

COMPOSITE MODEL PREDICTION OF ELASTIC MODULI FOR FLAKEBOARD

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ABSTRACT

The objective of this research was to develop a model to predict the flexural modulus of elasticity of oriented flakeboards. Variables incorporated into the model included flake geometry, flake orientation, density, resin content, and species. Verification of the model was accomplished by comparing predictions with flexural modulus of elasticity (MOE) values measured parallel and perpendicular to the flake alignment direction of 192 specimens made from two species (*Populus grandidentata*, Michx. and *Acer rubrum*) at 4 resin levels (3, 5, 7 and 9%) and 3 target densities (35, 42, and 52 pcf). Use of the longitudinal Halpin-Tsai equations in conjunction with measured and estimated wood and resin properties, predicted the moduli of aspen and red maple flakeboard specimens with a standard error of estimate of 93,985 psi, a coefficient of determination of 89.5%, and an average of 25% below experimental values. The statistical correlations were influenced by grouping of data using flake alignment direction and species. Important issues of vertical density gradient, inhomogeneities, and resin compatibility were not accounted for. The approach was easily computed and gave reasonably accurate predictions of elastic moduli of a single oriented flakeboard manufactured over a range of resin levels, density, alignment, flake geometry, and species conditions.

Keywords: Flakeboard, model prediction, MOE, resin content.

INTRODUCTION

An inherent advantage of the composite panel system is the relative ease with which material input properties are adjusted. Composite panel properties may be varied according to species mixtures, particle geometry, resin level, and panel density. Variations in these parameters influence the mechanical properties of the panel. Qualitative understandings of such parameter variations on final board properties have been extensively investigated. Quantitative relationships are typically empirical and are identified with extensive laboratory testing of each set of manufacturing variables.

Numerous investigators have modeled the elastic moduli of flakeboard. Hunt (1972) used a finite element approach to model the tensile moduli of a randomly oriented particleboard. Predictions of elastic moduli for two specimens were within 13 and 6% of measured values. Harris (1977) also used a finite element technique

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to predict tensile modulus and shear modulus of aspen and white pine at six levels of flake orientation. Corrections were necessary for specific gravity variations in the samples to obtain reasonable agreement between experimental and theoretical predictions.

Brown (1975) employed a laminate analogy to the particleboard panel, likening the vertical density gradient to layers of material with varying moduli. Predictions of layer properties were made by regression equations obtained from constant density gradient panels. Geimer (1986) has reported extensive regression equation compilations with the coefficients influenced by flake geometry, resin type, and flake orientation. Hunt et al. (1985) used the longitudinal rule of mixtures to predict flexural strength (MOR), apparent modulus of elasticity (MOE), and interlaminar shear strength of flake composite panels made from five species of equal portions. The values of each individual species and geometry variations were obtained by a regression analysis procedure. Predictions were typically within 10% for flexural performance. Interlaminar shear estimates were underestimated by up to 29%. Lau (1981) developed a regression model to represent the effect of flake alignment on MOE and shear moduli of waferboards made at different densities. A normal distribution was used to describe the flake alignment variation. He reported an exponential relationship between alignment and MOE, which was also influenced by the density of the boards.

COMPOSITE THEORIES

An alternate approach for modeling a composite is use of micromechanic equations to describe the net behavior after combining two or more materials together. Micromechanics refers to the determination of effective properties of a material based on the properties of constituent materials, their relative volumes, and their geometric arrangement. This approach allows great flexibility in modeling due to relative ease of computation (compared to the finite element method) and theoretical relationship of the materials joined together. An excellent survey of the application of micromechanics to prediction of the properties of composite materials is given by Hashin (1983).

Perhaps the most ubiquitous of all composite theories is that class known as the rule of mixtures. The necessary input parameters include: 1) modulus of matrix phase, 2) modulus of fiber phase, 3) volume fraction of matrix, and 4) volume fraction of fiber. The contribution of the fibers and matrix to the average composite properties is proportional to their volume fractions.

Two forms of the rule of mixtures exist. The first assumes a plane strain condition in the material. This is analogous to a uniaxial load applied parallel to the longitudinal axis of the fibers (Agarwal and Broutman 1980). This will be designated as the longitudinal rule of mixtures (LROM) and is given as:

$$E_c = V_f E_f + V_m E_m \quad (1)$$

where

- E_c = elastic modulus of the composite,
- E_f = elastic modulus of the fiber,
- E_m = elastic modulus of the matrix,
- V_f = volume fraction of the fiber, and
- V_m = volume fraction of the matrix.

Volume fraction is the proportion (decimal) of the total volume of the material. When a plane stress condition is assumed to exist in a composite, the following equation is developed:

$$E_c = E_f E_m / (E_f V_m + E_m V_f) \quad (2)$$

This condition arises when the direction of load application is perpendicular to the longitudinal axis of the fibers, and will be designated as the transverse rule of mixtures (TROM) estimate. The general form of the LROM and TROM equations are shown in Figs. 1 and 2, respectively. Numerous assumptions regarding material behavior and composite geometry are made in deriving the rule of mixtures equations. These include perfect adhesion at the phase interface, linear elastic material behavior, absence of residual stress, fibers and matrix that are isotropic, and fibers that are perfectly aligned (Jones 1972). The influence of these assumptions on the accuracy of predictions may be determined only by comparing calculated to experimental data.

Another important class of equations was developed by Halpin and Tsai (1969). The Halpin-Tsai equations are based on the rule of mixture formulation and account for fiber discontinuities through a slenderness ratio (length/thickness) and packing orientation parameter (Halpin and Kardos 1976). These equations have found wide usage because of their relative accuracy, especially in tensile loadings, and comparatively simple form (Agarwal and Broutman 1980). Salmen and deRuvo (1985) used the longitudinal Halpin-Tsai equation, along with estimates of the elastic moduli of lignin, hemicellulose, cellulose and laminate theory, to predict the longitudinal elastic moduli of individual wood fibers.

