Composition of Password-based Protocols

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Password-based protocols and Guessing attacks

Handshake protocol

A		B
new r	$\xrightarrow{\text{senc}_w(r)}$
	$\xleftarrow{\text{senc}_w(f(r))}$

Encrypted key exchange

A		B
new k	$\xrightarrow{\text{senc}_w(pk(k))}$
		new r
	$\xleftarrow{\text{senc}_w(aenc_{pk(k)}(r))}$

Guessing attack on w:

- Guess w
- Let $x = \text{sdec}_w(\text{senc}_w(r))$
- Let $y = \text{sdec}_w(\text{senc}_w(f(r)))$
- Confirm guess of w by checking $y = f(x)$

No guessing attack on w (assuming it is possible to encode $pk(k)$ so it looks indistinguishable from a random bitstring).

Password-based protocols and Guessing attacks

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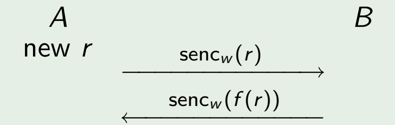
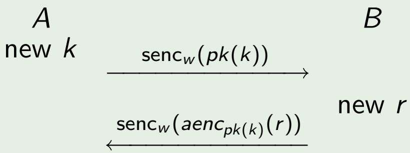


Encrypted key exchange



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new r

$A \xrightarrow{\mathsf{senc}_w(r)} B$

$A \xleftarrow{\mathsf{senc}_w(f(r))} B$

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Encrypted key exchange

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new r (B)

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No guessing attack on w (assuming it is possible to encode $pk(k)$ so it looks indistinguishable from a random bitstring).

Composing protocols

"EKE++"

A — new k | B

$A \xrightarrow{\text{senc}_w(pk(k))} B$

B: new r

$B \xrightarrow{\text{senc}_w(aenc_{pk(k)}(r))} A$

$A \xrightarrow{\text{senc}_r(w)} B$

"EKE+++"

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$A \xrightarrow{\text{senc}_w(pk(k))} B$

B: new r

$B \xrightarrow{\text{senc}_w(aenc_{pk(k)}(r))} A$

$\xrightarrow{x} B$

$B \xrightarrow{\text{sdec}_r(x)}$

- Each of them resists guessing attack separately
- Attack (even without guessing!) if they are run together: let $x = \text{senc}_r(w)$

Composing protocols

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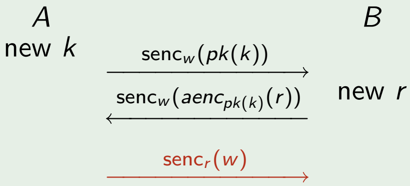

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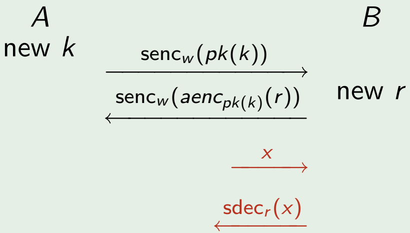

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Outline

- Define guessing attacks in the formal model
  - active and passive attacks
- Study composition of protocols that share the password
  - if the individual protocols resist guessing attacks, does the composed protocol also resist?

Terms and equational theories

Describe processes in a simple language inspired by applied pi calculus. Messages are modeled using terms.

- Abstract algebra given by a signature, *i.e.* a set of function symbols with arities
- Equivalence relation ($=_E$) on terms induced by an equational theory

Example (equational theory)

Consider the signature
$\Sigma_{\mathsf{enc}} = \{\mathsf{sdec}, \mathsf{senc}, \mathsf{adec}, \mathsf{aenc}, \mathsf{pk}, \langle\ \rangle, \mathsf{proj}_1, \mathsf{proj}_2\}$

$$\begin{array}{rcl@{\qquad}rcl} \mathsf{sdec}_y(\mathsf{senc}_y(x)) & = & x & \mathsf{adec}_y(\mathsf{aenc}_{\mathsf{pk}(y)}(x) & = & x \\ \mathsf{senc}_y(\mathsf{sdec}_y(x)) & = & x & \mathsf{proj}_i(\langle x_1, x_2\rangle) & = & x_i \quad i = 1, 2 \end{array}$$

