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Composite Quarks and Leptons Generated by Supersymmetric Quantum Chromodynamics with $N_{\text{“color”}} < N_{\text{“flavor”}}$

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Quarks and leptons together with other exotic fermions are regarded as quasi Nambu-Goldstone fermions and chiral fermions generated by supersymmetric quantum chromodynamics (SQCD) with $N_{\text{“color”}}(=N) < N_{\text{“flavor”}}(=M)$. A mass protection symmetry, H, contains a chiral $SU(M-N)$ [or $SU(M-N+1)$] symmetry as well as a chiral $U(1)$ symmetry supported "algebraically" by complementarity and "dynamically" by effective superpotential. The successive SUSY-breaking is constrained by the remnant of the QCD dynamics, i.e., the complete breakdown of chiral symmetries in H. The large mass splitting between $J=1/2$ and $J=0$ is shown to arise and masses for light composite fermions are specified by the SUSY-breaking scale M_{SS} and the confinement scale Λ_{sc}: $M_{SS}(M_{SS}/\Lambda_{sc})^{(M-N)/2\kappa}$ for $M_{SS} \ll \Lambda_{sc}$, where $2\kappa \leqq M-N$ is imposed by the consistency with the anomaly-matching.

§ 1. Introduction

Composite models of quarks and leptons have been discussed toward dynamical understanding of observed properties of quarks and leptons including their mass spectrum.[1] The masses of quarks and leptons, $m_{q,l}$, are much less than their inverse sizes $r_{q,l}^{-1}$, i.e., $m_{q,l}r_{q,l} \ll 1$. It is useful to consider massless composite fermions ($m_{q,l} = 0$) with the sizes $r_{q,l}$, giving $m_{q,l}r_{q,l}(=0) \ll 1$, that arise if underlying dynamics is equipped with a certain mass-protection mechanism.[2] Three types of mass-protection mechanisms utilizing chiral symmetry and/or supersymmetry (SUSY) have been known to work,*) which are all related to the Nambu-Goldstone theorem on spontaneous breaking of an global symmetry G to its subgroup H or of SUSY: (i) Massless chiral fermions (CF's)[2] are required to saturate anomalies of H while those of G/H are automatically saturated by the Nambu-Goldstone bosons;[3] (ii) massless quasi Nambu-Goldstone fermions (QNGF's)[4] are generated as superpartners of the Nambu-Goldstone bosons associated with $G \to H$ provided SUSY is not spontaneously broken; and (iii) massless Nambu-Goldstone fermions (NGF's)[5] appear as a result of spontaneous SUSY breaking. The most important issue is to induce "appropriate" explicit breakings of chiral symmetry and SUSY, which will generate relatively light masses for quarks and leptons and relatively heavy masses for exotic particles if they exist.

In supersymmetric theories such as supersymmetric quantum chromodynamics (SQCD), it seems plausible to select SUSY-preserving phase, in which quarks and leptons are born as QNGF's living in the coset space G/H that possesses enough

*) A local SUSY rather than the global one will also generate light composite fermions that become massless in the $M_{Pl} \to \infty$ limit.[6]

freedom to create quark-lepton quantum numbers. Since SUSY is exact, scalar partners of quarks and leptons, squarks and sleptons, show up as massless composite states and should be pushed up to sufficiently heavy masses. It can be achieved by introducing some sort of explicit SUSY breaking but only if massless fermions are further protected by chiral symmetries contained in the subgroup H. Then, SQCD as the simplest underlying dynamics for composite quarks and leptons should realize a specific phase that preserves SUSY and some of chiral symmetries.

If chiral symmetries are present in the remaining subgroup H, the anomaly-matching on H in general requires another class of massless composite fermions CF.[7] The spectrum of massless composite fermions is, thus, entirely determined by the coset space G/H and the anomalies of H. Since the symmetry breaking of $G \to H$ is so constrained to be compatible with the dynamics of SQCD, the subgroup H cannot be arbitrarily chosen to yield only favorite quantum numbers of massless composite fermions.

In the present article, we discuss the feasibility of SQCD with N-"colors" $<M$-"flavors" as underlying dynamics for composite quarks and leptons.*) The mass scale of SQCD, Λ_{sc}, is taken to be ~ 1 TeV in order to account for the electroweak scale of $G_F^{-1/2} \simeq 300$ GeV. The global symmetry is specified by $G = SU(M)_L \times SU(M)_R \times U(1)_V \times U(1)_A \times U(1)_{\text{anom}}$, in which $U(1)_{\text{anom}}$ is broken by instantons of $SU(N)_{sc}^{\text{loc}}$. Shown in Table I are the transformation properties of the (L-handed) gauge superfields $W_A{}^B (A, B = 1 \sim N)$ and the matter superfields $\Phi^{(1)}{}_i{}^A$ and $\Phi^{(2)}{}_A{}^i (A=1 \sim N; i = 1 \sim M)$ to be called subquarks (or preons). To incorporate quarks and leptons in the QNGF spectrum depends on the breaking patterns of G. To handle the spontaneous breakdown of G to H and the implementation of the SUSY-breaking effects, we rely upon an effective description of SQCD based on the effective superpotential (W_{eff}) of the Taylor-Veneziano-Yankielowicz type,[9] which will be extended to SQCD with $N < M$. Our results are as follows:

(1) The effective superpotential necessarily exhibits a singular behavior owing to the presence of chiral symmetry: $W_{\text{eff}} \sim \langle\lambda\lambda\rangle \ln(Z - \langle Z\rangle|_{\theta=0})$, where Z is the superfield made of composites. $Z|_{\theta=0} - \langle Z\rangle|_{\theta=0} (= \xi \ll 1)$ in the broken SUSY measures the breaking effect of the chiral symmetries, i.e., $\langle \phi_i{}^{(1)} \phi^{(2)i}\rangle$ $(i = n+1 \sim M) \sim \xi \Lambda_{sc}^2 \ll \Lambda_{sc}^2$ for $SU(M-n)_L \times SU(M-n)_R$.

(2) All remaining chiral symmetries are necessarily broken once SUSY is explicitly broken even if the SUSY-breaking itself preserves chiral symmetries. It is in accord with the result of QCD dynamics: The complete breakdown of chiral

Table I. Quantum numbers of $\Phi^{(1,2)}$ and W. (a, b) in $U(1)_A$ denotes the Q_A-charges of $(J=0, J=1/2)$ for $\Phi^{(1,2)}$ and $(J=1/2, J=1)$ for W.

superfields	$SU(N)_{sc}^{\text{loc}}$	$SU(M)_L$	$SU(M)_R$	$U(1)_V$	$U(1)_A$	$U(1)_{\text{anom}}$
$\Phi^{(1)}{}_i{}^A : (\phi^{(1)}, \psi^{(1)})_i{}^A$	$\boldsymbol{N^*}$	$\boldsymbol{M}$	1	1	$(N-M, N)$	1
$\Phi^{(2)}{}_A{}^i : (\phi^{(2)}, \psi^{(2)})_A{}^i$	$\boldsymbol{N}$	1	$\boldsymbol{M^*}$	-1	$(N-M, N)$	1
$W_B{}^A : (\lambda, G_{\mu\nu})_B{}^A$	$\boldsymbol{ADJ}$	1	1	0	$(-M, 0)$	0

*) For the $N=M$ model, see Ref. 8).

symmetries.

(3) The masses for light composite fermions including quarks and leptons are characterized by the SUSY-breaking scale M_{SS}: $M_F=\xi^{\alpha}M_{SS}$ with $\xi=(M_{SS}/\Lambda_{SC})^{(M-N)/\kappa}$ ($\alpha=1/2$ or 1), where M_{SS} controls masses of the scalar partners. In the $M_{SS}\rightarrow 0$ limit, the anomaly-matching is consistent with complementarity if $\alpha=1/2$ with $2\kappa\leqq M-N$.

(4) Phenomenologically viable model is based on SQCD with $SU(N-1)_{L+R}\times SU(M-N+1)_L\times SU(M-N+1)_R\times U(1)_w\times U(1)_c\times U(1)_\chi$ for $N(=1+$color degrees of freedom$)<M$ that accommodates quarks as QNGF's being two-body composites and leptons as CF's being N-body composites. One generation is provided by $N=4$ and $M=5$. Other models contain either light leptoquarks or the appreciable mixing of quarks and leptons with their mirrors.

In the next section 2, three types of SQCD with $N<M$ will be selected as quark-lepton models, which contain leptoquarks, color-octets, weak-triplets and neutrals as well as quarks and leptons. In § 3, the effective superpotential is constructed to yield the consistent symmetry breaking with the anomaly-matching on H. In § 4, the SUSY- and chiral symmetry-breaking are included in the effective lagrangian with a set of soft breaking terms arising from the gaugino-mass and scalar-subquark-masses, whose origin is beyond SQCD and is not discussed. In § 5, dynamical issues such as the mass spectrum of light composite fermions and bosons and the splitting of exotic fermions are discussed in the three types of quark-lepton models. The final section is devoted to a summary and discussion.

§ 2. Symmetry breaking and quark-lepton quantum numbers

The characteristic feature of SQCD is that massless SQCD cannot be defined as the massless limit of massive SQCD.[10] It can be translated in the SUSY-broken SQCD with two scales, the SUSY-breaking scale, M_{SS}, and the SUSY-mass, m_i, that the SUSY-limit maintaining $M_{SS}\gg m_i$ ($\rightarrow 0$ is allowed) leads to the massless SQCD while the limit of $M_{SS}\ll m_i$($\rightarrow 0$ is forbidden) leads to the massive SQCD. In the quark-lepton world one knows $M_{SS}\gg m_i$ since no scalar partners of quarks and leptons have been found. The massless SQCD is therefore relevant for underlying dynamics for composite quarks and leptons.

A. *Symmetry breaking of massless SQCD*

Properties of the massless SQCD have been studied by (i) the anomaly-relation for superpotential[11] requiring $\langle\lambda\lambda\rangle=0$, (ii) the instanton-induced amplitude[12] providing $\langle\det(\phi_i{}^{(1)}\phi^{(2)j})\rangle\neq 0$ for $N\geqq M$ and $\langle\lambda\lambda\rangle\neq 0$ for $N>M$ and (iii) complementarity.[13] The inconsistency between $\langle\lambda\lambda\rangle\neq 0$ for $N>M$ from the instanton-analysis and $\langle\lambda\lambda\rangle=0$ from the anomaly-relation is interpreted as a signal of spontaneous breakdown of SUSY in SQCD with $N>M$.

From complementarity,[2),14)] we expect in the Higgs and confining phases the same spectrum of massless particles and the same breaking pattern of $G\rightarrow H$ (more precisely $G\times SU(N)_{SC}^{\rm loc}\rightarrow H$ with $SU(N)_{SC}^{\rm loc}$ broken or confined).[15] In the Higgs phase, the symmetry-breaking of G to H is generated by $\langle\phi^{(1)}{}_i{}^A\rangle\propto\delta_i{}^A$ and $\langle\phi^{(2)}{}_A{}^i\rangle\propto\delta_A{}^i$, where A, $i=1\sim M$ for $N>M$ and $=1\sim N$ or $=1\sim N-1$ for $N\leqq M$. In the confining phase, the same symmetry-breaking is generated by 2-body condensations

$$\langle \phi_i^{(1)}\phi^{(2)j}\rangle \propto \delta_i^j\,, \tag{2·1}$$

where $i, j=1\sim M$ for $N>M$ and $=1\sim N$ or $=1\sim N-1$ for $N\leqq M$ as in the Higgs phase, supplemented by N-body condensations.*)

$$\langle[(\phi^{(a)})^N]\rangle \neq 0 \qquad (a=1, 2) \tag{2·2a}$$

for $i, j=1\sim N\leqq M$. Although this set of the condensations is consistent with complementarity, the case of $i, j=1\sim N-1$ will further utilize $2(N-1)$-body condensation

$$\langle \sum_{A=1}^{N}[(\phi^{(1)})^{N-1}]_A[(\phi^{(2)})^{N-1}]^A\rangle \neq 0\,, \tag{2·2b}$$

which is required to construct an effective superpotential. Since in the case of $N\leqq M$ the remaining symmetries contain chiral symmetries, the anomaly-matching should be satisfied. It turns out to be automatically satisfied because the anomaly-matching is trivial in the Higgs phase.

Collecting each result, one observes that the subgroup H consistent with the dynamics of the massless SQCD is as follows: (1) $SU(N)_{L+R}\times U(1)_V$ for $N>M$ with a signal of spontaneous SUSY-breaking; (2) $SU(M)_{L+R}\times U(1)_A$ for $N=M$; and (3) $SU(N)_{L+R}\times SU(M-N)_L\times SU(M-N)_R\times U(1)_V'\times U(1)_A'$ or $SU(N-1)_{L+R}\times SU(M-N+1)_L\times SU(M-N+1)_R\times U(1)_w\times U(1)_c\times U(1)_\chi$ for $N<M$. The QNGF-mechanism calls for the absence of spontaneous SUSY-breaking, which is possible in

Table II. (a) Transformation properties of fundamental superfields $\Phi^{(1,2)}$, W and massless superfields in the Higgs phase or in the confining phase under (A) $H=SU(N)_{L+R}\times SU(M-N)_L\times SU(M-N)_R\times U(1)_V'\times U(1)_A'$. (x, y) in $U(1)_A'$ denotes Q_A' denotes Q_A' for $(J=0, J=1/2)$ of $\Phi^{(1,2)}$ and composites and for $(J=1/2, J=1)$ of W.