The Halpin-Tsai equation for a plane strain condition (LHT) is:

$$E_c = E_m [(1 + 2SN_1 V_f) / (1 - N_1 V_f)] \quad (3)$$

where

$$N_1 = (E_f/E_m - 1) / (E_f/E_m + 2S)$$

S = slenderness ratio (length/thickness) of fiber.

The Halpin-Tsai equation for transverse (plane stress condition) moduli (THT) is:

$$E_c = E_m [(1 + 2N_t V_f) / (1 - N_t V_f)] \quad (4)$$

where

$$N_t = (E_f/E_m - 1) / (E_f/E_m + 2).$$

Composite moduli predicted using the LHT theory increase as fiber slenderness ratio (length/thickness) increases (Fig. 1). This reflects the increasing effectiveness of load transfer between matrix and fiber by means of shear stress. As the slenderness ratio increases, the LHT estimate approaches the LROM estimate.

The transverse (plane stress) composite equations reflect no fiber length influence due to the difference in assumed boundary conditions. Differences in predictions between the transverse Halpin-Tsai (THT) and transverse rule of mixtures (TROM) for ratios of fiber to matrix moduli of 10 and 20 are plotted in Fig. 2. It should be noted that use of a slenderness ratio of 1 results in equivalent predictions of the LHT and THT equations.

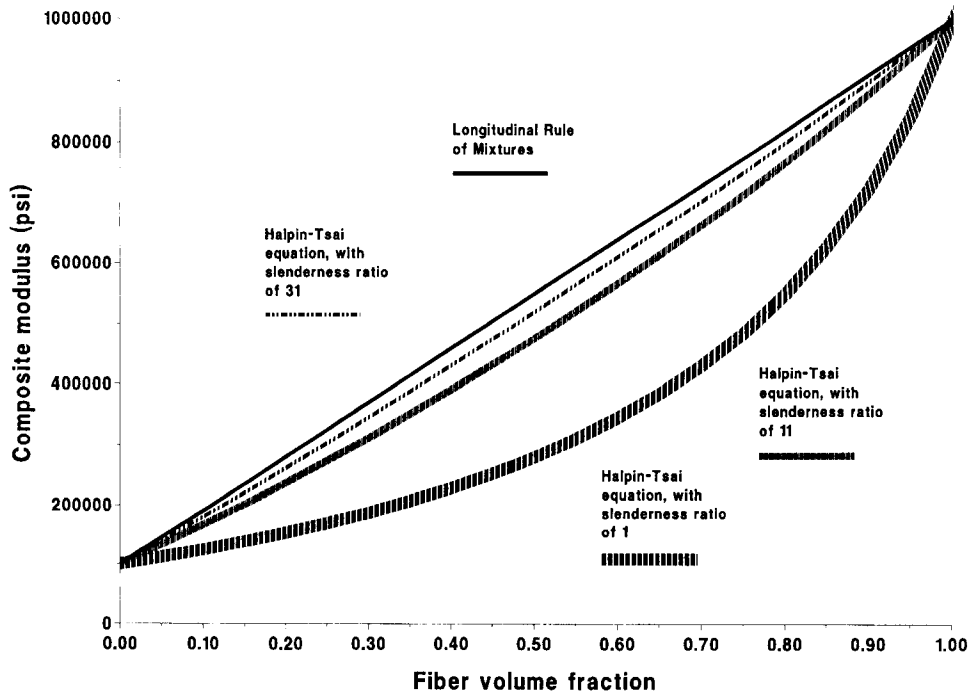


FIG. 1. Longitudinal composite modulus as a function of fiber volume fraction and slenderness ratio, a comparison of the rule of mixtures and Halpin-Tsai equations. (Matrix E = 100,000 psi, Fiber E = 1,000,000 psi.)

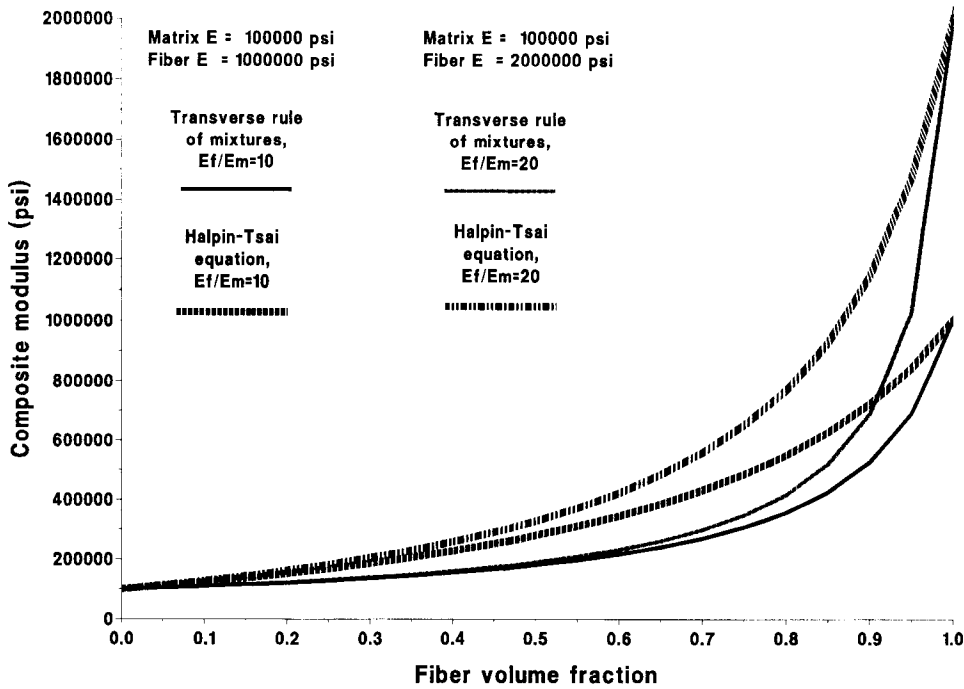


FIG. 2. Transverse composite modulus as a function of fiber volume fraction, a comparison of the rule of mixtures and Halpin-Tsai equations.