Frames and deduction

As a process evolves, it may output terms which are available to the attacker. The output of a process is called a frame: a set of secrets + a substitution:

$$\nu\tilde{n}.(\{^{M_1}/_{x_1}\} \mid \{^{M_2}/_{x_2}\} \mid \ldots \mid \{^{M_n}/_{x_n}\})$$

Example: $\phi = \nu k, s_1.\{^{\mathsf{senc}_k(\langle s_1, s_2\rangle)}/_{x_1}, {}^{k}/_{x_2}\}$

Definition (Deduction)

$\nu\tilde{n}.\sigma \vdash_{\mathsf{E}} M$ iff there exists N such that $fn(N) \cap \tilde{n} = \emptyset$ and $N\sigma =_{\mathsf{E}} M$. We call N a *recipe* of the term M.

	Recipe
$\phi \vdash_{\mathsf{E_{enc}}} k$	x_2
$\phi \vdash_{\mathsf{E_{enc}}} s_1$	$\mathsf{proj}_1(\mathsf{sdec}_{x_2}(x_1))$
$\phi \vdash_{\mathsf{E_{enc}}} s_2$	s_2

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Static equivalence

Definition (Static equivalence)

Two frames are statically equivalent if there is no "test" that tells them apart.

ϕ and ψ are statically equivalent, $\phi \approx_{\mathsf{E}} \psi$, when:

- $dom(\phi_1) = dom(\phi_2)$, and
- for all terms M, N such that $\tilde{n} \cap (fn(M) \cup fn(N)) = \emptyset$, $M\phi =_{\mathsf{E}} N\phi$ iff $M\psi =_{\mathsf{E}} N\psi$

Example

$$\phi = \nu k.\{^{\mathsf{senc}_k(s_0)}/_{x_1}, {}^{k}/_{x_2}\} \not\approx \nu k.\{^{\mathsf{senc}_k(s_1)}/_{x_1}, {}^{k}/_{x_2}\} = \phi'$$

because of the test $(\mathsf{sdec}_{x_2}(x_1), s_0)$

However,

$$\nu k.\{^{\mathsf{senc}_k(s_0)}/_{x_1}\} \approx \nu k.\{^{\mathsf{senc}_k(s_1)}/_{x_1}\}$$

Guessing attacks (passive case)

A passive guessing or dictionary attack consists of two phases

1. the attacker eavesdrops on one or several sessions of a protocol
2. the attacker tries offline each of the possible passwords (e.g. using a dictionary) on the data collected during the first phase

We suppose the eavesdropping phase results in a frame $\nu w.\phi$.

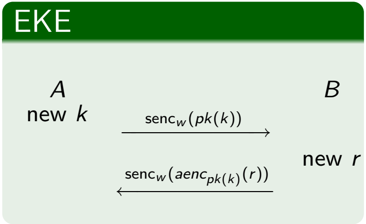
Definition (Passive guessing attacks)

$\nu w.\phi$ is resistant to guessing attacks against w iff

$$\nu w.(\phi \mid \{^{w}/_{x}\}) \approx \nu w.(\phi \mid \nu w'.\{^{w'}/_{x}\})$$

[Baudet05, Corin et al.03]

EKE resists guessing attacks?



EKE resists guessing attacks only if $pk(k)$ can be encoded indistinguishably from an arb. bitstring.