For $\Phi^{(1,2)}$ and W:

superfields	$SU(N)_{sc}^{\rm loc}$	$SU(N)_{L+R}$	$SU(M-N)_L$	$SU(M-N)_R$	$U(1)_V'$	$U(1)_A'$
$\Phi^{(1)A=1\sim N}_{\ \ a=1\sim N}$	$\boldsymbol{N^*}$	$\boldsymbol{N}$	1	1	0	$(0, M)$
$\Phi^{(1)A=1\sim N}_{\ \ i=N+1\sim M}$	$\boldsymbol{N^*}$	1	$\boldsymbol{M-N}$	1	1	$(-M, 0)$
$\Phi^{(2)i=1\sim N}_{\ \ A=1\sim N}$	$\boldsymbol{N}$	$\boldsymbol{N^*}$	1	1	0	$(0, M)$
$\Phi^{(2)i=N+1\sim M}_{\ \ A=1\sim N}$	$\boldsymbol{N}$	1	1	$\boldsymbol{M-N^*}$	-1	$(-M, 0)$
$W_A{}^B$	$\boldsymbol{ADJ}$	1	1	1	0	$(-M, 0)$

For composite superfields in the confining phase:

Higgs	confining	$SU(N)_{L+R}$	$SU(M-N)_L$	$SU(M-N)_R$	$U(1)_V'$	$U(1)_A'$
$\Phi^{(1)A=1\sim N}_{\ \ a=1\sim N}+\Phi^{(2)a=1\sim N}_{\ \ A=1\sim N}$	$\Phi^{(1)}{}_a{}^A\Phi^{(2)}{}_A{}^b$	$\boldsymbol{ADJ}$	1	1	0	$(0, M)$
$\Phi^{(1)A=1\sim N}_{\ \ i=N+1\sim M}$	$\Phi^{(1)}{}_i{}^A\Phi^{(2)}{}_A{}^a$	$\boldsymbol{N^*}$	$\boldsymbol{M-N}$	1	1	$(-M, 0)$
$\Phi^{(2)i=N+1\sim M}_{\ \ A=1\sim N}$	$\Phi^{(1)}{}_a{}^A\Phi^{(2)}{}_A{}^i$	$\boldsymbol{N}$	1	$\boldsymbol{M-N^*}$	-1	$(-M, 0)$
$\mathrm{Tr}(\Phi^{(1)}{}_a{}^A)$, $\mathrm{Tr}(\Phi^{(2)}{}_A{}^a)$	2-singlets	1	1	1	0	$(0, M)$

"2-singlets" are formed out of $\sum_{a=1}^N\Phi^{(1)}{}_a\Phi^{(2)a}$, $[(\Phi^{(1)}{}_a)^N]$ and $[(\Phi^{(2)a})^N]$. (continued)

*) [ ] denotes the antisymmetrization of the "colors": $[(\phi^{(1)})^N]=\det(\phi^{(1)}{}_i{}^A)\sim \varepsilon_{A_1\cdots A_N}\phi^{(1)A_1}_{i_1}\cdots\phi^{(1)A_N}_{i_N}$, $[(\phi^{(1)})^{N-1}]_B=\det_B(\phi^{(1)}{}_i{}^A)\sim\varepsilon_{BA_1\cdots A_{N-1}}\phi^{(1)A_1}_{i_1}\cdots\phi^{(1)A_{N-1}}_{i_{N-1}}$ and so on.

(b) The same as in Table II (a) but under (**B**) $H=SU(N-1)_{L+R}\times SU(M-N+1)_L\times SU(M-N+1)_R\times U(1)_w\times U(1)_c\times U(1)_x$.

For $\Phi^{(1,2)}$ and W:

superfields	$SU(N)_{sc}^{\rm loc}$	$SU(N-1)_{L+R}$	$SU(M-N+1)_L$	$SU(M-N+1)_R$	$U(1)_w$	$U(1)_c$	$U(1)_x$
$\Phi^{(1)A=1\sim N}_{\ a=1\sim N-1}$	$\boldsymbol{N}^*$	$\boldsymbol{N-1}$	$\boldsymbol{1}$	$\boldsymbol{1}$	0	1	$(0, M-N+1)$
$\Phi^{(1)a=1\sim N}_{\ i=N\sim M}$	$\boldsymbol{N}^*$	$\boldsymbol{1}$	$\boldsymbol{M-N+1}$	$\boldsymbol{1}$	-1	0	$(N-M, 1)$
$\Phi^{(2)a=1\sim N-1}_{\ A=1\sim N}$	$\boldsymbol{N}$	$\boldsymbol{N-1}^*$	$\boldsymbol{1}$	$\boldsymbol{1}$	0	-1	$(0, M-N+1)$
$\Phi^{(2)i=N\sim M}_{\ A=1\sim N}$	$\boldsymbol{N}$	$\boldsymbol{1}$	$\boldsymbol{1}$	$\boldsymbol{M-N+1}^*$	1	0	$(N-M, 1)$
$W_A{}^B$	$\boldsymbol{ADJ}$	$\boldsymbol{1}$	$\boldsymbol{1}$	$\boldsymbol{1}$	0	0	$(N-M-1, 0)$

For composite superfields in the confining phase:

Higgs	confining	$SU(N-1)_{L+R}$	$SU(M-N+1)_L$	$SU(M-N+1)_R$	$U(1)_w$	$U(1)_c$	$U(1)_x$
$\Phi^{(1)A=1\sim N-1}_{\ a=1\sim N-1} + \Phi^{(2)a=1\sim N-1}_{\ A=1\sim N-1}$	$\Phi^{(1)}{}_a{}^A\Phi^{(2)}{}_A{}^b$	$\boldsymbol{1+ADJ}$	$\boldsymbol{1}$	$\boldsymbol{1}$	0	0	$(0, M-N+1)$
$\Phi^{(1)A=1\sim N-1}_{\ i=N\sim M}$	$\Phi^{(1)}{}_i{}^A\Phi^{(2)}{}_A{}^a$	$\boldsymbol{N-1}^*$	$\boldsymbol{M-N+1}$	$\boldsymbol{1}$	1	1	$(N-M, 1)$
$\Phi^{(2)i=N\sim M}_{\ A=1\sim N-1}$	$\Phi^{(1)}{}_a{}^A\Phi^{(2)}{}_A{}^i$	$\boldsymbol{N-1}$	$\boldsymbol{1}$	$\boldsymbol{M-N+1}^*$	-1	-1	$(N-M, 1)$
$\Phi^{(1)A=N}_{\ i=N\sim M}$	$[(\Phi^{(2)a})^{N-1}]_A\Phi^{(1)}{}_i{}^A$	$\boldsymbol{1}$	$\boldsymbol{M-N+1}$	$\boldsymbol{1}$	1	$1-N$	$(N-M, 1)$
$\Phi^{(2)i=N\sim M}_{\ A=N}$	$[(\Phi^{(1)}{}_a)^{N-1}]^A\Phi^{(2)}{}_A{}^i$	$\boldsymbol{1}$	$\boldsymbol{1}$	$\boldsymbol{M-N+1}^*$	-1	$N-1$	$(N-M, 1)$

"1-singlet" in $1+\boldsymbol{ADJ}$ is formed out of $\sum_{a=1}^{N-1}\Phi^{(1)}{}_a\Phi^{(2)a}$ and $[(\Phi^{(1)}{}_a)^{N-1}(\Phi^{(2)a})^{N-1}]$.

SQCD with $N\leqq M$. Furthermore, SQCD with $N\leqq M$ possesses chiral symmetries that will work as the seed of large mass splitting between fermions and scalar partners. For the present discussion, we list in Table II massless superfields in SQCD with $N<M$. The anomaly-matching turns out to be realized by all QNGF's for SQCD with the chiral $SU(M-N)$ symmetry and by QNGF's and CF's for SQCD with the chiral $SU(M-N+1)$ symmetry.

B. *Quark-lepton models*

To see the feasibility of SQCD as underlying dynamics for composite quarks and leptons, we argue whether the spectrum of massless fermions includes the quark-lepton quantum numbers. The minimal set of the "flavor" superfields, i.e., subquarks, which provides one generation of quarks and leptons, will consist of three colors of $\hat{c}^A_{Li=1,2,3}=\Phi^{(1)A}_{i=1,2,3}$ and $\hat{c}^{Ci=1,2,3}_{RA}=\Phi_A{}^{(2)i=1,2,3}$, one $B-L$ of $\hat{c}^A_{L0}=\Phi^{(1)A}_{i=4}$ and $\hat{c}^{C0}_{RA}=\Phi_A{}^{(2)i=4}$ and two weak flavors of $\hat{w}^{CA}_{Ri=1,2}=\Phi^{(1)A}_{i=5,6}$ and $\hat{w}_{LA}^{i=1,2}=\Phi_A{}^{(2)i=5,6}$. The global symmetry $SU(6)_{L(R)}$ for $\Phi^{(1)}_{i=1\sim 6}$ $(\Phi^{(2)i=1\sim 6})$ is divided into the chiral version of the Pati-Salam $[SU(4)_C]_{L(R)}$ symmetry[16] and the weak-isospin $[SU(2)_W]_{L(R)}$ symmetry. The color $SU(3)_C{}^{\rm loc}$ symmetry is contained in the $[SU(4)_C]_{L+R}$ symmetry and the electroweak $[SU(2)_W]_L{}^{\rm loc}$ symmetry is identical to $[SU(2)_W]_L$. The quark-lepton superfields, $\hat{q}_a{}^i$ and $\hat{l}^{\,i}(a=1, 2, 3;\ i=1, 2)$, are made as

$$\hat{q}^{\,i}_{La}=\sum_{A=1}^{N}\hat{c}^A_{La}\hat{w}^i_{LA}\,,\qquad \hat{q}^{Ca}_{Ri}=\sum_{A=1}^{N}\hat{c}^{Ca}_{RA}\hat{w}^{CA}_{Ri}\,,\tag{2·3a, b}$$

$$\hat{l}_L{}^i=\sum_{A=1}^{N}\hat{c}^A_{L0}\hat{w}^i_{LA}\,,\qquad \hat{l}^{\,C}_{Ri}=\sum_{A=1}^{N}\hat{c}^{C0}_{RA}\hat{w}^{CA}_{Ri}\,,\tag{2·3c, d}$$

where $\hat{w}_L{}^{i=1,2}$ are assumed to exhibit the $V-A$ coupling to the weak bosons $W^{\pm}$. If

$N=4$, leptons can be made of $\hat{c}_{a=1,2,3}$ and $\hat{w}^{i=1,2}$ as

$$\hat{l}_L{}^i=\sum\varepsilon^{ABCD}\varepsilon_{abc}\hat{c}_{RA}^{Ca}\hat{c}_{RB}^{Cb}\hat{c}_{RC}^{Cc}\hat{w}_{LD}^{i}\,, \tag{2·4a}$$

$$\hat{l}_{Ri}^{C}=\sum\varepsilon_{ABCD}\varepsilon^{abc}\hat{c}_{La}^{A}\hat{c}_{Lb}^{B}\hat{c}_{Lc}^{C}\hat{w}_{Ri}^{CD}\,. \tag{2·4b}$$

There are three cases that at least contain one (or two) generation(s) of quarks and leptons in SQCD with $N<M$, which depends on the presence of $[SU(4)_c]_L\times[SU(4)_c]_R\rightarrow[SU(4)_c]_{L+R}$ and $[SU(2)_W]_L\times[SU(2)_W]_R\rightarrow[SU(2)_W]_{L+R}$. The symmetry breaking is given by (1) $N=4$ and $M=6$ with

$$H_1=[SU(4)_c]_{L+R}\times[SU(2)_W]_L\times[SU(2)_W]_R\times U(1)_{V'}\times U(1)_{A'}\,, \tag{2·5a}$$

(2) $N=4$ and $M=8$ with*)

$$H_2=[SU(4)_W]_{L+R}\times[SU(4)_c]_L\times[SU(4)_c]_R\times U(1)_{V'}\times U(1)_{A'} \tag{2·5b}$$

and (3) $N=4$ and $M=5$ with

$$H_3=[SU(3)_c]_{L+R}\times[SU(2)_W]_L\times[SU(2)_W]_R\times U(1)_w\times U(1)_c\times U(1)_\chi\,. \tag{2·5c}$$

$H_{1,2}$ arise from SQCD with the chiral $SU(M-N)$ symmetry while H_3 from SQCD with the chiral $SU(M-N+1)$ symmetry. Table III summarizes the spectra of light composite superfields and the condensations in terms of subquark fields. Weak interactions can be based on $[SU(2)_W]_L{}^{\mathrm{loc}}\times U(2)_Y{}^{\mathrm{loc}}$ with elementary or composite gauge bosons if the model allows $[SU(2)_W]_L\times[SU(2)_W]_R\rightarrow[SU(2)_W]_{L+R}$ that provides the weak triplet Nambu-Goldstone superfield (NGS) to be eliminated by the weak gauge fields. Otherwise, the weak bosons $W^\pm$ and Z should be created as ordinary composite particles and simulate almost everything of $W^\pm$ and Z of the gauge theory.

Phenomenologically dangerous particles are leptoquarks, $\hat{c}_{La}\hat{c}_R{}^{C0}$ and $\hat{c}_{L0}\hat{c}_R{}^{Ca}$ ($a=1,2,3$) and color-octets, $\hat{c}_{La}\hat{c}_R{}^{Cb}$ ($a,b=1,2,3$). Color-octets become harmless since they can develop huge dynamical masses ($\gtrsim 100$ GeV) due to QCD of the color $SU(3)_c{}^{\mathrm{loc}}$.[17] On the other hand, for leptoquarks present in the model with $N=4$ and $M=6$ (H_1), SQCD is the only dynamics to generate masses, which are thus at least required to be of order, say, 100 GeV once SUSY and chiral symmetries are explicitly broken. Thus, SQCD must generate relatively heavy leptoquarks while keeping quarks and leptons relatively light.