The objective of this research was to evaluate the suitability of the rule of mixtures (ROM) and Halpin-Tsai (HT) equations to predict flexural modulus of elasticity of oriented flakeboard. Use of information regarding species, resin type and amount, particle alignment, and overall density was used within the physical interpretation framework suggested by the polymer composite theories, to predict the flexural moduli.

PROCEDURES

The modeling process

An infinite number of assumptions regarding material and composite geometry are possible. The complete description of the composite would include the geometry of every particle, its orientation in space, the mechanical properties and their variation, resin distribution, moisture content, temperature, and material history. This is clearly unattainable. Various approaches have been investigated to simplify the actual conditions present in the flakeboard. The influence of these simplifications on predictive accuracy is described for selected modeling procedures.

The composite moduli of aligned flakeboard specimens was assumed to be influenced solely by selected physical and mechanical properties of the wood, the resin, and air. The specific factors investigated included: 1) elastic moduli of wood species, 2) elastic moduli of resin, 3) geometry of flakes (for Halpin-Tsai equations only), 4) alignment of wood flakes, and 5) volume fractions of the wood, resin, and air present in a specimen. Important issues of vertical density gradient, inhomogeneities, and resin compatibility were not accounted for in models.

Determination of input properties

Determination of elastic moduli of the wood species, resin, and flake geometry was accomplished prior to board manufacture. The elastic moduli of the flakes was estimated using established solid wood orthotropic relationships (Bodig and Goodman 1973). The flexural MOE of 42 specimens, using ASTM D-143 alternate procedures (ASTM 1987), was measured on material selected from the trees used to make the flakes. These flexural MOE values were adjusted for shear deformation and orthotropic relationships were applied (Bodig and Goodman 1973) to estimate all wood elastic constants (Table 1). Properties of the wood and flakeboard were adjusted to be at an 8% moisture content. The adjustment procedure assumed a 2% change in MOE for a 1% moisture change (Markwardt and Wilson 1935). The elastic constants of phenol-formaldehyde resin reported in Table 1 were obtained from the literature (Wellons 1983). The elastic constants of air were set to zero (no contribution to performance).

To describe flake geometry, the length and thickness of 192 flakes from each species were measured. The average length, width, and thickness of the aspen flakes were 1.502, 0.131, and 0.013 inch, respectively. For the red maple flakes, the length, width, and thickness were 1.503, 0.210, and 0.014 inch, respectively. The average calculated length to thickness ratio was 116 for the aspen flakes and 127 for the red maple flakes. The width to thickness ratios were 10 for aspen and 15 for red maple. The radial direction of the wood was assumed to be in the thickness of the panel, the tangential direction corresponding to the width of the

TABLE 1. *Input values for modeling of flakeboard modulus of elasticity (MOE).*

| Property | Species | | | |
|---------------------------|-------------------------------------------------------------------------|---------------|------------|---------------|
| | Aspen | | Red maple | |
| | Flake alignment direction | | | |
| | Parallel | Perpendicular | Parallel | Perpendicular |
| | ----- Before accounting for flake orientation ----- | | | |
| MOE | 1,899 ksi* | 58 ksi | 1,923 ksi* | 85 ksi |
| E/G** | 17.92 | 2.42 | 14.2 | 2.58 |
| G | 106 ksi | 24 ksi* | 135 ksi | 33 ksi |
| Length/thickness | 126 | 10 | 126 | 15 |
| | ----- Effective properties after accounting for flake orientation ----- | | | |
| MOE | 549 ksi | 180 ksi | 646 ksi | 233 ksi |
| E/G*** | 8.29 | 4.83 | 7.49 | 4.65 |
| G | 66 ksi | 37 ksi | 86 ksi | 50 ksi |
| Length/thickness | 104 | 66 | 106 | 70 |
| Phenol formaldehyde resin | | | | |
| E | 1,102 ksi | | | |
| V**** | 0.30 | | 0.30 | |
| G | 424 ksi | | 424 ksi | |

* Average flexural MOE value adjusted for shear deflection.

** Ratio obtained from Bodig and Goodman (1973).

*** Ratio calculated from estimated effective properties.

**** Poisson ratio.

panel, and the longitudinal material axes corresponding to the panel length (flake alignment direction).

Determination of flake alignment and volume fractions was accomplished on specimens used for verification studies. A wide range of panel compositions was desired to determine the limits of applicability of the composite theories. Experimental panels were made with properties given in Table 2. For each species, two panels at each of twelve density/resin level combinations were manufactured. The resin distributions in the panels were assumed to be uniform, but were not measured. A 12 KV DC electrostatic flake alignment device was used to create a unidirectional aligned mat. Specimens from each of the 48 panels were selected and tested for flexural properties, internal bond, vibrational properties, equilibrium moisture content, and density (Shaler 1986).

