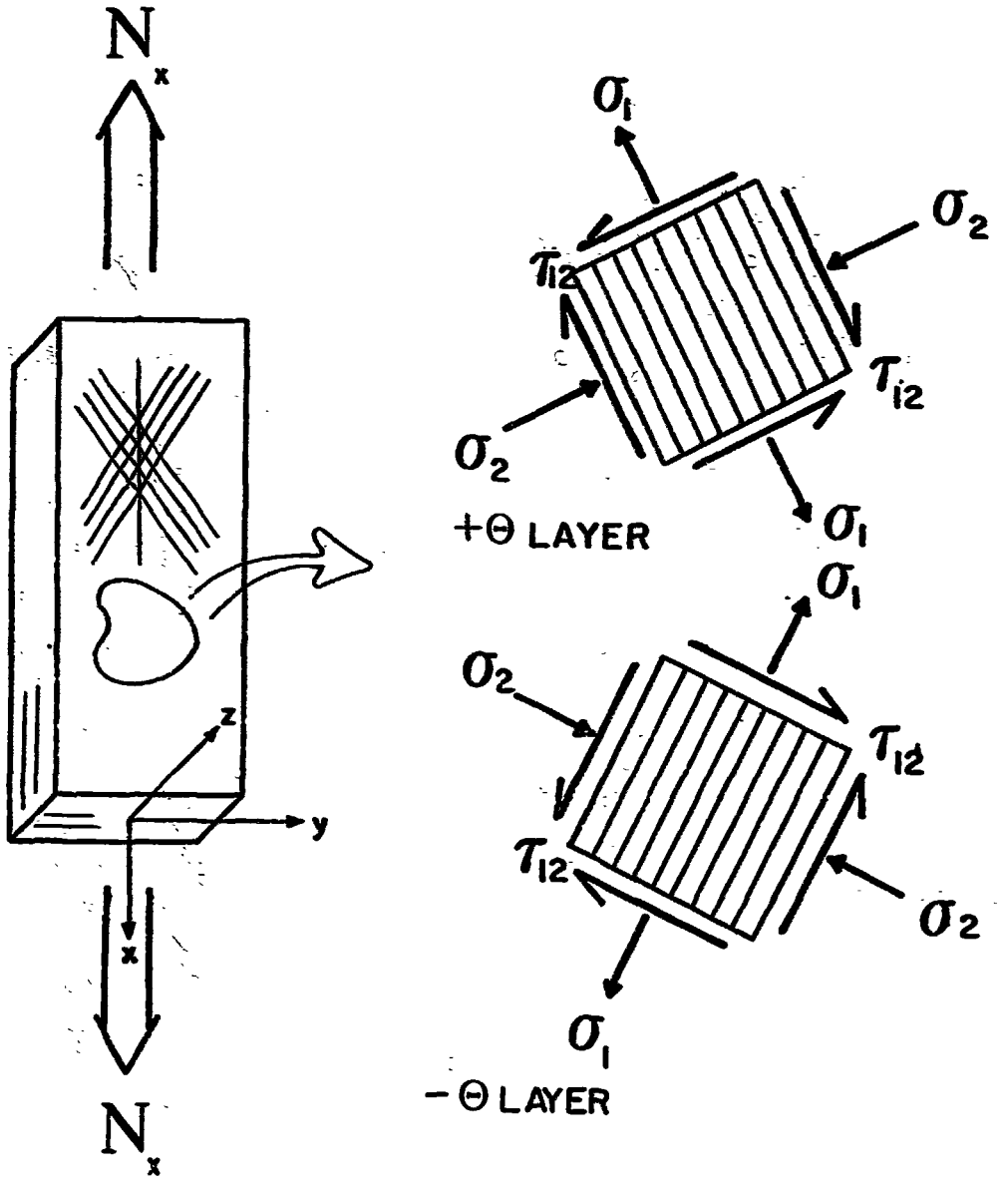

FIG.4. STRESS STATES IN ANGLE-PLY LAMINATES