In the remaining two models, i.e., $N=4$ with $M=5$ or 8 ($H_{2,3}$), the model with $N=4$ and $M=8$ deserves careful examination. Since there are two kinds of condensates, $\langle\tilde{w}_L\tilde{w}_R{}^C\rangle$ and $\langle[(\tilde{w}_L)^4]\rangle+\langle[(\tilde{w}_R{}^C)^4]\rangle$, that generate the "same" NGS's (with respect to the subgroup H), the physical NGS's are given by $\Lambda_{sc}^4\langle\tilde{w}_L\tilde{w}_R{}^C\rangle\hat{c}_{La}\hat{w}_L{}^i+\langle[(\tilde{w}_R{}^C)^4]\rangle\hat{c}_{La}[(\hat{w}_R{}^C)^3]^i$ ($a=0,1,2,3$; $i=1,2$) and those by $L\leftrightarrow R$. Since we are assuming that $\hat{w}_L{}^i$ have the $V-A$ coupling to the weak gauge bosons $W^\pm$, $\hat{c}_a\hat{w}^i$ are ordinary quarks and leptons while $\hat{c}_a[(\hat{w}^C)^3]^i$ turn out to be their mirrors exhibiting the $V+A$ interactions.[18] The model with $N=4$ and $M=8$ can be accepted as a realistic quark-lepton model only if SQCD ensures $\langle[(\tilde{w}_L)^4]\rangle/\langle\tilde{w}_L\tilde{w}_R{}^C\rangle\Lambda_{sc}^2\ll 1$ and $\langle[\tilde{w}_R{}^C)^4]\rangle/\langle\tilde{w}_L\tilde{w}_R{}^C\rangle\Lambda_{sc}^2\ll 1$. The same mixing occurs in the models with $N=4$ and M

*) Since $N\geq 3$ for a complex N representation, the simplest case with $N=2$ leading to $[SU(2)_W]_{L+R}\times[SU(4)_c]_L\times[SU(4)_c]_R\times U(1)_{V'}\times U(1)_{A'}$ is excluded.

Table III. Spectra of massless composite superfields including the quark-lepton ones. $\tilde{c}$ and $\tilde{w}$ are the scalar components of the subquark-superfields $\hat{c}$ and $\hat{w}$. Neutral fields are not listed.

N & M	Superfields / Condensations/Subgroup	Quarks/Leptons	Leptoquarks	Color-octet	Weak-triplet
$N=4$ $M=6$	$\hat{c}_{a=0,1,2,3}$; $\hat{w}^{i=1,2}$	$\hat{c}_a\hat{w}^i/\hat{c}_0\hat{w}^i$	$\hat{c}_a\hat{c}^{C0}+\hat{c}_0\hat{c}^{Ca}$	$\hat{c}_a\hat{c}^{Cb}$	$\cdots$
	$\langle\tilde{c}_{La}\tilde{c}_R{}^{Ca}\rangle$, $\langle[(\tilde{c}_L)^4]\rangle$, $\langle[(\tilde{c}_R{}^C)^4]\rangle$ $H_1=[SU(4)_C]_{L+R}\times[SU(2)_W]_L\times[SU(2)_W]_R\times U(1)_{V'}\times U(1)_{A'}$				
$N=4$ $M=8$	$\hat{c}_{a=0,1,2,3}$; $\hat{w}^{i=1\sim4}$	$\hat{c}_a\hat{w}^i/\hat{c}_0\hat{w}^i$	$\cdots$	$\cdots$	$\hat{w}^i\hat{w}_j{}^C$
	$\langle\tilde{w}_L{}^i\tilde{w}^C_{Ri}\rangle$, $\langle[(\tilde{w}_L)^4]\rangle$, $\langle[(\tilde{w}_R{}^C)^4]\rangle$ $H_2=[SU(4)_W]_{L+R}\times[SU(4)_C]_L\times[SU(4)_C]_R\times U(1)_{V'}\times U(1)_{A'}$				
$N=4$ $M=5$	$\hat{c}_{a=1,2,3}$; $\hat{w}^{i=1,2}$	$\hat{c}_a\hat{w}^i/[(\hat{c}^C)^3]\hat{w}^i$	$\cdots$	$\hat{c}_a\hat{c}^{Cb}$	$\cdots$
	$\langle\tilde{c}_{La}\tilde{c}_R{}^{Ca}\rangle$, $\langle[(\tilde{c}_L)^3(\tilde{c}_R{}^C)^3]\rangle$ $H_3=[SU(3)_C]_{L+R}\times[SU(2)_W]_L\times[SU(2)_W]_R\times U(1)_w\times U(1)_c\times U(1)_\chi$				

$=5, 6$ but between $\hat{c}_a\hat{w}^i$ and $[(\hat{c})^3][(\hat{c}^C)^2]_a\hat{w}^i$ for $M=5$ or $[(\hat{c}^C)^2\hat{c}_0{}^C]_a\hat{w}_i$ for $M=6$ that can be both taken as the ordinary quarks exhibiting the $V-A$ interactions.[19] Leptons in the model with $M=5$ are generated as CF's, $[(\hat{c}^C)^3]\hat{w}^i$, to saturate the anomalies of H.

To discuss these dynamical aspects of SQCD with $N<M$, we will rely upon the effective lagrangian approach to SQCD. In the next section, an effective superpotential is constructed to describe the symmetry breaking of SQCD.

§ 3. Effective description of SQCD

The symmetry breaking of SQCD should also be described by an effective lagrangian ($L_{\rm eff}$) for massless composite fields including quarks and leptons. The effective lagrangian contains an effective superpotential of the Taylor-Veneziano-Yankielowicz type.[9] It is invariant under $G=SU(M)_L\times SU(M)_R\times U(1)_V\times U(1)_A$ and accommodates the $U(1)_{\rm anom}$ transformation $\delta L_{\rm eff}=\delta(W_{\rm eff}|_{\theta\theta}+{\rm H.c.})=2M(g^2_{sc}/32\pi^2)G_{\mu\nu}\tilde{G}^{\mu\nu}$. The superpotential consists of two parts: $W_{\rm eff}=W^{(0)}_{\rm eff}+W^{(1)}_{\rm eff}$, where $W^{(0)}_{\rm eff}$ is responsible for $U(1)_{\rm anom}$. The relevant massless composite superfields are associated with **(A)** $G\to H=SU(N)_{L+R}\times SU(M-N)_L\times SU(M-N)_R\times U(1)_{V'}\times U(1)_{A'}$ and **(B)** $G\to H=SU(N-1)_{L+R}\times SU(M-N+1)_L\times SU(M-N+1)_R\times U(1)_w\times U(1)_c\times U(1)_\chi$ as shown in Table II.

The composite superfields required can be taken as:

$$T_i^j=\sum_{A=1}^{N}\Phi^{(1)}{}_i{}^A\Phi^{(2)}{}_A{}^j\,,\qquad (i,j=1\sim M) \tag{3·1a}$$

containing QNGF's associated with $\langle T_i{}^i\rangle|_{\theta=0}\sim\langle\phi_i{}^{(1)}\phi^{(2)i}\rangle$ ($i=1\sim N$ for **(A)** and $i=1\sim N-1$ for **(B)**),

$$Y^{(1)}_{[i_1\cdots i_N]}=\det(\Phi^{(1)}{}_i{}^A)\,, \tag{3·1b}$$

$$Y^{(2)[i_1\cdots i_N]}=\det(\Phi^{(2)}{}_A{}^i) \tag{3·1c}$$

with $\langle Y^{(1)}_{[1\cdots N]}\rangle|_{\theta=0}\sim\langle[(\phi^{(1)})^N]\rangle$ and $\langle Y^{(2)[1\cdots N]}\rangle|_{\theta=0}\sim[(\phi^{(2)})^N]\rangle$ for (**A**) containing QNGF's and with $\langle Y^{(1)}_{[1\cdots N]}\rangle|_{\theta=0}=\langle Y^{(2)[1\cdots N]}\rangle|_{\theta=0}=0$ for (**B**) containing CF's and

$$U^{[j_1\cdots j_{N-1}]}_{[i_1\cdots i_{N-1}]}=\sum_{A=1}^{N}\det{}_A(\phi^{(1)}{}_i{}^B)\det{}^A(\Phi^{(2)j}{}_B)\,, \tag{3·1d}$$

that will develop VEV as $\langle U^{[1\cdots N-1]}_{[1\cdots N-1]}\rangle|_{\theta=0}\sim\langle[(\phi^{(1)})^{N-1}(\phi^{(2)})^{N-1}]\rangle$ to yield $SU(N-1)_{L+R}$ for (**B**).*) An additional composite superfield,

$$S=\sum_{A,B=1}^{N}W_A{}^BW_B{}^A\,, \tag{3·2}$$

containing $\lambda\lambda$, will show up to accomodate the anomalous $U(1)_{\rm anom}$ transformation of $L_{\rm eff}$.

The superpotential $W^{(0)}_{\rm eff}$ can be constructed from T and S only. The $U(1)_{\rm anom}$ transformation of $\delta L_{\rm eff}=\delta(W^{(0)}_{\rm eff}|_{\theta\theta}+{\rm H.c})=2M(g^2_{sc}/32\pi^2)G\tilde{G}$ is translated into $\delta W^{(0)}_{\rm eff}=-2iMS$, leading to $T\partial W^{(0)}_{\rm eff}/\partial T=MS$. The $U(1)_A$-invariance of $W^{(0)}_{\rm eff}|_{\theta\theta}$ reads $[-M(S\partial/\partial S-1)+(M-N)T\partial/\partial T]W^{(0)}_{\rm eff}=0$. By considering other invariances, one obtains[9])

$$W^{(0)}_{\rm eff}=S\{\ln[S^{N-M}\det(T_i{}^j)/\Lambda^{3N-M}_{sc}]+M-N\}\,. \tag{3·3}$$

The remaining $W^{(1)}_{\rm eff}$ is arbitrary functions of $Z_Y=\det(T_i{}^j)/X_Y$ and $Z_U=\det(T_i{}^j)/X_U$:

$$W^{(1)}_{\rm eff}=S[f_Y(Z_Y)+f_U(Z_U)]\,, \tag{3·4}$$

where

$$X_Y=\varepsilon^{i_1\cdots i_M}\varepsilon_{j_1\cdots j_M}Y^{(1)}_{[i_1\cdots i_N]}Y^{(2)^{[j_1\cdots j_N]}}T^{j_{N+1}}_{i_{N+1}}\cdots T^{j_M}_{i_M}\,, \tag{3·5a}$$

$$X_U=(\varepsilon^{i_1\cdots i_M}\varepsilon^{i_1'\cdots i_M'}\varepsilon_{j_1\cdots j_M}\varepsilon_{j_1'\cdots j_M'}U^{[j_1'\cdots j_{N-1}']}_{[i_1\cdots i_{N-1}]}U^{[j_1\cdots j_{N-1}]}_{[i_1'\cdots i_{N-1}']}T^{j_N}_{i_N}\cdots T^{j_M}_{i_M}T^{j_N'}_{i_N'}\cdots T^{j_M'}_{i_M'})^{1/2}\,, \tag{3·5b}$$

up to normalization constants, where the repeated indices are all summed. Since det $(T_i{}^j)$ and $X_{Y,U}$ are both equivalent to $\prod_{i=1}^M(\Phi_i{}^{(1)}\Phi^{(2)i})$, the fields $Z_{Y,U}$ are neutral under $G\times U(1)_{\rm anom}$. The functions $f_{Y,U}(Z_{Y,U})$ will be determined by the consistent symmetry breaking with complementarity generated by the effective superpotential. The function $f_U(Z_U)$ is absent for (**A**) since the case (**A**) does not permit the massless U field. The condensations $\langle Z_Y\rangle|_{\theta=0}\neq0$ and $\langle Z_U\rangle|_{\theta=0}\neq0$ will fix $\langle T_i{}^i\rangle|_{\theta=0}$. The formation of $\langle Y^{(1)}_{[1\cdots N]}\rangle|_{\theta=0}$ and $\langle Y^{(2)[1\cdots N]}\rangle|_{\theta=0}$ spontaneously chooses $\langle T_i{}^j\rangle|_{\theta=0}\propto\delta_i{}^j$ $(i,j=1\sim N)$ leaving $SU(M-N)_L\times SU(M-N)_R$ unbroken while that of $\langle U^{[1\cdots N-1]}_{[1\cdots N-1]}\rangle|_{\theta=0}$ spontaneously chooses $\langle T_i{}^j\rangle|_{\theta=0}\propto\delta_i{}^j$ $(i,j=1\sim N-1)$ leaving $SU(M-N+1)_L\times SU(M-N+1)_R$ unbroken. Finally, the effective superpotential is given by

$$W_{\rm eff}=S\{\ln[S^{N-M}\det(T_i{}^j)]+f_Y(Z_Y)+f_U(Z_U)+M-N\}\,. \tag{3·6}$$

The possible condensations generated by $W_{\rm eff}$ are chosen to be:

$$\pi_i=\langle T_i{}^i\rangle|_{\theta=0}(\sim\langle\phi_i{}^{(1)}\phi^{(2)i}\rangle)\,, \tag{3·7a}$$