Actual flake alignment distribution of each manufactured panel was described by the von Mises distribution (Harris 1977) as determined from 100 flake angle measurements at random locations on each panel. The von Mises distribution

TABLE 2. *Experimental oriented flakeboard manufacturing conditions.*

| Property | Experimental condition |
|-------------------|----------------------------------------------|
| Panel size | 20 in. × 20 in., trimmed to 16 in. × 16 in. |
| Panel thickness | 0.25 in. |
| Species | Aspen or Red Maple |
| Resin type | Powdered Phenolic |
| Target density | 35, 42, and 52 pcf (at 12% EMC conditions) |
| Resin level | 3, 5, 7, and 9% (oven-dry wood weight basis) |
| Press temperature | 380 F |
| Total press time | 4 minutes |

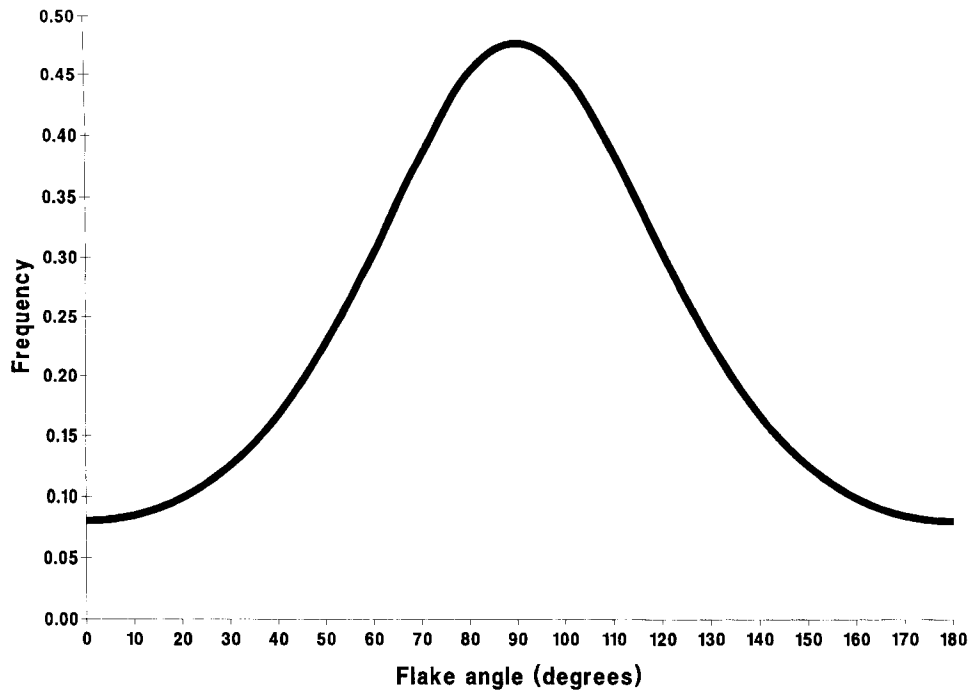


FIG. 3. Flake angle distribution as described by the von Mises distribution with a mean angle of 90.01 degrees and a concentration parameter of 0.88785.

parameters have been shown to be correlated with the more commonly reported average percent alignment (Shaler 1986). This alignment was tested and found to be consistent among all boards using a multi-sample Watson-Williams test (Zar 1984). The pooled data yielded a mean angle of 90.01 degrees and a concentration parameter of 0.88785. The fit distribution of flake alignment for all boards is given in Fig. 3.

This alignment was incorporated into the modeling procedure to generate effective flake properties parallel and perpendicular to the flake alignment direction. The effective flake dimensions in the parallel and perpendicular directions were determined using the geometrically developed Eq. (5).

$$l' = l \cos \theta + w \cos(\theta + 90) \quad (5)$$

where

l' = effective flake length,

l = average flake length,

w = average flake width, and

θ = flake angle relative to intended alignment direction (θ references alignment direction).

Flake thickness remained constant. A sample of 30,000 flake angles was selected from the von Mises distribution describing flake alignment. For each flake angle, an effective length was calculated from Eq. (5). The average of the simulated flakes was calculated to obtain the effective flake lengths (Table 1). The effective flake

length divided by the flake thickness yielded effective flake length/thickness ratios (Table 1).

A perfectly aligned flake ($\theta = 0$) was assumed to be oriented such that the longitudinal direction of the wood flake and panel were coincident. The tangential direction of the wood was then in-plane and perpendicular to the flake alignment axis. The radial direction of the wood was assumed to be aligned through the thickness of the board. The variation in elastic moduli of the flakes when the flake (material) axis formed an angle to the panel direction was assumed to follow Hankinson's formula. Using the parallel and perpendicular wood moduli values (Table 1), a simulation of effective flake modulus of elasticity and shear moduli in the board geometric directions were determined using angles predicted from the von Mises distribution. The average of 30,000 generated values was taken as the effective flake moduli in the board directions (Table 1). These values were used as elastic moduli of the wood flakes for modeling purposes. The effective moduli in the major alignment direction decreased with a concomitant increase predicted perpendicular to the intended flake alignment direction. No adjustment of resin properties was necessary because of assumed isotropic behavior.

The volume fraction of each component was important for application of the micromechanic composite theories. A combination of theoretical and experimental evidence was used to calculate volume fractions of the wood, resin, and air that comprised each specimen. The volume fraction of resin present in a specimen was determined from the nominal manufacturing conditions. No direct measurements on a specimen were taken. Based on a specific gravity of 1.4 for the polymerized resin (Wellons 1983), the volume was calculated as:

$$\text{Volume of resin} = \text{Wt. of resin} / \text{Specific gravity of resin.} \quad (6)$$

The volume fraction of resin was then calculated by dividing the resin volume by the target volume of each manufactured panel (20 in. \times 20 in. \times 0.25 in.).

Volume fractions of resin for the twelve combinations of resin level and panel target density were calculated (Table 3) and assumed no penetration of the resin into the wood flakes. It was of interest that volume fraction of resin appreciably increased with increasing board density, even though resin added on a weight basis remained constant.