Consider the equational theory:

$$
\begin{aligned}
\mathrm{sdec}_y(\mathrm{senc}_y(x)) &= x \\
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\mathrm{adec}_y(\mathrm{aenc}_{\mathrm{pk}(y)}(x)) &= x \\
\mathrm{proj}_i(\langle x_1, x_2 \rangle) &= x_i \ (i = 1, 2)
\end{aligned}
$$

We have

$$\nu w, k.(\{^{\mathrm{senc}_w(pk(k))}/_{x_1}\}, \{^{w}/_{x_2}\}) \approx \nu w, w', k.(\{^{\mathrm{senc}_w(pk(k))}/_{x_1}\}, \{^{w'}/_{x_2}\})$$

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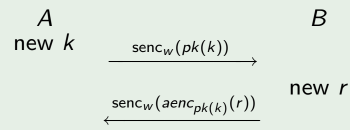
\mathrm{ispk}(\mathrm{pk}(x)) & = & \mathrm{true}
\end{array}
$$

EKE



We have

$$\nu w, k.(\{^{\mathrm{senc}_w(pk(k))}/_{x_1}\}, \{^{w}/_{x_2}\}) \not\approx \nu w, w', k.(\{^{\mathrm{senc}_w(pk(k))}/_{x_1}\}, \{^{w'}/_{x_2}\})$$

as witnessed by the test: $\mathrm{ispk}(\mathrm{sdec}_{x_2}(x_1)) = \mathrm{true}$.

Composing protocols that are resistant to passive guessing attacks

Proposition

The three following statements are equivalent:

1. $\nu w.\phi \mid \{^{w}/_{x}\} \approx \nu w.\phi \mid \nu w'.\{^{w'}/_{x}\}$ [Baudet05]
2. $\phi \approx \nu w.\phi$ [Corin et al.03]
3. $\phi \approx \phi\{^{w'}/_{w}\}$

Corollary

If $\nu w.\phi_1$ and $\nu w.\phi_2$ are resistant to guessing attacks against w then $\nu w.(\phi_1 \mid \phi_2)$ is also resistant to guessing attacks against w.

Thus, resistance to guessing attacks composes in the passive case. In particular, resistance for one session implies resitance for multiple sessions.

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Active case

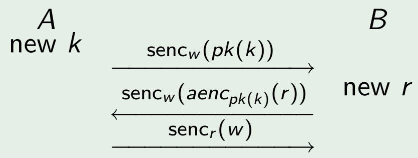
Syntax of the process language

$P, Q, R :=$ Plain processes

0	null process
$P \mid Q$	parallel composition
$\mathsf{in}(x).P$	message input
$\mathsf{out}(M).P$	message output
$\mathsf{if}\ M = N\ \mathsf{then}\ P\ \mathsf{else}\ Q$	conditional

Extended processes $A, B, C := P \mid A \mid B \mid \nu n.A \mid \{^M/_x\}$

Example: "EKE++"



$$
\begin{aligned}
&\nu w.(\\
&\quad \nu k.(\mathsf{out}(\mathsf{senc}_w(pk(k))).\mathsf{in}(x). \\
&\quad \mathsf{out}(\mathsf{senc}_{\mathsf{adec}_k(\mathsf{sdec}_w(x))}(w)) \\
&\mid \\
&\quad \mathsf{in}(y).\nu r.\mathsf{out}(\mathsf{senc}_w(\mathsf{aenc}_y(r))). \\
&\quad \mathsf{in}(z).\ldots \\
&)
\end{aligned}
$$

Semantics of the process language

Structural equivalence: the smallest equivalence relation closed by application of evaluation contexts and such that

PAR-0	$A \mid 0$	$\equiv$	A	NEW-PAR	$A \mid \nu n.B$	$\equiv$	$\nu n.(A \mid B)$
PAR-C	$A \mid B$	$\equiv$	$B \mid A$				$n \notin fn(A)$
PAR-A	$(A \mid B) \mid C$	$\equiv$	$A \mid (B \mid C)$	NEW-C	$\nu n_1.\nu n_2.A$	$\equiv$	$\nu n_2.\nu n_1.A$

Operational semantics: smallest relation between extended processes which is closed under structural equivalence ($\equiv$) and such that