*) [ ] denotes the antisymmetrization of the "colors": $[(\phi^{(1)})^N]=\det(\phi^{(1)}{}_i{}^A)\sim\varepsilon_{A_1\cdots A_N}\phi^{(1)A_1}_{i_1}\cdots\phi^{(1)A_N}_{i_N}$, $[(\phi^{(1)})^{N-1}]_B=\det{}_B(\phi^{(1)}{}_i{}^A)\sim\varepsilon_{BA_1\cdots A_{N-1}}\phi^{(1)A_1}_{i_1}\cdots\phi^{(1)A_{N-1}}_{i_{N-1}}$ and so on.

$$\pi_{y1}=\langle Y^{(1)}_{[1\cdots N]}\rangle|_{\theta=0}(\sim\langle\phi^{(1)}_{[1}\cdots\phi^{(1)}_{N]}\rangle)\,, \tag{3·7b}$$

$$\pi_{y2}=\langle Y^{(2)[1\cdots N]}\rangle|_{\theta=0}(\sim\langle\phi^{(2)[1}\cdots\phi^{(2)N]}\rangle)\,, \tag{3·7c}$$

$$\pi_{u}=\langle U^{[1\cdots N-1]}_{[1\cdots N-1]}\rangle|_{\theta=0}(\sim\langle\phi^{(1)}_{[1}\cdots\phi^{(1)}_{N-1]}\phi^{(2)[1}\cdots\phi^{(2)N-1]}\rangle)\,, \tag{3·7d}$$

$$\pi_{\lambda}=\langle S\rangle|_{\theta=0}(\sim\langle\lambda\lambda\rangle)\,. \tag{3·7e}$$

The SUSY vacuum is realized if $\partial W_{\text{eff}}/\partial\pi_I=0$ $(I=i,\,y1,\,y2,\,u,\,\lambda)$ with $\partial W_{\text{eff}}/\partial\pi_I=W_{;I}$ calculated to be:

$$W_{;\,i=1\sim N-1}=[1+\langle z_Y\rangle f_Y'(\langle z_Y\rangle)+\langle z_U\rangle f_U'(\langle z_U\rangle)](\pi_\lambda/\pi_i)\,, \tag{3·8a}$$

$$W_{;\,i=N}=[1+\langle z_Y\rangle f_Y'(\langle z_Y\rangle)](\pi_\lambda/\pi_i)\,, \tag{3·8b}$$

$$W_{;\,i=N+1\sim M}=\pi_\lambda/\pi_i\,, \tag{3·8c}$$

$$W_{;\,y1,y2}=-\langle z_Y\rangle f_Y'(\langle z_Y\rangle)(\pi_\lambda/\pi_{y1,y2})\,, \tag{3·8d}$$

$$W_{;\,u}=-\langle z_U\rangle f_U'(\langle z_U\rangle)(\pi_\lambda/\pi_u)\,, \tag{3·8e}$$

$$W_{;\,\lambda}=\ln[(\pi_\lambda/\Lambda^3_{sc})^{N-M}\prod_{i=1}^{M}(\pi_i/\Lambda^2_{sc})]+f_Y(\langle z_Y\rangle)+f_U(\langle z_U\rangle)\,, \tag{3·8f}$$

where $\langle z_Y\rangle=\langle Z_Y\rangle|_{\theta=0}=\prod_{i=1}^N\pi_i/\pi_{y1}\pi_{y2}$ and $\langle z_U\rangle=\langle Z_U\rangle|_{\theta=0}=\prod_{i=1}^{N-1}\pi_i/\pi_u$. The VEV's of π_I given by complementarity are

$$\pi_i\sim\Lambda^2_{sc}\qquad(i=1\sim N)\,,\quad \pi_{y1,y2}\sim\Lambda^N_{sc}\,,\quad \pi_{i,\lambda}=0\qquad(i=N+1\sim M)\,, \tag{3·9a}$$

for (**A**) and

$$\pi_i\sim\Lambda^2_{sc}\qquad(i=1\sim N-1)\,,\quad \pi_u\sim\Lambda^{N-1}_{sc}\,,\quad \pi_{i,y1,y2,\lambda}=0\qquad(i=N\sim M)\,, \tag{3·9b}$$

for (**B**). The form of $f_{Y,U}(Z_{Y,U})$ of $W^{(1)}_{\text{eff}}$ is so determined to provide these *VEV*'s.

One observes that $W^{(0)}_{\text{eff}}$ develops a singularity due to $\pi_\lambda=0$ and $\pi_i=0$ ($i=N+1\sim M$ for (**A**); $i=N\sim M$ for (**B**)). It can be seen from $W_{;\lambda}=0$ as

$$f_Y(\langle z_Y\rangle)+f_U(\langle z_U\rangle)=\ln[\pi_\lambda{}^{n-N}\prod_{i=1}^{M-n}(\pi_\lambda/\pi_{i+n})+\text{“const”}]\,, \tag{3·10}$$

where $n=N$ for (**A**) and $n=N-1$ for (**B**) and “const” represents the regular parts. This equation dictates[18),19)]

$$f_Y(Z_Y)=\rho\ln(Z_Y-\langle z_Y\rangle)\qquad(\rho>0) \tag{3·11a}$$

for (**A**) with $f_U(Z_U)$ being absent and

$$f_U(Z_U)=\rho\ln(Z_U-\langle z_U\rangle)\qquad(\rho>0) \tag{3·11b}$$

for (**B**) with a regular $f_Y(Z_Y)$. The singularity in $W^{(0)}_{\text{eff}}$ is transferred to the one at $Z_{Y,U}=\langle z_{Y,U}\rangle$ in $f_{Y,U}(Z_{Y,U})$. For the chiral fields $Y^{(1,2)}$ present in (**B**), $\langle Y^{(1,2)}\rangle|_{\theta=0}=0$ (thus $\langle Z_Y{}^{-1}\rangle|_{\theta=0}=0$) should be maintained even if SUSY is broken. The simplest choice is

$$f_Y(Z_Y)=hZ_Y{}^{-1}\,, \tag{3·12}$$

which effectively yields the Higgs-type coupling for CF's.

§ 4. Effective lagrangian for broken SQCD

The explicit breaking of SUSY and chiral symmetry is assumed to be caused by the following fundamental lagrangian $L_{\rm break}$ for scalar- and gaugino-mass:

$$-L_{\rm break}=\sum_{i=1}^{M}(\mu_{iL}^2|\phi_i^{(1)}|^2+\mu_{iR}^2|\phi^{(2)i}|^2)+\sum_{i=1}^{M}\mu_i{}^2(\phi_i^{(1)}\phi^{(2)i}+\text{H.c.})$$
$$+m_\lambda(\lambda\lambda+\text{H.c.})\,, \tag{4·1}$$

where $\mu_{iL,iR,i}$, $m_\lambda \ll \Lambda_{sc}$ for the approximate SUSY. The effective lagrangian, $L_{\rm mass}$, that possesses the same breaking effects on $SU(M)_L\times SU(M)_R\times U(1)_V\times U(1)_A$ takes the form of

$$-L_{\rm mass}=\{\sum_{i=1}^{M}\mu_{iL}^2\sum_{j=1}^{M}(\Lambda_{sc}^{-2}|T_i^j|^2+\Lambda_{sc}^{-(4N-6)}|U^{(1)}{}_i^j|^2+\Lambda_{sc}^{-2(N-1)}|Y^{(1)}{}_i^j|^2)$$
$$+\sum_{j=1}^{M}\mu_{jR}^2\sum_{i=1}^{M}(\Lambda_{sc}^{-2}|T_i^j|^2+\Lambda_{sc}^{-(4N-6)}|U^{(2)}{}_i^j|^2+\Lambda_{sc}^{-2(N-1)}|Y^{(2)}{}_i^j|^2)$$
$$+\sum_{i=1}^{M}\mu_i{}^2(T_i^i+T_i^{i*})+m_\lambda(S+S^*)\}|_{\theta=\bar\theta=0}\,, \tag{4·2}$$

where $U^{(1)}{}_i^j\sim\sum\varepsilon^{jk_1\cdots k_{N-2}}\varepsilon_{l_1\cdots l_{N-1}}U^{[l_1\cdots l_{N-1}]}_{[ik_1\cdots k_{N-2}]}$, $Y^{(1)}{}_i^j\sim\sum\varepsilon^{jk_1\cdots k_{N-1}}Y^{(1)}_{[ik_1\cdots k_{N-1}]}$ and similarly for $U^{(2)}$ and $Y^{(2)}$. The kinetic terms for composite superfields ($\Omega_I=T$, $Y^{(1,2)}$, U, S) are in general given by

$$L_{\rm kin}=\sum_I K_I(\Omega_I{}^*\Omega_I)|_{\theta\theta\bar\theta\bar\theta}\,, \tag{4·3a}$$

but K_I will be restricted to satisfy

$$\partial^2K_I(\Omega_I{}^*\Omega_I)/\partial\Omega_I{}^*\partial\Omega_J=\delta_{IJ}G_I{}^{-1}(\Omega_I{}^*\Omega_I)\,, \tag{4·3b}$$

where, as mass dimensions of G_I, $G_T\sim\Lambda_{sc}^2$, $G_{Y(a)}\sim\Lambda_{sc}^{2(N-1)}$, $G_U\sim\Lambda_{sc}^{4N-6}$ and $G_S\sim\Lambda_{sc}^4$. Then, our starting lagrangian is

$$L_{\rm eff}=L_{\rm kin}+(W_{\rm eff}|_{\theta\theta}+\text{H.c.})+L_{\rm mass}\,. \tag{4·4}$$

In the quark-lepton models, the mass parameters of $L_{\rm mass}$ are further constrained to be consistent with $SU(3)_C{}^{\rm loc}$ and $[SU(2)_W]_L{}^{\rm loc}\times U(1)_Y{}^{\rm loc}$ (or $U(1)_{em}^{\rm loc}$).

The effective potential ($V_{\rm eff}$) is derived as:

$$V_{\rm eff}=G_T\sum_{i=1}^{M}|W_{;i}|^2+\sum_{a=1}^{2}G_{Y(a)}|W_{;ya}|^2+G_U|W_{;u}|^2+G_S|W_{;\lambda}|^2+\sum_{i=1}^{M}[(\mu_{iL}^2$$
$$+\mu_{iR}^2)\Lambda_{sc}^{-2}|\pi_i|^2+\mu_i{}^2(\pi_i+\pi_i{}^*)]+\sum_{i=1}^{N}\Lambda_{sc}^{-2(N-1)}(\mu_{iL}^2|\pi_{y1}|^2+\mu_{iR}^2|\pi_{y2}|^2)$$
$$+\sum_{i=1}^{N-1}(\mu_{iL}^2+\mu_{iR}^2)\Lambda_{sc}^{-(4N-6)}|\pi_u|^2+m_\lambda(\pi_\lambda+\pi_\lambda{}^*)\,. \tag{4·5}$$

The functions $f_{Y,U}(z_{Y,U})$ contained in $W_{;I}$ are modified to

$$f_{Y,U}(z_{Y,U})=\ln\{(z_{Y,U}-\langle z_{Y,U}\rangle)^\rho[1+a_{Y,U}(z_{Y,U}-\langle z_{Y,U}\rangle)+\cdots]\}\,, \tag{4·6}$$

by the SUSY-breaking, where $z_{Y,U}=Z_{Y,U}|_{\theta=0}$ coincide with $\langle z_{Y,U}\rangle$ in the SUSY limit. The parameters $\xi_{Y,U}=z_{Y,U}-\langle z_{Y,U}\rangle$ measure the SUSY-breaking effect. The conditions of $\partial V_{\text{eff}}/\partial\pi_I=0$ are written as

$$G_T W_i^*(\pi_\lambda/\pi_i)=G_S W^*_{;\lambda}+M_i^2+G_T'|\pi_i|^2\sum_{j=1}^{M}|W_{;j}|^2 \tag{4·7a}$$

for $i=1\sim n$ ($n=N$ for (**A**); $N-1$ for (**B**)),

$$(1+\varepsilon)G_T W_i^*(\pi_\lambda/\pi_i)=\eta W+(1+\varepsilon)G_S W^*_{;\lambda}+M_i^2+G_T'|\pi_i|^2\sum_{j=1}^{M}|W_{;j}|^2 \tag{4·7b}$$

for $i=n+1\sim M$,

$$\varepsilon_Y G_{Y(a)}W^*_{;ya}(\pi_\lambda/\pi_{ya})=\eta_Y W_Y+\varepsilon_Y G_S W^*_{;\lambda}-M_{ya}^2-G'_{Y(a)}|\pi_{ya}|^2|W_{;ya}|^2\,, \tag{4·7c}$$

$$\varepsilon_U G_U W^*_{;u}(\pi_\lambda/\pi_u)=\eta_U W_U+\varepsilon_U G_S W^*_{;\lambda}-M_u^2-G_U'|\pi_u|^2|W_{;u}|^2\,, \tag{4·7d}$$

$$[G_T\sum_{i=1}^{M}W^*_{;i}(\pi_\lambda/\pi_i)+\varepsilon_Y W_Y+\varepsilon_U W_U+(N-M)G_S W^*_{;\lambda}]\pi_\lambda^{-1}+m_\lambda+\pi_\lambda^* G_S'|W_{;\lambda}|^2=0 \tag{4·7e}$$

with $\varepsilon_X=z_X f_X'(z_X)$ and $\eta_X=z_X(z_X f_X'(z_X))'$ for $X=Y$ or U; $\varepsilon=\varepsilon_Y+\varepsilon_U$; and $\eta=\eta_Y+\eta_U$, where

$$M_i^2=(\mu_{iL}^2+\mu_{iR}^2)\Lambda_{SC}^{-2}|\pi_i|^2+\mu_i^2\pi_i\,,\qquad M_{y1}^2=\sum_{i=1}^{N}\mu_{iL}^2\Lambda_{SC}^{-2(N-1)}|\pi_{y1}|^2\,, \tag{4·8a, b}$$

$$M_{y2}^2=\sum_{i=1}^{N}\mu_{iR}^2\Lambda_{SC}^{-2(N-1)}|\pi_{y2}|^2\,,\qquad M_u^2=\sum_{i=1}^{N-1}(\mu_{iL}^2+\mu_{iR}^2)\Lambda_{SC}^{-(4N-6)}|\pi_u|^2\,, \tag{4·8c, d}$$

$$W_Y=\sum_{i=1}^{N}G_T W^*_{;i}(\pi_\lambda/\pi_i)-\sum_{a=1}^{2}G_{Y(a)}W^*_{;ya}(\pi_\lambda/\pi_{ya})\,, \tag{4·8e}$$

$$W_U=\sum_{i=1}^{N-1}G_T W^*_{;i}(\pi_\lambda/\pi_i)-G_U W^*_{;u}(\pi_\lambda/\pi_u)\,. \tag{4·8f}$$