The volume of the air inclusions of oven-dry flakeboard as a function of specific gravity was determined by use of a mercury porosimeter (Shaler 1986). Measured void volume (porosity) included voids between flakes (interflake) as well as the void volume of the densified flakes themselves (intraflake). Regression analysis between total porosity (voids 0.012 microns and greater in diameter) and specific gravity (oven-dry weight/oven-dry volume basis) yielded a straight line relationship of:

$$\text{Porosity} = 1.02 - 0.713\text{SG} \quad (7)$$

where SG = specific gravity (oven-dry weight/oven-dry volume) of specimen.

Equation (7), based on 20 samples, had a coefficient of determination of 98.5% and was valid for all resin levels and both species. The empirical Eq. (7) was similar to the theoretical relationship for porosity with specific gravity presented in Siau (1984). However, equation (7) also included the effect of the applied resin on the porosity.

It would be more intuitively attractive to use the intraflake voids. This could only be partially measured from the porosimeter data. However, a distribution of void volume as a function of size was obtained. Difficulties due to the bottleneck effect or specimen crushing arise in trying to quantify the amount of voids at any one size (Schneider 1982). Schneider (1982) investigated the pore size distribution of flakes of several species before and after pressing in a mat. He found that the surface flakes changed in character primarily in voids larger than a 10 micron diameter. This indicated changes primarily in the lumens.

The percentage of total porosity above 10 microns (17.5 psia) in diameter (partial porosity) was determined for the 10 samples of each species. A second order polynomial model was fit to the data for each species. Models were significant in both cases with an R^2 of 98.6% for the red maple and 84.3% for the aspen. These relationships were used to adjust the void (air) volume fraction predicted by Eq. (7). This was accomplished by multiplying total predicted void content by predicted fractional void volume above 10 microns. The equations to predict air volume fraction above a 10 micron diameter in aspen and red maple flakeboard are given in Eqs. (8) and (9), respectively.

$$\text{aspen partial porosity} = 1.27 - 3.34SG + 2.92SG^2 - 0.846SG^2 \quad (8)$$

$$\text{maple partial porosity} = 1.358 - 3.543SG + 3.08SG^2 - 0.886SG^2 \quad (9)$$

These equations indicate that the partial porosity is a nonlinear function of species and board density. At high board densities, the predicted air volumes are approximately equal between the species at 0.05 of total porosity. The relationships, for a given board specific gravity, consistently predict more air space (partial porosity) in red maple compared to aspen flakeboard specimens. For a given board density, this lowers estimated composite moduli for red maple specimens more than aspen. The difference increased at lower densities.

RESULTS AND DISCUSSION

Application of composite theories

It is recognized that wood particle composites are a heterogeneous, multiphase composite with discontinuous gluelines. However, the use of relatively simple two-phase composite theories is of interest to develop a general, rational modeling procedure without the limitations inherent in empirical regression approaches. These simple two-phase composite theories may be used as a basis for forming more complex future models.

Several mathematical and physical models beyond those described herein were evaluated for their appropriateness (Shaler 1986). In addition to use of the rule of mixtures and Halpin-Tsai equations, theories proposed by Russel (1973) and Wu (1966) were evaluated. Results were similar to those obtained with the rule of mixtures and Halpin-Tsai equations. Theories by Russel (1973) and Wu (1966) were more computationally intensive than the ROM and HT theories. Since their added complexity did not improve the present predictive accuracy, they were not included in the following discussion.

Other modeling concerns considered selection of the resin and wood as fiber reinforcement and matrix material or vice versa. Again, reported results represent those evaluated combinations that proved superior. Evaluation of the modeling

TABLE 3. Weight of resin added and corresponding volume fraction for all combinations of resin level and target density.

| Resin level (%) (OD wood weight basis) | Panel target density (pcf) | | |
|-------------------------------------------|----------------------------|---------|----------|
| | 35 | 42 | 52 |
| 3 | 23.91 g ¹ | 28.67 g | 35.50 g |
| | 1.04% ² | 1.25% | 1.55% |
| 5 | 39.05 g | 46.91 g | 58.04 g |
| | 1.70% | 2.04% | 2.53% |
| 7 | 53.65 g | 64.40 g | 79.73 g |
| | 2.34% | 2.81% | 3.48% |
| 9 | 67.73 g | 79.00 g | 100.64 g |
| | 2.95% | 3.44% | 4.39% |

¹ Weight of resin added to manufacture laboratory board with dimensions of 20 in. × 20 in. × ¼ in.

² Percent volume fraction of composite represented by resin. Obtained by dividing total board volume by volume of resin, assuming a resin specific gravity of 1.4.

procedures was accomplished by comparing predicted composite moduli to flexural modulus of elasticity (MOE) values measured on 192 specimens obtained from 48 experimental panels (Shaler 1986). A regression analysis between predicted and experimental values was performed using a straight line model. A perfect modeling procedure would yield regression results with an intercept of 0, a slope of 1, and a coefficient of determination of 100%.

A two-stage approach was applied to the three-phase system of wood, resin, and air. The first stage involved combining the wood and resin. The resin was modeled as the matrix material and the wood flakes as the fiber. The resin matrix volume fractions in Table 3 and the effective mechanical properties in Table 1 were used as input to the equations.

Best results were obtained when the shear moduli of the flake and resin were used, rather than the modulus of elasticity values. This was apparent when one observes that the E of the resin was less than the effective E of the wood flakes in the alignment direction. Thus, an increase in resin content would lead to a predicted decrease in composite moduli. This was contrary to observed behavior. The rule of mixture and Halpin-Tsai equations were general and apply to all elastic moduli. Glueline stress distribution in lap joints such as occurs in flakeboard, was complex and contains both normal and shear stress components (Adams and Peppiatt 1974). However, stress transferral between flakes was primarily in shear when the flake composite system was stressed in-plane in tension or flexure.