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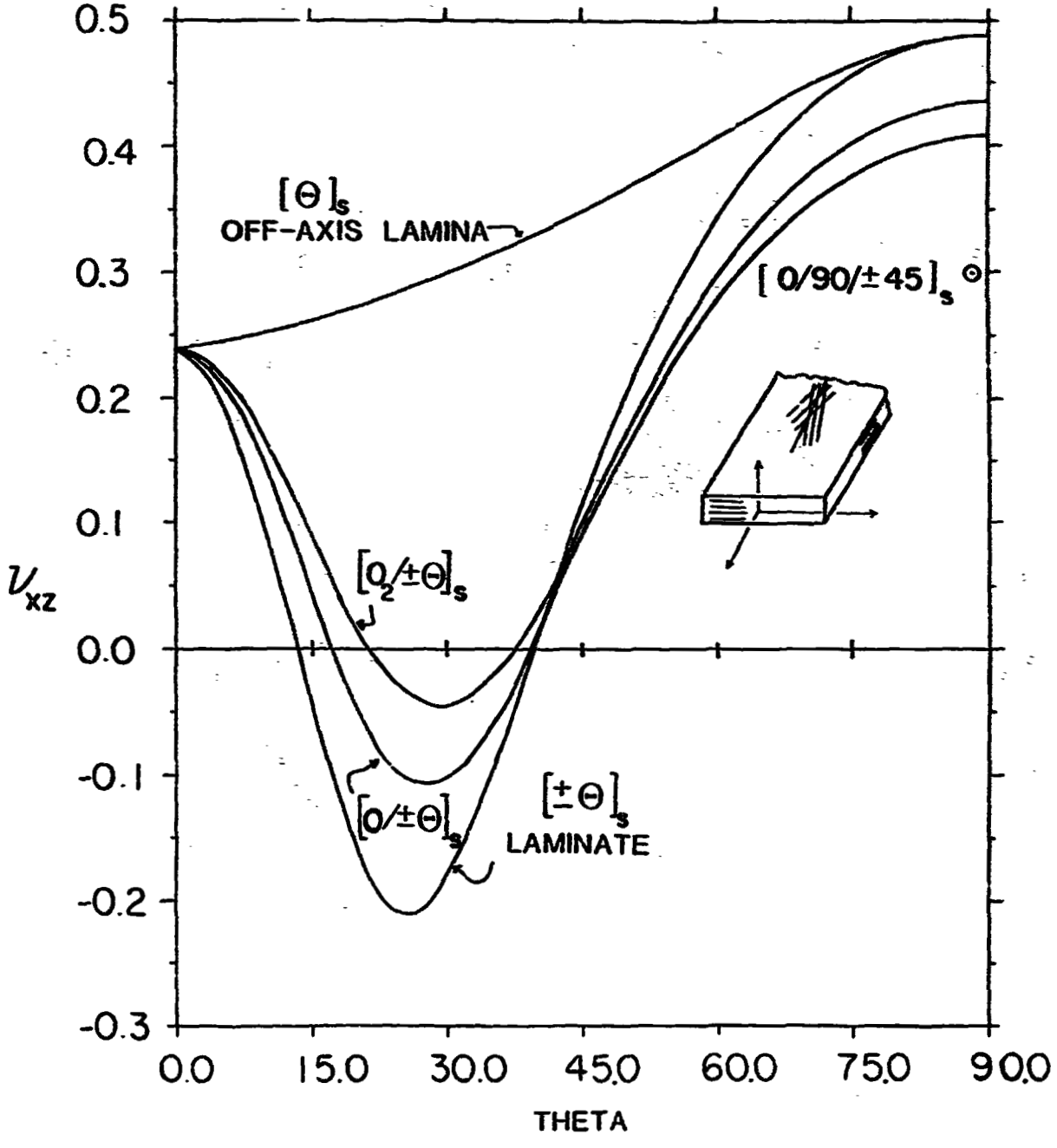