The factors ε and η depend on the symmetry-breaking patterns and are given by

$$\varepsilon_Y=\rho z_Y/\xi_Y\,,\qquad \eta_Y=-\rho z_Y\langle z_Y\rangle/\xi_Y^2\quad\text{and}\quad \varepsilon_U=\eta_U=0\,, \tag{4·9a}$$

from $f_Y(Z_Y)=\rho\ln(Z_Y-\langle z_Y\rangle)$ and $f_U(Z_U)=0$ in case (**A**) and

$$\varepsilon_Y=-hz_Y^{-1}\,,\quad \eta_Y=hz_Y^{-1}\,,\quad \varepsilon_U=\rho z_U/\xi_U\ \text{and}\ \eta_U=-\rho z_U\langle z_U\rangle/\xi_U^2\,, \tag{4·9b}$$

from $f_Y(Z_Y)=hZ_Y^{-1}$ and $f_U(Z_U)=\rho\ln(Z_U-\langle z_U\rangle)$ in case (**B**). One can observe that, for $f_Y(Z_Y)=hZ_Y^{-1}$, $\pi_{y1,y2}=0$ are solutions to $\partial V_{\text{eff}}/\partial\pi_{y1,y2}=0$ of Eq. (4·7c) because ε_Y,η_Y and $\pi_{ya}W_{;ya}=hz_Y^{-1}\pi_\lambda$ turn out to be all proportional to $\pi_{y1}\pi_{y2}$.

By keeping the leading terms of $\xi_{Y,U}^{-1}=(z_{Y,U}-\langle z_{Y,U}\rangle)^{-1}\gg 1$ and by using $|\pi_{i=1\sim n}|^2\gg|\pi_{i=n+1\sim M}|^2$, $|\pi_\lambda|^2/\Lambda_{SC}^2$, we find that

$$R[G_T|\pi_\lambda/\pi_i|^2-M_i^2-G_S W_{;\lambda}]=-n\Delta_T|\pi_i|^2\sum_{a=1}^{m}R_{Xa}(M^2_{\langle\,\rangle\neq 0}+M_{Xa}^2)\,,$$

$$(i=n+1\sim M) \tag{4·10a}$$

$$R[G_T|\rho\langle z\rangle\pi_\lambda/\xi\pi_i|^2-M_i^2+(M^2_{\langle\,\rangle\neq 0}/n)]$$

$$=[\rho R_{Ti}-(\sum_{a=1}^{m}R_{Xa})(1+\Delta_T M|\pi_i|^2)]G_S W_{;\lambda}$$

$$-\sum_{a=1}^{m}R_{Xa}[n^{-1}R_{Ti}(M^2_{\langle\,\rangle\neq 0}+nM^2_{xa})+\Delta_T|\pi_i|^2M^2_{\langle\,\rangle=0}]$$

$$+\Delta_T(n|\pi_i|^2-\sum_{j=1}^{n}|\pi_j|^2)[M^2_{\langle\,\rangle=0}+(M-n)G_S W_{;\lambda}]\,,\quad (i=1\sim n) \tag{4·10b}$$

$$RG_{Xa}|\rho(z)\pi_\lambda/\xi\pi_{xa}|^2=R_{Xa}\{[n+\Delta_T M(\sum_{j=1}^{n}|\pi_j|^2)-\rho]G_S W_{;\lambda}$$

$$+R_T(M^2_{\langle\,\rangle\neq 0}+nM^2_{xa})+\Delta_T(\sum_{j=1}^{n}|\pi_j|^2)M^2_{\langle\,\rangle=0}\}+\sum_{b=1}^{m}R_{Xb}(M^2_{xb}-M^2_{xa})\,, \tag{4·10c}$$

$$R(\rho\langle z\rangle)[(\rho-n+N)G_S W_{;\lambda}+M^2_{\langle\,\rangle=0}+m_\lambda\pi_\lambda]=\xi[nR_T(\sum_{a=1}^{m}R_{Xa}-\rho)M^2_{\langle\,\rangle\neq 0}$$

$$+(nR_T-\rho)\sum_{a=1}^{m}R_{Xa}M^2_{xa}] \tag{4·10d}$$

with $\Delta_{T,Y(a)}$, $R_{Ti,Xa}$ and R defined by

$$\Delta_T=G_T'/(G_T-G_T'\textstyle\sum_{i=1}^{M}|\pi_i|^2)\,, \tag{4·11a}$$

$$\Delta_{Y(a)}=G'_{Y(a)}/(G_{Y(a)}-G'_{Y(a)}|\pi_{ya}|^2)\,, \tag{4·11b}$$

$$R_{Ti}=1+n\Delta_T|\pi_i|^2\,,\quad R_T=1+\Delta_T\sum_{i=1}^{n}|\pi_i|^2=G_T|(G_T-G_T'\sum_{i=1}^{n}|\pi_i|^2)\,, \tag{4·11c, d}$$

$$R_{Xa}=1+\Delta_{Xa}|\pi_{xa}|^2=G_{Xa}/(G_{Xa}-G'_{Xa}|\pi_{xa}|^2)\,, \tag{4·11e}$$

$$R=\sum_{i=1}^{n}R_{Ti}-\sum_{a=1}^{m}R_{Xa},\ nR_T=\sum_{i=1}^{n}R_{Ti}\,, \tag{4·11f, g}$$

where $n=N$ for (**A**) and $n=N-1$ for (**B**) stand for the $SU(n)_{L+R}$ symmetry of the subgroup H; $Xa=Y^{(a)}$ $(a=1,2)$ with $m=2$ for (**A**) and $Xa=U$ ($a=1$ only) with $m=1$ for (**B**) are the freedom of the multi-body composites; $M_{xa}=M_{ya}$ $(a=1,2)$ for (**A**) and $=M_u$ ($a=1$ only) for (**B**); $|\pi_{\langle\,\rangle\neq 0}|^2=\sum_{i=1}^{n}|\pi_i|^2$ and $M^2_{\langle\,\rangle\neq 0}=\sum_{i=1}^{n}M_i^2$; $M^2_{\langle\,\rangle=0}=\sum_{i=n+1}^{M}M_i^2$ is the explicit breaking of the chiral $SU(M-n)$ symmetry; the parameters of z and ξ are $z=z_{Y(U)}$ and $\xi=\xi_{Y(U)}$ for (**A**) ((**B**)). Δ_T and $\Delta_{Y(a)}$ vanish for the canonical kinetic terms with $G_I'=0$ for composites.

From the conditions of (4·10 a~d), we find the following general features of symmetry breaking:

(1) The breaking from $M^2_{\langle\,\rangle\neq 0}$ and M^2_{xa},*) i.e., from $\mu_{iL,iR,i}$ $(i=1\sim n)$, is not compatible with Eqs. (4·10b, c) because $\sum_{i=1}^{n}RG_T(\rho(z)\pi_\lambda/\xi\pi_i)^2(>0)=-\sum_{a=1}^{m}\times RG_{Xa}(\rho\langle z\rangle\pi_\lambda/\xi\pi_{xa})^2$ (<0) results from $G_S W_{;\lambda}\sim\xi M^2_{\langle\,\rangle\neq 0,Xa}\ll M^2_{\langle\,\rangle\neq 0,Xa}$;

(2) the hierarchy of $\pi_{i=n+1\sim M}\ll\pi_{\langle\,\rangle\neq 0}\sim\Lambda^2_{sc}$ is dynamically ensured by $\xi\ll 1$ as

*) If all μ's and m_λ are the same order, then $M^2_{\langle\,\rangle\neq 0}$ and M^2_{xa} automatically dominate in the SUSY-breaking effect since $|\pi_i|$ $(i=n+1\sim M)\ll\Lambda^2_{sc}$ and $|\pi_\lambda|\ll\Lambda^3_{sc}$ as well as $|\pi_i|$ $(i=1\sim n)\sim\Lambda^2_{sc}$.

$$\pi_{i=n+1\sim M}{}^{\sim \xi\pi}{}_{\langle\rangle\neq 0}\ , \tag{4·12a}$$

which crucially depends on the singular behavior of $\ln(Z_{Y,U}-\langle z_{Y,U}\rangle)$ at $Z_{Y,U}=\langle z_{Y,U}\rangle$;

(3) a constraint on $G_S W_{;\lambda}$

$$G_S W_{;\lambda}>0\ , \tag{4·12b}$$

arises from $\sum_{i=1}^{n}$[Eq. (4·10b)]$+\sum_{a=1}^{m}$[Eq. (4·10c)]$=\rho R G_S W_{;\lambda}$.

Since the breaking by $\mu_{iL,iR,i}$ $(i=1\sim n)$ cannot be the main source, the splitting of π_i $(i=1\sim n)$ is suppressed. The VEV's can be approximated to be i-independent:

$$\pi_i=\pi_{\langle\rangle\neq 0}\ , \qquad (i=1\sim n) \tag{4·13}$$

by omitting the terms of M_i^2 $(i=1\sim n)$ and $M^2_{\langle\rangle\neq 0,xa}$. The four equations (4·10a~d) are further reduced to:

$$G_T|\pi_\lambda/\pi_i|^2=G_S W^*_{;\lambda}+M_i^2\ , \qquad (i=n+1\sim M) \tag{4·14a}$$

$$RG_T|\rho\langle z\rangle\pi_\lambda/\xi\pi_{\langle\rangle\neq 0}|^2=R_X\{[\rho(R_T/R_X)-1)]G_S W_{;\lambda}$$
$$-\Delta_T|\pi_{\langle\rangle\neq 0}|^2(MG_S W_{;\lambda}+M^2_{\langle\rangle=0})\}\ , \quad (i=1\sim n) \tag{4·14b}$$

$$RG_{Xa}|\rho\langle z\rangle\pi_\lambda/\xi\pi_{xa}|^2=R_{Xa}[(n-\rho)G_S W_{;\lambda}+\Delta_T|\pi_{\langle\rangle\neq 0}|^2(MG_S W_{;\lambda}+M^2_{\langle\rangle=0})]\ , \tag{4·14c}$$

$$(\rho-n+N)G_S W_{;\lambda}=-(M^2_{\langle\rangle=0}+m_\lambda\pi_\lambda)\ . \tag{4·14d}$$

By considering the M_{SS}- and ξ-dependence of these equations with $\pi_{i=n+1\sim M}\sim\xi\pi_{\langle\rangle\neq 0}$, one can find that

$$\pi_\lambda\sim\xi^2 M_{SS}\Lambda_{SC}^2 \tag{4·15a}$$

for M_{SS} provided by m_λ and $(\mu_{iL}^2+\mu_{iR}^2)^{1/2}$ in $M^2_{\langle\rangle=0}$ that preserve some of the remaining chiral symmetries and

$$\pi_\lambda\sim\xi^{3/2} M_{SS}\Lambda_{SC}^2 \tag{4·15b}$$

for M_{SS} provided by μ_i in $M^2_{\langle\rangle=0}$ that break all of the remaining chiral symmetries. The M_{SS}-dependence of ξ is determined by Eq. (4·14d) together with Eq. (3·9f) for $W_{;\lambda}$:*)

$$\xi\sim(M_{SS}/\Lambda_{SC})^{(M-N)/[\rho-(M-N)]}\ , \quad (M_{SS}=m_\lambda \text{ and } \mu_{iL,iR}) \tag{4·16a}$$

$$\xi\sim(M_{SS}/\Lambda_{SC})^{(M-N)/[\rho-(M-N)/2]}\ , \quad (M_{SS}=\mu_i) \tag{4·16b}$$

in SQCD with the chiral $SU(M-N)$ symmetry

$$\xi\sim(M_{SS}/\Lambda_{SC})^{(M-N)/[\rho+1-(M-N)]}\ , \quad (M_{SS}=m_\lambda \text{ and } \mu_{iL,iR}) \tag{4·17a}$$

$$\xi\sim(M_{SS}/\Lambda_{SC})^{(M-N)/[\rho+1-(M-N)/2]}\ , \quad (M_{SS}=\mu_i) \tag{4·17b}$$

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*) In the leading approximation with $a_{Y,U}=0$ in Eq. (4·6), the determined M_{SS}-dependence of ξ yields [RHS of Eq. (4·14d)]$=0$, which is actually of order $\mu^2_{\langle\rangle=0}\xi$ for $\mu_{\langle\rangle=0}\sim\mu_i$ $(i=n+1\sim M)$. The inclusion of the $a_{Y,U}$-term gives $a_{Y,U}\sim\mu^2_{\langle\rangle=0}\Lambda_{SC}^2/G_S$.

in SQCD with the chiral $SU(M-N+1)$ symmetry.