Since MOE values, rather than shear moduli, were measured on the specimens, it was necessary to convert the estimated composite shear moduli to MOE values. Predicted composite shear moduli were multiplied by the ratio of E to G values for the appropriate species and orientation (Table 1). The rationale for this approach was that the wood volume was over 50 times greater than the resin volume, and as such, its orthotropic character will be reflected in the composite orthotropic performance.

Interlaminar shear moduli were not measured, so no comparison with predicted performance was possible within this study. However, Geimer et al. (1974) measured interlaminar shear moduli (G_1) of aligned Douglas-fir flakeboards. They obtained average G_1 values of 102 ksi and 15 ksi parallel and perpendicular to the flake alignment direction, respectively. The degree of alignment of these panels

TABLE 4. Summary of regression results between measured flexural modulus of elasticity and composite modulus predicted using total porosity (Eq. 7).

| Independent variable ¹ | Regression equation | | R^2 (%) | SEE (psi) | Average predicted value (psi) ² |
|-----------------------------------|---------------------|-----------------|-----------|-----------|--------------------------------------------|
| | Slope | Intercept (psi) | | | |
| LROM/LROM | -110,844 | 1.803 | 93.4 | 72,474 | 357,387 |
| LROM/TROM | 151,730 | 33,955 | 6.0 | 274,397 | 11 |
| LROM/LHT | -108,377 | 1.836 | 93.3 | 73,113 | 349,586 |
| LROM/THT | -88,172 | 2.126 | 92.1 | 79,600 | 292,419 |
| TROM/LROM | -64,541 | 1.885 | 92.6 | 76,736 | 317,339 |
| TROM/TROM | 151,730 | 33,955 | 6.0 | 274,397 | 11 |
| TROM/LHT | -62,893 | 1.922 | 92.6 | 76,869 | 310,397 |
| TROM/THT | -48,745 | 2.244 | 92.1 | 79,763 | 259,541 |
| LHT/LROM | -110,442 | 1.803 | 93.4 | 72,442 | 357,157 |
| LHT/TROM | 151,730 | 33,955 | 6.0 | 274,397 | 11 |
| LHT/LHT | -107,982 | 1.837 | 93.3 | 73,078 | 349,361 |
| LHT/THT | -87,828 | 2.127 | 92.1 | 79,543 | 292,230 |
| THT/LROM | -98,182 | 1.835 | 93.5 | 72,167 | 344,262 |
| THT/TROM | 151,730 | 33,955 | 6.0 | 274,397 | 11 |
| THT/LHT | -95,977 | 1.870 | 93.4 | 72,654 | 336,743 |
| THT/THT | -77,675 | 2.170 | 92.4 | 78,136 | 281,645 |

¹ Indicates sequence in which equations are applied to model.

² Average measured value was 539,271 psi.

was not measured, so comparison of model to these literature values is difficult. However, predictions of G_1 using this model, for aspen, with similar density, resin contents, flake geometry, and a concentration parameter (k) of 2.0 yielded G_1 estimates of 88 ksi and 21 ksi in the parallel and perpendicular directions. This nominal agreement supports the two-stage approach used.

The intermediate composite modulus estimates from this first stage of the resin as the matrix and the wood flakes as the fiber were then combined with the air fraction of the wood composite (second stage of the approach). The wood/resin modulus estimate was assumed to be the matrix while the air voids acted as a fiber. Measured specific gravity (oven-dry weight/oven-dry volume) value of each specimen were used to determine the porosity using Eq. (7).

Evaluation of the effectiveness of the composite theories and physical model was accomplished by performing a regression analysis between predicted and experimental values. A total of 192 measured MOE values were obtained for two species (aspen and red maple), three target densities (35, 42, and 52 pcf), 4 resin levels (3, 5, 7, and 9% based on OD wood weight), and two flake directions (parallel and perpendicular to the intended orientation direction).

Since the modeling approach was a two-stage process (first stage—resin as matrix and wood as fiber, second stage—resin and wood as matrix and air as fiber) and there were four equations (LROM, TROM, LHT, and THT), a total of 16 equation mixes were evaluated (Table 4). For example, the LROM equation could be used to combine the resin and wood, while the TROM equation used to combine the wood/resin matrix and air fraction (partial porosity).

Coefficient of determination values (R^2) for all approaches ranged from 92.1 to 93.5%, with the exception of use of the TROM equation (R^2 of 6%) for combining the air with the intermediate composite estimate. Attendant standard error of estimate (SEE) ranged from 72 to 80 ksi. The best results were obtained using the