IN	$\text{in}(x).P \xrightarrow{in(M)} P\{^M/_x\}$	
OUT	$\text{out}(M).P \xrightarrow{out(M)} P \mid \{^M/_x\}$	where x is a fresh variable
THEN	if $M = N$ then P else $Q \xrightarrow{\tau} P$	where $M =_E N$
ELSE	if $M = N$ then P else $Q \xrightarrow{\tau} Q$	where $M \neq_E N$
CONT.	$\dfrac{A \xrightarrow{\ell} B}{C[A] \xrightarrow{\ell} C[B]}$	where C is an evaluation context if $\ell = in(M)$ then $\phi(C[A]) \vdash_E M$

Semantics of the process language

Structural equivalence: the smallest equivalence relation closed by application of evaluation contexts and such that

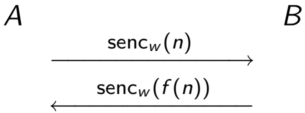
PAR-0	$A \mid 0 \equiv A$		NEW-PAR	$A \mid \nu n.B \equiv \nu n.(A \mid B)$		
PAR-C	$A \mid B \equiv B \mid A$			$n \notin fn(A)$		
PAR-A	$(A \mid B) \mid C \equiv A \mid (B \mid C)$		NEW-C	$\nu n_1.\nu n_2.A \equiv \nu n_2.\nu n_1.A$		

Operational semantics: smallest relation between extended processes which is closed under structural equivalence ($\equiv$) and such that

IN	$\mathrm{in}(x).P \xrightarrow{in(M)} P\{^M/_x\}$	
OUT	$\mathrm{out}(M).P \xrightarrow{out(M)} P \mid \{^M/_x\}$	where x is a fresh variable
THEN	if $M = N$ then P else $Q \xrightarrow{\tau} P$	where $M =_{\mathsf{E}} N$
ELSE	if $M = N$ then P else $Q \xrightarrow{\tau} Q$	where $M \neq_{\mathsf{E}} N$
CONT.	$$\frac{A \xrightarrow{\ell} B}{C[A] \xrightarrow{\ell} C[B]}$$	where C is an evaluation context if $\ell = in(M)$ then $\phi(C[A]) \vdash_{\mathsf{E}} M$

Example

Consider the handshake protocol. In our calculus it is modelled as:



- $A = \nu n.\mathsf{out}(\mathsf{senc}_w(n)).\ \mathsf{in}(x).\ \mathsf{if}\ \mathsf{sdec}_w(x) = f(n)\ \mathsf{then}\ P$
- $B = \mathsf{in}(y).\ \mathsf{out}(\mathsf{senc}_w(f(\mathsf{sdec}_w(y))))$

which admits the execution

$$
\begin{array}{ll}
\nu w.(A \mid B) & \\
\xrightarrow{out(\mathsf{senc}_w(n))} & \nu w.\nu n.(B \mid \{^{\mathsf{senc}_w(n)}/_{x_1}\} \mid \mathsf{in}(x).\ \mathsf{if}\ \mathsf{sdec}_w(x) = f(n)\ \mathsf{then}\ P) \\
\xrightarrow{in(\mathsf{senc}_w(n))} & \nu w.\nu n.(\mathsf{out}(M) \mid \{^{\mathsf{senc}_w(n)}/_{x_1}\} \mid \mathsf{in}(x).\ \mathsf{if}\ \mathsf{sdec}_w(x) = f(n)\ \mathsf{then}\ P) \\
\xrightarrow{out(M)} & \nu w.\nu n.(\{^{\mathsf{senc}_w(n)}/_{x_1}\} \mid \{^{M}/_{x_2}\} \mid \mathsf{in}(x).\ \mathsf{if}\ \mathsf{sdec}_w(x) = f(n)\ \mathsf{then}\ P) \\
\xrightarrow{in(\mathsf{senc}_w(f(n)))} & \nu w.\nu n.(\{^{\mathsf{senc}_w(n)}/_{x_1}\} \mid \{^{M}/_{x_2}\} \mid\ \mathsf{if}\ \mathsf{sdec}_w(\mathsf{senc}_w(f(n))) = f(n) \\
 & \qquad \mathsf{then}\ P) \\
\xrightarrow{\tau} & \nu w.\nu n.(\{^{\mathsf{senc}_w(n)}/_{x_1}\} \mid \{^{M}/_{x_2}\} \mid P)
\end{array}
$$

where $M = \mathsf{senc}_w(f(\mathsf{sdec}_w(\mathsf{senc}_w(n)))) =_{\mathsf{E}} \mathsf{senc}_w(f(n))$

Guessing attacks (active case)

Definition (Active guessing attacks)

A is resistant to guessing attack against w if, for every process B such that $A \rightarrow^* B$, we have that $\phi(B)$ is resistant to guessing attacks against w.