Consider the SUSY breaking by $\mu=(\mu_{iL}^2+\mu_{iR}^2)^{1/2}$ that is the singlet of all the chiral symmetries, which induces

$$\pi_\lambda(\approx\langle\lambda\lambda\rangle)\sim\Lambda_{SC}^2\mu(\mu/\Lambda_{SC})^{2(M-N)/\kappa}, \tag{4·18a}$$

$$\pi_{i=n+1\sim M}(\approx\langle\phi_i^{(1)}\phi^{(2)i}\rangle)\sim\Lambda_{SC}^2(\mu/\Lambda_{SC})^{(M-N)/\kappa}, \tag{4·18b}$$

where $\kappa=\rho-(M-N)$ or $\kappa=\rho+1-(M-N)$. These results show that the explicit SUSY breaking necessarily induces the breakdown of the chiral $SU(M-n)$ symmetry signalled by $\pi_{i=n+1\sim M}\neq 0$ and chiral $U(1)$ symmetry by $\pi_\lambda\neq 0$. The simultaneous breakdown of SUSY and chiral symmetries results from the QCD dynamics generating the complete breakdown of all chiral symmetries because SQCD effectively becomes QCD well below the scale μ.

§ 5. Mass spectrum of composite particles

The composite superfields are expressed by T, $Y^{(1,2)}$ and U. To define masses of composites, these fields should be replaced by canonical fields with the mass dimension=1:

$$\bar{\Omega}_I=\Omega_I/\sqrt{G_I(\langle 0|\Omega_I{}^*\Omega_I|0\rangle)}, \tag{5·1}$$

which yield $\bar{\Omega}_I{}^*\bar{\Omega}_I|_{\theta\theta\bar{\theta}\bar{\theta}}$ as kinetic terms. The massless fields required by complementarity do not exhaust all of components of T, $Y^{(1,2)}$ and U. The fields with the "same" quantum numbers get mixed. Some of them become massive in agreement with the anomaly-matching given by complementarity and decouple out of the low-energy physics.

A. *Heavier and lighter fields*

It should be noted that the masses for the decoupled fields behave as 0/0 in the SUSY limit. The number 0 will be replaced by the appropriate powers of the SUSY-breaking scale of M_{SS}. Consider the mixing between composite fermions in T and $X(=Y^{(1,2)}$ or $U)$ whose spectrum has been fixed by the anomaly-matching. There are $\pi_i=\langle T_i{}^i\rangle|_{\theta=0}\neq 0$ for $i=1\sim n$ and $\pi_x=\langle Y^{(a)}\rangle|_{\theta=0}\neq 0$ or $=\langle U\rangle|_{\theta=0}\neq 0$ that generate the "same" NGS's associated with broken generators of G/H. Let K be a broken generator, and then the physical states are classified as NGS's:

$$|\text{light}\rangle=(r\bar{T}+\bar{X})/\sqrt{1+r^2}, \qquad (r=\bar{\pi}_i/\bar{\pi}_x) \tag{5·2a}$$

where $\langle 0|[K,|\text{light}\rangle]|0\rangle\neq 0$ and as chiral fields:

$$|\text{heavy}\rangle=(\bar{T}-r\bar{X})/\sqrt{1+r^2}, \tag{5·2b}$$

where $\langle 0|[K,|\text{heavy}\rangle]|0\rangle=0$. $\bar{T}$ and $\bar{X}$ $(=Y^{(1,2)}$ or $U)$ are the same members of G/H: For $T_k{}^i$,

$$Y_k{}^i=\sum\varepsilon^{ii_1\cdots i_{N-1}}Y^{(1)}_{[ki_1\cdots i_{N-1}]}, \tag{5·3a}$$

$$U_k{}^i=\sum\varepsilon^{ii_1\cdots i_{N-2}}U^{[1\cdots N-1]}_{[ki_1\cdots i_{N-2}]}, \tag{5·3b}$$

and similarly for Y_i^k and U_i^k, where the indices i, $i_1, \cdots, _{n-1}$ from $SU(n)_{L+R}$ and the index k from $SU(M-n)_{L,R}$ ($n=N$ for $Y^{(1,2)}$ and $=N-1$ for U). The chiral fields should not be massless because their presence disturbs the anomaly-matching in a manner consistent with complementarity.

The M_{SS}-dependence of the masses for "light" and "heavy" fermions can be evaluated from $(\boldsymbol{M})_{IJ}=\partial^2 W_{\rm eff}/\partial\bar{\pi}_I\partial\bar{\pi}_J$:

$$\boldsymbol{M}=-(\pi_\lambda/\bar{\pi}_{\langle\rangle\neq0}\bar{\pi}_{\langle\rangle=0})\begin{pmatrix}1+(z)\rho/\xi & -\langle z\rangle\pi r/\xi \\ -\langle z\rangle\rho r/\xi & \langle z\rangle\rho r^2/\xi\end{pmatrix}, \quad (r=\bar{\pi}_{\langle\rangle\neq0}/\bar{\pi}_x) \tag{5·4}$$

on the $(\bar{T}, \bar{X})$-basis, where $\pi_{\langle\rangle=0}$ represents one of π_i for $i=n+1\sim M$ with $\pi_i=0$ in the SUSY limit; $\bar{\pi}_x=\bar{\pi}_y$ or $\bar{\pi}_u$; and $\bar{\pi}_{y1}=\bar{\pi}_{y2}=\bar{\pi}_y$. The masses are given by

$$m_{\rm light}=-r^2\pi_\lambda/(1+r^2)\bar{\pi}_{\langle\rangle\neq0}\bar{\pi}_{\langle\rangle=0}\,, \tag{5·5a}$$

$$m_{\rm heavy}=-(1+r^2)\rho\langle z\rangle\pi_\lambda/\xi\bar{\pi}_{\langle\rangle\neq0}\bar{\pi}_{\langle\rangle=0}\,. \tag{5·5b}$$

In the SUSY-limit with ξ, $\pi_{\langle\rangle=0}$, $\pi_\lambda\to0$, both $m_{\rm light}$ and $m_{\rm heavy}$ behave as $0/0$. The ratio $0/0$ should yield 0 for "light" and $\gtrsim\Lambda_{sc}$ for "heavy" so that "heavy" fields decouple of the low-energy physics. Using the ξ- and M_{SS}-dependence of π_λ and $\pi_{\langle\rangle=0}$, we find that

$$m_{\rm light}\sim\xi M_{SS} \quad\text{and}\quad m_{\rm heavy}\sim M_{SS}\,, \tag{5·6a}$$

for $M_{SS}=m_\lambda$ and $\mu_{iL,R}$ $(i=n+1\sim M)$ that preserve some of the chiral symmetries and

$$m_{\rm light}\sim\sqrt{\xi}M_{SS} \quad\text{and}\quad m_{\rm heavy}\sim M_{SS}/\sqrt{\xi}\,, \tag{5·6b}$$

for $M_{SS}=\mu_i$ $(i=n+1\sim M)$ that break all of chiral symmetries.

The SUSY-breaking by $M_{SS}=m_\lambda$ and $\mu_{iL,R}$ $(i=n+1\sim M)$ is not suitable because it yields the massless "heavy" fields at $M_{SS}=0$.*) The consistent SUSY breaking is then only possible for

$$M_{SS}=\mu_{\langle\rangle=0}\,, \tag{5·7}$$

where $\mu_{\langle\rangle=0}$ represents one of μ_i $(i=n+1\sim M)$, which induces

$$\pi_\lambda(\sim\langle\lambda\lambda\rangle)\sim\xi^{3/2}\mu_{\langle\rangle=0}\Lambda_{sc}^2\,, \tag{5·8a}$$

$$\pi_{\langle\rangle=0}(\sim\langle\phi^{(1)}\phi^{(2)}\rangle)\sim\xi\pi_{\langle\rangle\neq0}\,, \tag{5·8b}$$

$$\xi\sim(M_{SS}/\Lambda_{sc})^{(M-N)/\kappa} \tag{5·8c}$$

with $\pi_{\langle\rangle\neq0}\sim\Lambda_{sc}^2$, where $2\kappa=2\rho-(M-N)$ or $=2(1+\rho)-(M-N)$ as in Eqs. (4·16b) and (4·17b). From $m_{\rm heavy}\sim M_{SS}^{1-(M-N)/2\kappa}$, we finally find that the decoupling is realized if $2\kappa\leqq M-N$, i.e.,

$$\rho\leqq M-N\,, \tag{5·9a}$$

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*) If one abandons complementarity as a guiding principle, the contribution from the extra "heavy" fields will be cancelled by the one from their complex conjugates constructed by L-handed conjugate superfields[20] of $\bar{\boldsymbol{D}}\bar{\boldsymbol{D}}[\bar{\boldsymbol{W}}\bar{\boldsymbol{W}}\exp(g_{sc}V)\Phi^{(1,2)*}]=(G_{\mu\nu}G^{\mu\nu}\phi^{(1,2)*}+\cdots,\ \sigma_\rho\partial^\rho(G_{\mu\nu}\sigma^{\mu\nu}\lambda\phi^{(1,2)*})+\cdots)$, where $\boldsymbol{V}$ is the vector superfield giving $\bar{\boldsymbol{W}}\sim\boldsymbol{D}\boldsymbol{D}\bar{\boldsymbol{D}}\boldsymbol{V}$.

for SQCD with the chiral $SU(M-N)$ symmetry $(2\kappa=2\rho-(M-N))$ and

$$\rho\leqq M-N-1, \tag{5·9b}$$

for SQCD with the chiral $SU(M-N+1)$ symmetry $(2\kappa=2(\rho+1)-(M-N))$. Other massive fields such as T_i^j for i, j in $SU(M-n)_L\times SU(M-n)_R$ acquire masses typically of order $\pi_\lambda/\pi^2_{\langle\,\rangle=0}$, which gives $M_{SS}/\sqrt{\xi}\sim m_{\rm heavy}$ because $\pi_{\langle\,\rangle=0}\sim\xi\pi_{\langle\,\rangle\neq0}$. The "light" fields are then specified by the lighter mass scale of

$$\sqrt{\xi}M_{SS}\leqq M_{SS}^2/\Lambda_{sc}, \tag{5·10a}$$

while the "heavy" fields by the heavier scale of

$$M_{SS}/\sqrt{\xi}\geqq\Lambda_{sc}, \tag{5·10b}$$

since $\xi\sim(M_{SS}/\Lambda_{sc})^{(M-N)/\kappa}\leqq(M_{SS}/\Lambda_{sc})^2$ by $2\kappa\leqq M-N$.

The $\mu_{\langle\,\rangle=0}$-dominance does not mean the dominance in mass scales. Because the effect of the SUSY-breaking always comes in the form of $(\mu_{iL}^2+\mu_{iR}^2)\Lambda_{sc}^{-2}|\pi_i|^2$, $\mu_i{}^2\pi_i$ and $m_\lambda\pi_\lambda$, it is realized once $\mu_{iL,iR,i}$ $(i=1\sim n)$ are neglected. Owing to $\pi_i\sim\xi\Lambda_{sc}^2\gg|\pi^i|^2/\Lambda_{sc}^2$, and $\mu_i{}^2\pi_i\sim\xi\mu^2_{\langle\,\rangle=0}\Lambda_{sc}^2\gg m_\lambda\pi_\lambda\sim\xi^{3/2}m_\lambda\mu_{\langle\,\rangle=0}\Lambda_{sc}^2$, the $\mu_{\langle\,\rangle=0}$-dominance is possible even for $\mu_{\langle\,\rangle=0}\sim\mu_{iL,iR}\sim m_\lambda$.

B. *Masses for light composite fermions and bosons*

Let us now examine quark-lepton models characterized by $H_1=[SU(4)_C]_{L+R}\times[SU(2)_W]_L\times[SU(2)_W]_R\times U(1)_V'\times U(1)_A'$ with $N=4$ and $M=6$ $(n=N)$, $H_2=[SU(4)_W]_{L+R}\times[SU(4)_C]_L\times[SU(4)_C]_R\times U(1)_V'\times U(1)_A'$ with $N=4$ and $M=8$ $(n=N)$ and $H_3=[SU(3)_C]_{L+R}\times[SU(2)_W]_L\times[SU(2)_W]_R\times U(1)_w\times U(1)_c\times U(1)_x$ with $N=4$ and $M=5$ $(n=N-1)$. The SUSY-breaking is restricted to induce the breaking of all the remaining chiral symmetries given by the scalar masses of $\mu_i{}^2\phi_i{}^{(1)}\phi^{(2)i}$ $(i=n+1\sim M)$. We denote the breaking of $[SU(4)_C]_L\times[SU(4)_C]_R$ (or $[SU(3)_C]_L\times[SU(3)_C]_R$) by μ_{ca} $(a=1, 2, 3)$ and μ_l with $\mu_{ca}=\mu_c$ for $SU(3)_C{}^{\rm loc}$ and the breaking of $[SU(2)_W]_L\times[SU(2)_W]_R$ by μ_{wi}. Since for $H_{1,3}$ the SUSY-breaking necessarily induces the explicit breaking of $[SU(2)_W]_L\times[SU(2)_W]_R$, the weak interactions cannot be based on $[SU(2)_W]_L{}^{\rm loc}$

Table IV. Masses for quarks, leptons, leptoquarks, color-octets and weak-triplets and constraints on the models. The parameters $r_{c,l}$ are given as $r_{c,l}=\bar{\pi}_{c,l}/\bar{\pi}_x$.