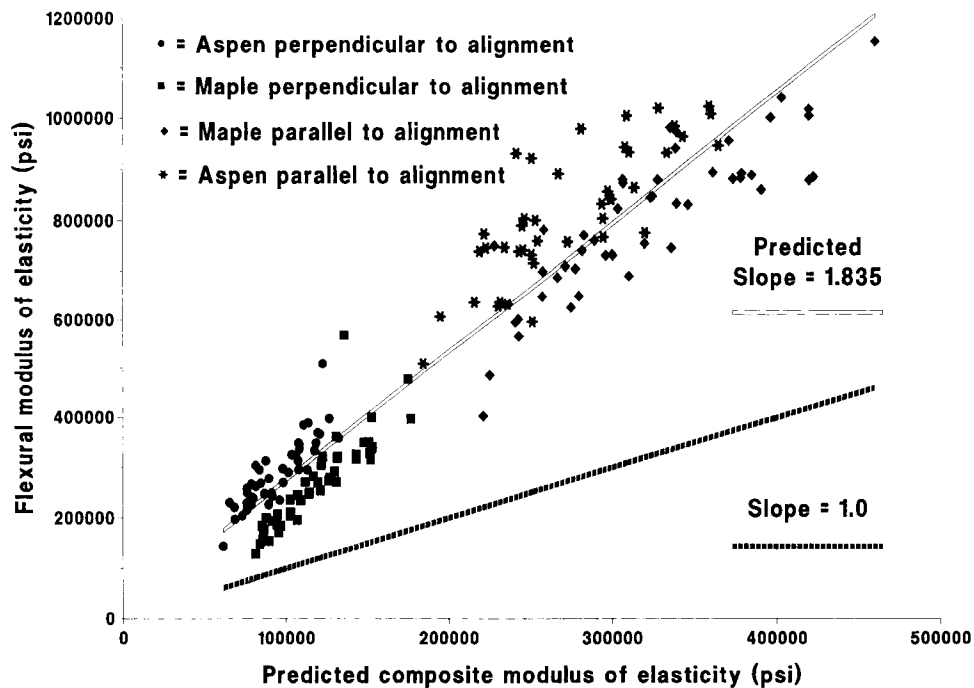


FIG. 4. Predicted and measured modulus of elasticity (MOE) using total air porosity and the LHT/LHT equation sequence.

THT/LROM sequence of equations. Little difference was observed for use of the LROM or LHT for second stage equations. Use of the TROM equation yielded results consistently poorer than other approaches. This was due to the low moduli of air that controls modulus estimates until fiber fractions reach 90% (Fig. 2). Predicted values averaged on the order of 65–70% of experimental values. This was possibly due to the inclusion of the entire void volume including the cell-wall capillaries. No significant differences in correlation were obtained using the different composite theories. For flexibility, it was decided to use the LHT/LHT equation sequence (Fig. 4). This was selected since the LHT equation can mimic the THT and LROM equation values through appropriate selection of slenderness ratio values.

Use of Eqs. (8) and (9) to include species specific partial porosity, (above 10 microns in diameter) in conjunction with the LHT/LHT equations was evaluated. Regression results are given in Table 5. Compared to use of total porosity, the coefficient of determination decreased from 93.3% to 89.5%, and the SEE increased from 73,078 psi to 93,985 psi. The averaged predicted value of the 192 specimens increased from 349,361 psi to 414,420 psi. Even with the increase in average predicted composite moduli due to use of partial porosity (above 10 microns in diameter), the average estimate was still 25% below the average measured MOE of 539,271 psi for all specimens.

A scatter diagram of predicted and measured moduli is given in Fig. 5. The data grouping on the upper right of Fig. 5 represents values parallel to flake alignment direction, while the group in the lower left represents specimens tested

TABLE 5. Summary of regression results between flexural modulus of elasticity and composite modulus predicted using partial porosity above 10 microns in diameter (Eqs. 8 and 9) and LHT/LHT equation sequence.

| Material grouping | Regression equation | | R^2 (%) | SEE (psi) | Average predicted value (psi) |
|---------------------|---------------------|-----------------|-----------|-----------|-------------------------------|
| | Slope | Intercept (psi) | | | |
| All Specimens | 1.37 | -28,497 | 89.5 | 93,985 | 414,420 |
| Aspen only | 1.47 | -14,786 | 93.9 | 70,546 | 380,290 |
| Maple only | 1.37 | -74,142 | 91.5 | 86,304 | 444,996 |
| Parallel only | 1.36 | -23,364 | 37.0 | 111,979 | 608,779 |
| Perpendicular only | 1.06 | 39,017 | 17.2 | 72,668 | 224,287 |
| Aspen/Perpendicular | 1.95 | -106,642 | 41.2 | 52,453 | 202,647 |
| Aspen/Parallel | 2.33 | -506,365 | 67.1 | 75,973 | 566,391 |
| Maple/Parallel | 2.44 | -775,694 | 68.3 | 92,267 | 645,869 |
| Maple/Perpendicular | 2.68 | -385,184 | 51.2 | 60,787 | 244,123 |

perpendicular to the flake alignment direction. The presence of two distinct groups of data always yields high R^2 values in a regression analysis and should be interpreted with caution. It is apparent that the trend between predicted and actual MOE is different in the parallel and perpendicular directions. An expansion of the data for parallel aligned specimens, identified by species is presented in Fig. 6.

The overall correlation between predicted and measured MOE values decreases in comparison to the correlation obtained with two disparate groups (parallel and