FIG.5. ν_{xz} FOR $[0_n/\pm\Theta]_s$ AND OTHER LAMINATES

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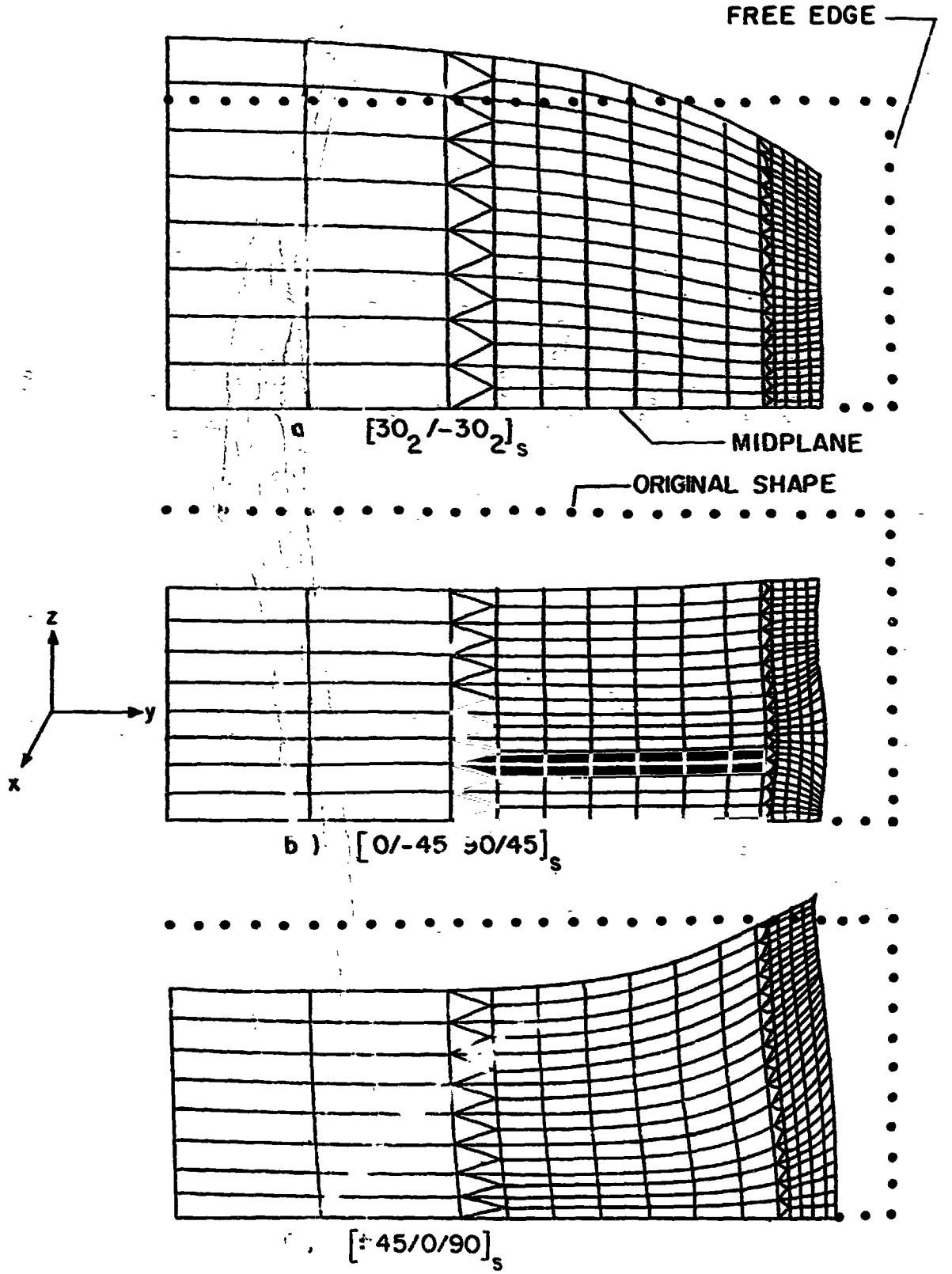

FIG.6. THICKNESS CHANGES IN BOUNDARY LAYER