SQCD with	$N=4, M=6$	$N=4, M=8$	$N=4, M=5$
VEV's of π_i	$\lvert\pi_{c,l}\rvert\gg\lvert\pi_{wi}\rvert$	$\lvert\pi_{c,l}\rvert\ll\lvert\pi_{wi}\rvert$	$\lvert\pi_c\rvert\gg\lvert\pi_{wi}\rvert$
Quarks	$\dfrac{r_c{}^2}{(1+r_c{}^2)}\dfrac{\lvert\pi_\lambda\rvert}{\lvert\bar{\pi}_c\bar{\pi}_{wi}\rvert}$		
Leptons	$\dfrac{r_l{}^2}{(1+r_l{}^2)}\dfrac{\lvert\pi_\lambda\rvert}{\lvert\bar{\pi}_l\bar{\pi}_{wi}\rvert}$		$\dfrac{h\sqrt{f_{Y(1)}f_{Y(2)}}\lvert\pi_\lambda\rvert}{\lvert(\bar{\pi}_c)^{N-1}\pi_{wi}\rvert}$
Leptoquarks	$\dfrac{\rho\langle z\rangle\lvert\pi_\lambda\rvert}{\xi\lvert\bar{\pi}_c\bar{\pi}_l\rvert}$	$\geq\Lambda_{sc}$	…
Color-octets	$\dfrac{\rho\langle z\rangle\lvert\pi_\lambda\rvert}{\xi\bar{\pi}_c{}^2}$	$\geq\Lambda_{sc}$	$\dfrac{\rho\langle z\rangle\lvert\pi_\lambda\rvert}{\xi\bar{\pi}_c{}^2}$
Weak-triplets	$\geq\Lambda_{sc}$	$\dfrac{\rho\langle z\rangle\lvert\pi_\lambda\rvert}{\xi\bar{\pi}_{wi}\bar{\pi}_{wj}}$	$\geq\Lambda_{sc}$
Constraints	$\langle z\rangle\gg1$	$r_{c,l}\gg1$	…

$\times U(1)_Y{}^{\mathrm{loc}}$ while for H_2 weak interactions can be introduced as the gauge interactions.

The spectrum of light composite fermions in each model is listed in Table III. Their masses shown in Table IV are calculated by $\partial^2 W_{\mathrm{eff}}/\partial\pi_I\partial\pi_J$ and expressed in terms of $\pi_c(\sim\langle\tilde{c}_{La}\tilde{c}_R{}^{Ca}\rangle)$ $(a=1, 2, 3)$, $\pi_l(\sim\langle\tilde{c}_{L0}\tilde{c}_R{}^{C0}\rangle)\rangle$ $\pi_{wi}(\sim\langle\tilde{w}_L{}^i\tilde{w}^C_{Ri}\rangle)$ and $\pi_\lambda(\sim\langle\lambda\lambda\rangle)$. The typical masses are given by $\pi_\lambda/\pi_{\langle\rangle\neq0}\pi_{\langle\rangle=0}\sim\mu_{\langle\rangle=0}(\mu_{\langle\rangle=0}/\Lambda_{sc})^{(M-N)/\kappa}$ from $\pi_\lambda\sim\xi^{3/2}\mu_{\langle\rangle=0}\Lambda_{sc}^2$ and $\pi_{\langle\rangle=0}\sim\xi\Lambda_{sc}^2$ with $\xi\sim(\mu_{\langle\rangle=0}/\Lambda_{sc})^{(M-N)/\kappa}$. Light composite fermions are characterized by the scale M_F:

$$M_F=M_{SS}(M_{SS}/\Lambda_{sc})^{(M-N)/2\kappa} \tag{5·11}$$

with $2\kappa\leqq M-N$ by the anomaly-matching, where $M_{SS}=\mu_{\langle\rangle=0}$. The parameter κ is: $\kappa=\rho-(M-N)/2$ for SQCD with the chiral $SU(M-N)$ symmetry and $\kappa=\rho+1-(M-N)$ for SQCD with the chiral $SU(M-N+1)$ symmetry. M_F is bounded as $M_F\leqq M_{SS}/\Lambda_{sc}^2$.

The masses of light composite scalars of $\partial^2 V_{\mathrm{eff}}/\partial\pi_I^*\partial\pi_J{}^{\pm}|\ \partial^2 V_{\mathrm{eff}}/\partial\pi_I\partial\pi_J|$ turn out to be consistent with diagrammatic argument: Attach $\mu^2_{iL(iR)}$ to the line for $|\phi_i{}^{(1)}|^2(|\phi^{(2)i}|^2)$ and $\mu_i{}^2$ to the line for $\phi_i{}^{(1)}\phi^{(2)i}$. We calculated their masses by assuming the canonical kinetic terms for composites with $G_I(0)=f_I=$const that simplified the conditions from $\partial V_{\mathrm{eff}}/\partial\pi_I=0$ into, for real VEV's,

$$f_T(\pi_\lambda/\pi_i)^2=f_S W_{;\lambda}+\mu_i{}^2\pi_i\,,\qquad (i=n+1\sim M) \tag{5·12a}$$

$$f_T(\rho\langle z\rangle\pi_\lambda/\xi\pi_{\langle\rangle\neq0})^2=(\rho-m)f_S W_{;\lambda}/(N-2)\,,\qquad (i=1\sim n) \tag{5·12b}$$

$$f_{Xa}(\rho\langle z\rangle\pi_\lambda/\xi\pi_{xa})^2=(n-\rho)f_S W_{;\lambda}/(N-2)\,,\qquad (a=1\sim m) \tag{5·12c}$$

Table V. Calculated VEV's of π_I. In the case $N<M, n=N$ and $\tau=\rho$ for SQCD with the chiral $SU(M-N)$ symmetry and $n=N-1$ and $\tau=\rho+1$ for SQCD with the chiral $SU(M-N+1)$ symmetry. The negative VEV's arise from $(\text{mass})^2>0$ for true NGB's and Eq. (4·12b) for $\pi_{\langle\rangle=0}<0$.

	$\mu=\mu_{i+n}(i=1\sim k)\gg\mu_{i+n}(i=k+1\sim M-n)$	
$\pi_{\langle\rangle\neq0}$	$\pi_i\approx-\Lambda_{sc}^2$	$(i=1\sim n)$
π_x	$\pi_{y1,y2}\approx-\Lambda_{sc}^N$, $\pi_u\approx-\Lambda_{sc}^{2(N-1)}$	
π_λ	$\pm A\xi^{3/2}\mu\Lambda_{sc}^2$	
$\pi_{\langle\rangle=0}$	$\pi_{i+n}=-\gamma\xi\lvert\pi_{\langle\rangle\neq0}\rvert$	$(i=1\sim k)$
	$\pi_{i+n}=\left(\dfrac{k-\tau}{k}\right)^{1/2}(-\gamma\xi\lvert\pi_{\langle\rangle\neq0}\rvert)$	$(i=k+1\sim M-n)$
ξ	$\xi^{[2\tau-(M-N)]}=C\left(\dfrac{\mu}{\Lambda_{sc}}\right)^{2(M-N)}$	
ρ	$\mathrm{Max}(2, \dfrac{M-N}{2})<\tau<\mathrm{Min}(N, k)^{*)}$ $\tau\leqq M-N$	
A	$\left(\dfrac{k-\tau}{\tau}\right)^{1/2}\left(\dfrac{\gamma\lvert\pi_{\langle\rangle\neq0}\rvert}{f_T}\right)^{3/2}$	
C	$\gamma^{2(N-n)}\left(\dfrac{\gamma(k-\tau)\lvert\pi_{\langle\rangle\neq0}\rvert}{\tau f_T}\right)^{M-n}\left(\dfrac{k}{k-\tau}\right)^{M-n-k}\left(\dfrac{\Lambda_{sc}^2}{\pi_{\langle\rangle\neq0}}\right)^{2N}$	
γ	$(\rho\langle z\rangle)^{-1}\left(\dfrac{k(\tau-2)}{(N-2)(k-\tau)}\right)^{1/2}$	

*) For $N=4$, $M-n\geqq k>2$ leads to $M>6$ $(n=N)$ and $M>5$ $(n=N-1)$.

$$(\rho-n+N)f_S W_{;\lambda}=-\sum_{i=n+1}^{M}\mu_i^2\pi_i \tag{5·12d}$$

with $n=N$ and $m=2$ for the chiral $SU(M-N)$ symmetry and $n=N-1$ and $m=1$ for the chiral $SU(M-N+1)$ symmetry. Our results are listed in Table V for $\mu_{i+n}^2=\mu^2$ ($i=1-k$) $\gg\mu_{i+n}^2$ ($i=k+1\sim M-n$).

We show masses for octets (m_8^2), squarks (m_{sq}^2) and sleptons (m_{sl}^2), which are the superpartners of QNGF's:

$$m_8^2=f_T\mu_c^2/|\pi_c|\,,\quad 2f_T C_S|M^2_{\langle\,\rangle=0}|/\pi_c^2\,, \tag{5·13a, b}$$

$$m_{sq,sl}^2=r^2 f_T\{C_{TX}\Lambda_{sc}^{-2}(\bar{\mu}_{c,l}^2+\bar{\mu}_{wi}^2)/2$$
$$\pm\pi_{\langle\,\rangle=0}^{-2}|\mu^2_{\langle\,\rangle=0}\pi_{\langle\,\rangle=0}-C_s M^2_{\langle\,\rangle=0}|\}/(1+r^2)\,, \tag{5·13c}$$

where $r=\bar{\pi}_i/\bar{\pi}_x$ ($i=1\sim n$); $C_s=(\rho-m)/(\rho+N-n)(N-2)$; $C_{TX}=1+(f_X/f_T r^2\Lambda_{sc}^{2D})$; $M^2_{\langle\,\rangle=0}=\sum_{i=n+1}^{M}\mu_i^2\pi_i$; $D=(N-1)$ for $X=Y^{(1,2)}$ and $D=2(N-2)$ for $X=U$; $\pi_{\langle\,\rangle=0}$ ($\mu_{\langle\,\rangle=0}$) denotes either $\pi_{c,l}(\mu_{c,l})$ or $\pi_{wi}(\mu_{wi})$ with VEV$=0$ in the SUSY limit. The positivity of the true NGB's for octets gives $\pi_c(=\pi_{i=1\sim n})<0$. For sleptons in H_3 being as the superpartners of CF's, their masses are calculated to be:

$$m_{sl}^2=f_Y[\bar{\mu}_{wi}^2/2\pm(\Lambda_{sc}^{2(N-1)}/\pi_c^{N-1})\mu_{wi}^2]/\Lambda_{sc}^{2(N-1)}\,. \tag{5·14}$$

We have taken the L-R symmetric situation with $\bar{\mu}_i^2=\mu_{iL}^2+\mu_{iR}^2=2\mu_{iL}^2=2\mu_{iR}^2$ and with $f_{Y(1)}=f_{Y(2)}=f_Y$ and used $\mu_{iL,iR,i}$ ($i=n+1\sim M$) $\gg\mu_{iL,iR,i}$ ($i=1\sim n$). The mass hierarchy between fermions and their partners is realized as

$$m_{J=1/2}\sim\mu_{\langle\,\rangle=0}(\mu_{\langle\,\rangle=0}/\Lambda_{sc})^{(M-N)/K}\ll m_{J=0}\sim\mu_{\langle\,\rangle=0}\,, \tag{5·15}$$

for $N<M$.

C. *Dynamical constraints on quark-lepton models*

Let us consider the possibility of splitting the quark and lepton masses and the potential difficulties in the quark-lepton phenomenology. To make arguments qualitative, we employ the canonical kinetic terms and the conditions given by Eqs. (5·12a ~ d).

First, we discuss the quark-lepton mass splitting. In the model with quarks being QNGF's and leptons being CF's for $N=4$ and $M=5$, the splitting is expected owing to the dynamical difference between QNGF's and CF's, which will reflect in the form of kinetic terms. The simplest canonical ones lead to

$$m_l/m_q=h\sqrt{f_{Y(1)}f_{Y(2)}}/f_T\langle\tilde{c}\,\tilde{c}^C\rangle^{N-2}\,, \tag{5·16}$$

although $m_l/m_q\ll1$ is not predictive.

For models with quarks and leptons both described by QNGF's, the quark-lepton masses are given by $m_{q,l}\propto\pi_{c,l}^{-1}$ and the ratio is

$$m_l/m_q=\pi_c/\pi_l\,. \tag{5·17}$$

The large splitting of $m_l/m_q\ll1$ at least requires $[SU(4)_c]_L\times[SU(4)_c]_R$ as an approximate symmetry, which allows the "flavor"-dependent explicit breaking and will generate $\pi_l\gg\pi_c$. It is because if $[SU(4)_c]_{L+R}$ is a symmetry for the SUSY-vacuum, then the explicit breaking of $[SU(4)_C]_{L+R}$ is dominated by the "flavor"-independent terms because $\mu_{iL,iR,i}$ $(i\neq SU(4)_C)\gg\mu_{iL,iR,i}$ $(i=SU(4)_C)$ are dynamically required, and will induce $\pi_l=\pi_c$. However, even in the presence of an approximate $[SU(4)_C]_L\times[SU(4)_C]_R$, $m_l/m_c\ll 1$ is not easily attained since $\pi_c\sim\pi_l$ is still driven by the "flavor"-independent $f_S W_{;\lambda}=-(n_c\mu_c{}^2\pi_c+n_l\mu_l{}^2\pi_l)/(\rho-n+N)$ as in Eq. (5·12a), where $n_{c(l)}$ denotes the number of colors (lepton numbers). This aspect of the "flavor"-independent VEV's is the remnant of the QCD-dynamics of the "flavor"-independent spontaneous breakdown of chiral symmetries. Therefore, unless some kind of fine tuning is invoked, the large mass splitting between quarks and leptons both as QNGF's is dynamically suppressed in SQCD.