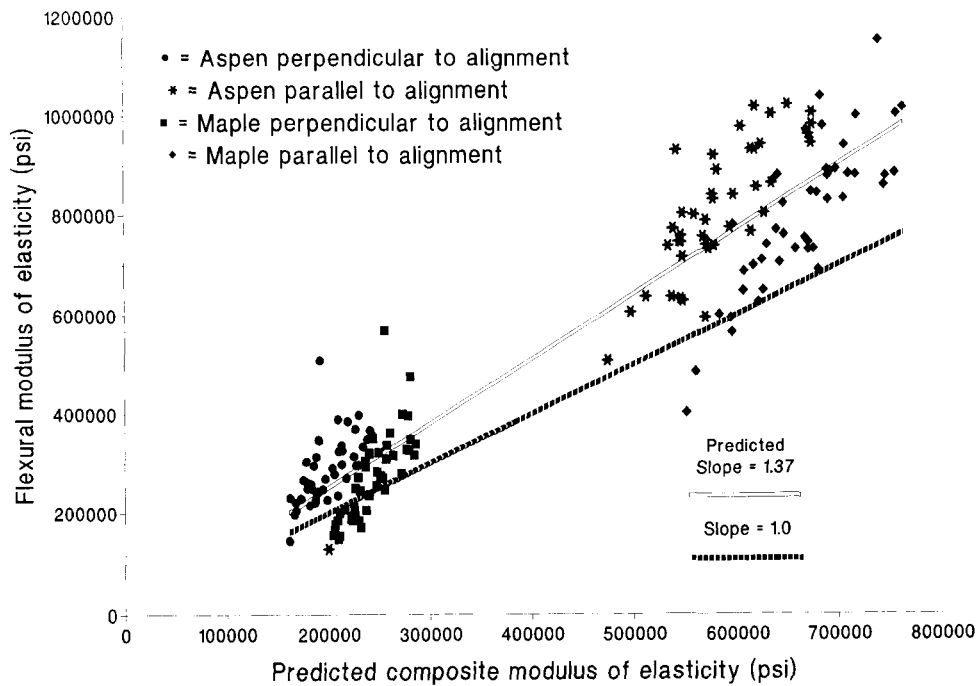


FIG. 5. Predicted and measured modulus of elasticity (MOE) of all specimens using air volume above 10 microns only, and the LHT/LHT equation sequence.

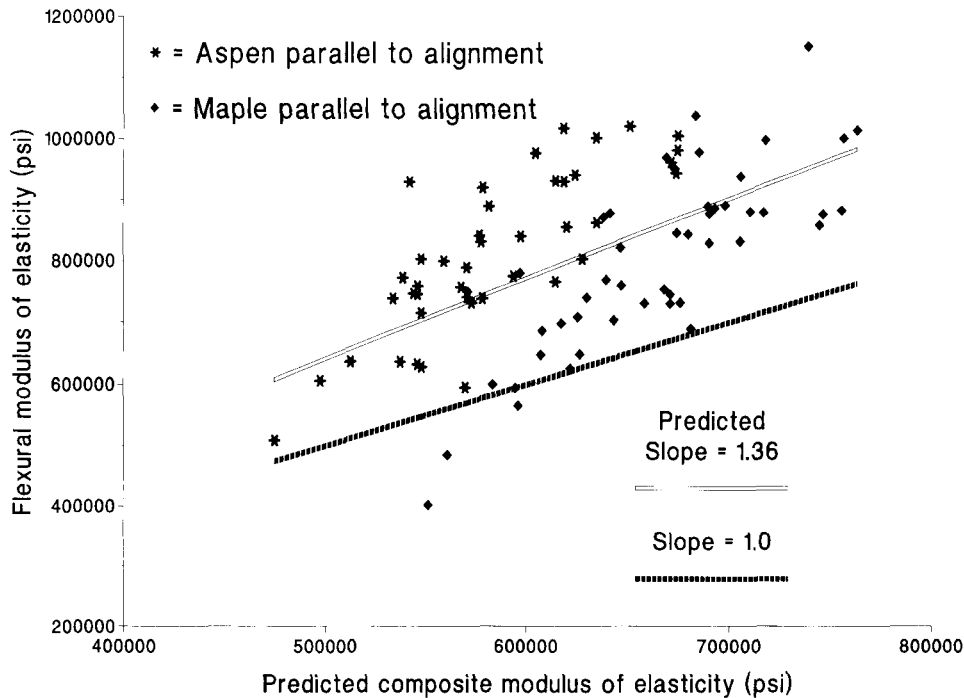


FIG. 6. Predicted and measured modulus of elasticity (MOE) parallel to flake alignment direction using air volume above 10 microns only, the LHT/LHT equation sequence.

perpendicular alignments). The use of two separate groups of data often yields a different correlation from the combined data set and should be interpreted with caution (Table 5). What is apparent from Fig. 6 is the grouping by species, with the aspen predictions being consistently lower in comparison with measured values than that of flakeboard specimens made with red maple flakes. Correlations improve when the data are further subset by species and alignment (Table 5). The average predicted MOE values in Table 4 using total porosity [Eq. (7)] as an input were lower than the measured average MOE value (539,271 psi) for all specimens. Using partial porosity above a 10 micron diameter increased the average predicted MOE to 414,420 psi. Further segregation by species and testing direction (parallel or perpendicular) using species specific partial porosity [Eqs. (8) and (9)] produced average predicted MOE values (Table 5) that were almost identical to the effective average MOE values in Table 1. However, perhaps the model is a unified approach but should be applied to a species at a given time.

Variation in the mechanical properties assumed (Table 1) the level of pore volume used (e.g., 15 microns, 25 microns, etc.) could be employed to make the predictions more closely resembling the experimental results. In addition, vertical density gradients were not accounted for in the present model. Accounting for a density gradient effect could be accomplished by using the developed model to obtain predicted values for the various levels of specific gravity. These laminae predictions could then be coupled with laminate theory to obtain better accuracy. These approaches have some merit and warrant additional investigation.

SUMMARY

A computationally simple, rational, and flexible approach toward prediction of flexural moduli of oriented flakeboard as a function of species, resin content, density, flake geometry, and flake alignment was developed. The physical model assumes the flakes have the longitudinal wood axis coincident with the flake length, tangential direction with flake width, and radial direction corresponding to flake thickness. Average flake elastic moduli in the board axes were generated by applying Hankinson's formula with the grain angle corresponding to flake angle. These effective elastic moduli, along with the resin and air were best combined in a two-step process using the longitudinal Halpin-Tsai equation. Partial porosity was a nonlinear function of measured specimen density and species. The process underestimated measured moduli by an average of 25%. The contribution of resin content to overall performance was obtained from a volume basis only (resin compatibility was assumed). While exact input model values were unknown, the approach seems valid over a wide range of resin levels, board densities, and flake orientations.

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