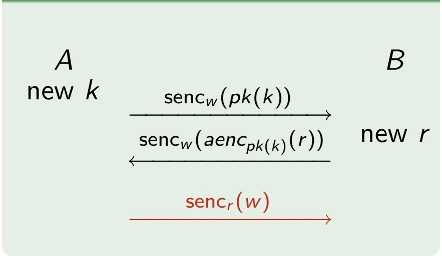
Frame of a process

$\phi(A)$ = result of replacing plain processes in A by 0.

Composing protocols that are resistant to active guessing attacks

Contrary to passive case, resistance does not compose in general.

“EKE++”



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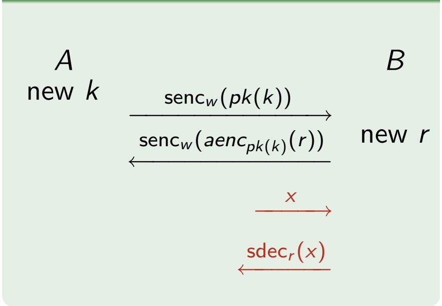

After the execution in which $x = \mathrm{senc}_r(w)$:

$$\phi = \nu w, k, r.(\quad \{^{\mathrm{senc}_w(pk(k))}/_{x_1}\}, \{^{\mathrm{senc}_w(aenc_{pk(k)}(r))}/_{x_2}\}, \{^{\mathrm{senc}_r(w)}/_{x_3}\}, \{^{w}/_{x_4}\})$$

Well-taged protocols and composition

Intuitively, a protocol is well-tagged w.r.t. a secret w if all the occurrences of w are of the form $\mathsf{h}(\alpha, w)$

Definition (well-tagged)

M is α-tagged w.r.t. w if there exists M' s.t. $M'\{^{\mathsf{h}(\alpha,w)}/_{w}\} =_{\mathsf{E}} M$.
A term is said well-tagged w.r.t. w if it is α-tagged for some name α.
A is α-tagged if any term occurring in it is α-tagged. An extended process is well-tagged if it is α-tagged for some name α.

Well-tagged processes compose!

Theorem (composition result)

*Let A_1 be α-tagged and A_2 be β-tagged w.r.t. w.
If $\nu w.A_1$ and $\nu w.A_2$ are resistant to guessing attacks against w then $\nu w.(A_1 \mid A_2)$ is also resistant to guessing attacks against w.*

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If $\nu w.A_1$ and $\nu w.A_2$ are resistant to guessing attacks against w then $\nu w.(A_1 \mid A_2)$ is also resistant to guessing attacks against w.

A secure transformation

Theorem

If $\nu w.A$ is resistant to guessing attacks against w then $\nu w.(A\{^{\mathsf{h}(\alpha,w)}/_{w}\})$ is also resistant to guessing attacks against w.

Easy, syntactic transformation: thumbrule for good design?

Remark on other transformations:

- replacing w by $\langle w, \alpha \rangle$ does not guarantee composition
- tagging encryptions (used in [CortierDelaitreDelaune07] to ensure composition of other properties) would add guessing attacks

Conclusion and future work

Passive guessing attacks do compose.

Active guessing attacks do not compose in general.

But for well-taged protocols:

Secure transformation to obtain well-tagged protocols

Future work

Avoid tags : are there (interesting) classes of protocols and equational theories for which guessing attacks compose?

Other forms of composition :

- composition for observational equivalence
- sequential composition