We next examine the potential difficulties present in two of the three types of models as have been discussed in § 2: The existence of light leptoquarks in SQCD with $N=4$ and $M=6$ (H_1) or the appearance of the mixing of quarks and leptons with their mirrors in SQCD with $N=4$ and $M=8$ (H_2).[18]

In SQCD with $[SU(2)_W]_L\times[SU(2)_W]_R$ for $N=4$ and $M=6$, leptoquarks appear as pure states in $\hat{c}\,\hat{c}^C$ and quarks and leptons as mixed states as in Eq. (5·2a). Their masses are given by $m_{ql}=\rho\langle z\rangle|\pi_\lambda|/|\xi|\,\bar{\pi}_c\bar{\pi}_l|$ and $m_{qi,l}=r^2|\pi_\lambda|/(1+r^2)|\,\bar{\pi}_{c,l}\bar{\pi}_{wi}|$, where $\pi_{wi}\sim\xi\pi_{c,l}$. The large splitting of $m_{ql}\gg m_q$ is possible if $\langle z\rangle\gg 1$, namely, $\prod_{i=1}^{N}\pi_i/\pi_{y1}\pi_{y2}$ $(=\pi_c{}^3\pi_l/\pi_{y1}\pi_{y2})\gg 1$. On the other hand, SQCD with $[SU(4)_C]_L\times[SU(4)_C]_R$ for $N=4$ and $M=8$ generates the mixing of quarks and leptons with their mirrors as in Eq. (5·2 a): $|\text{light}\rangle=(rT+X)/\sqrt{1+r^2}$ with $r=\bar{\pi}_i/\bar{\pi}_{ya}$ for $i=1\sim N$ and $a=1, 2$. The mixing is required to satisfy $r\gg 1$, namely, $\bar{\pi}_i/\bar{\pi}_{ya}(=\pi_{wi}/\pi_{ya})\gg 1$. To be phenomenologically consistent, both cases of SQCD are subject to the constraint of $\bar{\pi}_i\gg\bar{\pi}_{ya}$ $(i=1\sim N;\ a=1, 2)$. From Eq. (5·12 b, c) with $m=2$ and $n=N(=4)$, we obtain $\bar{\pi}_{\langle\,\rangle\neq 0}/\bar{\pi}_{ya}=\sqrt{N-\rho/\rho-2}$ $(2<\rho<N)$, where $\pi_{\langle\,\rangle\neq 0}$ represents π_i $(i=1\sim N)$. The large ratio is attained by the fine tuning $\rho\sim 2$, which shows from Eqs. (5·12b, c) that $\pi_{\langle\,\rangle\neq 0}$ gets larger as $1/\sqrt{\rho-2}$, i.e.,

$$\bar{\pi}_{\langle\,\rangle\neq 0}\gg\bar{\pi}_{ya}\sim\Lambda_{SC}\,. \tag{5·18}$$

The VEV of $\pi_{\langle\,\rangle\neq 0}$ describes $[SU(4)_C]_L\times[SU(4)_C]_R\to[SU(4)_C]_{L+R}$ for $N=4$ and $M=6$ and $[SU(4)_W]_L\times[SU(4)_W]_R\to[SU(4)_W]_{L+R}$ for $N=4$ and $M=8$. For $N=4$ and $M=8$, it seems unnatural to take $\pi_{\langle\,\rangle\neq 0}\gg\Lambda_{SC}\gtrsim 1$ TeV since $\pi_{\langle\,\rangle\neq 0}\sim\langle\hat{w}\hat{w}^C\rangle|_{\theta=0}$ in this case will provide the electroweak scale of $G_F{}^{-1/2}\simeq 300$ GeV. For $N=4$ and $M=6$, there is no restriction on $\pi_{\langle\,\rangle\neq 0}\sim\langle\hat{c}\,\hat{c}^C\rangle|_{\theta=0}$ although VEV's much larger than the scale of the theory is considered as a singnal of SQCD realized in the Higgs phase. This restriction is also valid for general kinetic terms with $G_I{}'\neq 0$ since $\pi^{-1}_{\langle\,\rangle\neq 0}$ or π_{ya}^{-1} can be made extremely small by adjusting the parameter ρ.

The remaining case with $[SU(2)_W]_L\times[SU(2)_W]_R$ for $N=4$ and $M=5$ contains neither light leptoquarks nor the mixing with mirror quarks and leptons.[19] This is the model without "tuning", in which the light fermions consist of color-octet and color-singlet as QNGF's in $\hat{c}\,\hat{c}^C$, quarks as QNGF's in $\hat{c}\hat{w}+[(\hat{c})^3(\hat{c}^C)^2\hat{w}]$ and leptons as CF's in $[(\hat{c}^C)^3\hat{w}]$.

§ 6. Summary and discussion

We have demonstrated that SQCD with N-"colors"$<M$-"flavors" generates massless quasi Nambu-Goldstone fermions and chiral fermions including quarks and leptons and their scalar partners including squarks and sleptons. Massless composite fermions are protected by

$$SU(N)_{L+R}\times SU(M-N)_L\times SU(M-N)_R\times U(1)_{A'}\times U(1)_{V'}\,, \qquad (n=N)$$

$$SU(N-1)_{L+R}\times SU(M-N+1)_L \times SU(M-N+1)_R\times U(1)_w\times U(1)_c\times U(1)_\chi\,, \qquad (n=N-1)$$

supported by complementarity and an effective superpotential.

The successive SUSY-breaking is provided by the gaugino mass term, $m_\lambda\lambda\lambda$ that breaks $U(1)_\chi$ and $U(1)_{A'}$, and the scalar-subquark mass terms, $\sum_{i=n+1}^{M}(\mu_{iL}|\phi_i^{(1)}|^2+\mu_{iR}|\phi^{(1)i}|^2)$ that preserve some of the chiral symmetries and $\sum_{i=n+1}^{M}\mu_i^2\phi_i^{(1)}\phi^{(2)i}$ that at least breaks the chiral $SU(M-n)$ symmetry. The breakdown of the remaining chiral symmetries by the SUSY-breaking reflects the QCD dynamics featured by the complete breakdown of all chiral symmetries. Even if the SUSY-breaking itself preserves some of chiral symmetries, the breakdown of all chiral symmetries is a must. In fact, the SUSY-breaking by $\mu_{iL,iR}=\mu$ $(i=n+1\sim M)$ that does not break any of them is shown to induce the breakdown of the chiral $U(1)$ symmetries signalled by

$$\langle\lambda\lambda\rangle\sim\xi^2\mu\Lambda_{sc}^2$$

and chiral $SU(M-n)$ symmetry by

$$\langle\phi_i^{(1)}\phi^{(2)i}\rangle\sim\xi\Lambda_{sc}^2 \qquad (i=n+1\sim M)$$

with $\xi\sim(\mu/\Lambda_{sc})^{(M-N)/\kappa}$ (see Eq. (4·18) for κ). However, the SUSY-breaking by $\mu_{iL,iR,i}$ $(i=1\sim n)$ alone, which are irrelevant for the remaining chiral symmetries, is dynamically forbidden. The appearance of the suppression factor ξ is linked to the singular behavior present in the effective superpotential as $\ln\xi$.

The large mass splitting between $J=1/2$ fermions and $J=0$ bosons is shown to arise as a result of the approximate chiral symmetries:

$$m_{J=1/2}\sim\xi^a M_{SS}\ll m_{J=0}\sim M_{SS}\,,$$

where $a=1$ or $1/2$. In case that the SUSY-breaking preserves some of the chiral symmetries, the masses for composite fermions are characterized by the two scales ξM_{SS} for "lighter" fermions $(a=1)$ and M_{SS} for "heavier" fermions. In the $M_{SS}\to 0$ limit, "heavier" fermions become massless and spoil the anomaly-matching realized in a manner consistent with complementarity. The consistent breaking is given by the SUSY-breaking from $\sum_{i=n+1}^{M}\mu_i^2\phi_i^{(1)}\phi^{(2)i}$ that always breaks the chiral $SU(M-n)$ symmetry and that yields

$$\langle\lambda\lambda\rangle\sim\xi^{3/2}M_{SS}\Lambda_{sc}^2\,, \qquad \langle\phi_i^{(1)}\phi^{(2)i}\rangle\sim\xi\Lambda_{sc}^2 \quad (i=n+1\sim M)$$

with $\xi \sim (M_{SS}/\Lambda_{SC})^{(M-N)/\kappa}$ (see Eq. (5·8) for κ). Since in this case the mass spectrum consists of $\sqrt{\xi} M_{SS}$ for "lighter" fermions ($\alpha=1/2$) and $M_{SS}/\sqrt{\xi}$ for "heavier" fermions, the decoupling of "heavier" fermions is realized if $2\kappa \leqq M-N$. The resulting masses for light composite fermions are controlled by M_F:

$$M_F = M_{SS}(M_{SS}/\Lambda_{SC})^{(M-N)/2\kappa}\,.$$

SQCD offers quark-lepton models based on $H_1=[SU(4)_C]_{L+R}\times[SU(2)_W]_L \times[SU(2)_W]_R \times U(1)_V' \times U(1)_A'$ ($N=4$ and $M=6$), $H_2=[SU(4)_W]_{L+R}\times[SU(4)_C]_L \times[SU(4)_C]_R \times U(1)_V' \times U(1)_A'$ ($N=4$ and $M=8$) and $H_3=[SU(3)_C]_{L+R}\times[SU(2)_W]_L \times[SU(2)_w]_R \times U(1)_w \times U(1)_c \times U(1)_\chi$ ($N=4$ and $M=5$). The models with $H_{1,2}$ are sufferring from potential difficulties due to the existence of massless leptoquarks as QNGF's present in H_1 and the mixing of quarks and leptons with their mirrors as QNGF's in H_2. The relatively heavy leptoquarks and the suppression of the mixing with mirrors are realized only if $\langle \hat{w}_L \hat{w}_R{}^C \rangle|_{\theta=0} \gg \Lambda_{SC}^2$ for H_1 and $\langle \hat{c}_L \hat{c}_R{}^C \rangle|_{\theta=0} \gg \Lambda_{SC}^2$ for H_2 although the VEV's much larger that the scale of the theory Λ_{SC} is a signal of SQCD falling in the Higgs phase. The model with H_3 is free from such difficulties and contains quarks as QNGF's and leptons as CF's. It is possible to accommodate the mass hierarchy between quarks and leptons by the difference between QNGF's and CF's.[19]

Generations of quarks and leptons can always be included by the copies of $\hat{c}_{a=1,2,3}$, $\hat{c}_0$ and/or $\hat{w}^{i=1,2}$. The number is constrained by $M<3N$ for the asymptotic freedom. For the model with H_3, N_g-generations are given by the N_g-copies of $\hat{w}^{i=1,2}$ for $N_g \leqq N=4$ and by $H_3=[SU(3)_C]_{L+R}\times[SU\ (2N)_g)_W]_L\times[SU\ (2N_g)_W]_R\times U(1)_W\times U(1)_C\times U(1)_\chi$. Once generations are included, one has to explain the suppression on flavor-changing processes such as $K\to\pi\mu e$, μe ; $\mu\to e\gamma$; K_0-$\bar{K}_0$.[21] In the present model, the suppression on $\mu\to e\gamma$ and K_0-$\bar{K}_0$ will be ensured by the (approximate) symmetries of $[SU\ (2N_g)_W]_{L,R}$ and $U(1)_{w,c}$, which should also be responsible for the interfamily mass hierarchies.

However, it is hard to suppress $K\to\pi\mu e$, μe by a possible symmetry-argument. The simple suppression mechanism[22] is based on the difference of subquarks, "chroms", responsible for $SU(3)_C$ and $U(1)_{B\text{-}L}$ such as "e"$\sim \hat{c}_a\hat{w}$ and "μ"$\sim \hat{s}_a\hat{w}$ or $\sim[(\hat{c}^C)^3]_a\hat{w}$. The SQCD-based model with two kinds of "chroms" is given by $N=7$ with $M=6+2N_g$ giving $H_3=[SU(6)]_C]_{L+R}\times[SU(2N_g)_W]_L\times[SU(2N_g)_W]_R\times U(1)_w\times U(1)_C\times U(1)_\chi$ ($N_g\leqq N=7$). For $N_g=3$, $\hat{c}_a$ and $\hat{s}_a$ ($a=1,2,3$) being two $SU(3)_C{}^{\text{loc}}$-triplets and $\hat{w}_i$, $\hat{u}_i$ and $\hat{v}_i$ ($i=1,2$) being three $[SU(2)_W]_{L+R}$-doublets yield 6-families of quarks$=(\hat{c}_a, \hat{s}_a)\otimes(\hat{w}_i, \hat{u}_i, \hat{v}_i)$ and 3-families of leptons$=(\hat{c}\hat{c}\hat{c}\hat{s}\hat{s}\hat{s})\times(\hat{w}_i, \hat{u}_i, \hat{v}_i)$. The excess of three families of quarks will not be comfortable. It seems that a key to the "strong" suppression of interfamily transitions is not found until the proper understanding of the "huge" splitting of interfamily masses is achieved.

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